THE CONFLUENCE OF THE YELLOW RIVER AND THE MEDITERRANEAN:

Synthesis of European mathematics and Chinese mathematics during the seventeenth, eighteenth and nineteenth centuries

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ABSTRACT

Transmission of mathematics between China and other parts of the world already went on in the ancient period. This presentation will however focus on a later period from the early seventeenth century onwards, when the transmission was mainly from the Western world into China.

1 Introduction

The title of this paper obviously carries a metaphoric rather than geographical meaning. The Yellow River in China does not flow into the Mediterranean, nor are they near to each other at all. It refers to the transmission of learning between the Eastern world and the Western world with a large span of land and sea in between. Such transmission has a very long history, with recorded accounts dating back at least to the Han Dynasty in China (the Former Han Dynasty from 202 B.C.E. to 9 C.E., and the Latter Han Dynasty from 25 C.E. to 220 C.E. with the Xin Dynasty of WANG Mang (王莽) from 9 C.E. to 23 C.E. in between). The famous Silk Road acted as the main trade route in Central Asia that established links between a cross-cultural mix of religions, civilizations and people of many different regions, and also enabled exchanges of learning and cultures of people of different races.

In mathematics transmission of learning, either directly or indirectly, between China and regions in Central Asia and the Middle East, India, the Islamic Empire and even Europe further to the West went on for many centuries from the Han Dynasty to the Yuan Dynasty (1279 to 1368). A well-known example often referred to is the Method of Double False Position, sometimes called by the name of "Rule of Khitai" (the term "Khitai", rendered as "Cathay" in English, means China, actually the Liao Dynasty in Northern China from the early 10th century to the early 12th century) (Shen, Crossley, & Lun, 1999). It should also be noted that the transmission of this method remains a debatable issue among historians of mathematics ever since the mid-20th century, with some historians putting it down to a linguistic misunderstanding in the Arabic term "hisab al-khata'ayn (reckoning from two falsehoods)". A more general view is that, despite the uncertainty about the time and way of its transmission, the origin of the method is that of *ying buzu* (盈不足 excess and deficit) explained in Chapter 7 of the Chinese mathematical classic *Jiuzhang Suanshu* [九章算術 The Nine Chapters on the Mathematical Art] that is believed to have been compiled between the 2nd century B.C.E and the 1st century C.E. In the 11th century it appeared in an anonymous

Latin book called *Book of Increase and Decrease*, and later in the famous treatise *Liber Abaci* of 1202 written by the Italian mathematician Fibonacci (also called Leonardo of Pisa). Another interesting item is a lost book with only its title remaining on record in the catalogue of the Library of the Astronomical Bureau of the Yuan Dynasty --- *Fifteen Books of Wuhuliedisi Baisuanfaduanshu* (兀忽烈的四擘算法段數十五部). Some historians surmise that this is a recension of Euclid's *Elements* by the title *Tahrir usul uqlidis* compiled by the Persian mathematician Nasīr al-Dīn Tūsī (1201-1274) in 1248. In the historical record of the Yuan Dynasty it is said that the Möngke Khan (蒙哥汗 1209-1259) of the Mogol Empire, a grandson of Genghis Khan (成吉思汗 1162-1227) and elder brother of Kublai Khan (忽必烈汗 1215-1294) who founded the Yuan Dynasty, understood some geometry through the study of this book. If this is indeed the case, then the book would mark the first transmission of Euclid's *Elements* into China, earlier than that through the Jesuits by three hundred and fifty years. Transmission of mathematical learning during that period is substantiated by the discovery in the late 1950s in the suburb of the city of Xian six iron plates that are 6 x 6 magic squares inscribed in Arabic numerals. More detailed information can be found in (Li 1999).

Obviously this long and intricate story of East-West transmission of mathematical learning is too vast a topic for a short presentation. We will therefore turn our attention to the latter part of the Ming Dynasty (1368 to 1644) when transmission of European mathematics into China on a much more systematic and larger scale began, and the subsequent three centuries. Even for that we can hardly offer a comprehensive account but can only sketch a few highlights.

