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Insights from students' private work in their notebooks: how do students learn from the teacher's examples?

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Abstract Students' seatwork plays an important part in their learning in their lessons, and very often, students record their private work in the notebooks during seatwork. The students' private work in their notebooks reflects students' learning and thinking, representing explicit learning outcomes. The students' private work in their notebooks of 14 mathematics lessons of an eighth-grade Hong Kong classroom was analyzed. The mathematical tasks used in the lessons were categorized with the Trends in International Mathematics and Science Study (TIMSS) cognitive domains framework. The implementation of the tasks was recorded in cycles of teacher's examples (TEs) and students' exercises (SEs). By comparing the methods employed by the students and the teacher, the students' methods were found to be mainly imitation or partial imitation regardless of the cognitive domains of the students' exercises. The students' perspectives on the instructional practice expressed in the post-lesson interviews were used as a triangulation for the results. The results showed that the students appreciated the teacher's explanation and demonstration in the teacher's exposition. Finally, the authors argue that the high percentages of imitation of teacher's methods not only are due to the students' choice, but also are influenced by the Confucian heritage cultures.

Keywords Students' private work · Learning · Cognitive domains · Imitation

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1 Introduction

Comparative studies such as Trends in International Mathematics and Science Study (TIMSS, Mullis, Martin, Foy & Arora, 2012) and Programme for International Assessment (PISA, OECD, 2010) have reported that students in East Asian regions such as Hong Kong, Korea, Singapore, and Taiwan have results outperform their counterparts in the non-Asian regions. As a result, much interest has been made in studies about East Asian classrooms and many studies

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of the instructional practices in East Asian regions such as Singapore, Shanghai and Hong Kong (Kaur, 2009; Leung, 2005; Mok & Lopez-Real, 2006; Mok, 2009), and Korea (Park & Leung, 2006) have been reported. The results of these studies show not only some similarities consistent with the teacher-led directive style but also unfolding, at a deeper level, some features conducive to learning in the cultural contexts of East Asian classrooms, hence, explaining, to a certain extent, the good performance of East Asian students. In general, students engage themselves in a lot of classroom activities under the teacher's instruction. This happens when students work individually or in a small group, and such organization of activities is called "seatwork" (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999, p.74). Students may produce different kinds of outcomes depending on the nature of the teacher's assigned tasks. In the case of Hong Kong, seatwork often serves the purpose for the students to practice what they have just learned by doing exercises in their notebooks privately. Such private work in the students' notebooks often matters to what the students have learned in that particular lesson and directly represents the explicit learning outcomes achieved by the students in the lesson (Fried & Amit, 2003; Jablonka, 2006). However, there are very few studies on the students' private work in their notebooks. The aim of this paper is to fill the gap with a case study in the context of Hong Kong mathematics lessons putting the focus on students' private work in their notebooks, hoping to provide a gateway for understanding the nature of the students' learning in the classrooms.

Learning activities in a mathematics classroom are usually organized via mathematical tasks. A mathematical task may be a set of problems or a single problem for drawing students' attention on a particular mathematical idea (Stein, Grover, & Henningsen, 1996). These tasks that include teachers' examples and students' exercises in the lessons may come directly from the textbooks or the teacher's improvisation depending on the teacher's enactment of the lesson. The lessons in this study demonstrated a very typical feature in East Asian mathematics lessons; that is, the teacher's expository explanation through the teacher's examples formed a very important component of the instructional practice. How did the students learn from the teacher's exposition? This study attempted to investigate the relationship between the teacher's examples in the teacher's exposition and the students' private work through a detailed examination of the students' private work. The analysis was carried out in four aspects: (1) the cognitive domains of mathematical tasks, (2) the pattern of the teacher's examples and the students' exercises in the lessons, (3) the degree of imitation of the teacher's methods in the students' private work, and (4) the students' perspectives on the instructional practice.

While filling in the literature gap on students' private work in their notebooks, this paper aims to contribute in several aspects: to show how the role of the students' notebooks may serve as a locus wherein the public world of the classroom may be transformed into students' own private world of engagement with mathematical materials; the potentials and pitfalls of cognitive import in imitation; and the cultural aspect of imitation with respect to teacher's authority and students' patterns of learning with respect to the Confucian tradition.

2 Theoretical perspectives and terminology

2.1 Cognitive domains of mathematical tasks

The mathematical tasks are important vehicles for students to develop their mathematical learning and thinking because, on the one hand, mathematical tasks and the teacher's

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interpretation of the tasks determine the students' experience in their lessons (Doyle, 1988; National Council of Teacher and Mathematics, 1991). Different attempts have been made to study the cognitive demand of mathematical tasks that plays a pertinent role in defining the premises of the students' work. Doyle (1988) discussed the cognitive demand of an academic task in terms of the cognitive process that varies from low level of memory such as multiplication tables, to high level of decisions in problem solving or more advanced mathematical work. Stein et al. (1996) defined mathematical tasks as a class activity focusing students' attention on a particular mathematical idea, which could be examined in the dimensions of task features and cognitive demands. Mathematical features were referred to aspects of tasks for engaging student thinking, reasoning, and sense making. The cognitive demand of the task-set-up phase referred to the kind of process entailed in the teacher's announcement, whereas the cognitive demands at the implementation stage in the classrooms referred to the actual cognitive processes in which the students engaged while carrying out the tasks, that is, whether the students actually recalled facts and formulas or engaged in high-level thinking and reasoning. Cognitive demand or level defined in such way referring the actual process of students' engagement is dynamic and difficult to measure. Nonetheless, for studying the students' learning outcomes, it is important to have indicators for measuring the potential cognitive demand of the mathematical problems that the students engage in. By classifying the assessment items, TIMSS attempts to assess students' understanding at multiple levels in three cognitive domains, namely, knowing, applying, and reasoning (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009), hence, giving a valid inference of how students may perform on specific tasks (Nixon & Barth, 2014).

