Hybridizing Basic Variable Neighbourhood Search with Particle Swarm Optimization for Solving Sustainable Ship Routing and Bunker Management Problem

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***Abstract*—This paper studies a novel sustainable ship routing problem considering a time window concept and bunker fuel management. Ship routing involves decisions corresponding to the deployment of vessels to multiple ports and time window concept helps to maintain the service level of the port. Reducing carbon emissions within the maritime transportation domain remains one of the most significant challenges as it addresses the sustainability aspect. Bunker fuel management deals with the fuel bunkering issues faced by different ships such as selection of bunkering ports and total bunkered amount at a port. A novel mathematical model is developed capturing the intricacies of the problem. A hybrid particle swarm optimization with basic variable neighbourhood search algorithm is proposed to solve the model and compared with the exact solutions obtained using Cplex and other popular algorithms for several problem instances. The proposed algorithm outperforms other popular algorithms for all the instances in terms of the solution quality and provides good quality solutions with an average cost deviation of 5.99% from the optimal solution.**

***Index Terms*— Ship Routing, Bunker Fuel Management, Mixed Integer Linear Programming model, Variable Neighbourhood Search algorithm**

I. Introduction

SEA-BORNE shipping is considered as environmentally efficient as it participates in carrying around 80% of the world trade [1]. Fuel consumption from maritime transportation is estimated about 279-400 million tons [2]. Short-sea shipping contributes about 25% of the overall Green House Gas emissions [3]. Carbon emissions from shipping industry are directly associated with the fuel consumption and minimizing the bunkering fuel cost is desired, as it accounts for more than 50% of the total operating costs [4]. It is essential to consider the optimal bunker fuel management strategy comprising of bunkering ports selection and amount of fuel bunkered for mitigating the fuel consumption and subsequently reduce the fuel cost. Routing and time window decisions are related to the bunker fuel management strategies; hence it is imperative to link the vessel routing decisions along with fuel bunker management strategies. Few researchers dealt with ship routing problems considering bunkering strategies based on varying bunker fuel prices at different ports, and also stated the importance of the carbon emission aspect [5]. Therefore, it is essential to investigate the possible ways in considering bunker fuel management strategies and reducing carbon emissions while designing the vessel routes to meet the demands of different ports. Bunker fuel management policies help to determine the bunkering port and fuel bunkering amount which in turn may lower the total cost incurred and improve the shipping company's profitability [5].

With the enhancement of global trade, maritime logistics became the principal mode of transportation. It is vital for the shipping companies to adopt proper bunker consumption optimization techniques to reduce the bunker cost while maintaining an appropriate service level [6]. Some studies addressed the importance of bunker cost and service quality by considering the port arrival times and fuel consumption [7]. Other studies stressed on the importance of maintaining the service level by considering time window concept while designing the vessel route networks [8].

A. Time Window Concept

Managing appropriate time window at the ports is a complex operation and helps to enhance the service level of the port. Ships may arrive early at the port and/or depart late leading to certain additional charges. Measures need to be incorporated to improve the service level at the port by explicitly performing the loading/unloading operation within a specified time window. Specific scenarios associated with the vessel failing to complete its loading/unloading operation within the prescribed time window can be countered by imposing a demurrage charge per hour for operating outside the time window. Moreover, ships may arrive early at the port and wait till the starting of the

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time window to begin its operation, and an inconvenience cost is incurred depending upon the total waiting time. Agra et al. [9] considered the penalty cost in their model to deal with failing to finish the port operation within the allotted time window, yet didn't address the complexities associated with the waiting time. Song et al. [10] considered penalty cost to counter the violation of time window and thereby managed to improve the port's service level. It is desired to investigate the demurrage charges to deal with the early arrival of the vessel at the port and thereby help to reduce the congestion level of the port and increasing port's service level.

B. Ship Routing Problem

The ship routing problem aims to design the routes for every vessel between several ports to reduce the cost of transportation and operation. Some studies have addressed vessel routing problems and developed models to minimize the total relevant costs [10]. Some earlier work can be extended by addressing the sustainability aspects such as estimating carbon emissions and incorporating bunker fuel management decisions. Agra et al. [9] studied the scheduling and routing of vessels carrying fuel oil products and considered varying production as well as consumption rate and loading/unloading operation at different ports. Agra et al. [11] presented a mathematical model with several real-time constraints for a practical short sea inventory routing problem. From the sustainability aspect, the model in [11] can be enhanced by incorporating carbon emissions and fuel consumption. Christiansen et al. [12] addressed the routing and scheduling of a fleet of fuel supply vessels to provide fuel to customer ships while minimizing the overall transportation costs, which did not consider fuel bunkering management and carbon emissions.

C. Bunker Fuel Management

Over the last decade, maritime transportation experienced an increased awareness about the effect of fuel consumption on the total operating cost and environmental emissions. Wang et al. [6] stated that bunker fuel cost constitutes about three-quarters of the overall operating cost of the shipping company and hence it is vital to adopt bunker fuel management policies to address the rise in fuel cost. Several researchers dealt with bunker fuel management problem and determined the selection of bunkering ports and bunkering amounts [6] [13]. The model in [13] assumed that the service routes, the number of ports of call, port time (arrival and departure time) for each vessel are known beforehand. Although, the design of the shipping network (vessel's port of call) need to be carried out while determining the bunkering ports, as ships prefer to perform its bunkering at a port with lower fuel price. Meng et al. [5] considered a joint tramp ship routing and bunkering problem to investigate the optimal routing decisions for vessels while determining the bunkering ports depending on the bunker fuel prices. Hence, ship routing problem needs to be merged with bunker fuel management problem as the order of port visits may affect the selection of bunkering ports due to the changing bunker fuel prices for different ports. Vessel route design need to be performed while keeping in mind of the selection of bunkering port as vessels prefer to perform the fuel bunkering at a port having lower fuel price. Aydin et al. [7] determined the bunkering decisions based on the bunker fuel prices. Hence, it is desired to integrate bunkering fuel management problem along with shipping network design problem to minimize the overall operational costs associated with transportation, service level, and bunkering at different ports.

D. Research Gap and Contribution

The research gaps in the literature are summarized below. First, the sustainability aspect and bunker fuel management have not been incorporated into the ship routing problem. Second, although time window concept was studied [9] [3], they did not consider penalty charges for the early arrival of vessels. Third, existing mathematical models have considered an assumption of a single vessel operating at a port in a given time period [9] [10] [3], which is not practical, as multiple vessels simultaneously perform loading/unloading operations at a port. Fourth, the bunker fuel management problem in the literature did not consider the design of vessel route. It is noted that combining the bunker fuel management problem with fleet deployment gives the shipping company the leverage for opting to visit ports having lower fuel price and later sail to the ports with higher fuel price.

The contributions of this paper are twofold. First, we address the research gaps above and study a novel ship routing problem with consideration of time window concept, loading/unloading operation, bunker fuel management and carbon emissions. Second, we propose a novel hybrid algorithm, i.e., basic variable neighbourhood search with particle swarm optimization (BVNS-PSO), to solve the proposed model. The BVNS-PSO is validated and compared with BVNS, PSO, PSO with differential evolution (PSO-DE) and genetic algorithm (GA) to highlight the superiority of the proposed algorithm.

The remaining of the paper is as follows. Sections II, III and IV present the problem description, mathematical model and linear reformulation respectively. Sections V and VI illustrate the solution methodology and the computational experiment. Conclusion and future scope are provided in section VII.

II. Problem Description

Sustainable ship routing with time window and bunker fuel management (SSRTWBM) problem is considered with the objective to minimize the overall cost of the shipping companies by designing appropriate schedules and routes for the vessels. The problem corresponds to the domain of short sea shipping where vessels operate between ports starting from an initial port position and sailing to several ports to meet the demand. The problem aims to design the routes for a fleet of ships for the transportation of containerized cargo between different loading and discharging ports while minimizing the cost related to transportation, service level, docking charges, loading/unloading operation, fuel bunkering, etc.

Each vessel performs its loading/unloading operation within an allotted time window, which is considered by the port authorities to improve the port's service level. A ship has to wait if it arrives before the starting of the time window and pay additional penalty charges. With the initiation of the time window, the vessel starts its port operation – loading and unloading of containers and once the vessel finishes its operation, it can depart from the port. Furthermore, a ship can arrive at a port after the beginning of the time window and start its service considering the availability of a berth. In such a scenario, the ship may fail to finish its port operation within the specified time window and end up with time window violation. For countering such unexpected delays, penalty costs are incurred depending on the number of hours operated by the vessel outside the time window. Penalties charges are considered to ensure the completion of the port operation within the allocated time window. Such operational measure improves ports' service level and provides robust schedules pertaining to vessels' arrival and departure time. Therefore, time window concept helps to improve the port's service level by imposing penalty charges on the vessel for two scenarios – early arrival of the vessel before the starting of the time window and the time window violation by failing to finish the port operation within the allotted time. Ports having multiple berths allow several vessels to carry out loading/unloading operations simultaneously and hence multiple vessels can arrive at the port in a given period leading to an increase in the port congestion. It is essential to penalize the early arrival of the vessel as it may help to lower the port congestion and permit the entry of a fixed number of ships depending on the number of available berths at the port. Port operation predominantly involves a fixed set up cost associated with the arrangement of the quay cranes for performing the loading and unloading of containers [14]. A variable cost is also incurred depending on the number of containers loaded and unloaded.