2 First wave of transmission

The story started with the Christian mission in China of the Jesuits in the late 16th century. As a by-product of the evangelical efforts of the missionaries an important page of intellectual and cultural encounter between two great civilizations unfolded in history, the two most important protagonists of that period being the Italian Jesuit Matteo Ricci (1552-1610) and the Chinese scholar-official of the Ming court XU Guang-qi (徐光啟 1562-1633).

Early contact with Europeans in the 16th century, the first being the Portuguese who came in the double capacity of pirates and merchants, had left the Chinese people with a feeling of distrust and resentment. The brutal behavior of the Dutch, the Spaniards and the English who followed aggravated this uneasy relationship. In 1557 the Portuguese gained a permanent foothold by occupying Macao which developed into a settlement and centre of trade, through which the Catholic missionaries entered China. After studying at Collegio Romano in Rome, Ricci was soon afterwards sent on his China mission. He reached Macao in August of 1582 and proceeded to move into mainland China and finally reach Peking (Beijing) in January of 1601. To ease the hostile feeling the Chinese harboured against foreigners, many missionaries tried to learn the Chinese language, dressed in Chinese clothes and as far as possible adopted the Chinese way of living. Ricci, who adopted a Chinese name LI Ma-dou (利瑪竇), was a brilliant linguist, so he not only learnt the Chinese language but mastered it to such an extent that he could study Chinese classics. Coupled with his knowledge of

Western science he soon impressed the Chinese intellectuals who came into contact with him as an erudite man of learning, thereby commanding their trust and respect, becoming the most prominent Catholic missionary in China.

To Ricci, who studied mathematics under Christopher Clavius (1538-1612) at Collegio Romano, the treatise *Elements* of Euclid was the basis of any mathematical study. He therefore suggested to his Chinese friend XU Guang-qi that *Elements*, based on the version compiled by Clavius in 1574 (with subsequent editions), a fifteen-book edition titled *Euclidis Elementorum Libri XV*, should be the first mathematical text to be translated. Xu set himself to work very hard on this project. He went to listen to Ricci's exposition of *Elements* every day in the afternoon (since he could not read Latin, while Ricci was well versed in Chinese) and studied laboriously for four hours at a stretch every day, and at night he wrote out in Chinese everything he had learnt by day. According to an account by Ricci:

> When he [XU Guang-qi] began to understand the subtlety and solidity of the book, he took such a liking to it that he could not speak of any other subject with his fellow scholars, and he worked day and night to translate it in a clear, firm and elegant style. […] Thus he succeeded in reaching the end of the first six books which are the most necessary and, whilst studying them, he mingled with them other questions in mathematics. […] He would have wished to continue to the end of the Geometry; but the Father [Matteo Ricci] being desirous of devoting his time to more properly religious matters and to rein him in a bit told him to wait until they had seen from experience how the Chinese scholars received these first books, before translating the others.

Ricci reported that Xu agreed and they stopped the translation. The six translated chapters were published in 1607 under the title *Jihe Yuanben* [幾何原本 Source of Quantity]. However, in his heart Xu wanted very much to continue the translation. In a preface to a revised edition of *Jihe Yuanben* in 1611 he lamented, "It is hard to know when and by whom this project will be completed." This deep regret of Xu was resolved only two and a half centuries later when the Qing mathematician LI Shan-lan (李善蘭 1811-1882) in collaboration with the English missionary Alexander Wylie (1815-1887) translated Book VII to Book XV in 1857 based on the English translation of *Elements* by Henry Billingsley published in 1570.

How did *Elements* blend in with traditional Chinese mathematics? Let us first look at what Ricci said about traditional Chinese mathematics:

> The result of such a system is that anyone is free to exercise his wildest imagination relative to mathematics, without offering a definite proof of anything. In Euclid, on the contrary, they recognized something different, namely, propositions presented in order and so definitely proven that even the most obstinate could not deny them.