The TIMSS categories of cognitive domains were applied in the analysis and recapitulated here (Mullis et al., 2009, pp. 40-46):

- Knowing: covers the facts, concepts, and procedures that students need to know. The subcategories are recall, recognize, compute, retrieve, measure, and classify/order.
- Applying: focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The subcategories are select, represent, model, implement, and solving routine problems.
- Reasoning: goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems. The subcategories include analyze, generalize/specialize, integrate/synthesize, justify, and solving non-routine problems

2.2 Students' seatwork and private work in the classrooms

When classroom activities are organized in such way that students may engage themselves in mathematical materials in their seats either individually or in small groups, such organization of activities is called seatwork (Stigler et al, 1999, p.74). Seatwork often occupies a significant portion of the mathematics lessons in different places in the world (Stigler et al., 1999), and quite a few researchers have attempted to study seatwork in different cultural contexts. For example, Hino (2006) studied the role of seatwork in Japanese classrooms and found that the placement of the seatwork prior to the presentation of the main content of the lessons provided opportunities for students to share and exchange their ideas, and the main content could make a connection to their seatwork in the earlier part of the lesson. Serrano (2012) compared

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the seatwork in Germany, Japan, and USA in the TIMSS videos to investigate the influence of seatwork activities on students' thinking in the lessons.

Fried (2008) discussed public domain and private domain in mathematics classroom practice. The same mathematical activity such as seatwork or writing in students' notebook can be termed as private or public depending on the pedagogical practice. In particular, Fried and Amit (2003) investigated students' notebooks, one of the products of the seatwork in the lessons, in two Israel eighth-grade mathematics classes and found that the work in the students' notebooks was the rehearsals for public display as the students' work was open for inspection There is a certain tension between the private domain and the public domain of the treatment of the notebooks, but the work in the notebooks becomes a finished product by public inspection (Fried, 2008). In the case of Hong Kong mathematics classrooms, occasionally, the teacher might select the students' work to show on the board, to show the students' ideas, and to share alternative solutions (Jablonka, 2006). However, for most of the cases in Hong Kong, students' notebooks were often individual and private although there might be limited sharing between students when they talked to their classmates sitting next to them (Lui & Leung, 2013).

2.3 The framework of the study

The process of teaching and learning in mathematics classrooms is complex in the socio-cultural context. According to Vygotsky (1978), the interpersonal (the interaction between the teacher and peers) process is transformed into an intrapersonal (the student) one. Within the zone of proximal development (ZPD), students may handle problems beyond the capability of their mental age when they are under guidance or in collaboration with peers. Activities in a lesson are arranged based on mathematical tasks that may appear in the form of a problem statement going through three stages: the text format of the tasks, the setting up by the teachers, and the implementation by the students in the classrooms (Henningsen and Stein, 1997). In the case of Hong Kong classrooms, the social space consists of the teacher-led whole class interaction and the seatwork period when the students may occasionally talk to the classmate sitting next to them. When students interact within the social space in the lesson, their learning takes place when observing and imitating of teacher's procedures. This imitation is not necessary a purely mechanical process. Students imitate the teacher's procedures and later become independent through their minds.

In the lessons in this study, the text format of the tasks might be either worksheets designed by the teacher or problems adapted from the textbooks. The mathematical tasks might be used for teacher's expository work or assigned exercises for student seatwork, which occupied a significant component of the lessons. The students' work during seatwork was directly influenced by the design of the tasks, the teacher's exposition and demonstration, and the students' own implementation of the tasks. The methods demonstrated in the teacher's examples often acted as a model for students to imitate in their work. Thus, the methods employed in the students' work might infer how students learnt from the teacher's exposition. The students' private work is the focus in the study. The key terms are defined below.

A *task/mathematical task* in this paper is defined as a mathematical problem, which can either be used as an example in the teacher's expository explanation or demonstration, known as *teacher's example* (TE), or an exercise assigned for students to work during seatwork, known as *students' exercise* (SE). The problem statements of TE and SE might appear in the text form of a mathematical problem in the textbooks, a teacher example shown on the board, or a problem in the teacher-designed

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worksheets. Consequently, a lesson can be represented as a sequence of TE episodes and SE episodes, forming *TE-SE cycles*.

Student's private work (SW) refers to the records of the students' work in their notebook during seatwork. A preliminary analysis showed that the students' private work contained some direct copies of the TEs shown on the board and the students' private work when they engaged in the exercises on their own. The students' private work also contained some incomplete items (including the unattempted items) and some complete items. The reason for incomplete items might be due to insufficient time to complete the assigned exercises during the lessons. The complete items of SW were further analyzed. To make a differentiation, SE referred to the task problem statement of the SE, whereas SW referred to the students' private work when they completed the exercises in their notebooks during seatwork. SW includes the students' own answers worked out by themselves for the teacher's assigned student exercises (SE) or the students working on extra exercises not assigned by the teacher.

The *cognitive domains* of the mathematical tasks, including both TE and SE, were analyzed according to the TIMSS cognitive domain categories. Some examples are shown in Table 1.

