

# Capacity of Hybrid Wireless Networks with Long-Range Social Contacts Behavior

Ronghui Hou, Yu Cheng, Jiandong Li, Min Sheng, and King-Shan Lui

**Abstract**—Hybrid wireless network is composed of both ad hoc transmissions and cellular transmissions. Under the  $L$ -maximum-hop routing policy, flow is transmitted in the ad hoc mode if its source and destination are within  $L$  hops away; otherwise, it is transmitted in the cellular mode. Existing works study the hybrid wireless network capacity as a function of  $L$  so as to find the optimal  $L$  to maximize the network capacity. In this paper, we consider two more factors: traffic model and base station access mode. Different from existing works which only consider the uniform traffic model, we consider a traffic model with social behavior. We study the impact of traffic model on the optimal routing policy. Moreover, we consider two different access modes: one-hop access (each node directly communicates with base station) and multi-hop access (node may access base station through multiple hops due to power constraint). We study the impact of access mode on the optimal routing policy. Our results show that: 1) The optimal  $L$  does not only depend on traffic pattern, but also the access mode; 2) One-hop access provides higher network capacity than multi-hop access at the cost of increasing transmitting power; 3) Under the one-hop access mode, network capacity grows linearly with the number of base stations; however, it does not hold with the multi-hop access mode, and the number of base stations has different effects on network capacity for different traffic models.

**Index Terms**—Hybrid wireless networks, throughput capacity, routing policy, social contact behavior, access mode.

## I. INTRODUCTION

Studying network capacity is a big challenge and very important work to deeply understand the service capability provided by wireless networks. Gupta and Kumar are the first to give an extensive research on the capacity of pure ad hoc networks [1]. In a network of  $n$  nodes with channel capacity  $W$ , the average node throughput capacity is  $\Theta(\frac{W}{\sqrt{n \log n}})$ <sup>1</sup> when each node randomly and independently chooses another node in the network as its destination. Gupta

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Manuscript received Jul 07, 2005; revised Jan 26, 2016 and Jun 14, 2016. The work of Ronghui Hou, Jiandong Li, and Min Sheng was supported in part by the National Natural Science Foundation of China (Grants Nos. 61231008 and 61571351), the important national science & technology specific projects 2015ZX03002006-003, Natural Science Basic Research Plan in Shaanxi Province of China (Program No. 2016JM6028), and Huawei Innovation Research Program.

The work of Yu Cheng was supported in part by National Science Foundation (NSF) under grants CNS-1053777, CNS-1320736, and ECCS-1610874.

<sup>1</sup>The following notations are used throughout the paper. For two positive functions  $f(x)$  and  $g(x)$ :

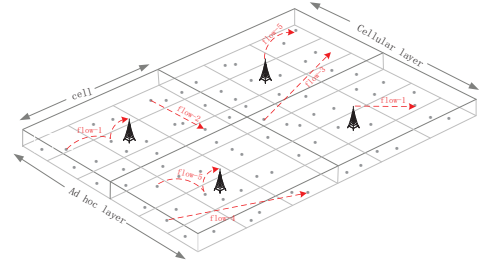


Fig. 1. A illustration for hybrid wireless network.

et al suggest to introduce base stations to the network in order to improve network capacity. When the path of a flow is long, the packets can be forwarded by base stations. Since the transmissions between two base stations are over the wired network, the wireless resources consumption would be reduced. The network composed by ad hoc transmission and cellular transmission is called a hybrid wireless network. Fig. 1 illustrates a hybrid wireless network which is borrowed from [2].

Routing policy is a dominant factor that affects network capacity. Given a flow, routing policy determines whether the flow is transmitted over the ad hoc layer or the cellular layer. Existing works apply two kinds of routing policies: *same cell routing policy* [3] and  *$L$ -maximum-hop routing policy* [4]. With the same cell routing policy, two nodes communicate in an ad hoc manner if they are in the same cell; otherwise, they communicate via the base stations. If the  $L$ -maximum-hop routing policy is applied, the flow is transmitted over the ad hoc layer if the source and the destination are within  $L$  hops away; otherwise, the flow is transmitted over the cellular layer. In this paper, we consider the  $L$ -maximum-hop routing policy.

Intuitively, different  $L$  would produce different network throughput. Only limited number of short ad hoc flows are allowed when  $L$  is too small, such that the ad hoc resources may not be fully utilized. On the other hand, if  $L$  is too large, many long flows are transmitted in the ad hoc layer, and more ad hoc resources are consumed for each flow due to interference. Therefore, there exists an optimal  $L$  to maximize the network throughput. This work aims at studying

- 1)  $f(n) = O(g(n))$  iff (if and only if) there exist constants  $N$  and  $C$  such that  $|f(n)| \leq C|g(n)|$  for all  $n > N$ ;
- 2)  $f(n) = \Omega(g(n))$  means that  $g(n) = O(f(n))$ ;
- 3)  $f(n) = \Theta(g(n))$  implies that  $f(n) = O(g(n))$  and  $g(n) = O(f(n))$ ;
- 4) If  $g(n) \neq 0$ ,  $f(n) = o(g(n))$  iff  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$ .
- 5) If  $g(n) = w(f(n))$ ,  $f(n) = o(g(n))$ .

the network throughput capacity as a function of  $L$  so as to find the optimal  $L$ .

Several metrics are used to study network capacity. Gupta and Kumar introduced two new notions of network capacity: *throughput capacity* and *transport capacity* [1]. The throughput capacity refers to the time average of the number of bits per second that can be transmitted by every source to its destination, while the network's transport capacity indicates the sum of products of bits and the distance over which they are carried. For instance, if two bits are carried over two meters in each second, the network's transport capacity is four. Both throughput capacity and transport capacity quantify the end-to-end throughput that can be achieved between source-destination pairs. The throughput capacity is useful to capture the impact of network topology, routing mechanism, and scheduling algorithm on network capacity. Another metric, *transmission capacity*, often used together with outage probability, quantifies the achievable single-hop rates in large wireless networks. Transmission capacity analysis normally assumes the one-hop flow traffic model, and focuses on the impact of physical layer details on the network capacity. In this work, we study the impact of routing policy on the capacity of hybrid wireless networks, and thus, we focus on the analysis of throughput capacity.

Most of the existing works on network capacity assume the uniform traffic model that each source randomly selects a node as its destination. Practically, traffic depends on the behavior of the users [5]. It is well known that small-world phenomenon is pervasive in a range of networks arising in nature and technology, and is a fundamental ingredient in the evolution of the World Wide Web. Watts and Strogatz proposed a model for the small-world phenomenon, in which the contacts of the social network are divided into *local* and *long-range* contacts [6]. A source node normally contacts its neighbors very often, but sometimes, the source node also contacts targets which are far away from the source node. In the model proposed by [6], a source node randomly selects its long-range contacts with a probability proportional to the inverse  $\alpha^{\text{th}}$ -power distribution, which is the long-range social contacts traffic model in our draft. Local contact denotes the communication from a source node to its neighbors. If all the traffics are local contact traffics, the base station would not be used for the data transmission, and the network capacity is the same as that in a pure ad hoc network. In this work, we study the capacity of hybrid wireless networks with the long-range social traffic model. We aim to capture the impact of joint routing policy and traffic model on the capacity of hybrid wireless networks.

## II. RELATED WORKS

Gupta and Kumar began the line of network capacity study [1], and their work shows the asymptotic behavior of per-node throughput capacity is  $\Theta(\frac{W}{\sqrt{n \log n}})$ . The work in [1] shows that pure wireless ad hoc network is not scalable with the number of nodes, and many following works study how to improve network capacity. Generally speaking, there are three directions to improve network capacity. One direction

is to apply advanced physical layer techniques, such as direct antennas [7], multi-packet reception (MPR) [8], and network coding [9]. The second direction is to exploit node mobility to increase network capacity [10]. The third direction is to augment infrastructures, such as base stations, to the pure ad hoc network, which is called hybrid wireless network [11], and is the focus of this paper. In a hybrid wireless network, data packets can be forwarded by base stations. Since the transmission between two base stations is over a wired network, the consumption of wireless resources would be reduced, and thus, the network throughput capacity is increased. The work in [12] provides a comprehensive overview of the throughput capacity in wireless networks including pure ad hoc networks and hybrid wireless networks. We will describe the very related works and present the significant differences between our work and the existing works.

Many works study the network capacity of hybrid wireless networks. The same cell routing policy was studied first. The work in [3] shows that if the number of base stations  $m$  grows asymptotically slower than  $\sqrt{n}$ , the maximum network throughput capacity is  $\Theta(\sqrt{\frac{n}{\log \frac{n}{m^2}} W})$ . On the other hand, if  $m$  grows asymptotically faster than  $\sqrt{n}$ , the maximum throughput capacity is  $\Theta(mW)$ . The work in [13] assumes that  $m$  grows asymptotically with  $n$ , and sets the transmission range of each node in a way that any two nodes in the same cell can directly communicate with each other. This implies that all the flows are transmitted by either one hop or two hops. The authors derive the network throughput capacity as  $\Theta(n \frac{W}{\log n})$ . The work in [14] considers a specific case that  $m = \Theta(n^d)$ , where  $0 < d < 1$ , and derive the hybrid network capacity under the same cell routing policy. Ref. [2] allows the source node to transmit to a base station using multiple hops. The authors derive the network capacity as  $\Theta(\sqrt{\frac{nm}{\log n}} W_a) + \Theta(\frac{m}{n} W_c)$ , where  $W_a$  and  $W_c$  are the bandwidth allocated for ad hoc mode and cellular mode, respectively. When  $m$  grows asymptotically slower than  $\frac{n}{\log n}$ , the maximum capacity is denoted by  $\Theta(\sqrt{\frac{nm}{\log n}} W)$ , which follows that if the number of base stations are increased by  $k$  times, we have a gain of  $\sqrt{k}$  on capacity.

The work in [4] applies the  $L$ -maximum-hop routing policy. If the source can reach to the destination within  $L$  hops, the packets are transmitted in the ad hoc manner. Otherwise, the packets are forwarded by the base stations. It is shown in [4] that when  $L = \Omega(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n})$ , the network capacity is  $\Theta(\frac{nW_a}{L \log n}) + \Theta(mW_c)$ . When  $L$  grows asymptotically slower than  $\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}$ , the network capacity becomes  $\Theta(L^2 \log n W_a) + \Theta(mW_c)$ . The work in [15] also applies the  $L$ -maximum-hop routing policy, and studies the hybrid network capacity with a function of  $L$ , base station number  $m$ , and the beamwidth of directional antenna  $\theta$ . The authors in [15] show that the throughput capacity of the hybrid directional wireless network is  $\Theta((\frac{nW_a}{\theta^2 L \log n}) W_a) + \Theta(mW_c)$ , if  $L = \Omega(\frac{n^{1/3}}{\theta^{4/3} \log^{2/3} n})$ ; and  $\Theta((\theta^2 L^2 \log n) W_a) + \Theta(mW_c)$ , if  $L = o(\frac{n^{1/3}}{\theta^{4/3} \log^{2/3} n})$ .

