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# Roughness-sublayer correction for the profiles of mean velocity and turbulence over urban areas

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## Background

- Monin-Obukhov similarity theory (MOST) applies in inertial sub-layer (ISL) but fails in roughness sub-layer (RSL) because the flow structure in RSL is highly inhomogeneous.
- Extrapolation of the conventional logarithmic law of wall into the RSL likely overlooks the inhomogeneity.
- Need for an analytical expression for mean velocity profile and ventilation estimate, including a new RSL correction, that is applicable over the urban boundary layer.

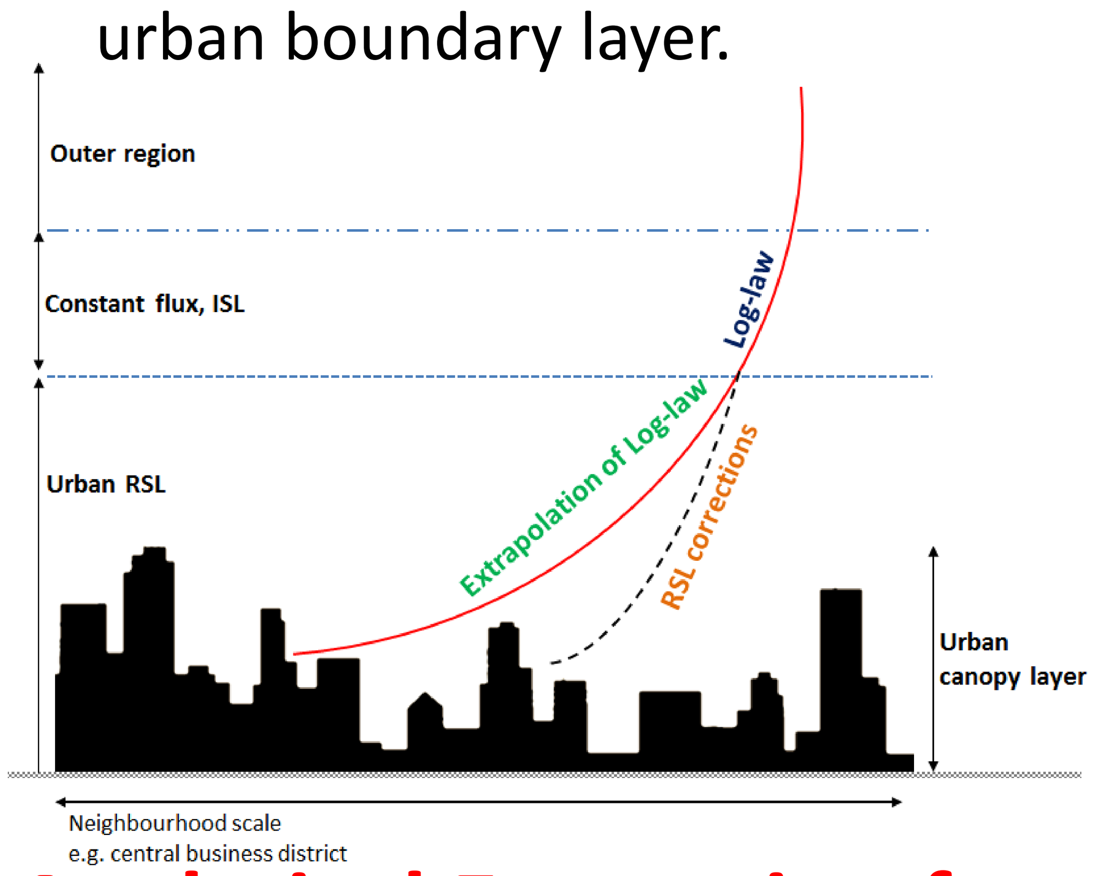


Figure 1: Schematic of different layers of the urban boundary layer, including the RSL and the ISL.

## Analytical Expression for RSL flow correction

**Assumptions:**

- $\Phi_m$ (= $\phi_m \hat{\phi}_m$) is a generalised similarity function of ISL & RSL
- Flows above urban canopy in neutral stratification ($\phi_m$ = 1)
- $\Phi_m$ is a function of the roughness elements that is independent from the MOST length scale $L$

$$\boxed{\Phi_m = \phi_m \hat{\phi}_m = \hat{\phi}_m\left(\frac{z}{z^*}\right)}$$

$z$ is the elevation & $z^*$ the RSL height.