The *degree of imitation* refers to the degree of similarity when the method employed in the complete items of students' private work (SW) was contrasted with the method employed in the TE. The degree of imitation of the SW thus gives an indicator on how the students learn from the teacher's expository demonstration.

3 Source of data: the LPS

The data consisted of 14 consecutive lessons of a Hong Kong school (HK3) taken from the Learner's Perspective Study (LPS) which was an international research collaboration to examine the patterns of participation in competently taught eighth-grade mathematics classrooms (Clarke, Keitel, & Shimizu, 2006). The 14 lessons covered two topics: slopes of lines and a system of simultaneous linear equations in two unknowns (Table 2). The class size was 40 and the mean International Benchmark Test (IBT)[1] scores of the class were 38.4 over 50 (77 %). The teacher had 12 years of secondary mathematics teaching experience and was identified as a competent teacher locally by the researchers and the school principal.

The data collection procedures followed the LPS design which aimed to collect a rich data set for allowing the researchers to reconstruct the lesson scenario from different perspectives including the learners' perspectives to make possible analysis under different themes and frameworks (Clarke et al., 2006). An integrated system of three cameras was used to collect data in which one was for the whole class, one was for the teacher, and one was for a group of two focus students. A total of 14 consecutive lessons of the same class were recorded. Two different students were chosen to be the focus for each lesson, and they were invited to take a post-lesson interview. All the lesson materials including the focus students were collected at the end of the lesson. The video-stimulated recall interview technique was used, and the students were asked to stop the video at episodes that they saw as important and explained why they saw the importance. The data used in this study consisted of the videos and transcripts, focus students' notebooks and worksheets, and interview transcripts.

[1] The International Benchmark Test for Mathematics (IBT) is norm-referenced and evaluates student achievement on mathematical content for eighth grade. Items are taken from the TIMSS Student Achievement Study (population 2).

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Table 1 The examples of cognitive domains of mathematical tasks in the lessons

t1.[illegible]	Cognitive domains	Teacher's examples (TE)	Students' exercises (SE)	Cognitive demands
t1.3	Knowing			
t1.4	Compute	Solving the simultaneous equations by the method of substitution $\begin{cases} 2x-3y=3 \\ 3x+4y=13 \end{cases}$	Use the method of elimination to solve the simultaneous equations $\begin{cases} 2x-5y=3 \\ 3x+5y=20 \end{cases}$	Carry out algebraic procedures in solving the equations
t1.5	Recognize	–	Compare the slopes of L1 and L2. What do you find?	Recognize the values of slopes
t1.6	Retrieve	–	Using the graph drawn in (a) to answer the following questions	Retrieve the information from the graph
t1.7	Applying			
t1.8	Select	A (6, 4) and B (2, −3) are two points on straight line L_1. Find the slope of L_1.	The vertices of a quadrilateral PQRS are P (2, 4), Q (3, 5), R (−3, 1), and S (−5, −1). Find the slope of each side of the quadrilateral PQRS.	Find the slope by using the formula of slope
t1.9	Model	The selling price of three tables and four chairs is \$6400. The selling price of four tables and one chair is \$5500. What are the selling prices of each table and each chair?	The cost of four apples and two oranges is \$14. The cost of six apples and five oranges is \$27. Find the cost of each fruit.	Generate the equations for solving the problem
t1.10	Implement	Draw the graph of the linear equation in two unknowns $y=2x+3$ where x takes values from −2 to 2.	Draw the graph of the linear equation $y=2-\frac{1}{2}x$ from $x=-2$ to $x=6$.	Implement a set of mathematical instruction in drawing the graph
t1.11	Reasoning			
t1.12	Analyze	–	(i) What is the size of ∠CAB and ∠ACB? (ii) What is the size of ∠AEF and ∠EAF? (iii)What can you say about ΔABC and ΔAEF?	(iii) Determine the relationships between two triangles according to the values of the angles from (i) and (ii).
t1.13	Generalize	–	From these results (in a), what conclusion can you make?	Generalize the results in (a) in a more general statement.
t1.14	Justify	Given that the vertices of ΔABC are A (−3, 2), B (−1, −2), and C (1, 4), prove that ΔABC is a right-angled triangle.	Given that the vertices of ΔABC are A (−1, 4), B (9, −11), and C (−16, −6), prove that ΔABC is a right-angled triangle.	Find the slope of the three lines, and connect the results to the properties of right-angled triangle for justification

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Table 2 The topics of the 14 lessons

Topics	Lessons
Slopes of lines	L01 to L04
A system of simultaneous linear equations in two unknowns (i) The graphical method	L05 to L07
(ii) The method of substitution	L08 to L09
(iii) The method of elimination	L10 to L11
(iii) The word problems	L11 to L14

4 Methods of analysis

4.1 The cognitive domains of mathematical tasks

A total of 116 mathematical tasks, which might be used as either a TE or a SE, were implemented in the 14 lessons. The cognitive domains of the mathematical tasks were classified into knowing, applying, or reasoning with their corresponding subcategories (Table 1).