Bunker fuel management strategy is integrated with the ship routing problem to determine the bunkering decision and designing the container shipping network with the aim of reducing the overall operation cost. Bunkering strategy involves two interrelated decisions of selection of bunkering ports and determining the bunkering amounts. Bunkering decisions (where to bunker, how much to bunker) depends on the bunker fuel prices at various ports and the amount of bunker fuel available in the ship fuel tank once the vessel arrives at the port. Meng et al. [5] stated the need to integrate ship routing and bunkering problem as the optimal bunkering strategies depends on bunker prices of different ports and vessel's port of call. Sustainability aspects are incorporated in the model by considering the impact of carbon emission on the total operating cost of the shipping company. Different fuel oils such as Heavy Fuel Oil (HFO) and Marine Diesel Oil (MDO) and their respective carbon emission coefficient are considered. The main engine of the vessel uses HFO while sailing in the sea and MDO is employed to run the auxiliary engine of the ship while operating at the port. The contribution presented in the paper integrates decisions on ship routing, time window concept for different ports, loading/unloading operation and bunker fuel management strategy. A mathematical formulation is developed considering different binary, continuous and integer variables and alongside conceiving several constraints.

III. Mathematical model

The assumptions of the model are as follows. (1) The number of containers loaded/unloaded at a specific port is constant. (2) Loading/unloading time for each type of containerized cargo is fixed. (3) Fuel prices at ports are known. Suppose, G is the set of container types, I is the set of ports, and V is the set of vessels. Let v represent vessel, g depict container type, (i, j) represent ports and i_v depict the initial port position for vessel v. Parameters of the mathematical are represented as follows. O_{iv} represents the docking cost of a vessel v at port i. P_i^E depicts the penalty cost for the time operated at port i outside the time window and P_i^W represents the penalty cost for waiting at port i before the starting of time window. Fixed cost for performing loading/unloading operation of container g at port i is represented as R_{ig} and variable cost for the time operated at port i inside the time window is expressed as S_i. H_{vi} depicts the bunker fuel consumption for vessel v at port i and D_{ijv} is the bunker fuel consumption rate for vessel v while sailing from port i to j. Bunker fuel capacity for vessel v (tons) is expressed as ω_v. Number of times bunkering can be performed for vessel v is expressed as δ_v. λ_i represents the bunker fuel price for port i and f_i depicts the fixed bunkering cost at port i. Carbon emission coefficient (KgCO_2/Kgfuel) at sea and port are expressed as $E_{CO_2}^{Sea}$ and $E_{CO_2}^{Port}$ respectively. $E_v^{CO_2}$ represents the maximum allowable limit of carbon emission for vessel v (KgCO_2). Travelling time of vessel v from port i to j is expressed as T_{ijv}. T_i^S, T_i^E represents the starting and ending of time window at port i and n_{iv}^S, n_{iv}^E depicts the earliest and latest arrival time of vessel v at port i. Time required to load/unload a container of type g at port i is expressed as ρ_{ig} and ϕ_{ig} represents the set up time for loading/discharging operation of a container of type g at port i. K_{vg} represents the maximum number of container of type g that vessel v can carry. η_i depicts the number of berths available at port i and M_{gi} represents the number of container of type g loaded/unloaded at port i. q_{ig} is 1, if port i is supplying container type g and -1, if port i has a demand of container type g. B is a large number and h is the number of hours in a day.

The decision variables of the mathematical model are expressed as follows. L_{vig} represents the number of container of type g on vessel v after leaving port i. A_{vi} depicts the bunker fuel level for vessel v at port i and μ_{vi} represents the bunker fuel inventory when vessel v arrives at port i. Arrival time at port i for vessel v is expressed as α_{vi} and the time operated by vessel v outside time window at port i is represented as β_{vi}. The starting and ending time of operation for vessel v at port i are expressed as τ_{vi}^{S} and τ_{vi}^{E} respectively. N_{vi} takes the value 1, if bunkering is performed by vessel v at port i, and 0, otherwise. z_{vi} takes the value 1, if vessel v ends its route at port i, and 0, otherwise. x_{ijv} takes the value 1, if the vessel v sails from port i to j and 0, otherwise. Assuming, $x_{ijv} = 0$, if $i = j$. u_{vig} takes the value 1, if container type g loaded/unloaded on ship v at port i, and 0, otherwise.

Objective function

Minimize

$$\sum_{i\in I}\sum_{v\in V}\sum_{g\in G} R_{ig}u_{vig} + \sum_{i\in I}\sum_{v\in V}\sum_{g\in G} O_{iv}u_{vig} + \sum_{i\in I}\sum_{j\in I}\sum_{v\in V} D_{ijv}x_{ijv}T_{ijv} + \sum_{i\in I}\sum_{v\in V} P_i^W\left[Max\left(0,\left(T_i^S - \alpha_{vi}\right)\right)\right] + \sum_{i\in I}\sum_{v\in V} P_i^E\beta_{vi} + \sum_{i\in I}\sum_{v\in V} S_i\left(\tau_{vi}^E - \tau_{vi}^S\right) + \sum_{i\in I}\sum_{v\in V}\left[\left(A_{vi} - \mu_{vi}\right)\lambda_i\right] + \sum_{i\in I}\sum_{v\in V} f_i N_{vi} \qquad (1)$$

Eq. (1) represents the objective function comprising of eight terms. The first term depicts the fixed cost for performing port's loading/unloading operation. The second term provides the port's docking charges. The third term interprets the bunker fuel consumption cost for every ship while sailing in sea. Fourth and fifth terms present the penalty charges incurred for waiting before the starting of the time window and operating outside the time window respectively. The sixth term depicts the variable cost for the total time operated at the port. The seventh and eighth terms represent the variable bunkering cost depending upon the amount of fuel bunkered and fixed bunkering cost at a port respectively.

$$\sum_{j\in I}\sum_{v\in V} x_{ijv} \le \eta_i \qquad \forall i\in I \qquad (2)$$

$$\sum_{i\in I} z_{vi} = 1 \qquad \forall v\in V \qquad (3)$$

$$\sum_{j\in I} x_{i_v jv} + z_{vi_v} = 1 \qquad \forall v\in V \qquad (4)$$

$$\sum_{j\in I} x_{jiv} - \sum_{j\in I} x_{ijv} - z_{vi} = 0 \qquad \forall i\in I,\ \forall v\in V \qquad (5)$$

$$u_{vig} \le \sum_{j\in I} x_{ijv} \qquad \forall i\in I,\ \forall v\in V,\ \forall g\in G \qquad (6)$$

Eq. (2) represents a scenario where the maximum numbers of vessels operating at a port depend on the number of berths available. For an individual ship, Eq. (3) ensures the termination of its route at a specified port. Eq. (4) depicts that a vessel may either, travel from its initial port to another port or, it may finish its journey at a particular port. Eq. (5) represents the flow conservation constraints. Eq. (5) states that the vessel may sail from one port to another, or it may end its route at the earlier port itself. Eq. (6) ensures that if the ship performs loading/unloading operation at a port, then the port must belong to ship's route.

$$\mu_{vj} = A_{vi} - H_{vi} - D_{ijv}T_{ijv}x_{ijv} \quad \forall i,j\in I,\ \forall v\in V \qquad (7)$$

$$A_{vi} - \mu_{vi} \le N_{vi}\omega_v \qquad \forall i\in I,\ \forall v\in V \qquad (8)$$

$$A_{vi} - \mu_{vi} \ge N_{vi}\,20\%\,\omega_v \qquad \forall i\in I,\ \forall v\in V \qquad (9)$$

$$\sum_{i\in I} N_{vi} \le \delta_v \qquad \forall v\in V \qquad (10)$$

$$N_{vi} \le \sum_{j\in I} x_{ijv} \qquad \forall i\in I,\ \forall v\in V \qquad (11)$$

$$\mu_{vi} \ge 5\%\,\omega_v \qquad \forall i\in I,\ \forall v\in V \qquad (12)$$

$$A_{vi} \le \omega_v \qquad \forall i\in I,\ \forall v\in V \qquad (13)$$

Eq. (7) depicts a fuel conservation constraint. It presents the relationship between the bunker fuel inventory for a vessel while arriving at a port and vessel's bunker fuel level and fuel consumed at its earlier port and fuel consumed by the vessel while sailing between the two ports. Eq. (8) ensures that bunkering of fuel can be performed up to the maximum bunker fuel capacity of the ship. Eq. (9) regulates the minimum bunkering amount on a ship at a port. Here, the minimum amount is arbitrarily considered as 20% of the total bunker fuel capacity of a ship as mentioned in Yao et al. [13]. Eq. (10) presents the upper limit on the number of times fuel bunkering can be performed on a vessel. Eq. (11) depicts that if fuel bunkering operation is performed at a port, then the port must belong to the ship's route. Eq. (12) depicts the minimum bunker fuel inventory on a vessel should be greater than 5% (as mentioned in Yao et al. [13]) of the fuel capacity of the vessel. Eq. (13) ensures that the bunker fuel level for a ship should be less than the maximum bunker fuel capacity.

