On connectivity of Post-earthquake Road Networks

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This paper proposes a novel stochastic mathematical framework to investigate the connectivity of road networks impacted by earthquakes. The concepts of "global connectivity" and "local connectivity" are defined and evaluated using percolation theory. Specifically, global connectivity measures the extent to which the whole network is connected, and local connectivity measures the distances between each node to its neighbors. Furthermore, the concept of efficiency is employed to integrate local connectivity. A new percolation process integrating the traditional localized attack and random failure is proposed, which sheds light on the application of percolation theory to practical networks when facing disruptions.

Key words: connectivity; percolation theory; transportation; robustness

1 Introduction

The human and financial toll of natural disasters have dramatically increased in recent decades. According to Statista (statista.com 2016), ten largest natural disasters in the history each caused more than 26,200 fatalities and 28 billion U.S. dollars of economic damage. The earthquake and tsunami in Japan in March 2011 resulted in economic losses of more than 210 billion U.S. dollars, and the earthquake and tsunami in Thailand in December 2004 killed more than 220,000 people. In 2014, China, India, and the United States were the three regions that were hit the hardest by natural disasters, and each country suffered economic losses of more than 16 billion U.S. dollars.

The connectivity of post-disaster road networks are very important since they act as lifelines that provide access to affected areas, and support evacuation, emergency response, and long-term recovery operations after a disaster (Altay and Green, 2006). Connectivity also frequently acts as the basis of the assessment of vulnerability, robustness, and resilience (Reggiani et al., 2015, Hong et al., 2017, Yan et al., 2017, Zhou et al., 2017, Wang et al., 2017, and Zhou and Wang, 2018, Zhou et al., 2018). Natural disasters, such as earthquakes or floods, may break or block some components of road networks. If some roads and intersections, represented by links and nodes respectively, are disrupted in a network, the connectivity of the network will change. The disruption of links may break the shortest paths between node pairs and increase the traveling distance between them. The failure of one node results into the disruption of all the links connected to it. Furthermore, if a disaster is very severe and many nodes and/or links are disrupted, the original network could be degraded into isolated subnetworks, and there will be no paths connecting some node pairs.

The concept of connectivity is widely studied in air transport networks and public transit networks. Airport networks are usually modelled as complex networks, and connectivity measures the degree to which an airport is connected to the other airports. The connectivity of public transit systems focuses on the quality of transferring services, especially the time spent on transferring between different transportation modes and accessibility of services. However, the studies on the connectivity of road road networks are very limited. To our knowledge, the connectivity of road networks impacted by disasters have not been systematically studied using analytical methods in the literature.

In this paper, we propose an analytical method to study the connectivity of earthquake-impacted road networks. The concepts of “global connectivity” and “local connectivity” are defined and evaluated using percolation theory. Specifically, global connectivity measures the extent to which the whole network is connected, and local connectivity measures the distances between each node to the others. Furthermore, the concept of efficiency is employed to integrate local connectivity. A unified model is proposed to capture the feature of earthquake-impacted road networks that the probability of component disruption varies with its distance to the epicenter.

The contributions of this study are twofold: (a) the concepts of global connectivity and local connectivity are defined and evaluated using percolation theory, especially the concept of efficiency is employed to integrate local connectivity, and (b) a unified model is proposed for modelling post-earthquake road networks.

The remainder of this paper is outlined as follows. Section 2 presents a review of the related literature. In Section 3, the concepts used in this paper are introduced. In Section 4, the mathematical framework used to study the global connectivity and local connectivity of road networks impacted by different magnitudes of earthquakes is proposed. Then, the proposed methodological framework is applied to a real-world transportation network for validation in Section 5, which is followed by conclusions in the last section.

2 Literature review

This paper focuses on the connectivity of post-earthquake road networks. The literature is divided into two parts: (1) connectivity of other transportation networks, including air transport networks, public transit networks, and logistic networks, and (2) connectivity of road networks, especially post-disaster road networks.

2.1 Connectivity of other transportation networks

Air transport networks. In air transport networks, connectivity measures the degree to which an airport is connected to the other airports. Airport networks are usually modelled as complex networks, and the metrics used to assess the connectivity of air transport networks include node degree, weighted node degree, shortest path length, quickest path length, and so on. Burghouwt and Redondi (2013) reviewed the connectivity measures for air transport networks used in the literature by then. Wei et al. (2014a) and Wei et al. (2014b) used algebraic connectivity to study the problems of route deletion/addition and weight assignment in air transport networks. Algebraic connectivity is defined as the second smallest Laplacian eigenvalue of a graph. Allroggen et al. (2015) proposed the Global Connectivity Index (GCI) to measure the quality and quantity of all available connections. Zhang et al. (2017) used the NetScan model to assess connectivity which considers both direct and indirect connections and travel time. The NetSacn model first assigns a quality index to each flight according to relative travel time and then brings these indexes together into one single connectivity index of each airport. Boonekamp and Burghouwt (2017) used the NetCargo model, which is based on NetScan model, to measure the connectivity of air freight networks at the regional level. Zhu et al. (2018) proposed a dynamic weighted model based on the NetScan model to quantify the quality of connections with consideration of flight capacity, in addition to flight time.