4.2 TE-SE cycles of the lessons

The mathematical tasks implemented in the lessons were identified as either TEs or SEs according to the lesson videos. The teacher usually demonstrated principles or procedures in

Lessons	Patterns of TE – SE cycles	Number of TE – SE cycles in the lessons
L01	TE (2) SE (5)	1
L02	SE (9)	0
L03	SE (11)	0
L04	SE (2) | TE (1) SE (7) | TE (1) SE (2) | TE (1) SE (5)	3
L05	TE (4) SE (2) | TE (1) SE (1)	2
L06	TE (1) SE (1)	1
L07	SE (2)	0
L08	TE (2) SE (2) | TE (1) SE (2) | TE (1) SE (2)	3
L09	TE (1) SE (6)	1
L10	TE (2) SE (4) | TE (1) SE (5) | TE (1) SE (5) | TE (1) SE (3)	4
L11	TE (2) SE (2) | TE (2) SE (1)	2
L12	TE (1) SE (1) | TE (1)	1
L13	TE (1) SE (2) | TE (1)	1
L14	TE (1) SE (2) | TE (2)	1

Legend:
| represents the border between each TE -SE cycle
TE: Teacher's example (number of task); **SE**: Students' exercise (number of task)
Note: The lengths of segments do not reflect the duration of the lessons.

Fig. 1 The structural patterns of teacher's examples and students' exercises in the lessons

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solving the TEs and then assigned exercises for student to practice forming a TE-SE cycle; hence, a lesson could be seen as a chain of TE-SE cycles (Fig. 1).

4.3 The degree of imitation when contrasting SW with the teacher's method in TE

After the classification of cognitive domains of mathematical tasks and the pattern of the TE-SE cycles, the students' private work in the students' notebooks done by the focus students was analyzed. A total of 252 items of SW in the students' notebooks were collected from 27 students in 14 lessons. Altogether, there were 136 complete exercise items (labeled as SW), which were further analyzed by comparing the methods employed by the students with the methods demonstrated in the TEs. For instance, Janice and Gary worked on the same SE and produced their own SW; therefore, the counting of SE was 1 and the counting of SW was 2 in this case (Fig. 2).

Very often, the teacher's demonstration of principles and procedures for solving a particular task in TE was prior to the SEs. Therefore, there was often a high degree of similarity between the TE and the SEs in a TE-SE cycle. Two examples of TE-SE cycles are given in Table 5. The methods employed in the students' private work (SW) were compared with the method in the TEs, the degree of imitation was categorized based on how closely the students imitated the teacher's methods, and the categories were as follows: imitation, partial imitation, and students' own method (Table 3).

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4.3.1 Examples of imitation and partial imitation

The students' private work (SW) by Gary and Janice (Fig. 2) is used here to illustrate the differentiation between imitation and partial imitation in the coding. The lesson (L06) was about graphical method for solving a pair of simultaneous linear equations. The teacher's method was to use three points with the values of x coordinates 1, 3, and 5 in two tables, respectively, to draw the two lines. Gary copied the TE in solving the equations ($4x-5y=2$, $7x-10y=2$); he imitated completely the teacher's method by using the same values of x coordinates (1, 3, 5, respectively) for plotting the two lines. His private work was coded as "imitation." In contrast, Janice also imitated the teacher's method of using three points, but she chose different values of x (1, 3, 5 for one equation and 1, 2, 3 for another equation). Janice's private work was classified as "partial imitation."

4.4 Students' perspectives on the instructional practice

Twenty-six student interview transcripts were analyzed to give the students' perspectives of the instructional practice. The stimulated-video-recall method was used in the post-lesson

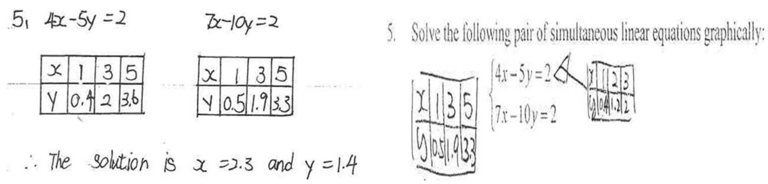

Fig. 2 Gary's private work (*left*) was classified as imitation, and Janice's private work (*right*) was classified as partial imitation

Table 3 The degree of imitation in the student's private work

Degrees of imitation	Descriptions
Imitation	Students reproduce the methods used in the teacher's example exactly in solving the task.
Partial imitation	Students imitate the teacher's methods incompletely, such as skip/miss some steps or use other values that were not same as in the teacher's examples.
Student's own method	Students use a method different from the teacher's examples or no corresponding teacher's example for imitation.

interviews. The students were invited to stop the lesson videos at moments where they saw as important and give their comments. In general, the instructional practice could be categorized into exposition, seatwork, and review, which could be further break down into subcategories (for details, Mok, Kaur, Zhu, & Yau, 2013). The lesson video segments and the students' attached importance for the video segment were coded by three sets of codes: (1) TE/SE, (2) exposition/seatwork/review, and (3) subcategories under exposition, seatwork, and review that are as follows:

- Exposition: teacher's explanation (EC), teacher's demonstration (D), new knowledge (NK), giving instruction (GI), and uses real-life examples during instruction (RE).
- Seatwork: students working individually/copying notes (IW), students working in groups/group discussion (GW), and material used as part of instruction (M).
- Review: reviews prior knowledge (PK), uses student's presentation or work to give feedback for in class work or homework (SP), gives feedback to individuals during lesson (IF), and gives feedback through grading of written assignments (GA).

4.5 Reliability and validity

Two researchers carried out the coding independently on the cognitive domains of mathematical tasks, the classification of TEs and SEs, the degree of imitation of teacher's methods in students' private work, and the exposition codes. The percentages of agreement were over 84 %.