$$\beta_{vi} \ge \tau_{vi}^E - T_i^E \qquad \forall i\in I,\ \forall v\in V \qquad (14)$$

$$\tau_{vi}^S \ge \alpha_{vi} \qquad \forall i\in I,\ \forall v\in V \qquad (15)$$

$$n_{iv}^S \le \alpha_{vi} \le n_{iv}^E \qquad \forall i\in I,\ \forall v\in V \qquad (16)$$

$$T_i^S \le \tau_{vi}^S \le T_i^E \qquad \forall i\in I,\ \forall v\in V \qquad (17)$$

$$\left(\tau_{vi}^E + T_{ijv} - h - \alpha_{vj}\right)x_{ijv} \le 0 \quad \forall i,j\in I,\ i\ne j,\ \forall v\in V \qquad (18)$$

$$\tau_{vi}^E = \tau_{vi}^S + \sum_{g\in G}\phi_{ig}u_{vig} + \sum_{g\in G}\rho_{ig}M_{gi} \quad \forall i\in I,\ \forall v\in V \qquad (19)$$

Eq. (14) presents the total time operated at a port by the vessel outside the time window. Eq. (15) depicts that the ship must start its operation only after arriving at the port. Eq. (16) ensures that the vessel arrives at the port within a particular interval of time and Eq. (17) provides the time window range for a port. Eq. (18) states that a vessel ends its port operation and then sails from the current port to the next port. The arrival time of the vessel at the new port has a relationship with the ending time of the operation at the earlier port and the sailing time between two ports. Eq. (19) presents the relationship between the ending time of a port operation with the starting time of the port operation and the total service time considering the setup time and the loading/unloading time.

$$E_{CO_2}^{Sea}\sum_{i\in I}\sum_{j\in I} D_{ijv}T_{ijv}x_{ijv} + E_{CO_2}^{Port}\sum_{i\in I} H_{vi}\left(\tau_{vi}^E - \alpha_{vi}\right) \le E_v^{CO_2}$$

$$\forall v \in V \quad (20)$$

Eq. (20) restricts the carbon emission level within a maximum allowable limit for each ship. The first term depicts the total CO_2 emission when the vessel is at sea and the second term addresses the ship's total CO_2 emission while operating at the port. For estimating, the total CO_2 emissions produced from the bunker fuel consumption, an appropriate carbon emission coefficient is considered for both types of fuel. For this problem, while sailing in the sea the main engine of the vessel employs Heavy Fuel Oil (HFO) and the auxiliary engine of the vessel uses Marine Diesel Oil (MDO) while operating at a port. Carbon emission coefficients considered for both Marine diesel Oil and Heavy Fuel Oil are 3.082 and 3.021 respectively [3].

$$M_{gi} \le K_{vg} u_{vig} \qquad \forall i \in I,\ \forall v \in V,\ \forall g \in G \quad (21)$$

$$L_{vig} \le K_{vg} \sum_{j \in I} x_{ijv} \qquad \forall i \in I,\ \forall v \in V,\ \forall g \in G \quad (22)$$

The port where the vessel performs its loading/unloading operation is depicted from the Eq. (21). Eq. (22) presents an upper bound on the maximum number of containers carried by a vessel.

$$\left(L_{vig} + q_{jg} M_{gj} - L_{vjg}\right) x_{ijv} = 0,\ \forall i, j \in I,\ \forall v \in V,\ \forall g \in G \quad (23)$$

Eq. (23) depicts a situation of a ship sailing from one port to another. The relationship between the total number of containers on-board the vessel while departing from the first port with the number of containers loaded/unloaded on second port and the number of containers on-board the vessel while departing from the second port.

$$x_{ijv} \in \{0,1\}, \qquad \forall i, j \in I,\ \forall v \in V,\ i \ne j \quad (24)$$

$$u_{vig} \in \{0,1\}, \qquad \forall i \in I,\ \forall v \in V,\ \forall g \in G \quad (25)$$

$$z_{vi} \in \{0,1\}, \qquad \forall i \in I,\ \forall v \in V \quad (26)$$

$$N_{vi} \in \{0,1\}, \qquad \forall i \in I,\ \forall v \in V \quad (27)$$

$$\tau^S_{vi},\ \tau^E_{vi},\ \beta_{vi},\ \alpha_{vi} \ge 0, \qquad \forall i \in I,\ \forall v \in V \quad (28)$$

$$\mu_{vi},\ A_{vi} \ge 0, \qquad \forall i \in I,\ \forall v \in V \quad (29)$$

$$L_{vig} \ge 0, \qquad \forall i \in I,\ \forall v \in V \quad (30)$$

Eqs. (24) – (27) are the binary variables and Eqs. (28) – (30) represents the non-negativity constraints. The mathematical formulation comprises of continuous and binary variables related to different operations such routing, loading/unloading, time window, fuel bunkering, etc.

IV. Linear Reformulation

In this section, some of the non-linear equations are linearized. The feasible region defined by equation (23) has following non-linear structure given as follows,

$$\{(y,z) \mid yf(z) = 0,\ y \in \{0,1\},\ z \in Z\} \quad (31)$$

Where $f(\cdot)$ is a function with domain *Z*. After comparing Eqs. (18) and (31), the following can be obtained, $y := \left(L_{vig}, M_{gj}, L_{vjg}\right), z := x_{ijv}$ and $f(z) := L_{vig} + q_{jg} M_{gj} - L_{vjg}$. Suppose, there is a set, $S := \{(y,z) \mid yf(z) = 0,\ y \in \{0,1\},\ z \in Z\}$, then $\{f(z) \mid z \in Z\}$ is compact or there exist certain bounds $[E, F]$, such that $E \le f(z) \le F,\ \forall z \in Z$. Therefore, set *S* can be expressed as,

$$S := \{(y,z) \mid B(1-y) \le f(z) \le C(1-y),\ y \in \{0,1\},\ z \in Z\} \quad (32)$$

For constraint (23), $f(z) := L_{vig} + q_{jg} M_{gj} - L_{vjg}$ is linear with $-K_{vg}$ and K_{vg} as the valid lower and upper bounds. So, constraint (23) can be replaced with following equations,

$$L_{vig} + q_{jg} M_{gj} - L_{vjg} + K_{vg} x_{ijv} \le K_{vg} \quad \forall i, j \in I, \forall v \in V, \forall g \in G \quad (33)$$

$$L_{vig} + q_{jg} M_{gj} - L_{vjg} - K_{vg} x_{ijv} \ge -K_{vg} \quad \forall i, j \in I, \forall v \in V, \forall g \in G \quad (34)$$

Note, that Eq. (18) represents the route and schedule compatibility constraint having the same structure as that of Eq. (31). Here, the following setting $y := \left(\tau^E_{vi}, T_{ijv}, \alpha_{vj}\right)$, $z := x_{ijv}$ and $f(z) := \tau^E_{vi} + T_{ijv} - h - \alpha_{vj}$ gives Eq. (18). Considering, the upper bound on $f(z)$ as *B* (a large number). Now as Eq. (18) is an inequality constraint, hence using Eq. (32), the following can be obtained,

$$\tau^E_{vi} + T_{ijv} - h - \alpha_{vj} + Bx_{ijv} \le B \quad \forall i, j \in I, i \ne j, \forall v \in V \quad (35)$$

Therefore, the non-linear constraints (18) and (23) can be replaced with linear constraints (33), (34) and (35) in the mathematical formulation. Linearization of non-linear constraints are performed to convert the mathematical model into a mixed integer linear programming model which would ensure that the model can be solved by optimization solver Cplex and the solution can be compared with BVNS-PSO. As the second contribution of the paper is the proposed BVNS-PSO algorithm and as the intelligent algorithm such as BVS-PSO provides only near-optimal solutions; hence, it is imperative to validate the solution quality of BVNS-PSO with the Cplex solution (which provides optimal solution) to justify the applicability of the proposed algorithm in solving real-life problems. The solution obtained by BVNS-PSO is also compared with solutions obtained by several benchmark algorithms such as BVNS, PSO, PSO-DE and GA.

V. Solution Methodology

The proposed model is complicated as the problem size depending upon the number of ports and ships increases. Many researchers have implemented meta-heuristic techniques to address complex problems in various fields [15], [16] and [17]. A hybrid algorithm, BVNS-PSO, is proposed to solve the proposed problem. It is validated and compared with an advanced version of PSO, i.e., PSO-DE, which was proved to be worked well to resolve sustainable maritime inventory routing problems [8].