The gradient of the wind profile in dimensionless form is,

$$\boxed{\frac{du}{dz} = \frac{u^*}{\kappa z}\hat{\phi}_m\left(\frac{z}{z^*}\right)}$$

$u$ is the wind speed, $u^*$ the friction velocity & $\kappa$ (= 0.41) the von Kármán constant.

Rearrange & integrate yields,

$$\boxed{\frac{\kappa}{u_*}u|_{z-d} = \ln\left(\frac{z-d}{z_0}\right) + \int_{z-d}^{\infty}\frac{1-\hat{\phi}_m}{z}\,dz}$$

$d$ is the displacement height & $z_0$ the roughness length scale.

We employ the (continuous) function of $\hat{\phi}_m$

$$\boxed{\hat{\phi}_m(z) = 1 - e^{-\mu(z/z^*)}}$$

$\mu$ is an empirical constant.

Use series expansion to calculate the exponential integral, an analytical expression for the urban RSL effects is formulated

$$\frac{u|_{z-d}}{u^*} = \frac{1}{\kappa}\left[\underbrace{\ln\left(\frac{z-d}{z_0}\right)}_{\text{ISL}} \underbrace{- \gamma - \ln\left(\mu\frac{z-d}{z^*}\right) - \sum_{n=1}^{\infty}\frac{(-1)^n\left(\mu\frac{z-d}{z_*}\right)^n}{n\cdot n!}}_{\text{RSL}}\right]$$

$\gamma$ ($\approx$ 0.57721): Euler constant.

## Wind Tunnel Measurements

- The open-circuit type wind tunnel at the Department of ME, HKU was used with neutral stratification and a reference wind speed of 9 m s$^{-1}$
- Idealised 2D-roughess elements with different aspect ratio (AR = $h/b$) were used to simulate the urban areas
- Cross-wire hot-wire measurements were performed

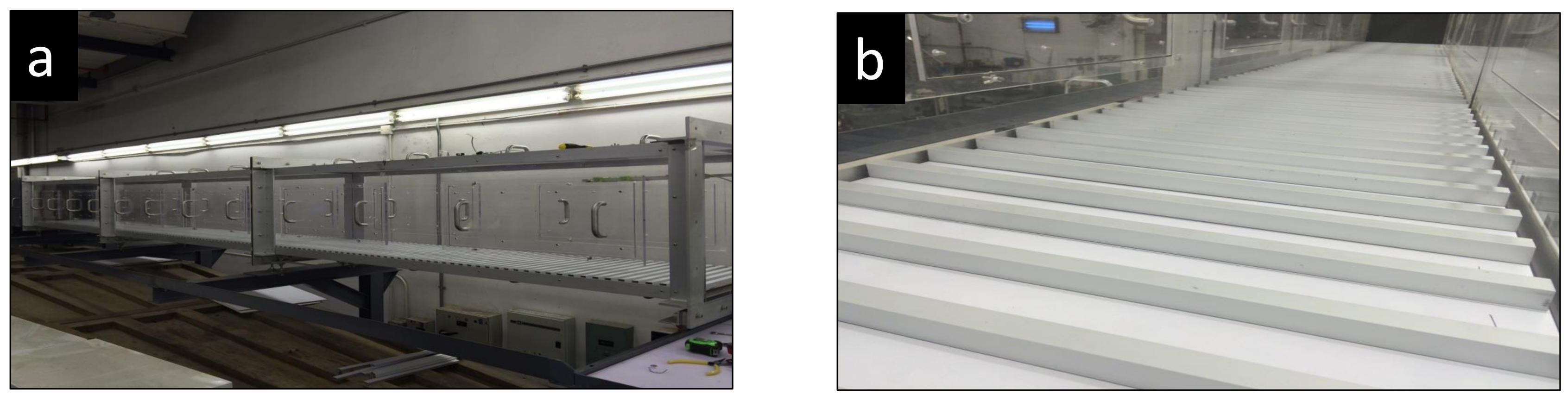


Figure 2. a) Open-circuit wind tunnel at the Department of Mechanical Engineering, HKU, b) Idealised urban area with aspect ratio ($h/b$ = 1/3)

## Flows and Ventilation Estimates over Idealised Urban Areas

- Flow inhomogeneity over idealised urban areas is revealed (Fig. 3a)
- RSL & ISL are clearly identified
- The newly proposed analytical expression performs well in both RSL & ISL for the prediction of velocity profiles over a wide range of aspect ratios, 0.5 < ARs < 0.083 (Fig. 3c)
- Friction factor $f$ & vertical velocity scale $\hat{w}$ are used to parameterise ventilation performance over urban areas with RSL corrections (Fig. 3b)