Public transit networks. The connectivity of public transit systems focuses on the quality of transferring services, especially the time spent on transferring between different transportation modes, and accessibility of services. Ceder et al. (2009) proposed a transit connectivity measure using eight quantitative attributes, such as average walking time, and three qualitative attributes, such as smoothness-of-transfer. Hadas and Ceder (2010) studied the connectivity reliability and comfort of transfers in multi-legged trips. Three different types of transfer were analyzed: nonadjacent transfer, adjacent transfer, and shared transfer. Hadas and Ranjitkar (2012) developed a model to measure the transit-network connectivity based on value of time and quality of transfer, and further used it to identify the inefficiencies in the public transit system. Hadas (2013) presented a method to extract, store, and analyze the public transit data and analyze the connectivity of the public transport using five different indicators. Mishra et al. (2012) proposed measures for determining the connectivity in different levels: node and line, transfer center, and region, for the purpose of prioritizing the allocation of funding to improve connectivity. Welch and Mishra (2013) proposed a method to assess transit equity using a quantity of attributes, including frequency, speed, capacity, and built environment. Mishara et al. (2015)

presented a graph theoretic method to assess transit connectivity at different levels and provided a platform for visualization. Zimmerman et al. (2015) studied the connectivity of rail and bus systems as alternative transportation modes when facing extreme events. Cheng and Chen (2015) developed an integrated conceptual measurement approach with Rasch model to find the perceived difficulties for public transportation users, and also investigated the relations among accessibility, mobility, and connectivity. Lowry et al. (2016) focused on prioritizing bicycle improvement projects to increase the connectivity between homes and important destinations, such as banks and restaurants. Psaltoglou and Calle (2018) proposed an enhanced connectivity index with consideration of the interaction between transportation networks and the urban environment, and further used it to detect critical nodes.

Logistic networks. Studies in logistic networks mainly focus on port connectivity. Jiang et al. (2015) proposed a minimum transportation time model and a maximum transportation capacity model to measure port connectivity and investigate its impact on the global container shipping network. Wang et al. (2016) developed an integrated port connectivity index, consisting of three layers, including international connectivity, inner bay connectivity, and hinterland connectivity. A thorough overview of port connectivity is also provided in that study.

2.2 Connectivity of post-disaster road networks

A related concept to connectivity that has been well studied in road networks is connectivity reliability. Connectivity reliability refers to the probability that the network nodes remain connected (Wakabayashi and Iida, 1992 and Iida, 1999). For a specific origin-destination pair, it represents the probability that there exists at least one path with available capacity between them. The connectivity reliability of a network entirely depends on the connectivity reliability of the links, and quantity of research has focused on calculating network reliability based on component reliability using different methods, such as statistical methods, simulation, and complex network theory (Muriel-Villegas et al., 2016, Hosseini and Wadbro, 2016). The concept of connectivity reliability is widely used in transportation planning and operations to involve the uncertainties in road connectivity (Peeta et al., 2010, Chu and Chen, 2015, and Wang et al., 2018). Chen et al. (1999) also expended the concept into capacity reliability to consider partial disruption of road links.

There are some studies on road networks which consider network connectivity as constraint in their model. Kasaei and Salman (2016) studied the arc routing problems for clearing blocked roads to restore network connection with two different objectives: to minimize the time and to maximize the benefit. Akbari and Salman (2017) provided a solution method to generate a schedule for the road clearing teams to restore network connectivity at post-disaster stage.

There are some metrics proposed in the literature to evaluate the connectivity of post-disaster road networks. Chang and Nojima (1995) employed three performance measures for the post-earthquake road network: total length of network open, total distance-based accessibility, and areal distance-based accessibility. Yang et al. (2018) defined network connectivity as the average connection state of any possible node pairs. Aydin et al. (2018) used the size of giant connected component to represent the connectivity of post-disaster networks. These connectivity metrics are applied to determined post-disaster networks or evaluated with simulations for different scenarios.

In summary, there is no analytical approaches in the literature which can be used to study the change of connectivity of road networks impacted by different magnitudes of disasters. In this paper, we present a framework to study the connectivity of road networks when facing one specific type of disaster, i.e. earthquake. We will first propose two concepts for post-earthquake road networks: global connectivity and local connectivity, and then present the method to evaluate these two properties using percolation theory. In the next section, the concepts to be used in this study and some background information of percolation theory will be given.

3 Percolation theory

3.1 Concepts

In this paper, we propose two important concepts to describe the property of road networks impacted by earthquakes, i.e. *global connectivity* and *local connectivity*. These two properties are quantified by several metrics using percolation theory. Global connectivity is quantified by three metrics: *threshold of the existence of giant subnetwork*, *the size of the giant subnetwork*, and *average sizes of* *small subnetworks*. Local connectivity is quantified by *number of neighbors*. Furthermore, the number of neighbors of different nodes are brought together into one single index for the whole network: *efficiency*. Next, we explain these metrics respectively.

Percolation theory was founded by Broadbent and Hammersley (1957) to study random physical processes, such as the flow of liquid through a disordered porous medium. Quantities of interest include (1) the critical probabilities of bond/site occupation, or alternatively link/node removal, for the existence of an infinite open cluster, and the phenomena near the critical point, and (2) the size and structure of the open cluster given a fraction of links/nodes removed. Percolation in different lattices is well studied by physicists and pure mathematicians. One important contribution recently made by Newman et al. (2001) is employing generating function formalism to study the percolation on random networks with arbitrary node degree distribution. This technique greatly broadens the application of percolation theory to different practical complex networks, such as epidemic spreading on networks, Internet networks, and interdependent power and communication networks. Li et al. (2015) and Wang et al. (2015) studied the percolation in urban road networks. A disrupted link is identified as a congested/overloaded link, which is different from that in this work, where a disrupted link is physically broken by disasters. By now there is no application of percolation to road networks accounting for the features of disasters in road networks.