A. BVNS

The VNS-based algorithm has been proved to be successful in solving a variety of combinatorial problems and dealing with large size problems as it becomes increasingly difficult to solve large problem instances with Cplex [18]. VNS algorithm is used with the motivation of obtaining a near-optimal solution with a better computational efficiency or providing efficient solutions for large instances in lesser computational time [19]. Application of hybrid VNS-based algorithm to deal with a ship routing and scheduling problem is present in shipping operation domain related literature [19].

The BVNS performs the local search within several neighbourhood structures to escape the local entrapment [18]. Components of BVNS include an initial feasible solution, shaking procedure, first improvement, neighbourhood change, and a terminating condition. Let $N_k(x)$ be a set of solutions in the k^{th} neighbour of the solution *x*. K_{max} is the maximum number of different neighbourhood structures generated in the shaking stage. In shaking procedure, a solution is randomly taken out from the k^{th} neighbourhood structure to be used as an initial solution for the first improvement local search. After an initial solution is found, the first improvement local search starts and rigorously searches the given neighbourhood structure and compares each solution with the given solution obtained in the shaking step and returns the best solution found. The solution obtained is compared with the overall best solution and accordingly neighbourhood change takes place. If an improvement is achieved, the local search returns to its first neighbourhood structure (k=1) and updates the best-known solution. Otherwise, the algorithm attempts to obtain a better solution from a different neighbourhood structure (k=k+1). After performing the local search in all the neighbourhood structures, the algorithm stores the best-known solution and moves on to the next iteration. BVNS terminates once it reaches the maximum number of the iterations. The number of iterations and the maximum number of neighbourhoods generated are the BVNS parameters.

B. Initial Solution and Neighbourhood Structure

Each algorithm requires an initial solution to begin its search procedure. It is essential to figure out the dependent and independent variables present in the mathematical formulation. The initial solution is generated considering the different types of variables and the constraints as presented in Table I. Table I highlights whether a variable is dependent or independent and in what way the variable is interacting with others depending on the constraints. After the initialization and the generated solution is fed into the neighbourhood structure.

TABLE I. DEPICTING THE GENERATION OF VARIABLES

Variables	Description
x_{ijv}	The value of this binary variable is obtained by satisfying equation (2). When $i = j$, the variable takes zero value. $x_{ijv} = 0$ for $i = j$
u_{vig}	At first, the value of the binary variable is obtained using the value of x_{ijv} and equation (6). Then the value of u_{vig} obtained is used to satisfy equation (21). This step helps in checking the feasibility of the values obtained for u_{vig} .
z_{vi}	This binary variable takes the value 0 or 1 by satisfying the equation (3). The values obtained for x_{ijv} and z_{vi} are used to satisfy the equation (5). Infeasible values are discarded, and feasible values are kept in this process.
N_{vi}	Values corresponding to the binary variable are generated using the values of x_{ijv} and equation (11). Then the value of N_{vi} obtained is used to satisfy the equation (10). Feasible values are stored and infeasible values are discarded.
τ_{vi}^S	The value of the continuous variable is estimated from the equation (17). τ_{vi}^S is considered zero if $u_{vig} = 0$ for any *v* or *i*.
τ_{vi}^E	The value of the continuous variable is calculated from equation (19) considering the values obtained for τ_{vi}^S and u_{vig} .
α_{vi}	The value of the continuous variable is generated using equation (16). The feasibility of the value found is validated using the value of τ_{vi}^S and satisfying equation (15).
β_{vi}	The value of the penalty variable for violating the time window is obtained from the equation (14) by considering the value of τ_{vi}^E .
L_{vig}	At first, the value is obtained for a given range. Now, considering the value of the binary variable x_{ijv} and equation (22) L_{vig} is validated. Equations (33) and (34) are used to check the feasibility of the value obtained.
A_{vi}	The value of A_{vi} is assumed to be zero when N_{vi} = 0. For N_{vi} = 1, the value of A_{vi} is generated using equation (13).
μ_{vi}	μ_{vi} is generated using the values of A_{vi} , x_{ijv} and equation (7). The feasibility of the value obtained is validated by satisfying equation (8) and (9). For N_{vi} = 0, μ_{vi} is assumed to be zero.

Neighbourhood structure is comprised of variables such as routing variables, bunkering variables, loading/unloading variables and time window variables. Fig. 1 presents a neighbourhood structure consisting of 100 feasible solutions and for illustrating the position of each variable, an example of four ports, three ships, and two containers groups is considered. The problem instance comprises of 192 variables, and their arrangements are presented in fig. 1. 100 solutions are considered in a neighbourhood structure and each solution presents the variable values and their positions. The algorithm is run for 100 iterations, and the constraints are checked and violations if any are adjusted by considering penalty values. The feasible solution obtained satisfies all the constraints. The best solution of the iteration is compared with the best fitness function value obtained in the previous iterations and accordingly, the best obtained value is stored. Result obtained after 100 iterations always satisfies the constraints and dependency relationships between different variables.

C. BVNS-PSO

The BVNS-PSO algorithm employs the capability of PSO to perform extensive exploration and deep exploitation. The PSO is widely known as it adapts to problem domains like ship routing and scheduling problem [3], [20] and provides superior results than other contemporary algorithms [15]. VNS algorithm carries out the local search procedure to find the best solution in each neighbourhood. Fig. 2 presents the pseudo-code of BVNS-PSO. The algorithm comprises of two stages - BVNS stage and PSO stage. The BVNS stage starts with an initial feasible solution and generates K_{max} number of neighbourhoods. Components of BVNS like shaking step, first improvement and neighbourhood change are elaborately described in sections above. The algorithm obtains a random solution belonging to a particular neighbourhood in the shaking step. Each solution obtained is improved by VNS using its local search procedure (First Improvement). If the local search technique finds a better solution, then it becomes the global best solution. Neighbourhood change takes place after the local search operation. In this way, the algorithm searches the neighbourhoods and obtains the best solution. Now, the concept of PSO is hybridized with BVNS. The PSO stage starts and the neighbourhood structure containing the best solution becomes the local best position of the iteration. The global best position takes the value of the global best solution. In Fig. 3, the global best solution is represented by x^*. K_{max} numbers of structures are initially generated. Velocity $V_j(x)$ is updated for each neighbourhood structure by Eq. (36). Each neighbourhood structure $N_j(x)$ is updated by Eq. (37). Both Eqs. (36) and (37) are the PSO equations.

$$V_j(x) = wV_j(x) + c_1 r_1 (L_{best\_pos} - N_j(x)) + c_2 r_2 (G_{best\_pos} - N_j(x)) \quad (36)$$

$$N_j(x) = N_j(x) + V_j(x) \quad (37)$$

Here, $V_j(x)$ is the velocity of the j^{th} neighbourhood structure. Parameters of PSO are inertia weight w and acceleration coefficients C_1 and C_2. r_1 and r_2 are random vectors. L_{best_pos} and G_{best_pos} are the local best position and global best position. $N_j(x)$ is the j^{th} neighbourhood structure.

$$\begin{bmatrix} Solution = 1 & 2 & \ldots & 100 \\ x_{ijv} \updownarrow_{48}^{1} & x_{ijv} \updownarrow_{48}^{1} & \ldots & x_{ijv} \updownarrow_{48}^{1} \\ u_{vig} \updownarrow_{72}^{49} & u_{vig} \updownarrow_{72}^{49} & \ldots & u_{vig} \updownarrow_{72}^{49} \\ z_{vi} \updownarrow_{84}^{73} & z_{vi} \updownarrow_{84}^{73} & \ldots & z_{vi} \updownarrow_{84}^{73} \\ N_{vi} \updownarrow_{96}^{85} & N_{vi} \updownarrow_{96}^{85} & \ldots & N_{vi} \updownarrow_{96}^{85} \\ \tau_{vi}^{S} \updownarrow_{108}^{97} & \tau_{vi}^{S} \updownarrow_{108}^{97} & \ldots & \tau_{vi}^{S} \updownarrow_{108}^{97} \\ \tau_{vi}^{E} \updownarrow_{120}^{109} & \tau_{vi}^{E} \updownarrow_{120}^{109} & \ldots & \tau_{vi}^{E} \updownarrow_{120}^{109} \\ \beta_{vi} \updownarrow_{132}^{121} & \beta_{vi} \updownarrow_{132}^{121} & \ldots & \beta_{vi} \updownarrow_{132}^{121} \\ \alpha_{vi} \updownarrow_{144}^{133} & \alpha_{vi} \updownarrow_{144}^{133} & \ldots & \alpha_{vi} \updownarrow_{144}^{133} \\ \mu_{vi} \updownarrow_{156}^{145} & \mu_{vi} \updownarrow_{156}^{145} & \ldots & \mu_{vi} \updownarrow_{156}^{145} \\ A_{vi} \updownarrow_{168}^{157} & A_{vi} \updownarrow_{168}^{157} & \ldots & A_{vi} \updownarrow_{168}^{157} \\ L_{vig} \updownarrow_{192}^{169} & L_{vig} \updownarrow_{192}^{169} & \ldots & L_{vig} \updownarrow_{192}^{169} \end{bmatrix}$$