The work in [16] assumes that a source node transmits a

portion of traffic to the destination using the ad hoc mode, and transmits another portion through the base station, so that their solutions are independent of routing policy. Their study shows that if  $m$  grows asymptotically slower than  $\sqrt{\frac{n}{\log n}}$ , adding base station does not take benefit according to the scaling law of network throughput capacity. If  $m$  grows asymptotically faster than  $\sqrt{\frac{n}{\log n}}$  and slower than  $\frac{n}{\log n}$ , the network capacity is  $\Theta(mW)$ . If  $m$  grows asymptotically faster than  $\frac{n}{\log n}$ , the network throughput capacity becomes  $\Theta(\frac{nW}{\log n})$ .

The work in [17] applies the same assumption as [13] that  $m$  grows linearly with  $n$ . This work shows that per-node throughput capacity  $\Theta(1)$  is achievable with the appropriate power control. The work in [18] considers that each base station is placed with multiple antennas, and the total number of antennas in the network scales linearly with  $n$ . The authors derive the network capacity, so as to find the optimal network configuration. All the flows in [17], [18] are transmitted over the cellular layer.

In addition to the number of nodes and number of base stations, some other factors impacting on the network capacity have received much attention, such as multi-channel multi-interface, channel model, network topology, traffic pattern, and so on [12]. The work in [19] shows that different hybrid network dimensions lead to significantly different capacity scaling laws. Specifically, for a one-dimensional network, augmenting base stations provides substantial capacity gain. The work in [19] also analyzes the capacity of 2-dimensional networks with square shape and strip shape.

As [12] mentioned, traffic pattern is an important factor that affects network capacity. The work in [20] discusses the impact of traffic model on network capacity. The probability that a source node communicates with a destination  $x$  distance units away is proportional with  $x^{-\alpha}$  [21]. The work in [20] derives the network capacity as a function of  $\alpha$ . Although the power law distribution considers the impact of distance on contact probability, this distribution does not consider the fact that a user usually has more than one long-range social contact in its social group [6]. The work in [5] studies the pure ad hoc network capacity with the long-range social behavior. The authors present the network capacity as a function of the social behavior parameter. The problem becomes more complicated when hybrid wireless network is considered because the network capacity not only depends on number of nodes and traffic model, but also depends on number of base stations and routing policy. Our work is the first work that studies the hybrid wireless network capacity when users exhibit social contact behaviors. Our results allow the optimal routing policy to be identified to maximize network capacity.

### III. HYBRID WIRELESS NETWORK MODEL

*Network Architecture* Following the works in [2], [3], [15], [16], [18], [22], we consider a two-dimensional hybrid wireless network with unit square area. The hybrid wireless network consists of two components: the ad hoc component and the infrastructure component. In the ad hoc component,  $n$  nodes, with the same transmission range  $r(n)$ , are uniformly and independently distributed on the square area. The  $n$  nodes

are assumed to be static with the same transmission range. In the infrastructure component,  $m$  base stations are regularly deployed in the network: dividing the area into  $m$  cells, and each cell has a size of  $\frac{1}{m}$ . The base stations are connected by a wired network, such that there is no bandwidth limit. In addition to the uniform node distribution, some works consider the homogeneous poisson node distribution. As the results of both distributions can be derived using similar approaches [23], due to space limitation, we assume uniform distribution in this paper.

*Network Connectivity* Following [5],  $r(n) = \Theta(\sqrt{\frac{\log n}{n}})$ , so that the probability that a node has no neighbor tends to zero as  $n$  goes to infinity. In other words, the setting of  $r(n)$  assures the connectivity of the network. This work applies the protocol interference model. Assume that node  $i$  transmits to node  $j$ , the packet can be correctly received by  $j$  if the following conditions are satisfied: 1) The distance between  $i$  and  $j$  is no larger than  $r(n)$ ; 2) no node inside the interference range of node  $j$  is transmitting concurrently, where the interference range is  $(1 + \Delta)r(n)$ , and  $\Delta > 0$  is a constant.

*Resource Allocation* A fixed spectral bandwidth is shared between the two components. Denote by  $W$  the total bandwidth,  $W_a$  the bandwidth for ad hoc transmission, and  $W_c$  the bandwidth for cellular transmission. We have  $W_a + W_c = W$ . Following many existing works, we consider the centralised TDMA-based access model. That is, the system is time-slotted, and each link is specified to transmit in a certain time slot. Any two interfering links cannot share the same time slot.

We apply the *L-maximum-hop routing policy*. If the source and the destination are no more than  $L$  hops away, the data is transmitted in an ad hoc manner; otherwise, the data is transmitted using cellular resources. In the ad hoc manner, the data is forwarded by the intermediate nodes between the source and the destination. When using the cellular resources, the source node first transmits the packet to the nearest base station, and then, the data is forwarded to the base station from which the destination can fetch the data. As the same as many existing works, each node is associated with its nearest BS. Each BS transmits directly to the users (downlink transmission).

We consider two kinds of uplink cellular transmissions: *one-hop access* and *multi-hop access* [2]. With one-hop access, each node directly sends data to base station, which may use more power than the ad hoc transmission. For multi-hop access, a source node may not directly access base station due to power limit. Instead, it sends its data in a hop-by-hop manner to another node, that can directly access the base station. Assume that a source node accesses its associated base station with  $k$  hops. The previous  $k - 1$  hops along the path consume ad hoc resources but not cellular resources, and the last hop transmission consumes cellular resources. In other words, a cellular flow may consume ad hoc resources, and thus, the average throughput of ad hoc flows would be reduced. We will study the impact of access mode on network capacity.

*Traffic Model* We assume users exhibit social behaviors that we apply the traffic model described in [5]. Let  $d_i^{-\alpha} \triangleq d_{s,i}^{-\alpha}$ , where  $d_{s,i}^{-\alpha}$  denotes the distance between  $s$  and  $i$ . The long-

range contacts are selected independently, but closer nodes to the source have a better chance of being selected as a LSC. Each source node has the same number of LSCs  $q$  selected in independent random trials. The probability that a particular  $q$  LSCs  $\{v_{i_1}, \dots, v_{i_q}\}$  can be written as

$$P(LSC = \{v_{i_1}, \dots, v_{i_q}\}) = \frac{d_{i_1}^{-\alpha} \dots d_{i_q}^{-\alpha}}{N_{\alpha, q}^M}, \quad (1)$$

where  $N_{\alpha, q} = \sum_{1 \leq i_1 < \dots < i_q \leq n} d_{i_1}^{-\alpha} \dots d_{i_q}^{-\alpha}$ . We have  $\sum_{1 \leq i_1 < \dots < i_q \leq n} P(LSC = \{v_{i_1}, \dots, v_{i_q}\}) = 1$ . The probability for node  $v_k$  to be a LSC of node  $s$  is

$$\begin{aligned} & P(v_k \in LSC) \\ &= \sum_{1 \leq i_1 < \dots < i_{q-1} \leq n, i_j \neq k} P(LSC = \{v_k, v_{i_1}, \dots, v_{i_{q-1}}\}) \\ &= \frac{\sum_{1 \leq i_1 < \dots < i_{q-1} \leq n, i_j \neq k} d_k^{-\alpha} d_{i_1}^{-\alpha} \dots d_{i_{q-1}}^{-\alpha}}{\sum_{1 \leq i_1 < \dots < i_q \leq n} d_{i_1}^{-\alpha} \dots d_{i_q}^{-\alpha}} \end{aligned} \quad (2)$$

Denote by  $\vartheta_s$  the destination of source node. Source node randomly selects one node from its LSCs as the destination, and thus the probability that  $v_k$  is the destination of the source node is given by

$$P(\vartheta_s = v_k) = \frac{1}{q} P(v_k \in LSC). \quad (3)$$

Our work presents a methodology to derive the capacity of hybrid wireless networks, and can be extended to consider other different traffic models. For instance, by considering the power law traffic model in [20], we would obtain the different  $P_{ad}$  and  $P_{cell}$ , while the derivation procedure is similar. In our technical report [24], we further consider another different way for selecting LSCs, and we show that the derived results can be easily extended to consider different ways of selecting LSCs.

#### IV. HYBRID WIRELESS NETWORK CAPACITY STUDY

In this section, we study the hybrid wireless network capacity with the concerned traffic model. We derive the network capacities for different  $L$ 's under both the one-hop access and multi-hop access modes, which facilitate us to find the optimal  $L$ .

##### A. Definitions and Notations

The per-node network throughput capacity of the network, denoted by  $\Lambda^0$ , is the average transmission rate, measured in bits or packets per unit time. The network throughput capacity is thus calculated as  $\Lambda = n \cdot \Lambda^0$ . The per-node throughput capacity is *feasible* if there exists a spatial and temporal scheme for scheduling transmissions, such that each node can transmit  $\Lambda^0$  bits per second on average to its destination node. The throughput capacity of the network is said of order  $\Theta(f(n))$  bits per second if there are deterministic constants  $c_1 > 0$  and  $c_2 < \infty$  such that

$$\begin{aligned} \lim_{n \rightarrow \infty} P(\Lambda = c_1 f(n) \text{ is feasible}) &= 1 \\ \lim_{n \rightarrow \infty} P(\Lambda = c_2 f(n) \text{ is feasible}) &< 1. \end{aligned}$$

An event  $A$  happens *with high probability* if  $\lim_{n \rightarrow \infty} A = 1$ . Therefore, the network throughput capacity is defined based on the definition of high probability.

As we said, the per-node throughput is the average transmission rate that can be supported uniformly for each node. In the hybrid wireless network, the flow is transmitted either in ad hoc mode or in cellular mode. Denote by  $\Lambda_{ad}^0$  and  $\Lambda_c^0$  the per-node throughput in ad hoc mode and in cellular mode, respectively. Denote by  $N_{ad}$  and  $N_c$  the number of flows over ad hoc layer and cellular layer, respectively. The network throughput is thus represented by  $N_{ad} \cdot \Lambda_{ad}^0 + N_c \cdot \Lambda_c^0$ .

##### B. Main Conclusions

We first present the main results in this subsection and provide the detailed proof in Sections IV.C-IV.G.

**Theorem 1.** Consider a hybrid wireless network with one-hop access mode.

Case I:  $q = n$ , traffic tends to be uniform.

When  $L = \Theta\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$ , the maximum network capacity is  $\Theta\left(\frac{n^{\frac{2}{3}}}{\log^{\frac{1}{3}} n} W_a\right) + \Theta(m W_c)$ .

Case II:  $\lim_{n \rightarrow \infty} q < \infty$ , traffic exhibits social behavior.

1)  $0 \leq \alpha < 2$ . When  $L = \Theta\left(\left(\frac{n-q+1}{n^2 a^{\frac{2-\alpha}{2}} (n)}\right)^{\frac{1}{3-\alpha}}\right)$ , where

$a(n) = \Theta\left(\frac{\log n}{n}\right)$ , we have the maximum network capacity  $\Theta\left(\frac{n^{\frac{6-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}} W_a\right) + \Theta(m W_c)$ .

2)  $2 \leq \alpha < 3$ . When  $L \geq 1$  and  $L = \Theta(1)$ , the maximum network capacity is  $\Theta\left(\frac{n}{\log n} W_a\right) + \Theta(m W_c)$ .