Theoretically, a pre-disaster road network should be well connected as one component, which means that from one node, users can reach any other node by following a set of links inside the network. Disasters may disrupt certain links/nodes in the network. If only small fraction of links were broken, the remaining links and nodes would remain connected. However, if a large fraction of links were broken, then these broken links may incur cascading failures, and the original network would be separated into many small *subnetworks*. Therefore, there should exist one critical point. In percolation theory, *giant component* is defined as "the mutually connected cluster spanning the entire network"[1] (page 1026, Buldyrev et al., 2010). If the network is disrupted, "the nodes belonging to the giant

[1] Percolation theory was originally founded on infinite networks, where giant component refers to the infinite open cluster which spans the network from one side to the opposite side. Since practical networks are always finite, giant component is usually regarded as the cluster with size of order of the whole network (Cohen and Havlin, 2009). However, we can still use the method derived from infinite networks to find the critical point for the existence of giant component in finite networks.

component connecting a finite fraction of the network are still functional, while the remaining small clusters become non-functional" (page 1026, Buldyrev et al., 2010). Thus, the threshold of the existence of the giant component can be one of the metrics of global connectivity in the network. *Giant component* is a term used in general networks, here we apply the same idea to road networks. In order to differentiate this, we name the mutually connected clusters spanning the entire disaster-impacted road network as *giant subnetwork*.

Given that there exists one giant subnetwork, the *size of the giant subnetwork* will vary with the magnitude of the disaster. We will want to know what fraction of nodes are included in it. For example, if 5% of links were disrupted, but all the nodes are still connected, then the disaster-impacted network can be viewed as unchanged from the perspective of general connectivity, because users can still reach any other node from any node inside the network. Similarly, if 10% of inks were disrupted, as a result, 98% of nodes remain connected as one giant subnetwork, with only 2% isolated as small subnetworks, then the disaster-impacted network can be viewed as nearly unchanged (98%). However, if there is no giant subnetwork, we will look into these separated small subnetworks, and get the *subnetwork sizes distribution*, which reflects the impact of the disaster on the global connectivity of the road network. Note that even if there is one giant subnetwork, there may still exist some small subnetworks, which contains very small fraction of nodes.

Local connectivity is quantified by *number of neighbors*. The number of neighbors with different distances and their changes with the magnitude of disasters are respectively investigated. Using the example in last paragraph, if 5% of links are disrupted, and all the nodes are still connected, the global connectivity of the network can be viewed as unchanged, while the local connectivity is not. The disruption of the link between one pair of nodes will increase the travel distance and in turn travel time between them. Furthermore, the number of neighbors of different nodes are brought together into one single index for the whole network: *efficiency*. If global connectivity determines whether the nodes can be reached, then local connectivity measures how easily these nodes can be reached.

3.2 Challenges

There are some difficulties in using current percolation methods to study the road networks impacted by earthquakes. Currently, in the application of percolation to other networks, two types of disasters are modelled: random failures and localized attacks. In a random failure, each node/link has the same probability of been disrupted. In a localized attack, a node is firstly affected, then its neighbors, and then the neighbors of neighbors, and so on. Finally, all the nodes with less than certain distance from the root one are disrupted and those which are out of the scope survive. An example of localized attack is the spreading of epidemic disease in the population. However, neither of these two can accurately model earthquakes in road networks. In an earthquake-impacted road network, indeed there is an epicentre, around which the magnitude of disaster is high. The difference from traditional localized attack models is that not all the nodes around epicentre are disrupted, though with higher probability. Furthermore, for those nodes not around the epicentre, the probability of being damaged is rather low, but not zero. In short, traditional percolation methods assume that all the nodes/links have the same probability of being broken. However, in earthquake-impacted road networks, the probability of being broken for a road decreases with its distance from the epicentre.

Therefore, in this paper, we will first propose an integrated model to describe the percolation process on earthquake-impacted road networks, and then investigate the change of global and local connectivity under different magnitudes of earthquakes.

4 Methodology

In this section, a mathematical framework is proposed to study the impact of earthquakes on the connectivity of road networks. To determine the impact of earthquakes on global connectivity, the percolation threshold and subnetwork sizes distribution are calculated. To study the impact of earthquakes on local connectivity, the number of neighbors with different distances and efficiency are calculated.

4.1 Earthquake modelling in road networks

As discussed in section 3.2, to capture the feature of earthquake-impacted road networks, we propose an integrated model which combines localized attacks and random failures. As shown in Figure 1 (middle figure), the earthquake-impacted road network is divided into two parts, namely inner network and outer network. The impact of the earthquake on the inner network is modelled as a localized attack. In the inner network, node disruption is dominant and all the links connected to disrupted nodes are also disrupted. There could be some nodes or groups of nodes which are not disrupted themselves, while all the nodes/links surrounding them are disrupted. These nodes are also regarded as disrupted as they are isolated from the other parts. In the outer network, link disruption is dominant, and we assume that all the links have the same probability of being disrupted.