Fig. 1. Neighbourhood structure for a given example

Function BVNS – PSO
N = *number of iterations*
$Kmax$ = *maximum number of neighbourhoods*
Tuning parameters for PSO (inertia weight (w), *acceleration coefficients* (C_1, C_2)) *are considered*
Two random numbers r_1 *and* r_2 *are generated*

Choose an initial feasible solution x *and compute* $f(x)$
———————— *BVNS Stage* ————————
Generate Kmax number of neighbourhoods $N_k(x)$
Iterations
for $n = 1$ *to* N
$k = 1$
  while $k < Kmax$
    Shaking: obtain a random solution x' *belongs to* $N_k(x)$
    Local search: Obtain a local minima x'' *from FirstImprovement* (x')
    Neighbourhood Change:
    if $f(x'') < f(x)$
      Set $k = 1$, $x = x''$, $f(x) = f(x'')$
      if $f(x'') < f(x^*)$
        Set $x^* = x''$, $f(x^*) = f(x'')$
      end
    else
      Set $k = k + 1$
    end
  end
  Search all the neighborhoods and obtained the best solution
  ———————— *PSO Stage* ————————
  L_{best_pos} = *neighbourhood containing the best solution*
  $G_{best\_pos} = x^*$
  for $j = 1$ *to Kmax*
    $V_j(x) = wV_j(x) + C_1 r_1 (L_{best\_pos} - N_j(x)) + C_2 r_2 (G_{best\_pos} - N_j(x))$
    $N_j(x) = N_j(x) + V_j(x)$
  end
end

Fig. 2. Pseudo code of BVNS-PSO algorithm

VI. Computational Experiment

The algorithm is coded on MATLAB, and all experiments are conducted on a computer with Intel Core i5, 2.90 GHz processor with 8 GB RAM on a Windows 7 environment. The parameters are tuned after preliminary tests to obtain the near optimal solution. Parameters of BVNS-PSO are inertia weight, acceleration coefficients and maximum number of neighbourhoods. 30 test runs are performed for each of the parameters. Maximum number of neighbourhoods K_{max} is considered as 5, inertia weight w as 0.9 and acceleration coefficient C_1 and C_2 as 0.1 and 0.98 respectively.

A. Data Collection

The computational experiment presented below is intended to fortify the purpose of proposing a new robust and flexible algorithm to facilitate solution generation for a complex problem in the domain of maritime transportation. The applicability of BVNS-PSO is demonstrated on a real-time problem of SSRTWBM. The used data is taken from secondary reliable sources in [2] [3]. Table II presents the data set for the parameters of the mathematical model. The problem instances presented in Table III are solved by BVNS-PSO and the solutions are compared with that of Cplex to determine the solutions gap. The optimal solutions for the problem instances are obtained by IBM ILOG Cplex V12.5 optimization studio. 30 problem instances are developed on the basis of the number of the ports, containers, and ships and Table III illustrates the complexities of the problem instances considered by providing the number of variables (binary, continuous and integer variables) and constraints (equality and inequality). The problem instances considering container-ships carrying two types of containers - 20 foot and 40 foot containers.

TABLE II: DATA SET FOR COMPUTATIONAL PURPOSE

Parameter or variable	Range	Units
Fixed set up cost, R_{ig}	(500, 1000)	USD/operation
Variable cost for time operated at port, S_i	(400, 500)	USD/hour
Penalty cost for violating time window, P_i^E	(400, 500)	USD/hour
Penalty cost for waiting time, P_i^W	(50, 100)	USD/hour
Docking charges, O_{iv}	(400, 500)	USD
Bunker fuel consumption at a port, H_{vi}	(10, 20)	Kg fuel/hour
Bunker fuel consumption rate on sea, D_{ijv}	(1, 2)	Kg fuel/hour
Start and end of time window, T_i^S, T_i^E	(6, 20)	Hours (real time)
Fixed bunkering cost at a port, f_i	(500, 600)	USD/bunkering operation
Bunker fuel price at a port, λ_i	(450,500)	USD/ton
Arrival time range at a port, n_{iv}^S, n_{iv}^E	(4, 8)	Hours (real time)
Time required to load/unload a container, ρ_{ig}	(0.1, 0.2)	Hours per container
Set up time for performing port operation, ϕ_{ig}	(0.5, 1)	Hours per operation
Travelling time of a vessel, T_{ijv}	(12, 18)	Hours
Number of containers carried by a vessel K_{vg}	(200, 300)	Units
Number of containers loaded/unloaded, M_{gi}	(20, 10)	Units
Number of berths at a port, η_i	(2, 4)	Berths

B. *Solving the Problem Instances*

The problem instances are solved by BVNS-PSO and Cplex and results are presented in Table III. Results obtained using BVNS, PSO, GA and PSO-DE help to validate the performance of BVNS-PSO. Table III shows the results of objective function value and computational time for 30 problem instances obtained by BVNS-PSO, BVNS, PSO, GA, PSO-DE algorithms and Cplex.

It is observed that BVNS-PSO yields better performance than BVNS for each instance. Moreover, BVNS-PSO outperforms PSO, GA and PSO-DE demonstrating its superiority over these algorithms. The computational efficiency of the PSO-based algorithms is better than BVNS-PSO. The involvement of multiple numbers of neighbourhoods in a single iteration of BVNS-PSO increases the computational time as it takes a longer time to perform the local search in the neighbourhoods. PSO-based algorithms deal with a single swarm (size of the swarm is considered similar to that of a neighbourhood) in each iteration. Cplex provides a better solution compared with BVNS-PSO. The difference in the total cost obtained by BVNS-PSO and Cplex is reported as the solution gap, which can be calculated as solution gap (%) = $\left(\left(TC_{BP} - TC_C\right)/TC_C\right) \times 100$, where TC_{BP} and TC_C are the total cost obtained by BVNS-PSO algorithm and Cplex. Figs. 3 and 4 present the visual illustration of the convergence graph of the algorithms for problem instance 18 (10-2-5) and 29 (15-2-10) respectively.

It is concluded that the BVNS-PSO is competitive for solving large size problems. The performance of the approach is measured in terms of the solution gap and computational time and from Table III, it is observed that BVNS-PSO produces solutions with an average, minimum and maximum solution gaps of 5.99%, 1.99% and 10.90% respectively within reasonable computational time. The change in solution gap with every problem instance is random and it doesn't follow any trend or pattern. The proposed BVNS-PSO shows exemplary performance for the instances regarding solution quality in comparison to PSO, GA, BVNS and PSO-DE. The manifested results highlight the efficient performance of BVNS-PSO for instances of higher complexity.

C. *Analysis of the Results*

Table IV presents the values pertaining to bunker fuel consumption costs, variable bunkering costs, port operation costs, waiting cost and time window violation cost. Results in Tables III and IV indicate that for large problem instances the majority of the shipping company's cost comprises of the bunker fuel consumption cost, which depends on the fuel consumption rate while sailing between ports. Although for small and medium-sized problem instances, the significant portion of the total cost of the shipping companies comprises of bunker consumption cost, variable bunkering cost and port operational cost. Penalty charges for waiting before the start of the time window and penalty costs for time window violation are negligible when compared with other cost components. Table V presents the results obtained after performing sensitivity analysis on bunker fuel price at different ports. The effect of the variation in bunker fuel price on the variable bunkering cost is investigated for the problem instances. It is observed that the increase in bunker price has a little influence on the total cost. For large problem instances, the variation in the bunker fuel price leads to a significant difference in the variable bunkering cost.

The problem instances are also solved without the carbon emission constraints. Table VI presents the detailed results highlighting the effect of the carbon emission constraints on the total cost and bunker fuel consumption cost for 15 problem instances. From the table, it is interpreted that the total cost and the bunker fuel consumption cost are less while solving the problem instances without considering the carbon emission constraints and BVNS-PSO provides total cost results with an average, maximum and minimum solution gaps of 6.05%, 11.64% and 2.46% respectively. Effect of neglecting the carbon emission constraints leads to an average 6.71% and 6.24% decrement in the total cost while solving with the BVNS-PSO and Cplex respectively. BVNS-PSO provides the values associated with bunker fuel consumption cost for the problem instances with an average, maximum and minimum solution gaps of 1.66%, 2.40% and 1.04% respectively. On an average 2.88% and 2.39% decrement in the values of the bunker fuel consumption cost is obtained by BVNS-PSO and Cplex respectively, while solving the problem instances for both the cases – with and without carbon emission constraints. Table VII presents the vessel routes, bunkering ports and amount of fuel bunkered for some of the problem instances. Voyage information in Table VII provides the output of the mathematical model. The results will assist the shipping companies to reconsider the impact of fuel bunkering decisions on the routing decision.