5 Results: how did the students learn from the teacher's exposition?

5.1 The cognitive domains of mathematical tasks

The distribution of the cognitive domains of the tasks in the 14 lessons is shown in Table 4. The ratio of SEs to TEs was about 2.6 (84:32). The distributions of the cognitive domains of the TEs were knowing (47 %), applying (50 %), and reasoning (3 %), whereas those of SEs were knowing (33 %), applying (45 %), and reasoning (21 %). Therefore, the students had more practice on the knowing and applying tasks in comparison with the reasoning tasks. The proportion of reasoning tasks for SE was greater than that for TE.

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t4.1 **Table 4** The cognitive domains of mathematical tasks

		Teacher's examples			Students' exercises			Total		
t4.2		Teacher's examples			Students' exercises			Total		
t4.3	Topics	Knowing	Applying	Reasoning	Knowing	Applying	Reasoning	Knowing	Applying	Reasoning
t4.4		CO/RC/RE	SE/IM/MO	JU/GE/AN	CO/RC/RE	SE/IM/MO	JU/GE/AN	CO/RC/RE	SE/IM/MO	JU/GE/AN
t4.5	Slope of the lines	0/0/0	4/0/0	1/0/0	1/1/0	21/0/0	12/2/4	1/1/0	25/0/0	13/2/4
t4.6	The graphical method	4/0/0	0/2/0	0/0/0	1/0/1	0/4/0	0/0/0	5/0/1	0/6/0	0/0/0
t4.7	The methods of substitution and elimination	11/0/0	0/1/0	0/0/0	24/0/0	6/1/0	0/0/0	35/0/0	6/2/0	0/0/0
t4.8	Word problems	0/0/0	0/0/9	0/0/0	0/0/0	0/0/6	0/0/0	0/0/0	0/0/15	0/0/0
t4.9	Total number of tasks in each subcategory	15/0/0	4/3/9	1/0/0	26/1/1	27/5/6	12/2/4	41/1/1	31/8/15	13/2/4
t4.10	Total number of tasks in each cognitive domain (%)	15 (47 %)	16 (50 %)	1 (3 %)	28 (33 %)	38 (45 %)	18 (21 %)	43 (37 %)	54 (47 %)	19 (16 %)
t4.11	Total number of mathematical tasks (%)	32 (28 %)			84 (72 %)			116 (100 %)		

CO knowing —compute, *RC* knowing—recognize, *RE* knowing—retrieve, *SE* applying—select, *IM* applying—implement, *MO* applying—Model, *JU* reasoning—Justify, *GE* reasoning—generalize, *AN* reasoning—analyze

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5.2 The TE-SE cycles in the lessons

The pattern of TE-SE cycles in each lesson is shown in Fig. 1. There were 11 lessons containing TE-SE cycles with different length of SE items, showing a variation in the emphasis of SEs in these cycles. Three lessons L02, L03, and L07 did not have TEs. When we examined the TEs and the SEs, the tasks used for TE and SE were very similar in each cycle. Using Q4 Lesson L08 as an example, the first cycle was TE(2)-SE(2). The teacher first introduced the lesson with one TE on the board; then, he used the first item (question 1 (a)) of his self-designed worksheet as the second TE. The worksheet consisted of 18 items that were grouped into six questions. All the items were very similar with minor changes, and the teacher gave emphasis in different part of the computation procedures in his explanation for different examples. The students were expected to use the teacher's methods in TE to complete the assigned SE (Table 5).

5.3 The students' private work in their notebooks

The students' private work (SW) in their notebooks was analyzed. One hundred thirty-six items of students' complete private work were coded for the degree of imitation. Among the items, 116 items belonged to teacher-assigned exercises and 20 items belonged to items that were not assigned by the teacher but completed on the students' self-initiative because they completed the assigned work early. The distribution of the different degrees of imitation in the students' private work is given in Table 6. Imitating from the teacher's method in the

t5.1 **Table 5** The teacher's examples and the students' exercises implemented in L08 (the first seven tasks)

	Mathematical tasks	Classification
t5.2	Mathematical tasks	Classification
t5.3	The first TE-SE cycle, TE(2)-SE(2)	
t5.4	$\begin{cases} y = x + 1 \\ 2x - y - 5 = 0 \end{cases}$	Teacher's example (written on the board)
t5.5	1(a) Solving the simultaneous equations by the method of substitution $\begin{cases} y = x + 1 \\ 3x + 4y = 11 \end{cases}$	Teacher's example (item on the worksheet)
t5.6	1(b) Solving the simultaneous equations by the method of substitution $\begin{cases} y = 3x + 1 \\ y = x + 7 \end{cases}$	Students' exercise
t5.7	1(c) Solving the simultaneous equations by the method of substitution $\begin{cases} x = 4y + 7 \\ x + 4y - 7 = 0 \end{cases}$	Students' exercise
t5.8	The second TE-SE cycle, TE(1)-SE(2)	
t5.9	2 (a) Solving the simultaneous equations by the method of substitution $\begin{cases} 2x + 3y = 5 \\ x - y = 5 \end{cases}$	Teacher's example (item on the worksheet)
t5.10	2 (b) Solving the simultaneous equations by the method of substitution $\begin{cases} 2x + y = 9 \\ x - y = 3 \end{cases}$	Students' exercise
t5.11	2 (c) Solving the simultaneous equations by the method of substitution $\begin{cases} 5x + 7y = 18 \\ x + y = 6 \end{cases}$	Students' exercise

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demonstration was the major feature in the students' private work. There were 60 items of imitation and 73 items of partial imitation, making up a total of 133 out of 136 items of SW regardless of the cognitive domain of the tasks. One possible reason for large number of imitation might be due to the TE-SE pattern in which the TEs were always arranged before the SEs and the TE and SE tasks for each cycle were similar in nature. Furthermore, the teacher demonstrated detailed procedures or instructions, giving a model for students to imitate. These features help the students to recognize and imitate the teacher's methods easily.