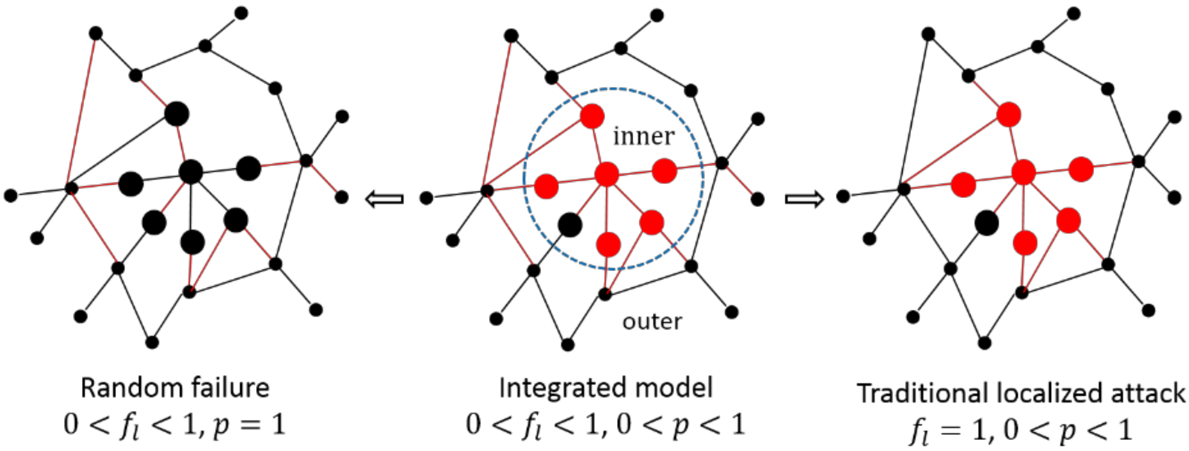

Figure 1. (Colour online) The integrated model, which combines the traditional random failure and localized attack in road networks. Red nodes/links are broken.

In percolation theory, generating function is used to characterize the node degree distribution of networks (Callaway et al., 2000, Newman et al., 2001, Buldyrev et al., 2010, Gao et al., 2012). We define

$$G_0(x) = \sum_{k=0}^{\infty} P(k)x^k, \quad (1)$$

where $P(k)$ is the probability that a randomly chosen node has degree k and x is an arbitrary complex variable. In generating function formalism, the probabilities of different node degrees are treated as the coefficients of a power series. This greatly simplifies the calculation and expression of $P(k)$.

Given that there are $1 - p$ percentage of nodes contained in the inner network, which are attacked.

The process of a localized attack is divided into two stages (Shao et al., 2015): (i) remove all the nodes belonging to the attacked area (inner network in our model) but keep the links connecting the removed nodes to the remaining nodes; (ii) remove those links. After the first stage, the generating function for the remaining network is (Shao et al., 2009)

$$G_a(x) = \frac{G_0(fx)}{G_0(f)}, \tag{2}$$

where $f = G_0^{-1}(p)$. After the second stage, the generating function becomes (Shao et al., 2015):

$$G_0^p(x) = G_a(1 - \tilde{p} + \tilde{p}x) = \frac{1}{G_0(f)} G_0\left[f + \frac{G_0'(f)}{G_0'(1)}(x - 1)\right], \tag{3}$$

where $\tilde{p} = G_0'(f)/G_0'(1)$.

By now, we have obtained the generating function for the node degree distribution after the localized attack in the inner network, as in Equation (3). Next we consider the random failure in the outer network. Given that the probability of being disrupted for each link in the outer network is $1 - f_l$, the generating function of the remaining network is (Appendix I)

$$G_0^{p,f_l}(x) = G_0^p(1 - f_l + f_l x) = \frac{1}{G_0(f)} G_0\left[f + \frac{G_0'(f)}{G_0'(1)} f_l(x - 1)\right]. \tag{4}$$

Therefore, the generating function of the remaining network with $1 - p$ percentage of nodes removed in the inner network and $1 - f_l$ percentage of links removed in the outer network is represented in Equation (4). By setting different values of f_l and p, earthquakes of different magnitudes and different areas being affected can be modelled. Specifically, this integrated model degrades into a traditional random failure model when $p = 1$; a traditional localized attack model when $f_l = 1$ (Figure 1).

4.2 Global connectivity

The original road network represented by Equation (1) should be a mutually connected network with no parts being isolated. While it is not true for the remaining network at post-earthquake stage as represented in Equation (4). The remaining network may consist of a giant subnetwork and several small subnetworks or only small subnetworks. In this section, we calculate (1) the percolation threshold for the existence of the giant subnetwork, (2) the size of the giant subnetwork, and (3) the sizes of the small subnetworks for the road networks impacted by different magnitudes of earthquakes.

4.2.1 Percolation threshold

Let $G_1(x)$ be the generating function for the number of outgoing links in the original network. Note that the number of outgoing links is averaged on randomly selected links, not randomly selected nodes. It holds

$$G_1(x) = \frac{\sum_{k=0}^{\infty} kP(k) \cdot x^{k-1}}{\sum_{k=0}^{\infty} kP(k)} = \frac{G_0{}'(x)}{G_0{}'(1)}. \tag{5}$$

It is proven that the percolation criterion for a network is that the average number of outgoing links equals 1 (Cohen et al., 2000), i.e. $G_1{}'(1) = 1$.