Certain managerial implications can be drawn. In this study, the early arrival of the vessel and violation of time window by failing to finish the port operation within the allotted time window is countered by considering penalty charges which help to improve ports' service level. The penalty costs incurred for the problem instances are less as compared to other cost components. This highlights the fact that early arrival of vessels and delay in finishing port operation is countered in most of the cases. Amount of fuel bunkered is affected by the variation in bunker fuel price leading to a significant change of the variable bunkering cost, which in turn affect the total cost. The bunker fuel prices vary at different ports making it imperative to design the vessel route in such a way that the overall bunkering cost is reduced. It is observed that the total cost and bunker fuel consumption cost doesn't increase substantially while considering the carbon emission restriction. Hence, carbon emission constraints can be added to the mathematical model to address the sustainability aspect.

VII. Conclusion and Future Scope

The paper investigates the SSRTWBM problem, formulates a mixed integer linear programming model, and presents a novel algorithm, BVNS-PSO. The characteristics of BVNS algorithm helps in intensifying the search procedure for obtaining a better solution and the diversification of PSO is responsible for escaping from the local entrapment and moving towards the unexplored areas of the solution space. The performance of BVNS-PSO is validated on several problem instances with a solution gap of less than 6% of average total cost deviation from the exact solutions obtained using Cplex. Computational experiment depicts the superiority of BVNS-PSO in terms of solution quality over benchmark algorithms like BVNS, PSO, GA and PSO-DE. It is observed that BVNS-PSO portrays greater potential and competitiveness in solving sustainable ship routing and scheduling problems and providing promising results pertaining to vessel route, bunker port selection and amount of fuel bunkered. The insights obtained from this research would help the shipping companies to readjust their vessel's route and bunkering decisions in an efficient way. In future, the model can be extended under probabilistic environment for dealing with the challenges associated with stochastic bunker fuel price.

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TABLE VII. SHIP ROUTES, BUNKERING PORTS AND BUNKERED AMOUNTS FOR FEW PROBLEM INSTANCES

Problem instances (ports, containers, ships)	Ship no.	Vessel route (ports visited by the ship)	Bunkering ports	Bunkered amounts (ton)
(9, 2, 5)	Ship 1	2, 5, 1, 3, 4, 8, 6, 7, 9	Port 3; Port 7	P3 – 674; P7 – 748
	Ship 2	1, 2, 3, 4, 5, 8, 7, 9, 6	Port 4; Port 9	P4 – 634; P9 – 765
	Ship 3	4, 5, 2, 3, 1, 6, 8, 7, 9	Port 3; Port 7	P3 – 621; P7 – 630
	Ship 4	5, 3, 2, 1, 4, 9, 8, 7, 6	Port 1; Port 7	P1 – 618; P7 – 702
	Ship 5	4, 5, 2, 1, 3, 9, 8, 7, 6	Port 1; Port 7	P1 – 607; P7 – 674
(10, 2, 6)	Ship 1	3, 1, 2, 5, 4, 8, 10, 9, 7, 6	Port 5; Port 9	P5 – 616; P9 – 741
	Ship 2	5, 1, 4, 3, 2, 9, 10, 6, 7, 8	Port 3; Port 6	P3 – 637; P6 – 694
	Ship 3	2, 5, 3, 1, 4, 8, 6, 9, 7, 10	Port 1; Port 9	P1 – 622; P9 – 704
	Ship 4	2, 1, 5, 4, 3, 9, 10, 8, 6, 7	Port 4; Port 8	P4 – 597; P8 – 721
	Ship 5	4, 2, 3, 5, 1, 8, 7, 9, 6, 10	Port 5; Port 9	P5 – 643; P9 – 687
	Ship 6	2, 4, 3, 1, 5, 7, 10, 6, 8, 9	Port 1; Port 6	P1 – 605; P6 – 669
(12, 2, 8)	Ship 1	4, 3, 6, 1, 2, 5, 11, 9, 10, 12, 8, 7	Port 1; Port 9	P1 – 654; P9 – 688
	Ship 2	2, 3, 6, 5, 1, 4, 12, 10, 8, 7, 11, 9	Port 5; Port 10	P5 – 691; P10 – 642
	Ship 3	3, 6, 1, 2, 4, 5, 12, 7, 8, 9, 10, 11	Port 2; Port 7	P2 – 609; P7 – 722
	Ship 4	4, 5, 1, 6, 2, 3, 7, 12, 10, 11, 9, 8	Port 6; Port 12	P6 – 654; P12 – 681
	Ship 5	4, 1, 6, 3, 5, 2, 7, 11, 10, 12, 9, 8	Port 3; Port 11	P3 – 636; P11 – 701
	Ship 6	2, 1, 4, 3, 5, 6, 9, 10, 11, 7, 8, 12	Port 3; Port 10	P3 – 611; P10 – 640
	Ship 7	6, 1, 2, 3, 4, 5, 10, 8, 11, 9, 12, 7	Port 5; Port 8	P5 – 621; P8 – 686
	Ship 8	4, 2, 3, 5, 6, 1, 10, 7, 8, 9, 11, 12	Port 5; Port 7	P5 – 634; P7 – 691