For example, the teacher used two lessons (L08 and L09) for teaching the method of substitution and he based his lessons on a self-designed worksheet. The worksheet consisted of six questions of different variations of the coefficients and forms of the equations. Each question consisted of three similar items, making up a total of 18 items of very similar format. Each item was a pair of simultaneous equations that might either be a TE or SE. The lesson pattern of L08 consisted three TE-SE cycles (TE(2)-SE(2), TE(1)-SE(2), and TE(1)-SE(2)) where each cycle had two assigned items for SEs. L09 was the second lesson for the topic aiming to give more practice on the method with only one TE-SE cycle, TE(1)-SE(6). That is, in L09, the teacher used one item in the worksheet as TE and assigned six items as SEs. Joanne's notebook was collected by the end of L09, therefore, contained her private work for both L08 and L09. When we examined Joanne's notebook in L09 in details, she did the assigned SE selectively. In L08, she did only one SE (producing one SW) in each TE-SE, and in L09, she produced three SWs out of six SEs in her notebook. In the post-lesson interview, she explained that she discerned between similar methods and seemed to be reluctant to do items with repetitive calculation methods. She said, "The same calculation method, but not the same numbers, just for familiarizing, see whether you understand it or not." Her private work in L09 was coded as imitation (1), partial imitation (1), and students' own method (1). By partial imitation, there were some skipping steps in the students' private work, but these skipping steps did not hinder the students to get the correct answers while repeating the teacher's method. These skipping steps such as missing labels of some equations during substitution sometimes might cause some ambiguity in the presentation of answers.

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Table 6 The relationship between the cognitive domains of the students' exercises and the degrees of imitation of the students' private work

Cognitive domains of the students' exercises		Degrees of imitation in students' private work		
	Subcategories	Imitation	Partial imitation	Student's own method
Knowing	Compute	4	32	1
	Recognize	2	0	0
	Retrieve	2	0	0
Applying	Select	32	30	1
	Implement	4	5	0
	Model	6	4	0
Reasoning	Justify	6	2	0
	Generalize	0	0	0
	Analyze	4	0	1
Total number of SW in different degree of imitation (%)		60 (44 %)	73 (54 %)	3 (2 %)
Total number of completed students' private work		136		

The analysis showed that 20 out of 136 items of students' private work were items not assigned by the teacher. Six students in three different lessons worked on extra tasks after they had completed the teacher-assigned exercises. The extra tasks were similar to those assigned SE in nature and belonged to the same topic. For instance, the 14 tasks in worksheets used in Lesson L02 were about the slopes of parallel lines. The first eight assigned SEs were to prove a pair of parallel lines or four points forming a trapezium. Shown in Helen's private work, the six extra tasks demanded the students to solve similar problems (Fig. 3). In the post-lesson interview, Helen explained why she carried out the six extra tasks after completing the eight assigned SEs. She said, "I worked on the later questions in this worksheet because I know how to do it." When the interviewer asked whether she could do all these, Helen said, "Yes, I can." Upon further probing, Helen added, "If I can't, I will ask my neighbor, because she is strong in calculation. So, I ask her most of the time." Helen's case, unfolded how the student might work through the ZPD. At the beginning, her work was mostly imitating the TEs under the teacher's guidance, supplemented with interaction with a more capable peer. Achieving the skills, the student developed her confidence and motivation to do additional exercises on her own. Such phenomenon might happen for students of different degree of fondness for mathematics.

5.4 What were the students' perspectives on the instructional practice?

Table 7 summarizes the number of video segments at which the students stopped the video to say the instructional practice at that moment was important. Forty-two percent of the video segments were TE, and 55 % of the video segments were SE; therefore, both TE and SE were important while SE was slightly more important than TE. The teacher's exposition was the most important when comparing with seatwork and review. In the further breakdown of the subcategories for exposition, the teacher's demonstration of procedures (D) in TE and SE (16 segments in TE and 15 segments in SE) was important. The fact that the teacher demonstrated

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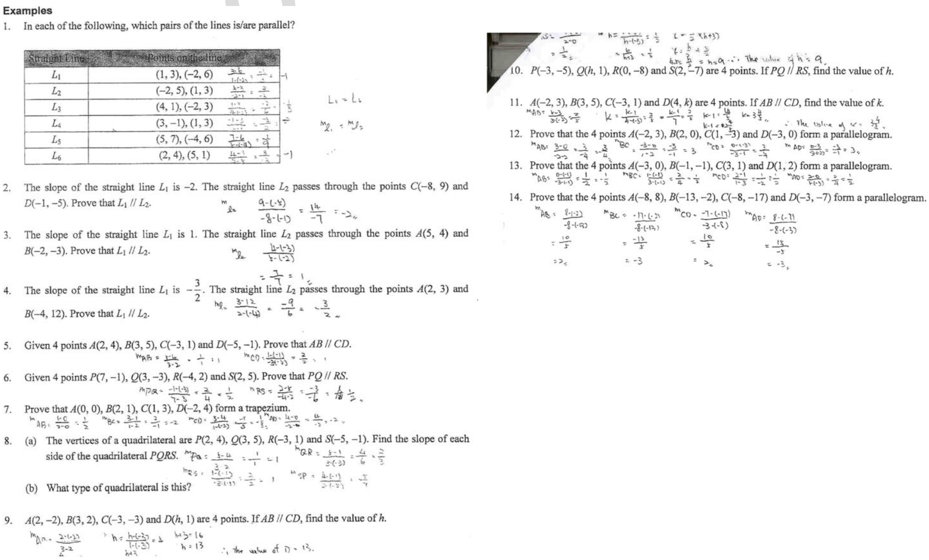
Examples