3)  $\alpha \geq 3$ . The network capacity is  $\Theta\left(\frac{n}{\log n} W_a\right) + \Theta(m W_c)$ , which is independent of  $L$ .

□

When  $\alpha$  increases, the probability for selecting a destination far away from the source node reduces, and so, the average hop count of ad hoc flows would be reduced. Thus, more ad hoc resources would be allocated for each flow, and the ad hoc network capacity would be increased. Therefore, we observe that the order of ad hoc network capacity increases with larger  $\alpha$ . When  $\alpha$  is large enough, most of the destinations are near to their sources, and the average hop count of ad hoc flows is no more than a constant large value. At this time, the ad hoc network capacity is independent of  $\alpha$ . In Theorem 1, we observe that the ad hoc network capacity is independent of  $\alpha$  when  $\alpha > 2$ .

Intuitively, the traffic is more uniform as  $q$  increases. For instance, it becomes the uniform traffic model when  $q = n$ . Larger  $q$  implies that the average hop count of ad hoc flows is higher, and thus, the ad hoc network capacity is reduced. We should set smaller  $L$  with larger  $q$  to limit the long-range ad hoc flows. Therefore, the optimal  $L$  and  $q$  are interdependent. When  $\alpha$  is large enough, say  $\alpha > 2$ . Most of the flows are short-range. A constant  $L$  can limit the average hop count of ad hoc flows, so that the network capacity is maximized. In

this case, the average hop count of ad hoc flows is independent of  $q$ , and thus, the optimal  $L$  is independent of  $q$ .

**Theorem 2.** Consider a hybrid wireless network with multi-hop access node.

When  $m = w(\frac{n}{\log n})$ , each source node can access the base station directly with high probability. The network throughput capacity is the same as that with one-hop access, and so we have the same results as those in Theorem 1.

When  $m = o(\frac{n}{\log n})$ , we have the follows.

Case I:  $q = n$ , traffic tends to be uniform.

When  $L = \Theta(\frac{n^{\frac{1}{2}}}{(\log n)^{\frac{1}{2}} m^{\frac{1}{6}}})$ , the maximum network capacity is  $\Theta(\frac{n^{\frac{1}{2}} m^{\frac{1}{6}}}{(\log n)^{\frac{1}{2}}} W_a + m W_c)$

Case II:  $\lim_{n \rightarrow \infty} q < \infty$ , traffic exhibits social behavior.

- 1)  $0 \leq \alpha < 2$ . When  $L = \Theta(\frac{n^{\frac{1}{2}}}{(\log n)^{\frac{1}{2}} m^{-\frac{1}{2(3-\alpha)}}})$ , the maximum network capacity is  $\Theta(\frac{n^{\frac{1}{2}}}{(\log n)^{\frac{1}{2}} m^{\frac{1}{2(3-\alpha)}}} W_a) + \Theta(m W_c)$ .
- 2)  $2 \leq \alpha < 3$ . When  $L = \Theta(\frac{n^{\frac{1}{2}}}{(\log n)^{\frac{1}{2}} m^{-\frac{1}{2}}})$ , the maximum network capacity is  $\Theta(\frac{n^{\frac{\alpha-1}{2}}}{(\log n)^{\frac{\alpha-1}{2}} m^{\frac{3-\alpha}{2}}} W_a) + \Theta(m W_c)$ .
- 3)  $\alpha \geq 3$ . When  $L = \Omega((\frac{n}{m \log n})^{\frac{1}{2(\alpha-2)}})$ , the maximum network capacity is  $\Theta(\frac{n}{\log n} W_a) + \Theta(m W_c)$ .

We then consider the case that  $m = \Theta(\frac{n}{\log n})$ . When  $m \geq \frac{1}{2r^2(n)}$ , the average hop count for each source node to access base station is one with high probability, and thus, the network throughput capacity is the same as that with one-hop access mode. We consider the case that  $m < \frac{1}{2r^2(n)}$  in the following.

Case I:  $\lim_{n \rightarrow \infty} q = \infty$ . The average hop count of each source node to access base station is  $\Theta(1)$ . When  $L =$

$\Theta(\frac{n^{\frac{1}{3}}}{\log^{\frac{1}{3}} n})$ , the maximum network throughput capacity is  $\Theta(\frac{n^{\frac{1}{3}}}{\log^{\frac{1}{3}} n} W_a) + \Theta(m W_c)$ .

Case II:  $\lim_{n \rightarrow \infty} q < \infty$ .

- 1)  $0 \leq \alpha < 2$ . When  $L = \Theta((\frac{n}{\log n})^{\frac{2-\alpha}{6-2\alpha}})$ , the maximum network capacity is  $\Theta((\frac{n}{\log n})^{\frac{4-\alpha}{6-2\alpha}} W_a) + \Theta(m W_c)$ .
- 2)  $2 \leq \alpha < 3$ . When  $L = \Theta(1)$ , the maximum network capacity is  $\Theta(\frac{n}{\log n} W_a) + \Theta(m W_c)$ .
- 3)  $\alpha \geq 3$ . The network capacity is represented by  $\Theta(\frac{n}{\log n} W_a) + \Theta(m W_c)$ , which is independent of  $L$ .

□

In the one-hop access mode, number of base stations  $m$  does not affect the number of ad hoc flows and the average hop count of ad hoc flows, that is to say, the ad hoc network capacity is independent of  $m$ , and the optimal  $L$  is independent of  $m$ . In the multi-hop access mode,  $m$  determines how

many ad hoc resources allocated for cellular flows, so that the optimal  $L$  depends on  $m$ .

In the multi-hop access, we observe that the optimal  $L$  is independent of  $q$  when  $0 \leq \alpha < 2$ . Let us consider a specific case that every cellular flow consumes ad hoc resources for multi-hop access. In other words, all flows consume ad hoc resources. The average hop count for each flow consuming ad hoc resources is determined by both  $L$  and  $m$ , but not  $q$ . Intuitively, the optimal  $L$  is set to obtain an appropriate average hop count, and so the optimal  $L$  would depend on  $m$  but not  $q$ .

In the multi-hop access mode, a cellular flow may consume ad hoc resources, and thus, the ad hoc resources allocated for ad hoc flows would be reduced as compared with one-hop access mode. Therefore, the ad hoc network capacity with multi-hop access mode is less than that with one-hop access mode given a specific traffic model. More base stations would reduce the average hop count for each node to access base station, and thus, more ad hoc resources can be allocated for ad hoc flows. Therefore, the ad hoc network capacity increases as  $m$  increases. We can observe these conclusions from Theorem 2.

### C. Procedure of Network Capacity Derivation

The capacity in the cellular layer denotes the throughput contributed by cellular transmission. Assume that there are at most  $\Delta_c$  cells interfering with a given cell. The bandwidth allocated for each cell is lower bounded by  $\frac{W_c}{\Delta_c}$ . On the other hand, the bandwidth for each cell is upper bounded by  $W_c$ .  $\Delta_c$  is independent of  $n$  and  $m$  [4], and thus, the throughput capacity for each cell is  $\Theta(W_c)$ . Since there are totally  $m$  cells, the throughput capacity contributed by the cellular layer is denoted by

$$\Lambda_c = \Theta(m W_c). \quad (4)$$

In the following, we focus on analyzing the throughput capacity contributed by the ad hoc layer. Before we present the details, we describe our derivation process.

We divide the network area into multiple subcells, as illustrated in Fig. 2 which is borrowed from [5]. Each subarea has four neighbor subareas. Let  $a(n)$  be the area size, such that the edge length is  $\sqrt{a(n)}$ . We should have  $\sqrt{a(n)} = \Theta(r(n))$ , so that any two nodes in neighboring subareas can communicate with each other, where  $r(n)$  is the communication range. In order to assure the network connectivity, we let  $a(n) = \Theta(\frac{\log n}{n})$ , since  $r(n) = \Theta(\sqrt{\frac{\log n}{n}})$ . Note that a cell may consist of several subcells. Fig. 2 illustrates four cells divided by dash lines. Denote by  $P_{ad}$  the probability for each source node to transmit to the destination in an ad hoc manner. Based on  $P_{ad}$ , we calculate the number of flows transmitted in the ad hoc layer, denoted by  $N_{ad}$ . Afterwards, we calculate the expected hop count for each ad hoc flow, denoted by  $E[h]$ , and then, we calculate the total number of hops for all the ad hoc flows,  $H = N_{ad} \cdot E[h]$ . Since nodes are randomly deployed in the network area, following [25], we calculate the average number of flows going through a

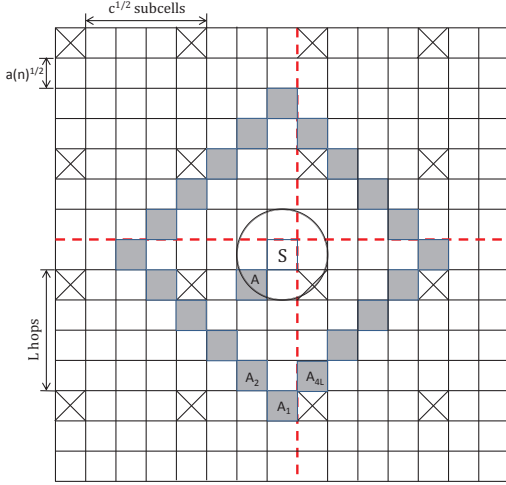


Fig. 2. A illustration for network division.

certain subcell, denoted by  $E[Z] = H \cdot a(n)$ . As referred to [2], it is shown that there is at most  $c = O((2 + \Delta)^2)$  number of interfering subcells of any given subcell. In Fig. 2, only one subcell in each  $c$  subcells can be active at the same time. Since  $c$  is independent of  $n$  and  $m$ , the bandwidth allocated for each subcell is asymptotically proportional with  $W_a$ . We thus calculate the bandwidth allocated for each hop in a given subcell as  $\Lambda_{\text{ad}}^0 = \Theta(\frac{W_a}{E[Z]})$ . Finally, we discuss the network throughput contributed by ad hoc layer as  $\Lambda_{\text{ad}} = N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0$ .

#### D. Calculating Total Number of Ad hoc Flows

As illustrated in Fig. 2, when the destination is located at the gray areas, the number of hops from the source to the destination is  $L$ . We can observe that are  $4x$  subcells in which the destination is  $x$  hops away from the source node. Following [5], we do not consider the edge issue. If the destination is located in the area surrounded by the grey subcells, the flow is transmitted in the ad hoc layer. Denote by  $P(X = x)$  the probability that the destination is  $x$  hops away from the source node. We thus have the following

$$P(X = x) = \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \quad (5)$$

$A_l$  represents a subcell  $l$  hops away from the source node. The probability for a destination located in  $A_l$  is calculated by  $\sum_{v_k \in A_l} P(\vartheta_s = v_k)$ , that is, the sum of the probabilities that each node in  $A_l$  is the destination. Since the nodes are randomly deployed in the network area, the probability for any node located in  $A_l$  is  $a(n)$ , and the number of nodes contained in  $A_l$  is thus  $n \cdot a(n)$ . We thus have

$$P(X = x) = \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) = \sum_{l=1}^{4x} n \cdot a(n) P(\vartheta_s = v_k) \quad (6)$$

In Fig. 2, if the destination is located in subcell  $A$ , we consider that it takes two hops from the source to the destination, and actually the destination may be in the transmission

range of the source node. This assumption would not affect the analysis on the scaling behavior, as referred to [5].