Next, we apply this percolation criterion to the earthquake-impacted road network. Let $G_1^{p,f_l}(x)$ be the generating function for the number of outgoing links in the network impacted by the earthquake. We have

$$G_1^{p,f_l}(x) = \frac{G_0^{p,f_l{}'}(x)}{G_0^{p,f_l{}'}(1)} = \frac{G_0'\left[f + \frac{G_0'(f)}{G_0'(1)} f_l(x-1)\right]}{G_0'(f)}. \tag{6}$$

With $G_1^{p,f_l{}'}(1) = 1$, we find the percolation threshold $(f_l, p)^*$ satisfies

$$f_l G_0''\left(G_0^{-1}(p)\right) = G_0'(1). \tag{7}$$

4.2.2 Size of giant subnetwork

Let $H_0(x)$ be the generating function for the subnetwork sizes distribution in the original network. According to Newman et al. (2001), it holds $H_0(x) = xG_0\big(H_1(x)\big)$, where $H_1(x)$ satisfies the transcendental equation $H_1(x) = xG_1\big(H_1(x)\big)$. The size of the giant subnetwork can be represented as $1 - H_0(1)$. In the original network, we have $1 - H_0(1) = 1$.

Next, we apply the calculation to the earthquake-impacted road network. Letting S^{p,f_l} be the size of the giant subnetwork in the road network impacted by the earthquake, we have

$$S^{p,f_l} = 1 - H_0^{p,f_l}(1), \tag{8}$$

where

$$H_0^{p,f_l}(x) = xG_0^{p,f_l}\left(H_1^{p,f_l}(x)\right). \tag{9}$$

And $H_1^{p,f_l}(x)$ satisfies the transcendental equation

$$H_1^{p,f_l}(x) = xG_1^{p,f_l}\left(H_1^{p,f_l}(x)\right). \tag{10}$$

The equations can be numerically solved.

4.2.3 Average size of small subnetworks

Although it is difficult to find a closed-form expression for the subnetwork sizes distribution as shown in the last subsection, the closed-form expressions for the average size of subnetworks, can be derived. Newman et al. (2001) studied the average component size of a pre-disaster network, $\langle s \rangle$, and found

$$\langle s \rangle = \frac{1}{H_0(1)}\left[G_0\big(H_1(1)\big) + \frac{G_0{}'\big(H_1(1)\big)G_1\big(H_1(1)\big)}{1 - G_1{}'\big(H_1(1)\big)}\right]. \tag{11}$$

Similarly, the average size of small subnetworks in the road network impacted by the earthquake, $\langle s^{p,f_l} \rangle$, is given by

$$\langle s^{p,f_l} \rangle = \frac{1}{H_0^{p,f_l}(1)}\left[G_0^{p,f_l}\left(H_1^{p,f_l}(1)\right) + \frac{G_0^{p,f_l{}'}\left(H_1^{p,f_l}(1)\right)G_1^{p,f_l}\left(H_1^{p,f_l}(1)\right)}{1 - G_1^{p,f_l{}'}\left(H_1^{p,f_l}(1)\right)}\right]. \tag{12}$$

4.3 Local connectivity

Global connectivity measures the impact of earthquakes on the connection of the whole network, while it does not reflect the change of local connections between nodes. In other words, global connectivity only reflect whether nodes pairs are connected, but cannot answer how close they are connected. Assume that a small proportion of links are disrupted by the earthquake and the whole network still keeps connected, the global connectivity will remain unchanged. However, the disruption of links will absolutely increase the distances between certain node pairs. We use number of neighbors with different distances as a metric to measure local connectivity. The average number of neighbors are calculated to give a more comprehensive assessment. Furthermore, to bring number of neighbors with different distances into a single index for the local connectivity of the whole network, we combine them and employed the concept of efficiency.

4.3.1 Number of neighbors

In a road network, the first-nearest neighbors to one node are the nodes that are directly connected to it. Similarly, the second-nearest neighbors are those that can be reached by following at least two road links. The generating functions for the probability distribution of the first-nearest and the second-nearest neighbors in a pre-disaster network are $G_0(x)$ and $G_0(G_1(x))$. By extension, the generating function for *r*th-nearthest neighbors, $G_{(r)}(x)$, satisfies $G_{(r)}(x) = G_{(r-1)}\big(G_1(x)\big)$. And the average number of *r*th-nearest neighbors, z_r, satisfies $z_r = G_0{}'(1)[G_1{}'(1)]^{r-1}$ (Newman et al., 2001).

In the earthquake-impacted road networks, let $G_{(r)}^{p,f_l}(x)$ be the generating function for the distribution of *r*th-nearthest neighbors in the road network impacted by the earthquake, and z_r^{p,f_l} be the average number of *r*th-nearest neighbors in the road network impacted by the earthquake. We have

$$G_{(r)}^{p,f_l}(x) = \begin{cases} G_0^{p,f_l}(x), & r = 1, \\ G_{(r-1)}^{p,f_l}\left(G_1^{p,f_l}(x)\right), & r \geq 2. \end{cases} \tag{13}$$

and

$$z_r^{p,f_l} = \left[G_1^{p,f_l\prime}(1)\right]^{r-1} G_0^{p,f_l\prime}(1). \tag{14}$$