1. In each of the following, which pairs of the lines is/are parallel?

Straight Line	Points on the line
L_1	(1, 3), (−2, 6)
L_2	(−2, 5), (1, 3)
L_3	(4, 1), (−2, 3)
L_4	(3, −1), (1, 3)
L_5	(5, 7), (−4, 6)
L_6	(2, 4), (5, 1)

2. The slope of the straight line L_1 is −2. The straight line L_2 passes through the points $C(-8, 9)$ and $D(-1, -5)$. Prove that $L_1 // L_2$.
3. The slope of the straight line L_1 is 1. The straight line L_2 passes through the points $A(5, 4)$ and $B(-2, -3)$. Prove that $L_1 // L_2$.
4. The slope of the straight line L_1 is $-\frac{3}{2}$. The straight line L_2 passes through the points $A(2, 3)$ and $B(-4, 12)$. Prove that $L_1 // L_2$.
5. Given 4 points $A(2, 4)$, $B(3, 5)$, $C(-3, 1)$ and $D(-5, -1)$. Prove that $AB // CD$.
6. Given 4 points $P(7, -1)$, $Q(3, -3)$, $R(-4, 2)$ and $S(2, 5)$. Prove that $PQ // RS$.
7. Prove that $A(0, 0)$, $B(2, 1)$, $C(1, 3)$, $D(-2, 4)$ form a trapezium.
8. (a) The vertices of a quadrilateral are $P(2, 4)$, $Q(3, 5)$, $R(-3, 1)$ and $S(-5, -1)$. Find the slope of each side of the quadrilateral $PQRS$.
   (b) What type of quadrilateral is this?
9. $A(2, -2)$, $B(3, 2)$, $C(-3, -3)$ and $D(h, 1)$ are 4 points. If $AB // CD$, find the value of h.
10. $P(-3, -5)$, $Q(h, 1)$, $R(0, -8)$ and $S(2, -7)$ are 4 points. If $PQ // RS$, find the value of h.
11. $A(-2, 3)$, $B(3, 5)$, $C(-3, 1)$ and $D(4, k)$ are 4 points. If $AB // CD$, find the value of k.
12. Prove that the 4 points $A(-2, 3)$, $B(2, 0)$, $C(1, -3)$ and $D(-3, 0)$ form a parallelogram.
13. Prove that the 4 points $A(-3, 0)$, $B(-1, -1)$, $C(3, 1)$ and $D(1, 2)$ form a parallelogram.
14. Prove that the 4 points $A(-8, 8)$, $B(-13, -2)$, $C(-8, -17)$ and $D(-3, -7)$ form a parallelogram.

Fig. 3 Helen's private work. Tasks 1 to 8 were the assigned SE items, and tasks 9 to 14 were the extra items that Helen worked by her own detailed procedures in TE and gave detailed instructions prior to students working on SEs had a strong impact on how the students learned. Referring to what the students said in the post-lesson interviews, the students appreciated and learned from the teacher's explanation and demonstration. For example, Iris in L04 said, "Before here I didn't quite understand, after listening the teacher's explanation, I started to understand a little bit." Joanne in L09 thought "The teacher was doing the example. I don't know how to do it without examples. Example is for you to see how to do it." Students believed doing the SEs independently (IW) was important for their learning, for example, "Do it yourself, don't know if you don't do it." (Janice in L06) and "Because you have to work. You have to work it out for sure after the teacher has taught you things." (Gordon in L07). These results showing the strong students' appreciation for teacher's demonstration and explanation and working on SEs were consistent with the results for other East Asian classrooms reported in the work of Mok and others (Mok et al., 2013)

6 Discussion and conclusions

In our study, the students' private work of an eighth-grade mathematics classroom in Hong Kong was analyzed. The cognitive domains of TEs and SEs were mainly belonged to the knowing and applying, whereas relatively fewer tasks belonged to the domain of reasoning. In the 14 consecutive lessons, the mathematical tasks were arranged as TE-SE cycles of TEs and SEs. Regardless of the cognitive domains of SEs, the methods employed by the students in their private work were mainly the imitation of teacher's methods. This imitation was not only simply determined by the students' choice in learning mathematics, but also influenced by the TE-SE arrangement and the similarities of tasks in each TE-SE cycle. In the students' perspectives, the teacher's demonstration was the most important.

The finding of high proportion of imitation of teacher's methods was not a surprise because education in Hong Kong and other East Asian regions is often reported to be much influenced by the Confucian philosophy (e.g., Watkins & Biggs, 2001), emphasizing that the teachers are the role models of subject matter (Leung, 2001). Very often, teachers play a significant guiding role in the mathematics classrooms (e.g., Mok, 2009; Leung & Park, 2002). The teacher facilitated the role of learning by demonstrating the TEs or giving hints before the SEs. Moreover, with the image of scholar-teacher deeply rooted in Confucian culture, it is very likely that the students believed that the methods used in solving the TEs were the best. In East Asian classrooms, the emphasis on practice is an important feature in the pedagogical philosophy. The traditional Chinese beliefs of "practice makes perfect" (Li, 2006) and memorization which could come before understanding (Cai & Wang, 2010) may explain for the high percentage of SEs (72 % of total tasks) in the lessons. However, practice is not equivalent to repetition by rote. The variations embedded in the TEs and SEs, in fact, help students to experience the object of learning in a deep sense leading to an understanding of the mathematical concepts and procedures from multiple perspectives (e.g., Gu, Huang & Marton, 2004; Huang & Leung, 2004; Wong, 2006). Huang and Leung (2004) studied the mathematical tasks in Shanghai and Hong Kong classrooms and found that the tasks might serve the purpose of consolidation and help developing proficiency and understanding of the topic.