When the destination is located in an area which is  $x \leq L$  hops away from the source node, the flow is an *ad hoc flow*. The probability that a flow is an ad hoc flow is calculated as:

$$P_{\text{ad}} = \sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \quad (7)$$

When the destination is located in an area which is  $x > L$  hops away from the source node, the flow is a *cellular flow*. The hop count of each flow is upper bounded by  $\frac{2}{\sqrt{a(n)}}$ . Similar to (7) we calculate the probability that a flow is a cellular flow as

$$P_{\text{cell}} = \sum_{x=L+1}^{\frac{2}{\sqrt{a(n)}}} \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \quad (8)$$

When  $f(n) = \Theta(g(n))$ , it is denoted by  $f(n) \equiv g(n)$ . In general, we should consider two cases:  $\lim_{n \rightarrow \infty} q = \infty$  and  $\lim_{n \rightarrow \infty} q < \infty$ . However, in case that  $\lim_{n \rightarrow \infty} q = \infty$ , it is difficult to rigorously analyze  $P(\vartheta_s = v_k)$ , as explained in our technical report [24]. It is obvious that  $P(\vartheta_s = v_k) = \frac{1}{n}$  when  $q = n$ , and we thus consider two cases:  $q = n$  and  $\lim_{n \rightarrow \infty} q < \infty$ . As said in [26], each node normally has constant LSCs from the social perspective, and [26] only considers the case that  $q$  is constant, which is our second case. In our technical report, we show that if we modify the way for selecting LSCs, we can rigorously calculate  $P(\vartheta_s = v_k) = \Theta(\frac{1}{n})$  when  $\lim_{n \rightarrow \infty} q = \infty$ . This means that our results for  $q = n$  is suitable for the case  $\lim_{n \rightarrow \infty} q = \infty$  when modifying the LSCs selection scheme.

*Case I:  $q = n$ .* We calculate

$$\begin{aligned} P_{\text{ad}} &= \sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \\ &\equiv \sum_{x=1}^L \sum_{l=1}^{4x} n \cdot a(n) \cdot \frac{1}{n} \\ &\equiv \Theta(1) \cdot a(n) \int_0^L x dx = L^2 a(n) \end{aligned} \quad (9)$$

$$N_{\text{ad}} = n P_{\text{ad}} = \Theta(n L^2 a(n)). \quad (10)$$

$$\begin{aligned} P_{\text{cell}} &= \sum_{x=L+1}^{\frac{2}{\sqrt{a(n)}}} \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \\ &\equiv \sum_{x=L+1}^{\frac{2}{\sqrt{a(n)}}} \sum_{l=1}^{4x} n \cdot a(n) \cdot \frac{1}{n} \\ &\equiv \cdot a(n) \int_0^{\frac{2}{\sqrt{a(n)}}} x dx - P_{\text{ad}} = 1 - L^2 a(n) \end{aligned} \quad (11)$$

$$N_{\text{cell}} = n P_{\text{cell}} = \Theta(n(1 - L^2 a(n))). \quad (12)$$

Since each flow is transmitted using either ad hoc resources or cellular resources, we should have  $P_{\text{cell}} = 1 - P_{\text{ad}}$ . According to (9) and (11), we have  $P_{\text{cell}} + P_{\text{ad}} = \Theta(1)$ . For instance, let  $L = \frac{1}{\sqrt{a(n)}}$ , and the source node is located on the upper-left corner. We can easily verify that  $P_{\text{ad}} = \frac{1}{2}$  and  $P_{\text{cell}} = \frac{1}{2}$ , which is independent of the number of nodes. By (9) and (11),  $P_{\text{ad}} = \Theta(1)$  and  $P_{\text{cell}} = \Theta(1)$ . As mentioned in [4] and [5], the order calculation does not have to consider the edge issue.

Case II:  $\lim_{n \rightarrow \infty} q < \infty$ . We have Lemma 1.

**Lemma 1.** *The probability for each source node to transmit the flow using ad hoc mode is*

$$P_{\text{ad}} = \begin{cases} \Theta\left(\frac{n}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{n}{n-q+1} \frac{\ln L}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n}{n-q+1}\right) & \alpha > 2 \end{cases} \quad (13)$$

*The probability for each source node to transmit the flow using cellular mode is*

$$P_{\text{cell}} = \begin{cases} \Theta\left(\frac{n}{n-q+1} (2 - a^{1-\frac{\alpha}{2}}(n)) L^{2-\alpha}\right) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{n}{n-q+1} \frac{\ln \frac{2}{\sqrt{a(n)}} - \ln(L+1)}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n}{n-q+1} \frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}\right) & \alpha > 2 \end{cases} \quad (14)$$

*Proof.* By (2) and (3), we have

$$P(\vartheta_s = v_k) = \frac{\sum_{1 \leq i_1, \dots, i_{q-1} \leq n, i_j \neq k} d_k^{-\alpha} d_{i_1}^{-\alpha} \dots d_{i_{q-1}}^{-\alpha}}{q \sum_{1 \leq j_1, \dots, j_q \leq n} d_{j_1}^{-\alpha} \dots d_{j_q}^{-\alpha}} \quad (15)$$

Let  $\tau = (\tau_1, \dots, \tau_n)$  represent  $(d_1^{-\alpha}, \dots, d_n^{-\alpha})$ ,  
 $v_{q,n}(\tau) = \sum_{1 \leq i_1 \leq i_2 \leq \dots \leq i_p \leq n} \tau_{i_1} \dots \tau_{i_p}$ , and  
 $v_{q,n-1}^k(\tau) = v_{q,n-1}(\tau_1, \dots, \tau_{k-1}, \tau_{k+1}, \dots, \tau_n)$ . We then have

$$P(\vartheta_s = v_k) = \frac{d_k^{-\alpha} v_{q-1,n-1}^k(\tau)}{q v_{q,n}(\tau)} \quad (16)$$

(17)-(22) are borrowed from [5]. In (19),  $B_1$  and  $B_2$  are constants.  $d_k$  is the distance from a node in  $A_l$  to  $s$ , and so we have  $B_1 x \sqrt{a(n)} \leq d_k \leq B_2 x \sqrt{a(n)}$ , where  $x$  denotes the hop count from the node to  $s$ . In (20),  $\gamma \leq 1$  and  $d_{\text{max}}$  denotes the maximum distance between any two nodes in the network.

$$\begin{cases} P(\vartheta_s = v_k) \geq \frac{d_k^{-\alpha} v_{q-1,n}(\tau) - d_k^{-\alpha} v_{q-2,n}(\tau)}{q v_{q,n}(\tau)} \\ P(\vartheta_s = v_k) \leq \frac{d_k^{-\alpha} v_{q-1,n}(\tau)}{q v_{q,n}(\tau)} \end{cases} \quad (17)$$

$$\frac{v_{1,n}(\tau) v_{q-1,n}(\tau)}{q v_{q,n}(\tau)} = \Theta\left(\frac{n}{n-q+1}\right) \quad (18)$$

$$\begin{cases} \sum_{l=1}^{4x} \sum_{v_k \in A_l} d_k^{-\alpha} \geq \sum_{l=1}^{4x} \sum_{v_k \in A_l} (B_1 x \sqrt{a(n)})^{-\alpha} \\ \sum_{l=1}^{4x} \sum_{v_k \in A_l} d_k^{-\alpha} \leq \sum_{l=1}^{4x} \sum_{v_k \in A_l} (B_2 x \sqrt{a(n)})^{-\alpha} \end{cases} \quad (19)$$

$$\begin{aligned} v_{1,n}(\tau) &= \sum_{v_k} d_k^{-\alpha} \equiv \int_{\sqrt{a(n)}}^{\gamma d_{\text{max}}} n x^{1-\alpha} dx \\ &= \begin{cases} \Theta(n) & 0 \leq \alpha < 2 \\ \Theta(n \ln a^{-\frac{1}{2}}(n)) & \alpha = 2 \\ \Theta(n a^{1-\frac{\alpha}{2}}(n)) & \alpha > 2 \end{cases} \end{aligned} \quad (20)$$

$$\frac{v_{1,n}(\tau) v_{q-2,n}(\tau)}{(q-1) v_{q-1,n}(\tau)} = \Theta\left(\frac{n}{n-q+1}\right) \quad (21)$$

$$\begin{aligned} \frac{v_{q-2,n}(\tau)}{q v_{q,n}(\tau)} &= \Theta\left(\frac{(q-1)n^2}{(n-q+1)(n-q+2)v_{1,n}^2(\tau)}\right) \\ &= \begin{cases} \Theta\left(\frac{q-1}{(n-q+1)(n-q+2)}\right) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{q-1}{(n-q+1)(n-q+2) \ln^2 a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{q-1}{(n-q+1)(n-q+2)a^{2-\alpha}(n)}\right) & \alpha > 2 \end{cases} \end{aligned} \quad (22)$$

Since the probability that each node  $v_k$  is located in the subcell  $A_l$  is  $a(n)$ , we now calculate

$$\begin{aligned} &\sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} d_k^{-\alpha} \\ &= n a^{1-\frac{\alpha}{2}}(n) \sum_{x=1}^L x^{1-\alpha} \\ &\equiv n a^{1-\frac{\alpha}{2}}(n) \int_1^L x^{1-\alpha} dx \\ &= \begin{cases} \Theta\left(n a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) & 0 \leq \alpha < 2 \\ \Theta\left(n \ln L\right) & \alpha = 2 \\ \Theta\left(n a^{1-\frac{\alpha}{2}}(n)\right) & \alpha > 2 \end{cases} \end{aligned} \quad (23)$$

$$\begin{aligned} &\frac{v_{q-2,n}(\tau)}{q v_{q,n}(\tau)} \sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} d_k^{-2\alpha} \\ &= \begin{cases} \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)} a^{1-\alpha}(n) L^{2-2\alpha}\right) & 0 \leq \alpha < 1 \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)} \ln L\right) & \alpha = 1 \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)} a^{1-\alpha}(n)\right) & 1 < \alpha < 2 \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2) \ln^2 a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)a(n)}\right) & \alpha > 2 \end{cases} \end{aligned} \quad (24)$$