It is debatable whether it is true the notion of a mathematical proof was completely absent from ancient Chinese mathematics as Ricci remarked. We shall look at one example, which would have made Ricci think otherwise, had he the opportunity of having access to the commentaries of LIU Hui (劉徽) on *Jiuzhang Suanshu* in the third century. This is the following problem: Given a right-angled triangle *ABC* with *AC* as its hypotenuse, inscribe a square in it, that is, construct a square *BDEF* with *D* on *AB*, *E* on *AC*, and *F* on *BC*?

This problem does not appear in Euclid's *Elements*. Were it there, the solution would have probably looked like this: Bisect $\angle ABC$ by *BE* (*E* on *AC*) [Book I, Proposition 9]. Drop perpendiculars *ED*, *EF* (*D* on *AB*, *F* on *BC*) [Book I, Proposition 12]. Prove that *BDEF* is the inscribed square we want. The problem (in a more general version) appears as Added Proposition 15 of Book VI in *Euclidis Elementorum Libri XV*, which was translated by Ricci and Xu: Divide *AB* at *D* such that $AD : DB = AB : BC$ [Book VI, Proposition 10]. Draw *DE* parallel to *BC* and *EF* parallel to *AB*, (*E* on *AC*, *F* on *BC*). *DBFE* is the inscribed square we want.

Now that we know such an inscribed square exists we can ask what the length of its side is. It can be shown from the construction that the side x of the inscribed square in a right-angled triangle with sides of length a, b containing the right angle is given by $x = ab/(a+b)$.

The same problem appears as Problem 15 of Chapter 9 in *Jiuzhang Suanshu,* which says: "Now given a right-angled triangle whose *gou* is 5 *bu* and whose *gu* is 12 *bu*. What is the side of an inscribed square? The answer is 3 and 9/17 *bu*. Method: Let the sum of the *gou* and the *gu* be the divisor; let the product of the *gou* and the *gu* be the dividend. Divide to obtain the side of the square." (See Figure 1.)

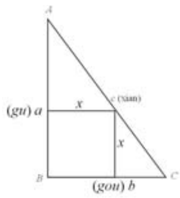

Figure 1. Problem 15 of Chapter 9 of *Jiuzhang Suanshu*

The line of thinking and style of presentation of the explanation by LIU Hui are quite different from that in *Elements*. Liu gave a "visual proof" of the formula $x = ab/(a+b)$ by dissecting and re-assembling coloured pieces. (See Figure 2.). Liu's commentary actually describes the coloured pieces so that were the original diagram extant it would provide the making of a useful teaching aid!

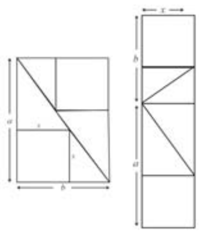

Figure 2. Explanation by LIU Hui

How did XU Guang-qi perceive Euclidean geometry which he newly learnt from Clavius' rendition of Euclid's *Elements*, and to what extent did he understand the thinking, approach and presentation of the book, which are so very different from those of traditional

Chinese mathematics that he was familiar with? Despite Xu's emphasis on utility of mathematics, he was sufficiently perceptive to notice the essential feature about *Elements*. In a preface to *Jihe Yuanben* of 1607 he wrote:

> As one proceeds from things obvious to things subtle, doubt is turned to conviction. Things that seem useless at the beginning are actually very useful, for upon them useful applications are based. […] It can be truly described as the envelopment of all myriad forms and phenomena, and as the erudite ocean of a hundred schools of thought and study.

In a preface to another book *Celiang Fayi* [測量法義 Methods and Principles in Surveying] of 1608, which is an adapted translation by Matteo Ricci and XU Guang-qi of parts of *Geometria practica* compiled by Christopher Clavius in 1606, he wrote:

> It has already been ten years since Master Xitai [西泰子 that is, Matteo Ricci] translated the methods in surveying. However, only started from 1607 onwards the methods can be related to their principles. Why do we have to wait? It is because at that time the six books of *Jihe Yuanben* were just completed so that the principles could be transmitted. […] As far as the methods are concerned, are they different from that of measurement at a distance in *Jiuzhang* [*Suanshu*] and *Zhoubi* [*Suanjing*]? They are not different. If that is so, why then should they be valued? They are valued for their principles.