Another factor shaping the students' learning in mathematics is the cognitive domains of tasks. Examining the cognitive domains of tasks in the SEs across 14 consecutive lessons showed that the majority was knowing and applying tasks, with relatively lower percentage of

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t7.1 **Table 7** The students' perspectives of the instructional practice in post-lesson student interviews

		Teacher's examples			Students' exercises			Off tasks			Total
t7.2		Teacher's examples			Students' exercises			Off tasks			Total
t7.3		Exposition	Seatwork	Review	Exposition	Seatwork	Review	Exposition	Seatwork	Review	
t7.4		EC/D/NK/GI/RE	IW/GW/M	PK/SP/IF/GA	EC/D/NK/GI/RE	IW/GW/M	PK/SP/IF/GA	EC/D/NK/GI/RE	IW/GW/M	PK/SP/IF/GA	
t7.5	No. of segments	4/16/7/0/0	1/0/1	0/1/2/0	2/15/3/0/0	9/4/0	1/5/3/0	0/0/0/2/0	0/0/0	0/0/0/0	
t7.6	Total no. of segments	32/76 (42 %)			42/76 (55 %)			2 (3 %)			76

EC explains/explains clearly, *D* demonstrates a procedure: "teaches the method" or shows using manipulative a concept/relationship, *NK* introduces new knowledge, *GI* gives instructions (assigning homework/how work should be done/when work should be handed in for grading, etc.), *RE* uses real-life examples during instruction, *IW* students working individually/copying notes, *GW* students working in groups/group discussion, *M* material used as part of instruction (worksheet or any other print resource), *PK* reviews prior knowledge, *SP* uses student's presentation or work to give feedback for in class work or homework, *IF* gives feedback to individuals during lesson, *GA* gives feedback through grading of written assignments

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reasoning tasks. One of the possible reasons for the phenomenon may be due to the nature of the topics. Another possible reason may be the teacher's expectation of students' ability and pedagogical style. Although the opportunity for practice in reasoning was relatively fewer, the proportion of reasoning items in SE was greater than that in TE and also sufficient practice on all three domains was guaranteed by the high amount of exercises. The findings were consistent with the TIMSS 2011 results in which Hong Kong students (eighth grade) got the mathematics high average scores of 591, 587, and 580 for in knowing, applying, and reasoning domains, respectively (Mullis et al., 2012, p. 150).

Seventy-one percentage of students' private work (136 out of 190 items of students' private work, excluding the copies of TEs) were completely recorded, showing students' motivation of engaging themselves in the tasks. This might be due to the belief in effort and illustrated the Chinese dictum "diligence could remedy mediocrity." A high expectation of parents and the competitiveness of examination cultures strongly influence students' belief in working hard as the route of success. Students' conceptions of mathematics and mathematics learning are obviously shaped by their experience of learning (Bishop, 1991). In the student interviews, students showed appreciation for how they learned from the teacher's exposition. They believed that the imitation of the teacher's methods with correct answers in their private work was the key of success in learning mathematics. So, they focused on the methods or procedures in solving the tasks. Although students in East Asian classrooms might have interpreted as passive at the surface, they might have been active in their minds (Biggs, 1998). In our study, six students showed their motivation to work on extra tasks; their private work and the students' post-lesson interviews showed how the teacher's demonstration in the public domain of the lesson might possibly be internalized in the students' learning outcomes. While some celebrated the mastering of skills, in some cases, the partial imitation instances indicated the pitfall. The missing steps might not hinder the students from getting the answers, but the students might lose the chance in developing the mathematical connections. Putting an overemphasis on the teacher's methods as the role models, the motivation for exploring new methods might be lost. To conclude, imitation that might be seen often in East Asian classrooms does not necessarily imply mechanical learning. Suitable use of imitative work, the students might possibly extend their mental capacity under the teacher's guidance and peer influence in the zone of proximal development (Vygotsky, 1978), developing a confidence and motivation in the work and possibly a "deep" approach that brings about understanding beyond memorization (Biggs, 1998).

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AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES.

Q1. Author names and affiliation are taken from the manuscript draft. Please check if captured correctly.

Q2. The sentence “Given that the vertices of ΔABC are A (−3, 2)…” has been edited for clarity. Please check that the intended meaning was retained.

Q3. The sentence “Given that the vertices of ΔABC are A (−1, 4), B (9, −11)…” has been edited for clarity. Please check that the intended meaning was retained.

Q4. The sentence “Using Lesson L08 as an example…” has been edited for clarity. Please check that the intended meaning was retained.

Q5. The sentence “That is, in L09, the teacher used…” has been edited for clarity. Please check that the intended meaning was retained.

Q6. The sentence “The finding of high proportion…” has been edited for clarity. Please check that the intended meaning was retained.

Q7. The sentence “The traditional Chinese beliefs of “practice makes perfect”…” has been edited for clarity. Please check that the intended meaning was retained.

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