We now consider the case of  $0 \leq \alpha < 1$ . In order to compare the order of  $\frac{n}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}$  with that of  $\frac{(q-1)n}{(n-q+1)(n-q+2)} a^{1-\alpha}(n) L^{2-2\alpha}$ , we calculate  $\lim_{n \rightarrow \infty} \frac{\frac{n}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}}{\frac{(q-1)n}{(n-q+1)(n-q+2)} a^{1-\alpha}(n) L^{2-2\alpha}} = \infty$  based on  $a(n) = \Theta\left(\frac{\log n}{n}\right)$ . Thus, we know that the order of  $\frac{n}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}$  is higher than that of  $\frac{(q-1)n}{(n-q+1)(n-q+2)} a^{1-\alpha}(n) L^{2-2\alpha}$ . With the same method, we can verify that  $\sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} d_k^{-\alpha} \frac{v_{q-1,n}(\tau)}{q v_{q,n}(\tau)} = \Omega\left(\sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} d_k^{-2\alpha} \frac{v_{q-2,n}(\tau)}{q v_{q,n}(\tau)}\right)$ . Therefore, we have

$$\begin{aligned} P_{\text{ad}} &= \sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \\ &= \Theta\left(\frac{n}{n-q+1}\right) \sum_{x=1}^L \sum_{l=1}^{4x} \sum_{v_k \in A_l} \frac{d_k^{-\alpha}}{v_{1,n}(\tau)} \end{aligned} \quad (25)$$

$$\begin{aligned} P_{\text{cell}} &= \sum_{x=L+1}^{\frac{2}{\sqrt{a(n)}}} \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \\ &= \Theta\left(\frac{n}{n-q+1}\right) \sum_{x=L+1}^{\frac{2}{\sqrt{a(n)}}} \sum_{l=1}^{4x} \sum_{v_k \in A_l} \frac{d_k^{-\alpha}}{v_{1,n}(\tau)} \end{aligned} \quad (26)$$

Combining (20), (23), and (25), we obtain (13). Similarly, we also obtain (14).  $\square$



By Lemma 1, we calculate the total number of ad hoc flows and the total number of cellular flows as

$$N_{\text{ad}} = \begin{cases} \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{n^2}{n-q+1} \frac{\ln L}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n^2}{n-q+1}\right) & \alpha > 2 \end{cases} \quad (27)$$

$$N_{\text{cell}} = \begin{cases} \Theta\left(\frac{n^2}{n-q+1} (2 - a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha})\right) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{n^2}{n-q+1} \frac{\ln \frac{2}{\sqrt{a(n)}} - \ln(L+1)}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n^2}{n-q+1} \frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}\right) & \alpha > 2 \end{cases} \quad (28)$$

(27) and (28) show that the number of ad hoc flows is affected by the routing policy  $L$ . Due to  $L = O\left(\frac{n^{\frac{1}{2}}}{(\log n)^{\frac{1}{2}}}\right)$  and  $a(n) = \Theta\left(\frac{n}{\log n}\right)$ , we can verify that  $N_{\text{ad}} + N_{\text{cell}} = \Theta(n)$ . When  $\alpha > 2$ ,  $N_{\text{ad}}$  increases linearly with  $n$ , while the order of  $N_{\text{cell}}$  depends on  $L$ . As  $\alpha$  increases, more destinations are near to the sources, and the number of ad hoc flows increases faster as  $n$  increases. Our derivation results show that when  $\alpha > 2$ , the order of  $N_{\text{ad}}$  is the highest, which is  $\Theta(n)$  in case that  $q$  is a constant. Note that larger  $L$  may yield larger  $N_{\text{ad}}$ , but the order of  $N_{\text{ad}}$  is independent of  $L$ .

### E. Calculating Number of Flows in Each Subcell

In the following, we describe how to calculate the total number of flows going through a given subcell, which is used to derive the average throughput of each ad hoc flow. We should consider two cases: *one-hop access* and *multi-hop access*.

1) *One-hop access mode*: Denote by  $H$  the total hops of the ad hoc flows. Let  $h_i$  be the number of hops of ad hoc flow  $i$ . We have

$$E[H] = E\left[\sum_{i=1}^{N_{\text{ad}}} h_i\right] = \sum_{i=1}^{N_{\text{ad}}} E[h_i] \quad (29)$$

Let  $Z_i^j = 1$  represent that flow  $i$  goes through subcell  $j$ ; otherwise,  $Z_i^j = 0$ . Following [2], we have  $E[Z_i^j] = a(n)$ , where  $a(n)$  is the size of each subcell. The average number of ad hoc flows going through a certain subcell can be calculated as follows [25].

$$\begin{aligned} E[Z] &= E_H[E[Z|H]] = E_H[HE[Z_i^j]] \\ &= E[H] \cdot E[Z_i^j] = N_{\text{ad}} \cdot E[h_i] \cdot a(n) \end{aligned} \quad (30)$$

By (30), in order to calculate  $E[Z]$ , we need to calculate  $E[h_i]$ .

$$\begin{aligned} E[h_i] &= \sum_{x=1}^L x P(X=x | \text{ad hoc flow}) \\ &= \sum_{x=1}^L x \frac{P(X=x, \text{ad hoc flow})}{P(\text{ad hoc flow})} = \sum_{x=1}^L x \frac{P(X=x)}{P_{\text{ad}}} \end{aligned} \quad (31)$$

We also have

$$\sum_{x=1}^L x P(X=x) = \sum_{x=1}^L x \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_d = v_k). \quad (32)$$

Calculating  $E[h_i]$  also needs to consider two cases:  $q = n$  and  $\lim_{n \rightarrow \infty} q < \infty$ .

*Case I:  $q = n$ .*

Since  $P(\vartheta_d = v_k) = \frac{1}{n}$ , we have

$$\sum_{x=1}^L x P(X=x) = \frac{2L(1+L)(2L+1)a(n)}{3}. \quad (33)$$

Therefore,

$$E[h_i] = \frac{2L+1}{3}. \quad (34)$$

*Case II:  $\lim_{n \rightarrow \infty} q < \infty$ .*

Similar to the calculation of  $P_{\text{ad}}$ , we have

$$\begin{aligned} E[h_i] &= \sum_{x=1}^L x \frac{P(X=x)}{P_{\text{ad}}} \\ &= \frac{1}{P_{\text{ad}}} \cdot \Theta\left(\frac{n}{n-q+1}\right) \sum_{x=1}^L x \sum_{l=1}^{4x} \sum_{v_k \in s_l} \frac{d^{-\alpha}}{v_{1,n}(\tau)} \\ &\equiv \frac{1}{P_{\text{ad}}} \cdot \Theta\left(\frac{n}{n-q+1}\right) n a^{1-\frac{\alpha}{2}}(n) \cdot \frac{1}{v_{1,n}(\tau)} \int_1^L x^{2-\alpha} dx \end{aligned} \quad (35)$$

Combining (13), (20), and (35), we have Lemma 2.

**Lemma 2.** *The expected value for the number of hops of each ad hoc flow is*

$$E[h_i] = \begin{cases} \Theta\left(\frac{L^{3-\alpha}}{L^{2-\alpha}}\right) = \Theta(L) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{L}{\ln L}\right) & \alpha = 2 \\ \Theta\left(L^{3-\alpha}\right) & 2 < \alpha < 3 \\ \Theta\left(\ln L\right) & \alpha = 3 \\ \Theta\left(1\right) & \alpha > 3 \end{cases} \quad (36)$$

(36) shows that when  $0 \leq \alpha \leq 3$ , the average hop count of ad hoc flows increases with  $L$ , and the growth rate reduces as  $\alpha$  increases. When  $\alpha$  is larger, more destinations are located close to the source node, and thus, the impact factor of  $L$  on the scaling law of  $E[h_i]$  is less significant. When  $\alpha > 3$ , we find that the scaling law of  $E[h_i]$  is independent of  $L$ .

2) *Multi-hop access mode*: The derivation procedure is the same as the previous one. We first calculate the total hop counts of all the cellular flows transmitted using ad hoc resources, denoted by  $E[H']$ , and then, we calculate the total number of cellular flows going through a certain subcell with consuming the ad hoc resources, denoted by  $E[Z']$ .  $E[Z] + E[Z']$  denotes the total number of flows going through a given subcell with consuming ad hoc resources.

Denote by  $E[h']$  the average number of hops for each source node to access base station. For instance, if a source accesses the base station by three hops, the previous two hops consume ad hoc resource, and the last hop consumes the uplink cellular resource.



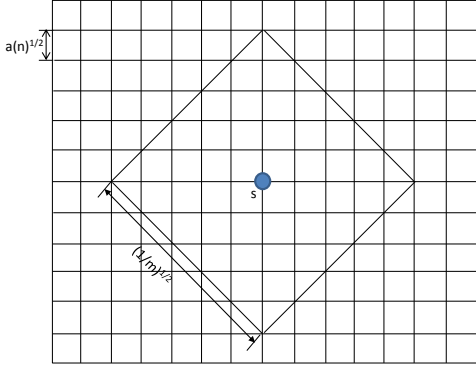


Fig. 3. A illustration for multihop access.

Similar to most existing works, each node is associated with its nearest base station. Fig. 3 shows an area with the size of  $\frac{1}{\sqrt{m}} \cdot \frac{1}{\sqrt{m}}$  centers around the source node  $s$ . There must exist a base station in this area, and thus, the maximum hop counts from the source node to the base station is  $\frac{1}{\sqrt{2m}}$ . We calculate  $E[h']$  as

$$\begin{aligned} E[h'] &= \sum_{x=1}^{\frac{1}{\sqrt{2ma(n)}}} \frac{4x}{1/m} a(n)(x-1) \\ &= \frac{2(\frac{1}{\sqrt{2ma(n)}} - \sqrt{2ma(n)})}{3} \end{aligned} \quad (37)$$

The total area for base station located  $x$  hops away from the source node is  $4xa(n)$ , and thus  $\frac{4xa(n)}{1/m}$  denotes the probability that the base station in this area is  $x$  hops away.  $x-1$  is the number of hops consuming ad hoc resource. When the flow is determined to be transmitted in cellular mode,  $E[h']$  only depends on the distance between the base station and the source node, which is independent of the specific traffic model and routing policy.

Similar to (30), we calculate the total hop counts of cellular flows going through a certain subcell with consuming the ad hoc resources:

$$\begin{aligned} E[Z'] &= E_{H'}[E[Z'|H']] = E_{H'}[H'E[Z'_i^j]] \\ &= E[H'] \cdot Z[Z'_i^j] = N_{\text{cell}} \cdot E[h'] \cdot a(n) \\ &= N_{\text{cell}} \frac{2 \cdot a(n) (\frac{1}{\sqrt{2ma(n)}} - \sqrt{2ma(n)})}{3} \end{aligned} \quad (38)$$

We thus calculate the total number of flows going through a certain subcell and consuming ad hoc resources as

$$E[Z_{\text{total}}] = E[Z] + E[Z']. \quad (39)$$

#### F. Network Capacity Analysis with One-hop Access

In this section, we study the network capacity when each node accesses base station with one-hop. We calculate the throughput of each ad hoc flow

$$\Lambda_{\text{ad}}^0 = \Theta\left(\frac{W_a}{E[Z]}\right). \quad (40)$$

In case that  $E[Z] = O(1)$ , we should set  $\Lambda_{\text{ad}}^0 = \Theta(W_a)$ , since the average throughput of ad hoc flows should not be

larger than  $W_a$ . In the following, we analyze whether  $E[Z] = O(1)$  given a specific traffic model.