He elaborated this point in an introduction to his 1608 book *Celiang Yitong* [測量異同 Similarities and Differences in Surveying] by saying:

> In the chapter on *gougu* of *Jiuzhang Suanshu* there are several problems on surveying using the gnomon and the trysquare, the methods of which are more or less similar to those in the recently translated *Celiang Fayi* (Methods and Principles in Surveying). […] The y*i* [義 principles] are completely lacking. Anyone who studies them cannot understand where they are derived from. I have therefore provided new *lun* [論 proofs] so that examination of the old text becomes as easy as looking at the palm of your hand.

In connection with this he wrote in a memorial submitted to the Emperor in 1629 in his capacity as the official in charge of the Astronomical Bureau:

> [not knowing that] there are *li* [理 theory], *yi* [義 principle], *fa* [法 method] and *shu* [數 calculation] in it. Without understanding the theory we cannot derive the method; without grasping the principle we cannot do the calculation. It may require hard work to understand the theory and to grasp the principle, but it takes routine work to derive the method and to do the calculation.

With this perception Xu tried hard to assimilate Western mathematics and to synthesize it with Chinese traditional mathematics. One example is his work on Problem 15 in Chapter 9 of *Jiuzhang Suanshu* and Added Proposition 15 of Book VI in *Euclidis Elementorum Libri XV* reported in his 1609 book *Gougu Yi* [勾股義 Principle of the Right-angled Triangle]. He explained this as Problem 4, which involves complicated reasoning that may seem rather round-about and unnecessary. Perhaps it indicates a kind of incompatibility between the two styles of doing mathematics so that it would be unnatural to force one into the mould of the other. However, despite its shortcomings this also indicates an admirable attempt of Xu to synthesize Western and Chinese mathematics. A more detailed discussion on this topic can be found in (Siu, 2011).

Despite the enthusiasm on the part of XU Guang-qi to introduce *Elements* into China, the Chinese in the seventeenth and eighteenth centuries did not seem to feel the impact of the essential feature of Western mathematics exemplified in *Elements* as strongly as he. The influence of the newly introduced Western mathematics on mathematical thinking in China was not as extensive and as direct as he had imagined. However, unexpectedly the fruit was brought forth not in mathematics, but in a domain of perhaps even higher historical importance. Study of Western science in general, and Western mathematics in particular, attracted the attention of some active liberal intellectuals of the time, among whom three prominent figures KANG You-wei (康有為 1858-1927), LIANG Qi-chao (梁啟超 1873-1929) and TAN Si-tong (譚嗣同 1865-1898) played an important role in the history of modern China as leading participants in the episode of "Hundred-day Reform" of 1898. The "Hundred-day Reform" ended in failure with Tan being arrested and executed in that same year, while Kang and Liang had to flee the country and went to Japan. This was one important step in a whole series of events that culminated in the overthrow of Imperial Qing and the establishment of the Chinese Republic in 1911.

Together with LI Zhi-zao (李之藻 1565-1630) and YANG Ting-jun (楊廷筠 1557-1627), colleagues and friends of XU Guang-qi, the three scholar-officials of high ranking in the Ming Court were hailed as the "three pillars of the Catholic Church in China". When Li got acquainted with Ricci in 1601 in Nanjing. he was deeply impressed by the map of the world that Ricci prepared, the *Kunyu Wanguo Quantu* [坤輿萬國全圖 Complete Map of the Myriad Countries of the World]. Li himself had prepared a map of the fifteen provinces of China at the age of twenty and thought at the time he had well mastered the knowledge of cartography so that he was all the more amazed by this work of Ricci.