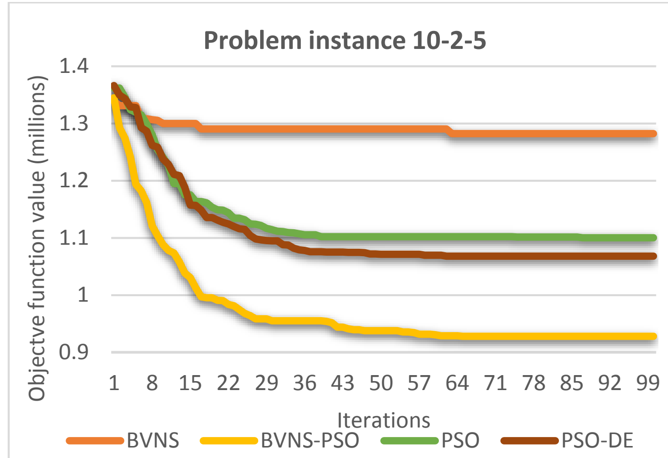

Fig. 3. Convergence graph of problem instance 10-2-5 for all the algorithms

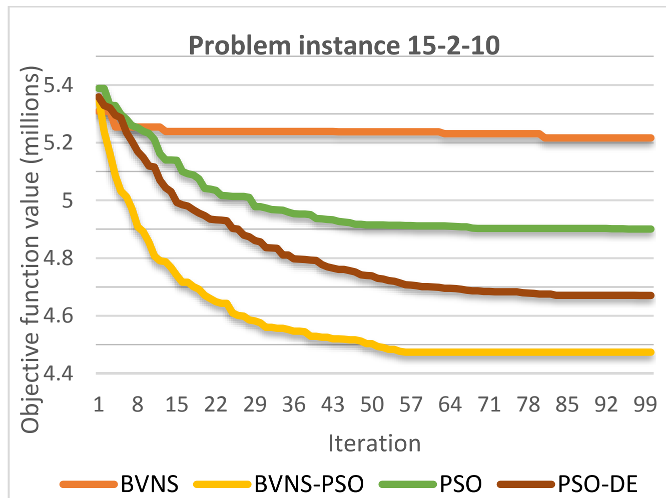

Fig. 4. Convergence graph of problem instance 15-2-10 for all the algorithms

TABLE III. TOTAL COST INCURRED AND COMPUTATIONAL TIME REPORTED FOR ALL THE 30 PROBLEM INSTANCES

Problem instances (ports, containers, ships)	Total cost (USD)					Computational time (sec)					CPLEX performance	Solution gap of BVNS-PSO with CPLEX (%)
	BVNS-PSO	BVNS	PSO	GA	PSO-DE	BVNS-PSO	BVNS	PSO	GA	PSO-DE	Total cost (USD)	
(3, 2, 2)	3.022×10^4	6.913×10^4	5.391×10^4	5.452×10^4	4.393×10^4	45	72	19	25	35	2.891×10^4	4.53
(3, 2, 3)	3.406×10^4	9.763×10^4	9.299×10^4	9.369×10^4	7.256×10^4	57	84	23	30	42	3.162×10^4	7.71
(4, 2, 2)	4.729×10^4	9.942×10^4	9.019×10^4	9.194×10^4	7.441×10^4	58	83	23	29	43	4.368×10^4	8.26
(4, 2, 3)	7.373×10^4	1.225×10^5	1.186×10^5	1.197×10^5	1.124×10^5	66	97	31	38	55	7.196×10^4	2.45
(5, 2, 2)	5.758×10^4	1.574×10^5	1.180×10^5	1.202×10^5	1.027×10^5	62	93	28	37	52	5.389×10^4	6.84
(5, 2, 3)	1.139×10^5	1.954×10^5	1.770×10^5	1.832×10^5	1.607×10^5	79	108	37	49	68	1.097×10^5	3.82
(5, 2, 4)	1.674×10^5	2.819×10^5	2.662×10^5	2.754×10^5	2.527×10^5	96	129	45	58	83	1.512×10^5	10.71
(6, 2, 3)	1.568×10^5	3.345×10^5	2.331×10^5	2.492×10^5	2.031×10^5	97	128	46	62	83	1.521×10^5	3.09
(6, 2, 4)	1.742×10^5	6.418×10^5	3.384×10^5	3.496×10^5	3.147×10^5	122	169	58	79	106	1.708×10^5	1.99
(7, 2, 3)	1.976×10^5	6.583×10^5	3.489×10^5	3.593×10^5	3.186×10^5	125	174	59	77	108	1.885×10^5	4.82
(7, 2, 4)	2.498×10^5	7.132×10^5	4.621×10^5	4.882×10^5	4.263×10^5	149	198	71	96	132	2.371×10^5	5.35
(7, 2, 5)	3.821×10^5	8.947×10^5	6.013×10^5	6.271×10^5	5.454×10^5	179	223	87	112	157	3.687×10^5	3.63
(8, 2, 4)	3.747×10^5	8.379×10^5	5.777×10^5	5.918×10^5	5.506×10^5	168	211	85	109	126	3.595×10^5	4.22
(8, 2, 5)	5.329×10^5	9.964×10^5	7.671×10^5	7.889×10^5	6.947×10^5	213	261	105	134	182	5.177×10^5	2.93
(9, 2, 4)	5.793×10^5	9.431×10^5	7.377×10^5	7.565×10^5	6.334×10^5	209	259	102	129	187	5.541×10^5	4.54
(9, 2, 5)	7.237×10^5	1.169×10^6	9.426×10^5	9.719×10^5	8.965×10^5	241	298	125	151	209	7.014×10^5	3.17
(9, 2, 6)	9.114×10^5	1.475×10^6	1.138×10^6	1.296×10^6	1.104×10^6	267	310	148	175	253	8.751×10^5	4.14
(10, 2, 5)	9.279×10^5	1.282×10^6	1.100×10^6	1.244×10^6	1.068×10^6	271	312	149	180	252	8.812×10^5	5.29
(10, 2, 6)	1.169×10^6	1.845×10^6	1.425×10^6	1.657×10^6	1.332×10^6	288	327	175	215	282	1.083×10^6	7.94
(10, 2, 7)	1.454×10^6	2.175×10^6	1.720×10^6	1.915×10^6	1.657×10^6	339	356	202	249	313	1.311×10^6	10.90
(11, 2, 6)	1.231×10^6	1.963×10^6	1.535×10^6	1.691×10^6	1.392×10^6	346	391	203	254	315	1.118×10^6	10.10
(11, 2, 7)	1.670×10^6	2.359×10^6	1.955×10^6	2.183×10^6	1.786×10^6	359	413	239	287	335	1.528×10^6	9.29
(11, 2, 8)	1.832×10^6	2.674×10^6	2.221×10^6	2.476×10^6	2.045×10^6	431	492	305	352	414	1.691×10^6	8.33
(12, 2, 8)	2.115×10^6	2.943×10^6	2.610×10^6	2.825×10^6	2.343×10^6	462	549	348	406	454	1.936×10^6	9.24
(12, 2, 9)	2.358×10^6	3.342×10^6	2.956×10^6	3.172×10^6	2.764×10^6	457	537	392	447	486	2.274×10^6	3.69
(13, 2, 9)	2.871×10^6	3.632×10^6	3.384×10^6	3.537×10^6	3.178×10^6	574	637	463	521	557	2.602×10^6	10.33
(14, 2, 9)	3.327×10^6	4.149×10^6	3.858×10^6	3.962×10^6	3.615×10^6	621	701	516	573	654	3.186×10^6	4.42
(14, 2, 10)	3.846×10^6	4.673×10^6	4.269×10^6	4.383×10^6	4.095×10^6	682	742	566	629	660	3.693×10^6	4.14
(15, 2, 10)	4.473×10^6	5.216×10^6	4.900×10^6	5.108×10^6	4.670×10^6	748	797	646	701	722	4.228×10^6	5.79
(16, 2, 10)	5.000×10^6	5.915×10^6	5.527×10^6	5.704×10^6	5.363×10^6	816	883	720	765	796	4.621×10^6	8.20

TABLE IV. PROBLEM COMPLEXITY AND DIFFERENT COST COMPONENTS OF THE OBJECTIVE FUNCTION FOR ALL THE PROBLEM INSTANCE

Problem instances (ports, containers, ships)	Variables			Total number of variables	Constraints		Total number of constraints	Bunker fuel cost (USD)	Variable bunkering cost (USD)	Operational time cost (USD)	Penalty charges for waiting (USD)	Penalty charges for violating (USD)
	Binary	Continuous	Integer		Equality	Inequality						
(3, 2, 2)	42	36	12	90	70	109	179	6.841 x 10^{3}	4.578 x 10^{3}	1.365 x 10^{4}	1.040 x 10^{3}	1.496 x 10^{2}
(3, 2, 3)	63	54	18	135	105	162	267	7.054 x 10^{3}	4.192 x 10^{3}	1.736 x 10^{4}	1.138 x 10^{3}	1.087 x 10^{3}
(4, 2, 2)	64	48	16	128	116	152	268	1.082 x 10^{4}	4.485 x 10^{3}	2.274 x 10^{4}	1.360 x 10^{3}	1.982 x 10^{3}
(4, 2, 3)	96	72	24	192	174	226	400	1.426 x 10^{4}	5.696 x 10^{3}	3.290 x 10^{4}	2.180 x 10^{3}	2.486 x 10^{3}
(5, 2, 2)	90	60	20	170	174	199	373	1.203 x 10^{4}	5.432 x 10^{3}	3.014 x 10^{4}	1.475 x 10^{3}	1.835 x 10^{3}
(5, 2, 3)	135	90	30	255	261	296	557	2.672 x 10^{4}	1.912 x 10^{4}	4.820 x 10^{4}	1.931 x 10^{3}	4.288 x 10^{3}
(5, 2, 4)	180	120	40	340	348	393	741	4.277 x 10^{4}	2.875 x 10^{4}	5.962 x 10^{4}	2.870 x 10^{3}	7.954 x 10^{3}
(6, 2, 3)	180	108	36	324	366	372	738	4.021 x 10^{4}	3.323 x 10^{4}	5.018 x 10^{4}	2.660 x 10^{3}	5.513 x 10^{3}
(6, 2, 4)	240	144	48	432	488	494	982	5.352 x 10^{4}	3.388 x 10^{4}	5.102 x 10^{4}	3.254 x 10^{3}	6.614 x 10^{3}
(7, 2, 3)	231	126	42	399	489	454	943	6.141 x 10^{4}	4.696 x 10^{4}	5.708 x 10^{4}	3.647 x 10^{3}	5.974 x 10^{3}
(7, 2, 4)	308	168	56	532	652	603	1255	7.996 x 10^{4}	5.272 x 10^{4}	8.309 x 10^{4}	4.293 x 10^{3}	1.061 x 10^{4}
(7, 2, 5)	385	210	70	665	815	752	1567	1.053 x 10^{5}	1.014 x 10^{5}	1.259 x 10^{5}	4.967 x 10^{3}	1.649 x 10^{4}
(8, 2, 4)	384	192	64	640	840	720	1560	1.325 x 10^{5}	7.149 x 10^{4}	1.313 x 10^{5}	5.217 x 10^{3}	1.657 x 10^{4}
(8, 2, 5)	480	240	80	800	1050	898	1948	1.882 x 10^{5}	1.093 x 10^{5}	1.702 x 10^{5}	6.395 x 10^{3}	1.301 x 10^{4}
(9, 2, 4)	468	216	72	756	1052	845	1897	1.816 x 10^{5}	8.906 x 10^{4}	1.746 x 10^{5}	5.826 x 10^{3}	1.872 x 10^{4}
(9, 2, 5)	585	270	90	945	1315	1054	2369	2.803 x 10^{5}	1.758 x 10^{5}	2.204 x 10^{5}	6.749 x 10^{3}	2.329 x 10^{4}
(9, 2, 6)	702	324	108	1134	1578	1263	2841	3.788 x 10^{5}	1.974 x 10^{5}	2.647 x 10^{5}	7.678 x 10^{3}	2.059 x 10^{4}
(10, 2, 5)	700	300	100	1100	1610	1220	2830	4.091 x 10^{5}	1.225 x 10^{5}	2.463 x 10^{5}	7.452 x 10^{3}	2.463 x 10^{4}
(10, 2, 6)	840	360	120	1320	1932	1462	3394	4.737 x 10^{5}	2.214 x 10^{5}	3.091 x 10^{5}	9.296 x 10^{3}	2.355 x 10^{4}
(10, 2, 7)	980	420	140	1540	2254	1704	3958	5.686 x 10^{5}	2.591 x 10^{5}	3.502 x 10^{5}	9.667 x 10^{3}	2.785 x 10^{4}
(11, 2, 6)	990	396	132	1518	2322	1673	3995	5.839 x 10^{5}	2.499 x 10^{5}	3.312 x 10^{5}	1.009 x 10^{4}	2.712 x 10^{4}
(11, 2, 7)	1155	462	154	1771	2709	1950	4659	7.841 x 10^{5}	2.495 x 10^{5}	3.831 x 10^{5}	1.159 x 10^{4}	3.414 x 10^{4}
(11, 2, 8)	1320	528	176	2024	3096	2227	5323	8.770 x 10^{5}	3.246 x 10^{5}	4.403 x 10^{5}	1.355 x 10^{4}	3.265 x 10^{4}
(12, 2, 8)	1536	576	192	2304	3664	2524	6188	8.839 x 10^{5}	3.894 x 10^{5}	4.879 x 10^{5}	1.475 x 10^{4}	4.191 x 10^{4}
(12, 2, 9)	1728	648	216	2592	4122	2838	6960	9.017 x 10^{5}	4.216 x 10^{5}	5.506.x 10^{5}	1.652 x 10^{4}	4.410 x 10^{4}
(13, 2, 9)	1989	702	234	2925	4815	3190	8005	9.239 x 10^{5}	4.061 x 10^{5}	5.932 x 10^{5}	1.754 x 10^{4}	5.156 x 10^{4}
(14, 2, 9)	2268	756	252	3276	5562	3560	9122	2.009 x 10^{6}	4.694 x 10^{5}	6.376 x 10^{5}	1.824 x 10^{4}	5.280 x 10^{4}
(14, 2, 10)	2520	840	280	3640	6180	3954	10134	2.227 x 10^{6}	5.808 x 10^{5}	7.112 x 10^{5}	2.082 x 10^{4}	5.956 x 10^{4}
(15, 2, 10)	2850	900	300	4050	7070	4385	11455	2.444 x 10^{6}	6.462 x 10^{5}	7.615 x 10^{5}	2.184 x 10^{4}	6.429 x 10^{4}
(16, 2, 10)	3200	960	320	4480	8020	4836	12856	2.680 x 10^{6}	6.634 x 10^{5}	8.221 x 10^{5}	2.360 x 10^{4}	7.431 x 10^{4}