In the case that  $q = n$ , we have

$$\begin{aligned} \Lambda_{\text{ad}}^0 &= \Theta\left(\frac{W_a}{E[Z]}\right) = \Theta\left(\frac{W_a}{N_{\text{ad}} \cdot E[h_i] \cdot a(n)}\right) \\ &= \Theta\left(\frac{W_a}{nL^3 a^2(n)}\right) \end{aligned} \quad (41)$$

In case that  $L = \Omega\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$ , we have

$$\begin{aligned} \Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 = N_{\text{ad}} \cdot \Theta\left(\frac{W_a}{N_{\text{ad}} \cdot E[h_i] \cdot a(n)}\right) \\ &= \Theta\left(\frac{nW_a}{L \log n}\right) \quad (a(n) = \Theta(\frac{\log n}{n})) \end{aligned} \quad (42)$$

When  $L = O\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$ , we have  $\Lambda_{\text{ad}}^0 = \Omega(W_a)$ . That is, per node ad hoc capacity grows asymptotically faster than a constant. On the other hand, the maximum capacity for each ad hoc flow must be less than  $W_a$ , and thus we have  $\Lambda_{\text{ad}}^0 = O(W_a)$ . Finally, we have  $\Lambda_{\text{ad}}^0 = \Theta(W_a)$  under this situation. The network ad hoc capacity is calculated as

$$\begin{aligned} \Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 = N_{\text{ad}} \cdot \Theta(W_a) \\ &= \Theta\left(L^2 \log n W_a\right) \quad (a(n) = \Theta(\frac{\log n}{n})) \end{aligned} \quad (43)$$

Combining (42) and (43), the ad hoc network capacity is maximized when  $L = \Theta\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$ , which is

$$\Lambda_{\text{ad}}^* = \Theta\left(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}} W_a\right) \quad (44)$$

We now consider the network capacity in the case that  $\lim_{n \rightarrow \infty} q < \infty$ . By (27), (30), (36), and (40), we calculate

$$\Lambda_{\text{ad}}^0 = \begin{cases} \Theta\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n) L^{3-\alpha}} W_a\right) & 0 \leq \alpha < 2 \\ \Theta\left(\frac{(n-q+1) \ln a^{-\frac{1}{2}}(n)}{n^2 L a(n)} W_a\right) & \alpha = 2 \\ \Theta\left(\frac{n-q+1}{n^2 a(n) L^{3-\alpha}} W_a\right) & 2 < \alpha < 3 \\ \Theta\left(\frac{n-q+1}{n^2 a(n) \ln L} W_a\right) & \alpha = 3 \\ \Theta\left(\frac{n-q+1}{n^2 a(n)} W_a\right) & \alpha > 3 \end{cases} \quad (45)$$

(a). When  $0 \leq \alpha < 2$ , we should discuss the per node ad hoc capacity when  $L = \Omega\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$  and  $L = O\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$ , separately. In case that  $L = O\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$ , we have  $\Lambda_{\text{ad}}^0(n) = \Omega(W_a)$ ; at the same time, there is the constant that the per node ad hoc capacity must be no greater than  $W_a$ , we therefore have  $\Lambda_{\text{ad}}^0(n) = \Theta(W_a)$ . In summary, we have

$$\Lambda_{\text{ad}}^0 = \begin{cases} \Theta\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)L^{3-\alpha}} W_a\right) & L = \Omega\left(\left(\frac{n-q+1}{n a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right) \\ \Theta\left(W_a\right) & L = O\left(\left(\frac{n-q+1}{n a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right) \end{cases} \quad (46)$$

When  $L = \Omega\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$ , the network capacity is written as

$$\begin{aligned} \Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) \cdot \Theta\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)L^{3-\alpha}}\right) \\ &= \Theta\left(\frac{W_a}{a(n)L}\right) \end{aligned} \quad (47)$$

When  $L = O\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$ , the network capacity is written as

$$\begin{aligned} \Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) \cdot \Theta\left(W_a\right) \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha} W_a\right) \end{aligned} \quad (48)$$

By (47) and (48), we have the maximum ad hoc network capacity when  $L = \Theta\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$ , which is

$$\begin{aligned} \Lambda_{\text{ad}}^* &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) \cdot \Theta\left(W_a\right) \\ &= \Theta\left(\frac{n^{\frac{6-\alpha}{2-\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}}(n-q+1)^{\frac{1}{3-\alpha}}} W_a\right) \quad (a(n) = \Theta\left(\frac{\log n}{n}\right)) \end{aligned} \quad (49)$$

Assume that  $q$  is a constant. When  $\alpha = 0$ , we have  $\Lambda_{\text{ad}}^* = \Theta\left(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{2}{3}}} W_a\right)$  according to (49), and it is the same as (44) derived by assuming the uniform traffic model. When  $\alpha = 0$ , each node randomly selects  $q$  constant long-range social contacts from all the nodes in the network, and then randomly chooses the destination from the  $q$  LSCs. This implies that the destination is uniformly selected by the source node, and thus, the traffic model is basically the same as the uniform traffic model. Therefore, the capacity scaling behavior with  $\alpha = 0$  and constant  $q$  is the same as that with the uniform traffic model.

(b). When  $2 < \alpha < 3$ , by (45), the capacity increases as  $L$  decreases. Since  $L$  must be larger than 1, we have the maximum capacity when  $L = \Theta(1)$ . By (45) and (27), the maximum network capacity is written as

$$\begin{aligned} \Lambda_{\text{ad}}^* &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1}\right) \cdot \Theta\left(\frac{n-q+1}{n^2 a(n) \cdot \Theta(1)} W_a\right) \\ &= \Theta\left(\frac{n}{\log n} W_a\right) \quad (a(n) = \Theta\left(\frac{\log n}{n}\right)) \end{aligned} \quad (50)$$

(c). When  $\alpha > 3$ , by (45), the per node capacity is independent of  $L$ , and the maximum network capacity is

$$\begin{aligned} \Lambda_{\text{ad}}^* &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1}\right) \cdot \Theta\left(\frac{n-q+1}{n^2 a(n)} W_a\right) \\ &= \Theta\left(\frac{n}{\log n} W_a\right) \quad (a(n) = \Theta\left(\frac{\log n}{n}\right)) \end{aligned} \quad (51)$$

When  $\alpha \geq 3$ , the destination is probably located locally around the source node, and therefore, the flow probably has small hop count and is transmitted in the ad hoc layer. That is why the network capacity is independent of  $L$ .

### G. Network Capacity Analysis with Multi-hop Access

In this section, we study network capacity when node accesses base station with multiple hops. In the previous section, we calculate the total number of flows going through a given subcell with consuming ad hoc resources  $E[Z_{\text{total}}]$ . We then have the average throughput of each ad hoc flow as

$$\Lambda_{\text{ad}}^{0'} = \Theta\left(\frac{W_a}{E[Z_{\text{total}}]}\right) \quad (52)$$

Denote by  $N_{\text{hybrid}}$  the number of cellular flows which consume ad hoc resources. The network throughput can be calculated as

$$\begin{aligned} \Lambda' &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0'} + \Lambda_{\text{ad}}^{0'} \cdot N_{\text{hybrid}} \\ &\quad + (\kappa \cdot m W_c - \Lambda_{\text{ad}}^{0'} \cdot N_{\text{hybrid}}) \\ &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0'} + \kappa \cdot m W_c \end{aligned} \quad (53)$$

The first part in (53) represents the total throughput of the ad hoc flows, the second part represents the total throughput of the cellular flows which consume some ad hoc resources, called *hybrid flows*, and the third part represents the total throughput of the cellular flows only consuming the cellular resources. We mentioned before that each cell is allocated for  $\kappa \cdot m W_c$  cellular resources, where  $\kappa$  is a constant, and the total cellular resources become  $m \cdot \kappa \cdot W_c$ . Each cellular flow consumes the same uplink and downlink cellular resources, and so the total throughput for the cellular flows should be  $m \cdot \kappa \cdot W_c$ . The total cellular throughput should be divided into two parts, One part is the hybrid flows with the throughput  $\Lambda_{\text{ad}}^{0'} \cdot N_{\text{hybrid}}$ , and another part is the pure cellular flows, which thus have the total throughput of  $m \cdot \kappa \cdot W_c - \Lambda_{\text{ad}}^{0'} \cdot N_{\text{hybrid}}$ .

When  $m = w\left(\frac{n}{\log n}\right)$  or  $m = c\frac{n}{\log n}$ ,  $c \geq 1$ , there is at least a base station in each subcell with high probability. Each source node can access base station with one hop, and thus, the network throughput is the same as that with one-hop access mode. We are going to study the network capacity when  $m = o\left(\frac{n}{\log n}\right)$  and  $m = c\frac{n}{\log n}$ ,  $c < 1$ .

*Case I:  $q = n$ .*

We first consider the case that  $m = c\frac{n}{\log n}$ ,  $c < 1$ . By (37), we have  $E[h'] = \Theta(1)$ . Based on (10), (12), (30), (34), and (38), we calculate

$$E[Z] = \frac{2nL(1+L)(2L+1)a^2(n)}{3} = \Theta(nL^3 a^2(n)) \quad (54)$$

$$\begin{aligned}
E[Z'] &= (2 - 2L(1 + L)a(n))na(n) \\
&= \Theta((1 - L^2a(n))na(n)) \\
&= \Theta(na(n)) \quad (\because L = O(\sqrt{\frac{n}{\log n}}) \quad L^2a(n) = O(1))
\end{aligned} \tag{55}$$

$$E[Z_{\text{total}}] = E[Z] + E[Z'] = \Theta(nL^3a^2(n)) + \Theta(na(n)) \tag{56}$$

When  $L = \Omega(\frac{n^{\frac{1}{3}}}{\log^{\frac{1}{3}}n})$  and  $L = O(\frac{n^{\frac{1}{2}}}{\log^{\frac{1}{2}}n})$ ,  $E[Z_{\text{total}}] = \Theta(nL^3a^2(n))$ , we calculate the network throughput as

$$\begin{aligned}
\Lambda' &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0'} + \kappa \cdot mW_c \\
&= 2nL(L + 1)a(n) \cdot \Theta(\frac{W_a}{nL^3a^2(n)}) + \Theta(mW_c) \\
&= \Theta(\frac{1}{La(n)}W_a) + \Theta(mW_c)
\end{aligned} \tag{57}$$

When  $L = O(\frac{n^{\frac{1}{3}}}{\log^{\frac{1}{3}}n})$  and  $L \geq 1$ ,  $E[Z_{\text{total}}] = \Theta(na(n))$ , we calculate the network throughput as

$$\begin{aligned}
\Lambda' &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0'} W_a + \kappa \cdot mW_c \\
&= 2nL(L + 1)a(n) \cdot \Theta(\frac{W_a}{na(n)})W_a + \Theta(mW_c) \\
&= \Theta(L^2W_a) + \Theta(mW_c)
\end{aligned} \tag{58}$$

Based on both (57) and (58), the network throughput with  $m = c\frac{n}{\log n}$ ,  $c < 1$  is maximized when  $L = \Theta(\frac{n^{\frac{1}{3}}}{\log^{\frac{1}{3}}n})$ , and the maximum network throughput is

$$\Lambda'_{\text{max}} = \Theta(\frac{n^{\frac{2}{3}}}{\log^{\frac{2}{3}}n}W_a) + \Theta(mW_c). \tag{59}$$