Li collaborated with Ricci to compile the treatise *Tongwen Suanzhi* [同文算指, literally meaning "rules of arithmetic common to cultures"], which first transmitted into China in a systematic and comprehensive way the art of *bisuan* (筆算 written calculation) that had been in common practice in Europe since the sixteenth century. This treatise, accomplished in 1613, was a compilation based on the 1583 European text *Epitome Arithmeticae Practicae* (literally meaning "abridgement of arithmetic in practice") of Clavius and the 1592 Chinese mathematical classics *Suanfa Tongzong* [算法統宗, literally meaning "unified source of computational methods"] of CHENG Da-wei (程大位 1533-1606). In accord with a prevalent intellectual trend of the time known as *zhongxi huitong* (中西會通, literally meaning "synthesis of Chinese and Western [learning]) started by the dedicated work of the translation of *Elements* in 1607, Li also attempted to synthesize European mathematics with traditional Chinese mathematics by treating problems taken out of Chinese mathematical texts by the newly introduced method of written calculation. A more detailed discussion on this topic can be found in (Siu, 2015b). We give only one example on division here.

In traditional Chinese mathematics, calculation in arithmetic was performed using counting rods since very early times. The arithmetical operations were explained in mathematical classics such as *Sunzi Suanjing* (孫子算經 Master Sun's mathematical manual) of the fourth/fifth century. In the Western world there was a movement of contest in efficiency of reckoning between the so-called "abacists" and "algorists" towards the latter part of medieval time. In particular, a method known as the *gelosia* method, coming from the Islamic world, was commonly used at the time. (See Figure 3.) Written calculation did appear in some Chinese texts even before *Tongwen Suanzhi*, but not in a way as systematic and as comprehensive as in *Tongwen Suanzhi*. The *gelosia* method introduced into China in those texts was given a picturesque name of *pudijin* (鋪地錦, literally meaning "covering the floor with a glamorous carpet") by CHENG Da-wei.

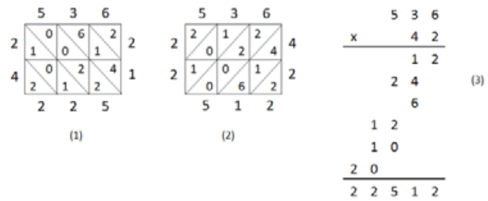

Figure 3. *Gelosia* method of multiplication

LI Zhi-zao seemed to prefer the more modern method to this pictureqe *pudijin*. In *Tongwen Suanzhi* division is performed by the *galley* method, which was already quite well-known in the Western world, for instance, in the *Treviso Arithmetic* of 1478. (See Figure 4, with the last item in modern notation inserted for comparison.)

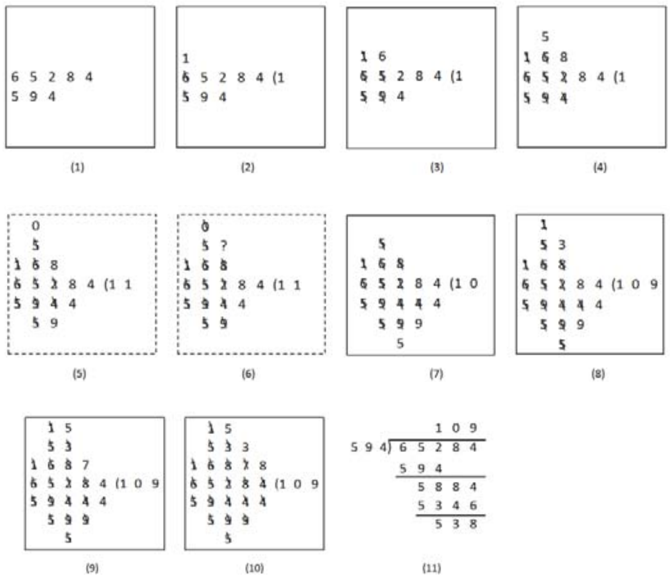

Figure 4. *Galley* method of division

3 Second and third waves of transmission

The translation of *Elements* by XU Guang-qi and Matteo Ricci led the way of the first wave of transmission of European science into China, with a second wave (or a wake of the first wave as some historians would see it) and a third wave to follow in the Qing Dynasty (1644-1911), but each in a rather different historical context with quite different mentality. The gain of this first wave seemed momentary and passed with the downfall of the Ming Dynasty. Looking back we can see its long-term influence, but at the time this small window which opened onto an amazing outside world was soon closed again, only to be forced open as a wider door two hundred years later by Western gunboats that inflicted upon the ancient nation a century of exploitation and humiliation, thus generating an urgency to know more about and to learn with zest from the Western world.