4.3.2 Integrated local connectivity: efficiency

In section 4.3.1, we have calculated the distribution of number of neighbors and average number of neighbors with different distances. Next, we will bring them together into one single index for the local connectivity of the whole network. Let us first consider the number of neighbors with different distances of one specific node i in the pre-disaster network, and let $n_{i,r}$ represent the number of *r*th-nearthest neighbors of node i. We have

$$\sum_{r=1}^{r_m} n_{i,r} = N - 1, \qquad (15)$$

where N is the overall number of nodes in the network, and r_m is the distance between node i and its farthest neighbour in the network. Obviously, the integer r_m can be determined by solving $\sum_{r=1}^{r_m-1} n_{i,r} \leq N - 1 \leq \sum_{r=1}^{r_m} n_{i,r}$. Here we have the fact that if node j is the *r*th-nearthest neighbor of node i, then the length of the shortest path between node i and j is r, if we neglect the different lengths of links. Next, we will regard the shortest path length between two nodes as the distance between them. Let D_i be the sum of the distances between node i and all the other nodes in the network, then we have

$$D_i = \sum_{r=1}^{r_m} r \cdot n_{i,r} \qquad (16)$$

Then the sum of the distances between each node to all the other nodes in the network should be

$$D = \sum_{i=1}^{N} D_i = \sum_{i=1}^{N} \sum_{r=1}^{r_m} r \cdot n_{i,r} = \sum_{r=1}^{r_M} \sum_{i=1}^{N} r \cdot n_{i,r} = \sum_{r=1}^{r_M} r \sum_{i=1}^{N} n_{i,r} = \sum_{r=1}^{r_M} r \cdot z_r \qquad (17)$$

where r_M is the largest distance between any possible node pairs in the network, and z_r is the average number of *r*th-nearest neighbors in the original network as defined in section 4.3.1. r_M can be determined by solving $\sum_{r=1}^{r_M-1} z_r \leq N - 1 \leq \sum_{r=1}^{r_M} z_r$. By now, we obtain a single index D to measure the local connectivity of the whole network.

Similarly as what we do to the above metrics, the total distance D can be calculated when the road network is impacted by different magnitudes of earthquakes, and the local connectivity can be assessed. However, there comes the problem when the whole network is divided into different parts. For the node pair that each of them belongs to different isolated subnetworks, the distance between them is infinite, and equations (15-17) do not work anymore. To overcome this problem, we replace r with $1/r$ in equations (16-17) and denote the new metrics as E_i and E. We have

$$E_i = \sum_{r=1}^{r_m} \frac{1}{r} \cdot n_{i,r} \qquad (18)$$

and

$$E = \sum_{i=1}^{N} E_i = \sum_{i=1}^{N}\sum_{r=1}^{r_m} \frac{1}{r} \cdot n_{i,r} = \sum_{r=1}^{r_M} \frac{1}{r} \cdot z_r. \tag{19}$$

In the new metric, local connectivity is actually quantified by the sum of the inverse of the distances between node pairs. The proposed metric E is very similar to the metric called "efficiency" in complex networks, where the efficiency of a graph G is defined as $\frac{1}{N(N-1)}\sum_{i\neq j\in G}\frac{1}{d_{ij}}$, and d_{ij} is the shortest path between node i and j. Therefore, we also call E as the efficiency of the pre-disaster road network.

The efficiency of a post-earthquake road network should be

$$E^{p,f_l} = \sum_{i=1}^{N\cdot S^{p,f_l}} E_i^{p,f_l} = \sum_{r=1}^{r_M^{p,f_l}} \frac{1}{r} \cdot z_r^{p,f_l}, \tag{20}$$

where S^{p,f_l} is the size of the giant subnetwork in the road network impacted by the earthquake as defined in equation (8), z_r^{p,f_l} is the average number of *r*th-nearest neighbors in the road network impacted by the earthquake as defined in equation (14), and r_M^{p,f_l} is the largest distance between any possible node pairs in the post-earthquake road network, which can be determined by solving $\sum_{r=1}^{r_M^{p,f_l}-1} z_r^{p,f_l} \leq N \cdot S^{p,f_l} - 1 \leq \sum_{r=1}^{r_M^{p,f_l}} z_r^{p,f_l}$. The total number of nodes in the pre-disaster network (N) is replaced by the total number of nodes in the giant subnetwork in the post-earthquake network ($N \cdot S^{p,f_l}$), because in post-earthquake stage, only the nodes in the giant subnetwork can remain functionality. The nodes contained in the small subnetworks are disconnected from the others.

Table 1 presents a summary of the methodology to show the inputs and outputs of this framework step by step. Our contributions in the methodology are (1) modelling of the structure of the post-earthquake road network, i.e. $G_0^{p,f_l}(x)$, (2) defining the concepts of global connectivity and local connectivity and choose the most appropriate metrics from the literature to quantify them in the post-earthquake stage, and (3) bringing number of neighbors with different distances together into one single index, efficiency, for the assessment of local connectivity of the whole post-earthquake road network. It is worth mentioning that most of the metrics are only used to evaluate pre-disaster networks in the literature, and we are the first to apply them to post-earthquake road networks.