We then consider the case that  $m = o(\frac{n}{\log n})$ . When  $m = o(\frac{n}{\log n})$ , since  $\sqrt{ma(n)} = o(\frac{1}{\sqrt{ma(n)}})$ , we have  $E[h'] = \Theta(\frac{1}{\sqrt{ma(n)}})$  according to (37). We have  $E[Z'] = \Theta(\frac{na(n)}{\sqrt{ma(n)}})$ , and the network capacity is calculated as

$$\begin{aligned}
\Lambda' &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0'} W_a + \kappa \cdot mW_c \\
&= 2nL(L + 1)a(n) \cdot \Theta(\frac{W_a}{nL^3a^2(n) + \frac{na(n)}{\sqrt{ma(n)}}})W_a \\
&\quad + \Theta(mW_c) \\
&= \Theta(\frac{L^2}{L^3a(n) + \frac{1}{\sqrt{ma(n)}}}W_a) + \Theta(mW_c)
\end{aligned} \tag{60}$$

When  $L = O(\frac{1}{m^{1/6}a^{1/2}(n)})$ , we have  $L^3a(n) = O(\frac{1}{\sqrt{2ma(n)}})$ , and thus

$$\Lambda' = \Theta(\sqrt{2ma(n)}L^2W_a + mW_c). \tag{61}$$

When  $L = w(\frac{1}{m^{1/6}a^{1/2}(n)})$  and  $L = O(\sqrt{\frac{n}{\log n}})$ ,  $\frac{1}{\sqrt{2ma(n)}} = O(L^3a(n))$ , and the network throughput is represented by

$$\Lambda' = \Theta(\frac{1}{La(n)}W_a + mW_c). \tag{62}$$

According to both (61) and (62), the network is maximized when  $L = \Theta(\frac{1}{m^{1/6}a^{1/2}(n)})$ , and the maximum network throughput is

$$\Lambda'_{\text{max}} = \Theta(\frac{m^{1/6}n^{1/2}}{\log^{1/2}n}W_a + mW_c). \tag{63}$$

*Case II:*  $\lim_{n \rightarrow \infty} q < \infty$ .

We first consider the case that  $m = c\frac{n}{\log n}$ ,  $c < 1$ . We have  $E[h'] = \Theta(1)$ . Combining (27) and (28), (30), (36), and (38), we have (64).

(1)  $0 \leq \alpha < 2$

Since  $L^2a(n) = O(1)$ ,  $E[Z_{\text{total}}] = \Theta(\frac{n^2}{n-q+1}a(n)(a^{1-\frac{\alpha}{2}}(n)L^{3-\alpha} + 2))$ .

When  $L = \Omega(a^{\frac{\alpha-2}{6-2\alpha}}(n))$ ,  $E[Z_{\text{total}}] = \Theta(\frac{n^2}{n-q+1}a^{2-\frac{\alpha}{2}}(n)L^{3-\alpha})$ . In this case, we can verify that  $E[Z_{\text{total}}] = \Omega(\log n)$ . The network capacity is

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^{0'}N_{\text{ad}} + mW_c) = \Theta(\frac{1}{a(n)L}W_a + mW_c) \tag{66}$$

When  $L = O(a^{\frac{\alpha-2}{6-2\alpha}}(n))$ ,  $E[Z_{\text{total}}] = \Theta(\frac{n^2}{n-q+1}a(n))$ . The network capacity is calculated as

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^{0'}N_{\text{ad}} + mW_c) = \Theta(\frac{L^{2-\alpha}}{a^{\frac{\alpha}{2}}(n)}W_a + mW_c) \tag{67}$$

According to both (66) and (67), the network capacity is maximized when  $L = \Theta(a^{\frac{\alpha-2}{6-2\alpha}}(n))$ , and the maximum network capacity is

$$\Lambda'_{\text{max}} = \Theta((\frac{n}{\log n})^{\frac{4-\alpha}{6-2\alpha}}W_a + mW_c) \tag{68}$$

(2)  $2 < \alpha < 3$ .

$E[Z_{\text{total}}] = \Theta(\frac{n^2}{n-q+1}a^{\frac{\alpha}{2}}(n)(a^{1-\frac{\alpha}{2}}(n)L^{3-\alpha} + a^{1-\frac{\alpha}{2}}(n)L^{2-\alpha} - 2)) = \Theta(\frac{n^2}{n-q+1}a(n)L^{3-\alpha})$  due to  $a^{1-\frac{\alpha}{2}}(n)L^{3-\alpha} = \Omega(1)$ . The network capacity is calculated as

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^{0'}N_{\text{ad}} + mW_c) = \Theta(\frac{n}{L^{3-\alpha}\log n}W_a + mW_c) \tag{69}$$

The network capacity is maximized when  $L = \Theta(1)$ , and the maximum network capacity is

$$\Lambda'_{\text{max}} = \Theta(\frac{n}{\log n}W_a + mW_c). \tag{70}$$

(3)  $\alpha > 3$ .

$E[Z_{\text{total}}] = \Theta(\frac{n^2}{n-q+1}a^{\frac{\alpha}{2}}(n)(a^{1-\frac{\alpha}{2}}(n) + a^{1-\frac{\alpha}{2}}(n)L^{2-\alpha} - 2)) = \Theta(\frac{n^2}{n-q+1}a(n))$ . The network capacity is independent of  $L$ , and the maximum network capacity is

$$\Lambda'_{\text{max}} = \Theta(\frac{n}{\log n}W_a + mW_c). \tag{71}$$

We then consider the case that  $m = o(\frac{n}{\log n})$ . When  $m = o(\frac{n}{\log n})$ , we mentioned before that  $E[h'] = \Theta(\frac{1}{\sqrt{ma(n)}})$ . Combining (27) and (28), (30), (36), and (38), we have (65).

(1)  $0 \leq \alpha < 2$ .

Since  $a^{1-\frac{\alpha}{2}}(n)(L + 1)^{2-\alpha} = O(1)$ , we can easily verify that  $E[Z_{\text{total}}] = \Theta(\frac{n^2}{n-q+1}a^{\frac{3-\alpha}{2}}(n)(a^{\frac{3-\alpha}{2}}(n)L^{3-\alpha} + \frac{1}{\sqrt{m}}))$ . When  $L = O(\frac{n^{\frac{1}{2}}}{\log^{\frac{1}{2}}n}m^{-\frac{1}{2(3-\alpha)}})$ ,  $a^{\frac{3-\alpha}{2}}(n)L^{3-\alpha} + \frac{1}{\sqrt{m}} = \Theta(\frac{1}{\sqrt{m}})$ , the average throughput of ad hoc flow is

$$E[Z_{\text{total}}] = \begin{cases} \Theta\left(\frac{n^2}{n-q+1}a^{2-\frac{\alpha}{2}}(n)L^{3-\alpha} + \frac{n^2}{n-q+1}(2 - a^{1-\frac{\alpha}{2}}(n)(L+1)^{2-\alpha})a(n)\right), 0 \leq \alpha < 2 \\ \Theta\left(\frac{n^2}{n-q+1}\frac{La(n)}{\ln a^{-\frac{1}{2}}(n)} + \frac{n^2}{n-q+1}\frac{\ln \frac{2}{\sqrt{a(n)}} - \ln(L+1)}{\ln a^{-\frac{1}{2}}(n)}a(n)\right), \alpha = 2 \\ \Theta\left(\frac{n^2}{n-q+1}a(n)L^{3-\alpha} + \frac{n^2}{n-q+1}\frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}a(n)\right), 2 < \alpha < 3 \\ \Theta\left(\frac{n^2}{n-q+1}a(n)\ln L + \frac{n^2}{n-q+1}\frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}a(n)\right), \alpha = 3 \\ \Theta\left(\frac{n^2}{n-q+1}a(n) + \frac{n^2}{n-q+1}\frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}a(n)\right), \alpha > 3 \end{cases} \quad (64)$$

$$E[Z_{\text{total}}] = \begin{cases} \Theta\left(\frac{n^2}{n-q+1}a^{2-\frac{\alpha}{2}}(n)L^{3-\alpha} + \frac{n^2}{n-q+1}(2 - a^{1-\frac{\alpha}{2}}(n)(L+1)^{2-\alpha})\frac{a(n)}{\sqrt{ma(n)}}\right), 0 \leq \alpha < 2 \\ \Theta\left(\frac{n^2}{n-q+1}\frac{La(n)}{\ln a^{-\frac{1}{2}}(n)} + \frac{n^2}{n-q+1}\frac{\ln \frac{2}{\sqrt{a(n)}} - \ln(L+1)}{\ln a^{-\frac{1}{2}}(n)}\frac{a(n)}{\sqrt{ma(n)}}\right), \alpha = 2 \\ \Theta\left(\frac{n^2}{n-q+1}a(n)L^{3-\alpha} + \frac{n^2}{n-q+1}\frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}\frac{a(n)}{\sqrt{ma(n)}}\right), 2 < \alpha < 3 \\ \Theta\left(\frac{n^2}{n-q+1}a(n)\ln L + \frac{n^2}{n-q+1}\frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}\frac{a(n)}{\sqrt{ma(n)}}\right), \alpha = 3 \\ \Theta\left(\frac{n^2}{n-q+1}a(n) + \frac{n^2}{n-q+1}\frac{1-2(a(n)L^2)^{\frac{\alpha}{2}-1}}{L^{\alpha-2}}\frac{a(n)}{\sqrt{ma(n)}}\right), \alpha > 3 \end{cases} \quad (65)$$

$$\Lambda_{\text{ad}}^0 = \frac{W_a}{E[Z_{\text{total}}]} = \Theta\left(\frac{n-q+1}{n^2}a^{-\frac{1}{2}}(n)m^{\frac{1}{2}}\right) \\ = \Theta\left(\frac{n-q+1}{n^{\frac{3}{2}}\log^{\frac{1}{2}}n}m^{\frac{1}{2}}\right) = O(1) (\because m = o(\frac{n}{\log n})) \quad (72)$$

We thus calculate the network throughput as:

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^0 N_{\text{ad}} + mW_c) \\ = \Theta(a^{\frac{1-\alpha}{2}}(n)m^{\frac{1}{2}}L^{2-\alpha}W_a + mW_c) \quad (73)$$

When  $L = \Omega\left(\frac{n^{\frac{1}{2}}}{\log^{\frac{1}{2}}n}m^{-\frac{1}{2(3-\alpha)}}\right)$ ,  $a^{\frac{3-\alpha}{2}}(n)L^{3-\alpha} + \frac{1}{\sqrt{m}} = \Theta(a^{\frac{3-\alpha}{2}}(n)L^{3-\alpha})$ . Since  $E[Z_{\text{total}}] = \Theta(na^{2-\frac{\alpha}{2}}L^{3-\alpha}) = \Omega(1)$ , we calculate the network throughput as

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^0 N_{\text{ad}} + mW_c) = \Theta\left(\frac{1}{a(n)L}W_a + mW_c\right) \quad (74)$$

According to (73) and (74), the network throughput is maximized when  $L = \Theta\left(\frac{n^{\frac{1}{2}}}{\log^{\frac{1}{2}}n}m^{-\frac{1}{2(3-\alpha)}}\right)$ , and the maximum network throughput is

$$\Lambda'_{\text{max}} = \Theta\left(\frac{n^{\frac{1}{2}}}{\log^{\frac{1}{2}}n}m^{\frac{1}{2(3-\alpha)}}W_a + mW_c\right) \quad (75)$$

2)  $2 < \alpha < 3$ .