The main features and the mentality of the three waves of transmission of Western learning into China can be summarized in the prototype slogans of the three epochs. In the late-sixteenth to mid-seventeenth centuries (during the Ming Dynasty) the slogan was: "In order to surpass we must try to understand and to synthesize (欲求超勝必須會通)." In the first part of the eighteenth century (during the Qing Dynasty) the slogan was: "Western learning has its origin in Chinese learning (西學中源)." In the latter part of the nineteenth century (during the Qing Dynasty) the slogan was: "Learn the strong techniques of the '[Western] barbarians' in order to control them (師夷長技以制夷)." It is interesting to note the gradual and subtle change in the attitude and mentality on the Chinese side in learning from the Western world, from an open-minded enthusiasm with self-confidence to a strange mix of self-arrogance and resistance and finally to a feeling of urgency in the face of the precarious fate of their mother country.

The second wave came and lasted from the mid-seventeenth century to the mid-eighteenth century. Instead of Chinese scholar-officials the chief promoter was Emperor Kangxi (康熙) of the Qing Dynasty (reigned 1662-1722). Instead of Italian and Portuguese Jesuits the Western partners were mainly French Jesuits, the so-called "King's Mathematicians" sent by Louis XIV, the "Sun King" of France (reigned 1643-1715), in 1685.

This group of Jesuits led by Jean de Fontaney (1643-1710) reached Peking in 1688. An interesting account of their lives and duties in the Imperial Court was recorded in the journal written by one of the group, Joachim Bouvet (1656-1730), and published in 1697. Bouvet recounted how he and the other Jesuits conducted lessons in science and mathematics in the Imperial Court and how Emperor Kangxi studied with enthusiasm and diligence. More information and an in-depth analysis of this episode can be found in (Jami, 2012).

A main outcome was the compilation of a monumental one-hundred-volume treatise *Lüli Yuanyuan* [律曆淵源 Origins of Mathematical Harmonics and Astronomy] commissioned by Emperor Kangxi, worked on by a large group of Jesuits, Chinese scholars and official astronomers. The project started in 1713 and the treatise was published in 1722/1723, comprising three parts: *Lixiang Kaocheng* [曆象考成 Compendium of Observational Computational Astronomy], *Shuli Jingyun* [數理精蘊 Collected Basic Principles of Mathematics] and *Lülü Zhengyi* [律呂正義 Exact Meaning of Pitchpipes]. The treatise *Shuli Jingyun* includes both traditional Chinese mathematics, the part that was still extant and was understood at the time, as well as Western mathematics, highly likely from the "lecture notes" prepared by the missionaries for Emperor Kangxi.

Books 2 to 4 of *Shuli Jingyun* are on geometry, which is believed to be based on *Elémens de géométrie* by Ignace Gaston Pardies (1636-1673), first published in 1671 with a sixth edition in 1705. Books 31 to 36 are on solving algebraic equations known by the name of *jiegenfang* (借根方 borrowed root and powers), taught to Emperor Kangxi by the Belgian Jesuit Antoine Thomas (1644-1709), who had studied at University of Coimbra in Portugal and compiled *Synopsis mathematica*, based on the 1600 book *De numerosa potestatum ad exegesim resolution* (On the numerical resolution of powers by exegetics) of François Viète (1540-1603). Thomas later revised it as *Suanfa Zuanyao Zonggang* (算法纂要總綱 Outline of the essential calculations) and *Jiegenfang Suanfa* (借根 方算法 Method of borrowed root and powers), to be used as lecture notes for the mathematics lessons on solving algebraic equations in the Imperial Court.