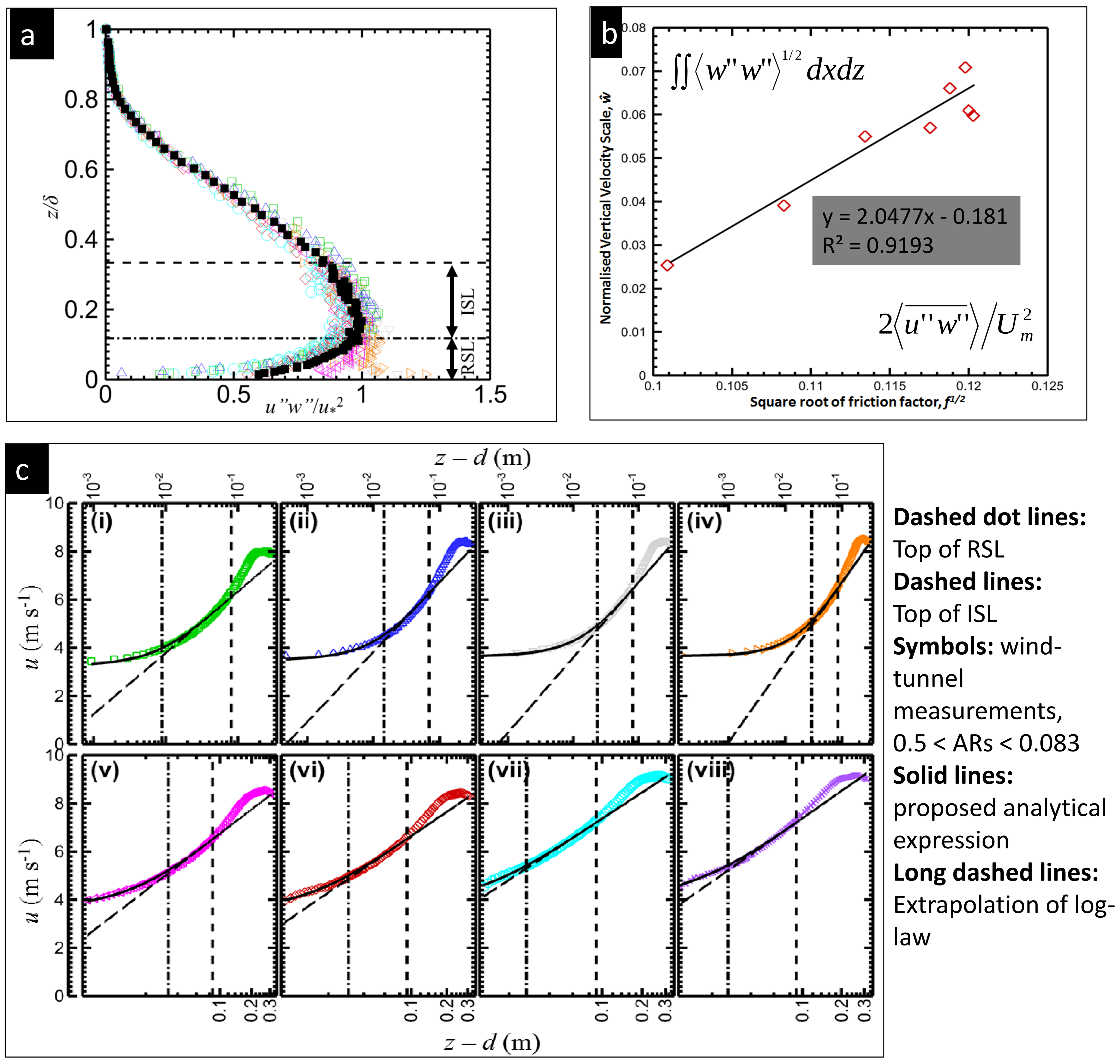


Figure 3. a) Normalised Reynolds stress profiles for AR = 0.25, b) $f^{1/2}$ against $\hat{w}$, c) Comparison of velocity profiles between wind tunnel measurements the newly proposed analytical expression.

## Next steps

- Tests with additional roughness elements of different forms using wind tunnel experiments, i.e. cube roughness, building height variability or realistic city models.
- Quantify the effect of aerodynamic roughness on RSL flows.
- Examine the RSL turbulence using mixing length models.

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