Since  $L = O\left(\sqrt{\frac{n}{\log n}}\right)$ ,  $(a(n)L^2)^{\frac{\alpha}{2}-1} = O(1)$ . We thus calculate  $E[Z_{\text{total}}] = \Theta\left(\frac{n^2}{n-q+1}a^{\frac{1}{2}}(n)(a^{\frac{1}{2}}(n)L^{3-\alpha} + L^{2-\alpha}m^{-\frac{1}{2}})\right)$ .

When  $L = O\left(a^{-\frac{1}{2}}(n)m^{-\frac{1}{2}}\right)$ ,  $\Theta(a^{\frac{1}{2}}(n)L^{3-\alpha} + L^{2-\alpha}m^{-\frac{1}{2}}) = \Theta(L^{2-\alpha}m^{-\frac{1}{2}})$ ,  $E[Z_{\text{total}}] = \Theta\left(\frac{n^2}{n-q+1}a^{\frac{1}{2}}(n)L^{2-\alpha}m^{-\frac{1}{2}}\right)$ . When  $L = \Theta\left(a^{-\frac{1}{2}}(n)m^{-\frac{1}{2}}\right)$ , we calculate  $E[Z_{\text{total}}] = \Omega(1)$ . Since  $E[Z_{\text{total}}]$  becomes larger with the smaller  $L$ ,  $E[Z_{\text{total}}] = \Omega(1)$  when  $L = O\left(a^{-\frac{1}{2}}(n)m^{-\frac{1}{2}}\right)$ . Therefore,

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^0 N_{\text{ad}} + mW_c) \\ = \Theta\left(\frac{n^{\frac{1}{2}}}{\log^{\frac{1}{2}}n}m^{\frac{1}{2}}L^{\alpha-2}W_a + mW_c\right) \quad (76)$$

When  $L = \Omega\left(a^{-\frac{1}{2}}(n)m^{-\frac{1}{2}}\right)$ ,  $\Theta(a^{\frac{1}{2}}(n)L^{3-\alpha} + L^{2-\alpha}m^{-\frac{1}{2}}) = \Theta(a^{\frac{1}{2}}(n)L^{3-\alpha})$ . We have  $E[Z_{\text{total}}] = \Theta(na(n)L^{3-\alpha})$ . We can easily verify that  $E[Z_{\text{total}}] = \Omega(1)$ , and then

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^0 N_{\text{ad}} + mW_c) = \Theta\left(\frac{1}{a(n)L^{3-\alpha}}W_a + mW_c\right) \quad (77)$$

According to (76) and (77), the network capacity is maximized when  $L = \Theta\left(\frac{n^{-\frac{1}{2}}}{\log^{\frac{1}{2}}n}m^{-\frac{1}{2}}\right)$ , and the maximum network capacity is

$$\Lambda'_{\text{max}} = \Theta\left(\frac{n^{\frac{\alpha-1}{2}}}{\log^{\frac{\alpha-1}{2}}n}m^{\frac{3-\alpha}{2}}W_a + mW_c\right). \quad (78)$$

3)  $\alpha > 3$

Similar to the case with  $2 < \alpha < 3$ , we calculate  $E[Z_{\text{total}}] = \Theta\left(\frac{n^2}{n-q+1}a^{\frac{1}{2}}(n)(a^{\frac{1}{2}}(n) + L^{2-\alpha}m^{-\frac{1}{2}})\right)$ . When  $L = O\left((a(n)m)^{-\frac{1}{2(\alpha-3)}}\right)$ ,  $\Theta(a^{\frac{1}{2}}(n) + L^{2-\alpha}m^{-\frac{1}{2}}) = \Theta(L^{2-\alpha}m^{-\frac{1}{2}})$ ,  $E[Z_{\text{total}}] = \Theta(na^{\frac{1}{2}}(n)L^{2-\alpha}m^{-\frac{1}{2}})$ . When  $L = \Theta\left((a(n)m)^{-\frac{1}{2(\alpha-3)}}\right)$ ,  $E[Z_{\text{total}}] = \Theta(\log n)$ . Since smaller  $L$  produces larger  $E[Z_{\text{total}}]$ , we have  $E[Z_{\text{total}}] = \Omega(1)$ . The network capacity is calculated as

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^0 N_{\text{ad}} + mW_c) = \Theta\left((a(n)m)^{\frac{1}{2}}L^{\alpha-2}W_a + mW_c\right). \quad (79)$$

When  $L = \Omega\left((a(n)m)^{-\frac{1}{2(\alpha-3)}}\right)$ ,  $\Theta(a^{\frac{1}{2}}(n) + L^{2-\alpha}m^{-\frac{1}{2}}) = \Theta(a^{\frac{1}{2}}(n))$ , and  $E[Z_{\text{total}}] = \Theta\left(\frac{n^2}{n-q+1}a^{\frac{1}{2}}(n)a^{\frac{1}{2}}(n)\right)$ . The network capacity is calculated as

$$\Lambda' = \Theta(\Lambda_{\text{ad}}^0 N_{\text{ad}} + mW_c) = \Theta\left(\frac{n}{\log n}W_a + mW_c\right). \quad (80)$$

Thus, the network is maximized when  $L = \Omega\left((a(n)m)^{-\frac{1}{2(\alpha-3)}}\right)$ , and the maximum network capacity is

$$\Lambda'_{\text{max}} = \Theta\left(\frac{n}{\log n}W_a + mW_c\right). \quad (81)$$

For simplicity, we did not consider the cases of  $\alpha = 2$  and  $\alpha = 3$  in the above discussion, and the analysis is actually very similar as before. Due to space limitation, we omit the boundary issue for  $\alpha$ .

#### H. Maximum Network Capacity Comparison

Generally speaking, the maximum network capacity with one-hop access is shown in (82); the maximum network capacity with multi-hop access and  $m = c \frac{n}{\log n}$ ,  $c < 1$  is shown in (83); and the maximum network capacity with multi-hop access and  $m = o(\frac{n}{\log n})$  is shown in (84).

Based on the derived results, we have the following observations.

- 1) *With the uniform traffic model, we can easily verify that the network capacity with one-hop access grows with the fastest speed. With multi-hop access mode, the network capacity with  $m = c \frac{n}{\log n}$ ,  $c < 1$  grows more quickly than that with  $m = o(\frac{n}{\log n})$ . The derived results align with our expectations. In the hybrid wireless network with one-hop access, the long-range flow would not consume the ad hoc resources, such that the average ad hoc network capacity is higher. On the other hand, with multi-hop access mode, even the number of ad hoc flows is reduced, some cellular flows still need to consume ad hoc resources, and thus, the ad hoc network capacity is reduced as compared with one-hop access.*
- 2) *In multi-hop access mode with  $m = \Theta(1)$ , the network capacity with uniform traffic model is the same as that in the pure ad hoc network. With the constant number of base stations, the order of the hop count for accessing base station is  $\Theta(\sqrt{\frac{n}{\log n}})$ . This implies that  $m = \Theta(1)$  does not improve the scaling behavior of network capacity under the multi-hop access mode. The results tell us that only augmenting base stations would not help to improve network scaling law performance, and we should increase the transmitting power of each node.*
- 3) *With the traffic model  $\lim_{n \rightarrow \infty} q < \infty$  and  $\alpha \geq 3$ , although the network capacity is the same for one-hop access and multi-hop access, optimal  $L$  is different. In the one-hop access and multi-hop access with  $m = c \frac{n}{\log n}$ ,  $c < 1$ , the network capacity is independent of  $L$ . However, in the multi-hop access mode with  $m = o(\frac{n}{\log n})$ ,  $L$  should be set as  $\Omega((\frac{n}{m \log n})^{\frac{1}{2(\alpha-2)}})$ . When  $L$  is too small, many short-range flows are transmitted through the cellular layer. Since each source node accesses base station with multiple hops, it is possible that the number of hops for a certain flow is increased when transmitting in cellular layer than that in ad hoc layer. Therefore, setting optimal  $L$  is important to improve network capacity performance.*

#### V. CONCLUSION

This work studies the scaling capacity of hybrid wireless networks with social behavior. The considered traffic model is represented by three parameters:  $n$ ,  $\alpha \geq 0$ , and  $q$ , in which  $n$  is the number of nodes,  $q$  denotes the number of long-range

social contacts (LSCs) for a source node, and  $\alpha$  is a factor to affect the probability for selecting LSC. Larger  $\alpha$  implies a source node prefers a near node to be a LSC. When  $q = n$ , the concerned traffic model is actually the uniform traffic model. In hybrid wireless networks, routing policy selects a transmission mode for a flow. We apply the  $L$ -maximum-hop routing policy, that is, the distance between source and destination is less than  $L$  hops, the flow is transmitted in ad hoc mode; otherwise, it is forwarded by base stations.

We derive the network capacity as a function of  $n$ ,  $\alpha$ ,  $q$ ,  $L$ , and  $m$ , where  $m$  is the base station number. Our derived results identify the optimal  $L$  to maximize network capacity. Moreover, we consider the case that a source node accesses base station via multiple hops. The multi-hop access would consume ad hoc resources, and our derivation results show that the optimal  $L$  depends on  $m$ . Although our analysis assumes a specific traffic model and node distribution model, our derivation method can be extended to consider other different traffic models and node distribution models.

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$$\Lambda_{\text{onehop}}^* = \Lambda_{\text{ad}}^* + \Lambda_c = \begin{cases} \Theta\left(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q = \infty \\ \Theta\left(\frac{n^{\frac{6-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, 0 \leq \alpha < 2 \\ \Theta\left(\frac{n}{\log n} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, \alpha \geq 2 \end{cases} \quad (82)$$

$$\Lambda_{\text{multihop}, m=c\frac{n}{\log n}}^* = \Lambda_{\text{ad}}^* + \Lambda_c = \begin{cases} \Theta\left(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{2}{3}}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q = \infty \\ \Theta\left(\left(\frac{n}{\log n}\right)^{\frac{4-\alpha}{6-2\alpha}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, 0 \leq \alpha < 2 \\ \Theta\left(\frac{n}{\log n} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, \alpha \geq 2 \end{cases} \quad (83)$$

$$\Lambda_{\text{multihop}, m=o\left(\frac{n}{\log n}\right)}^* = \Lambda_{\text{ad}}^* + \Lambda_c = \begin{cases} \Theta\left(\frac{m^{\frac{1}{6}} n^{\frac{1}{2}}}{(\log n)^{\frac{1}{2}}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q = \infty \\ \Theta\left(\left(\frac{n}{\log n}\right)^{\frac{1}{2}} m^{\frac{1}{2(3-\alpha)}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, 0 \leq \alpha < 2 \\ \Theta\left(\frac{n^{\frac{\alpha-1}{\alpha-2}}}{\log^{\frac{\alpha-1}{2}} n} m^{\frac{3-\alpha}{2}} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, 2 \leq \alpha < 3 \\ \Theta\left(\frac{n}{\log n} W_a\right) + \Theta(mW_c) & \lim_{n \rightarrow \infty} q < \infty, \alpha \geq 2 \end{cases} \quad (84)$$

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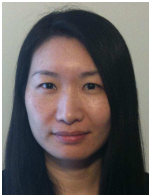


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