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Boundary behavior of holomorphic maps into bounded symmetric domains and applications to geometric problems

Abstract

Let $\Omega \Subset \mathbb{C}^n$ be a bounded symmetric domain in its Harish-Chandra realization and equipped with the Bergman metric ds^2_Ω. Let $b \in \partial\Omega$ a boundary point and S a germ of complex submanifold of $\mathbb{C}^n$ at b. We are interested to study the boundary behavior of $(S \cap \Omega, ds^2_\Omega|_{S\cap\Omega})$ near b as a Kähler submanifold of (Ω, ds^2_Ω). By way of examples we will illustrate in special cases how the knowledge of such boundary behavior implies solutions to geometric problems.

We will illustrate this with two examples. The first example is the case where Ω is the complex unit ball. In this case, slightly perturbing the base point $b \in S \cap \partial B^n$ if necessary, as is well-known S is of asymptotically constant holomorphic sectional curvature $-\frac{2}{n+1}$. This implies the following geometric statement. Given a finite-volume quotient $X = B^n/\Gamma$ and a germ of complex geodesic submanifold $E \subset X$ at some point $x \in X$, the Zariski closure of E in X is totally geodesic. In particular, it implies that the Gauss map on any irreducible subvariety $Z \subset X$ (with respect to the projective structure on X inherited from B^n) is generically finite, a result first established by Jun-Muk Hwang. The second example is the case where Ω is an irreducible bounded symmetric domain, $b \in \partial\Omega$ is a smooth point of $\partial\Omega$, and S is a germ of holomorphic curve at b. In this case we show that, slightly perturbing the base point b if necessary, the second fundamental form is asymptotically zero when one approaches $\partial S \cap \mathrm{Reg}(\partial\Omega)$. (A precise rate of decay of the second fundamental form is also obtained.) The asymptotic behavior gives an alternative proof of the characterization of measure-preserving holomorphic maps for Ω other than the unit disk granted the algebraic extendibility of such maps (as established by Mok-Ng). Both examples are prototypes of much more general phenomena with interesting geometric consequences.