The Chinese mathematician MEI Jue-cheng (梅瑴成 1681-1763) told the story on how he learnt this new method from Emperor Kangxi, who told him that the Westerners called it *aerrebala* (阿爾熱巴拉 algebra) that means "Method from the East". Mei suspected that the method resembled that of a traditional Chinese method of *tianyuan* (天元 celestial unknown) and studied it to clarify the matter, coming to the conclusion that despite the terminologies the two methods were the "the same, not just a mere resemblance." This would explain how the saying "Western learning has its origin in Chinese learning" got promulgated in those days. This is probably a tactic on the part of Emperor Kangxi to make his subjects willing to learn it and would not regard it as something opposing traditional value. Or, maybe he really thought that the method originated in older Chinese learning, without knowing that the art of solving algebraic equations was developed by Islamic mathematicians in medieval time. Indeed, similar methods were explained in Chinese mathematical classics of earlier days, most of which became less known by the Ming and early Qing period.

But when the French Jesuit Jean-François Foucquet (1665-1741) lectured on the "new method of *aerrebala*", which is symbolic algebra as explained in the 1591 book *Artem Analyticem Isagoge* (Introduction to the analytical art) of Viète, Emperor Kangxi reacted to it with strong resistance. I tend to believe that Emperor Kangxi was very diligent, determined and bright, but also studied hard not without vanity and intention to show off his knowledge with a political motive. Much as he left us with several sets of monumental compendia and a collection of books that benefit posterity, it has to be admitted (sadly) that owing to the limitation in his scope and motive this period of transmission was also a "missed opportunity" for China, because, being confined to a small group within the Imperial Court, it failed to exert the influence that would help the country to move forward and catch up with the Western world which had moved forward by leaps and bounds by the seventeenth century.

The third wave came in the last forty years of the nineteenth century in the form of the so-called "Self-strengthening Movement" after the country suffered from foreign exploitation during the First Opium War (1839-1842) and the Second Opium War (1856-1860). This time the initiators were officials led by Prince Gong (恭親王 1833-1898) with contribution from Chinese scholars and Protestant missionaries coming from England or America, among whom were LI Shan-lan and Alexander Wylie who completed the translation of *Elements*. In 1862 *Tongwen Guan* [同文館 College of Foreign Languages] was established by decree, at first serving as a school for studying foreign languages to train interpreters but gradually expanded into a college of Western learning, along with the establishment of other colleges of similar nature that sprouted in other cities like Shanghai, Guangzhou, Fuzhou, Tianjin, as well as the establishment of arsenals, shipyards and naval schools during the period of “Self-strengthening Movement” as a result of the fervent and urgent desire of the Chinese government to learn from the West in order to resist the foreign exploitation the country went through in the first and second Opium Wars. The slogan of the day, “learn the strong techniques of the ‘[Western] barbarians’ in order to control them”, reflected the purpose and mentality during that period. In 1866 the School of Astronomy and Mathematics was added to *Tongwen Guan*, with LI Shan-lan as its head of department. In 1902 *Tongwen Guan* became part of Peking Imperial University, which later became what is now Beijing University. A more detailed discussion on this topic can be found in (Chan & Siu, 2012).

4 An Epilogue in Montpellier

In 1810 the French mathematician Joseph Diaz Gergonne (1771–1859) established his own mathematics journal, officially called the *Annales de mathématiques pures et appliquées* but more popularly known as *Annales de Gergonne,* which was the first privately run journal wholly on mathematical topics. Geometry figured most prominently in this journal with many famous mathematicians of the time publishing papers there until the journal was discontinued in 1832 after Gergonne became the Rector of the University of Montpellier. To facilitate a dialogue between the Editor and the readership the journal posed problems regularly besides publishing papers. In the first volume of *Annales de Gergonne* the following problem was posed: “Given any triangle, inscribe three circles in such a way that each of them touches the other two and two sides of the triangle.”

Soon after the problem was posed a solution appeared in a later issue of the journal and referred to a letter from a reader, the Italian mathematician Giorgio Bidone (1781-1839) in Turin, who pointed out that the original problem was posed by his compatriot Gianfrancesco Malfatti (1731-1807). Malfatti in 1803 asked, “Given a right triangular prism of any sort of material, such as marble, how shall three circular cylinders of the same height as the prism and of the greatest possible volume of material be related to one another in the prism and leave over the least possible amount of material?” Malfatti thought that the three non-overlapping circles inside the triangle occupying optimal space would be three “kissing circles”. Actually this is never the solution, but it was only realized with the optimality problem fully settled as late as in 1994!

