On the Zariski closure of a germ of totally geodesic complex submanifold on an arithmetic variety

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Abstract

Let Ω be a bounded symmetric domain, $\Gamma \subset \mathrm{Aut}(\Omega)$ be a torsion-free lattice, $X := \Omega/\Gamma$. Let $Z \subset X$ be an irreducible quasi-projective variety such that Z is the Zariski closure of the germ of a totally geodesic complex submanifold $S \subset Z$ at some point $p \in Z$. Under certain non-degeneracy conditions one expects Z to be also totally geodesic, so that Z is in particular again uniformized by a bounded symmetric domain.

We explain first of all how this can be established in the special case of the complex unit ball. In this case, Z is proven to be totally geodesic without any additional hypothesis. Writing $\dim_{\mathbb{C}}(S) = d$, the idea is to generate an s-dimensional holomorphic family $\mathcal{A}$ of d-dimensional totally geodesic complex submanifolds $S_\alpha, \alpha \in \mathcal{A}$, on the universal covering ball $\mathbb{B}^n$, so that the $(s+d)$-dimensional set Σ swept out by $\mathcal{A}$ contains an open subset of an irreducible component $\widetilde{Z}$ of $\pi^{-1}(Z)$, $\pi : \mathbb{B}^n \to X$ being the universal covering map, and such that Σ can be extended holomorphically across $\partial\mathbb{B}^n$ at some boundary point $b \in \partial\mathbb{B}^n \cap \overline{\Sigma}$. Properties of Z are then derived from the asymptotic behavior of Σ as points approach b. A strengthening of the argument solves the problem in special cases such as the case where Ω is any bounded symmetric domain and Z is a complex surface.