Table 1. Summary of the methodology

	Pre-disaster road network		Post-earthquake road network	
Network structure	$G_0(x)$		$G_0^{p,f_l}(x)$	
Outgoing links	$G_1(x) = \frac{G_0{}'(x)}{G_0{}'(1)}$		$G_1^{p,f_l}(x) = \frac{G_0^{p,f_l{}'}(x)}{G_0^{p,f_l{}'}(1)}$	
Percolation threshold	$G_1{}'(1) = 1$		$G_1^{p,f_l{}'}(1) = 1 \rightarrow f_l G_0''\left(G_0^{-1}(p)\right) = G_0'(1)$	
Subnetwork sizes distribution	$\begin{cases} H_0(x) = xG_0\left(H_1(x)\right) \\ H_1(x) = xG_1\left(H_1(x)\right) \end{cases}$		$\begin{cases} H_0^{p,f_l}(x) = xG_0^{p,f_l}\left(H_1^{p,f_l}(x)\right) \\ H_1^{p,f_l}(x) = xG_1^{p,f_l}\left(H_1^{p,f_l}(x)\right) \end{cases}$	
Global Connectivity	Size of giant subnetwork	Average size of small subnetworks	Size of giant subnetwork	Average size of small subnetworks
	$S = 1 - H_0(1)$	$\langle s \rangle = H_0{}'(1)$	$S^{p,f_l} = 1 - H_0^{p,f_l}(1)$	$\langle s^{p,f_l} \rangle = H_0^{p,f_l{}'}(1)$
Number of neighbors	$G_{(r)}(x) = \begin{cases} G_0(x), & r = 1, \\ G_{(r-1)}\left(G_1(x)\right), & r \geq 2. \end{cases}$		$G_{(r)}^{p,f_l}(x) = \begin{cases} G_0^{p,f_l}(x), & r = 1, \\ G_{(r-1)}^{p,f_l}\left(G_1^{p,f_l}(x)\right), & r \geq 2. \end{cases}$	
Local connectivity	Average number of neighbors	Efficiency	Average number of neighbors	Efficiency
	$z_r = G_0{}'(1)[G_1{}'(1)]^{r-1}$	$E = \sum_{i=1}^{N} E_i = \sum_{r=1}^{r_M} \frac{1}{r} \cdot z_r$	$z_r^{p,f_l} = \left[G_1^{p,f_l{}'}(1)\right]^{r-1} G_0^{p,f_l{}'}(1)$	$E^{p,f_l} = \sum_{i=1}^{N \cdot S^{p,f_l}} E_i^{p,f_l} = \sum_{r=1}^{r_M^{p,f_l}} \frac{1}{r} \cdot z_r^{p,f_l}$

5 Case study

According to Statista.com, the 2008 Sichuan earthquake is one of the ten most significant natural disasters in terms of death toll around the world in the past three decades. This disaster was also the strongest earthquake since 1950 and the deadliest earthquake in China since the 1976 Tangshan earthquake. More than 69,000 people died in the 2008 Sichuan earthquake, and approximately 4.8 million people were left homeless. Infrastructure systems, including transportation systems, were greatly damaged by the earthquake and aftershocks. In the road network, 21 existing highways and 5 highways that were under construction were partially disrupted. In addition, five national roads and 11 principle roads were severely destroyed.

The specific network that we analyse is the national and provincial road network in the affected area, including Mianyang, Ngawa, Deyang, Guangyuan, and Chengdu (Figure 2). After defining the specific network, we calculate the node degree distribution in the network (Table 2). This network contains 109 nodes and 219 links. Each node represents one junction, and each link represents one road between two neighbouring junctions. According to Table 2, the generating function for the probability distribution of node degrees should be $G(x) = \sum_{k=0}^{\infty} P(k) \cdot x^k = 0.37x^3 + 0.52x^4 + 0.08x^5 + 0.03x^6$.