In the latter part of the 19th century some foreign missionaries, along with spreading Christian faith, worked hard to propagate Western learning in old imperial China through various means, one of which was publishing periodicals. The monthly periodical *Zhongxi Wenjian Lu* [中西聞見錄 Record of News in China and West] with English title *Peking Magazine*, founded in 1872, announced in the first issue that it adopted the practice and format of newspapers in the Western world in publishing international news and recent happenings in different countries, as well as essays on astronomy, geography and *gewu* [格物

science, literally meaning “investigating things”]. The fifth issue (December, 1872) of this magazine carried the following posed problem:

> A plane triangle (acute, right or obtuse) contains three circles of different radii that touch each other. We want to fix the centres of the three circles. What is the method? All students in *Tongwen Guan* retreated from trying this problem. Whoever can solve the problem should send the diagram [of the solution] to the School of Astronomy and Mathematics and would be rewarded with a copy of *Jihe Yuanben* [Chinese translation of Euclid’s *Elements*]. The diagram [of the solution] would be published in this magazine so that the author would gain universal fame.

A solution submitted by a reader was published in the eighth issue (March, 1873), followed by a comment by another reader in the twelfth issue (July, 1873) together with an acknowledgement of the error and a further comment by the School of Astronomy and Mathematics of *Tongwen Guan*.

This kind of fervent exchange of academic discussion carried on in public domain was a new phenomenon of the time in China. In 1897 a book on homework assignments by students of *Longcheng Shuyuan* [龍城書院 Academy of the Dragon City], which was a private academy famous for its mathematics curriculum, contained two articles that gave different solutions to the Malfatti Problem with accompanying remarks by the professor. One solution is particularly interesting because it made use of a hyperbola, which is a mathematical object that was totally foreign to Chinese traditional mathematics and was newly introduced in a systematic way only by the mid-nineteenth century. It is not certain when the Malfatti Problem was first introduced into China. Apparently it was introduced by Westerners into China only two to three decades after the problem became well-known in the West, at a time when the Chinese were just beginning to familiarize themselves with Euclidean geometry, which was not part of their traditional mathematics. A more detailed discussion on this topic can be found in (Siu, 2015a). It is worth noting, from the active discussion generated around the Malfatti Problem, how enthusiastic the Chinese were in learning mathematics from Westerners in the late nineteenth century.

5 Endnote

In the preface as well as in two forewords to *Tongwen Suanzhi*, LI Zhi-zao and his friends and fellow official-scholars XU Guang-qi and YANG Ting-jun stressed the meaning of *tongwen* (literally meaning “common cultures”), adopted as part of the title of the book, which exhibits their open mind and receptive attitude to foreign learning, at the same time indicating a deep appreciation of the common cultural roots of mathematics despite different mathematical traditions. Let us look at some of their sayings to further illustrate this point.

XU Guang-qi said in the Preface at the printing of *Tongwen Suanzhi* (1613):

> The origin of numbers, could it not be at the beginning of human history? Starting with one, ending with ten, the ten fingers symbolize them and are bent to calculate them, [numbers] are of unsurpassed utility! Across the five directions and myriad countries, changes in customs are multitudinous. When it comes to calculating

numbers, there are none that are not the same; that all possess ten fingers, there are none that are not the same.

(In my primary school days we were discouraged from making use of this "unsurpassed utility" to aid in doing arithmetic. When the teacher spotted such an attempt of using the fingers to count, the pupil would be reprimanded for doing so. In order not to get a reprimand I did my finger-counting by hiding the hand in the pocket of my pants. My good friend and a mathematics educator in Hong Kong, LAW Huk-Yuen, jokingly dubbed this act the pre-historic version of a genuine "pocket calculator"!)

LI Zhi-zao said in the Preface to the reprinting of *Tianzhu Shiyi* (天主實義 The True Meaning of the Lord of Heaven, written by Matteo Ricci and printed in 1603 in Peking): "Across the seas of the East and the West the mind and reasoning are the same [同 *tong*]. The difference lies only in the language and the writing."

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