TABLE V. SENSITIVITY ANALYSIS WITH RESPECT TO BUNKER FUEL PRICES AT DIFFERENT PORTS

Problem instance (ports, containers, ships)	Percentage of increase/decrease in the value of the parameter	Total cost (USD), Percentage difference	Variable Bunkering cost (USD), Percentage difference	Port operational cost (USD), percentage difference
(16, 2, 10)	50% (increase)	5.045×10^6, 0.90% (increase)	9.403×10^5, 41.79% (increase)	8.345×10^5, 1.50% (increase)
	25% (increase)	5.018×10^6, 0.36% (increase)	7.922×10^5, 19.41% (increase)	8.273×10^5, 0.63% (increase)
	25% (decrease)	4.936×10^6, 1.28% (decrease)	5.357×10^5, 19.24% (decrease)	8.181×10^5, 0.48% (decrease)
	50% (decrease)	4.863×10^6, 2.74% (decrease)	3.584×10^5, 45.97 % (decrease)	8.051×10^5, 2.06% (decrease)
(14, 2, 10)	50% increase	4.000×10^6, 4.0% (increase)	8.273×10^5, 42.44% (increase)	7.249×10^5, 1.88% (increase)
	25% increase	3.988×10^6, 3.69% (increase)	7.113×10^5, 22.46% (increase)	7.178×10^5, 0.92% (increase)
	25% decrease	3.768×10^6, 2.02% (decrease)	4.663×10^5, 19.17% (decrease)	7.007×10^5, 1.47% (decrease)
	50% decrease	3.643×10^6, 5.27% (decrease)	3.167×10^5, 45.47% (decrease)	6.973×10^5, 1.95% (decrease)
(10, 2, 6)	50% increase	1.233×10^6, 5.47% (increase)	3.254×10^5, 47.87% (increase)	3.197×10^5, 3.42% (increase)
	25% increase	1.193×10^6, 2.05% (increase)	2.693×10^5, 21.63% (increase)	3.113×10^5, 0.71% (increase)
	25% decrease	1.121×10^6, 4.10% (decrease)	1.779×10^5, 19.64% (decrease)	3.018×10^5, 2.36% (decrease)
	50% decrease	1.095×10^6, 6.33% (decrease)	1.313×10^5, 40.69% (decrease)	2.971×10^5, 3.88% (decrease)

TABLE VI. EFFECT OF CARBON EMISSION RELATED CONSTRAINT ON THE PROBLEM INSTANCES

Problem instances (ports, containers, ships)	Total cost (USD) considering carbon emission constraint		Bunker cost (USD) considering carbon emission constraint		Total cost (USD) without considering carbon emission constraint			Bunker fuel cost (USD) without considering carbon emission constraint			Percentage decrease in total cost (%)		Percentage decrease in Bunker fuel cost (%)	
	BVNS-PSO	CPLEX	BVNS-PSO	CPLEX	BVNS-PSO	CPLEX	Sol. Gap (%)	BVNS-PSO	CPLEX	Sol. Gap (%)	BVNS-PSO	CPLEX	BVNS-PSO	CPLEX
(9, 2, 5)	7.237×10^5	7.014×10^5	2.803×10^5	2.732×10^5	6.847×10^5	6.632×10^5	3.24	2.728×10^5	2.684×10^5	1.63	5.38	5.44	2.67	1.75
(9, 2, 6)	9.114×10^5	8.751×10^5	3.788×10^5	3.731×10^5	8.586×10^5	8.344×10^5	2.90	3.691×10^5	3.633×10^5	1.59	5.79	4.65	2.56	2.62
(10, 2, 5)	9.279×10^5	8.812×10^5	4.091×10^5	3.963×10^5	8.612×10^5	8.405×10^5	2.46	3.884×10^5	3.817×10^5	1.75	7.18	4.61	5.05	3.68
(10, 2, 6)	1.169×10^6	1.083×10^6	4.737×10^5	4.658×10^5	1.014×10^6	9.767×10^5	3.81	4.611×10^5	4.529×10^5	1.81	13.25	15.35	2.65	2.76
(10, 2, 7)	1.454×10^6	1.311×10^6	5.686×10^5	5.591×10^5	1.313×10^6	1.176×10^6	11.64	5.546×10^5	5.473×10^5	1.33	9.69	10.29	2.46	2.11
(11, 2, 6)	1.231×10^6	1.118×10^6	5.839×10^5	5.722×10^5	1.104×10^6	9.982×10^5	10.59	5.703×10^5	5.611×10^5	1.63	10.31	10.71	2.32	1.93
(11, 2, 7)	1.670×10^6	1.528×10^6	7.841×10^5	7.709×10^5	1.513×10^6	1.401×10^6	7.99	7.676×10^5	7.591×10^5	1.11	9.40	8.31	2.10	1.53
(11, 2, 8)	1.832×10^6	1.691×10^6	8.770×10^5	8.653×10^5	1.707×10^6	1.560×10^6	9.42	8.618×10^5	8.529×10^5	1.04	6.82	7.74	1.73	1.43
(12, 2, 8)	2.115×10^6	1.936×10^6	8.839×10^5	8.720×10^5	2.042×10^6	1.871×10^6	9.13	8.698×10^5	8.583×10^5	1.33	3.45	3.35	1.59	1.57
(12, 2, 9)	2.358×10^6	2.274×10^6	9.017×10^5	8.884×10^5	2.187×10^6	2.096×10^6	4.34	8.795×10^5	8.641×10^5	1.78	7.25	7.82	2.46	2.74
(13, 2, 9)	2.871×10^6	2.602×10^6	9.239×10^5	9.014×10^5	2.683×10^6	2.508×10^6	6.97	8.939×10^5	8.798×10^5	1.60	6.54	3.61	3.24	2.39
(14, 2, 9)	3.327×10^6	3.186×10^6	2.009×10^6	1.931×10^6	3.198×10^6	3.047×10^6	4.95	1.914×10^6	1.869×10^6	2.40	3.87	4.36	4.72	3.21
(14, 2, 10)	3.846×10^6	3.693×10^6	2.227×10^6	2.173×10^6	3.672×10^6	3.561×10^6	3.11	2.151×10^6	2.112×10^6	1.84	4.52	3.57	3.41	2.80
(15, 2, 10)	4.473×10^6	4.228×10^6	2.444×10^6	2.383×10^6	4.268×10^6	4.161×10^6	2.57	2.365×10^6	2.319×10^6	1.98	4.58	1.58	3.23	2.68
(16, 2, 10)	5.000×10^6	4.621×10^6	2.680×10^6	2.614×10^6	4.865×10^6	4.517×10^6	7.70	2.598×10^6	2.543×10^6	2.16	2.70	2.25	3.05	2.71