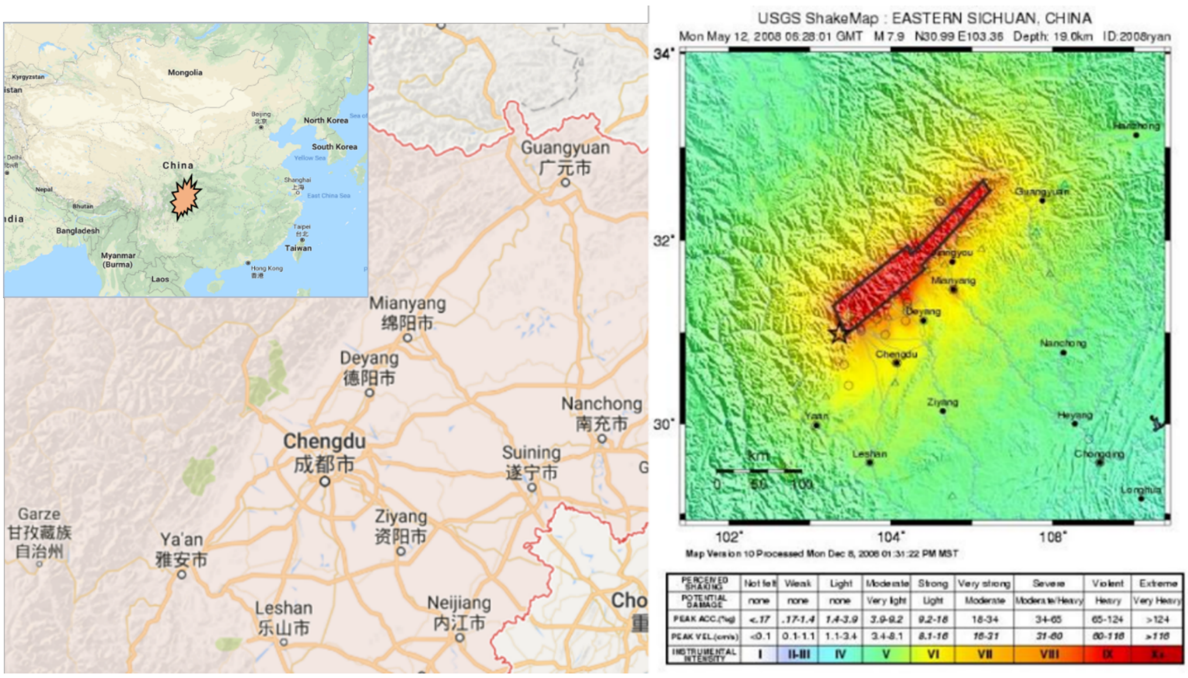

Figure 2. (a) The analysed affected area, including all of the national and provincial roads in Mianyang, Ngawa, Deyang, Guangyuan, and Chengdu. (b) Instrumental intensity of the Sichuan earthquake (earthquake.usgs.gov).

Table 2. Node degree distribution of the national and provincial road network.

Node degree (k)	3	4	5	6
Number of nodes	40	57	9	3
Probability ($P(k)$)	0.37	0.52	0.08	0.03

5.1 Global connectivity

5.1.1 Percolation threshold

The magnitude of earthquake is described by two variables, i.e. the fraction of nodes not contained in the inner network, p, and the probability that one link is not disrupted in the outer network, f_l. Using Equation (7), we find the percolation threshold of f_l for a fixed p, and the percolation threshold of p for a fixed f_l (Figure 3). The region is divided into two parts. There will exist one giant subnetwork in the post-earthquake network only if the value of (p, f_l) falls above the threshold line.

From two perspectives, we can find that the random failure in the outer network (f_l) have larger impact on percolation threshold than the localized attack in the inner network (p). Firstly, if there is no random failure in the outer network $(f_l = 1)$, the threshold is $p^* = 0.14$. It represents that the giant subnetwork exists even if up to 86% nodes are damaged by localized attacks. On the contrary, if there is no localized attack in the inner network $(p = 1)$, the threshold is ${f_l}^* = 0.34$, which indicates that there will be no giant subnetwork if over 66% links are disrupted in the network. Secondly, it can be verified by the gradients of the threshold line. When p decrease from 1 to 0.5, f_l increase from 0.34 to 0.5, while when f_l decrease from 1 to 0.5, p increase from 0.14 to 0.5. The reason why the pure localized attack have smaller impact is that all the removed nodes are originally connected as a "component". There is a large overlapping among the removed links. After the removal of these nodes, only these links connected to the boundary of this "component" are affected. We can see from the right graph in Figure 1, even if a large proportion of nodes are removed from the centre, the remaining nodes in the outer network can keep connected.

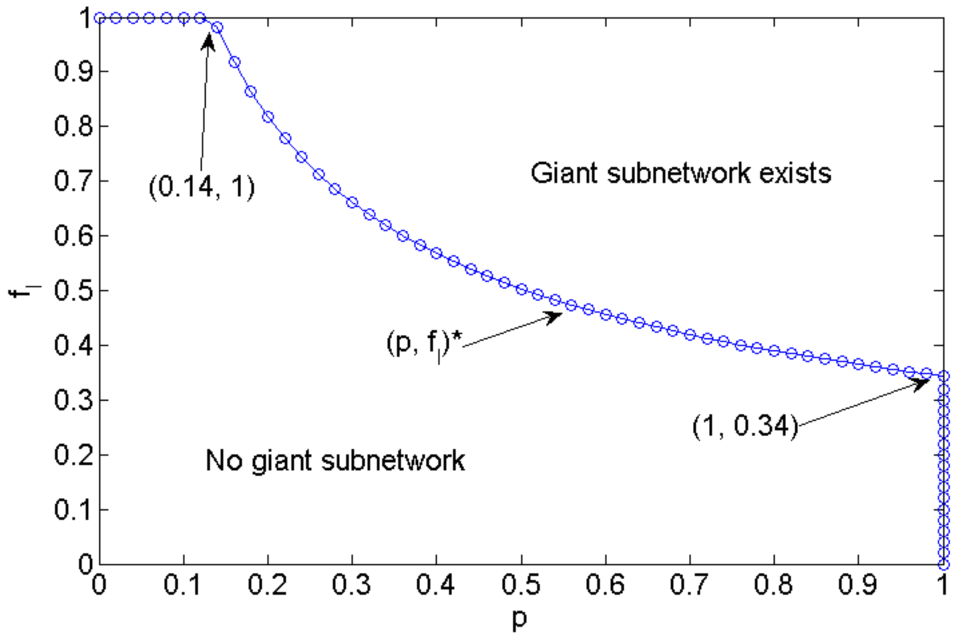

Figure 3. Percolation threshold of $(p,\ f_l)^*$ in the earthquake-impacted road network.

5.1.2 Size of giant subnetwork

Using Equation (8-10), the size of the giant subnetwork in the remaining road network impacted by different magnitudes of earthquakes can be calculated. Figure 4 shows the size of the giant subnetwork with respect to different values of f_l and p. Figure 5 shows the changing trend of the size of the giant subnetwork with respect to different value of f_l under six representative value of p. In Figure 4, we find that, for a fixed f_l, the size of the giant subnetwork approximately increases linearly with p. The value of p determines how many nodes are removed by the localized attack in the inner network. The situation is different when we fix p and consider the impact of f_l, in which case there exists an apparent phase transition. As shown in Figure 5, after the removal of nodes in the inner network, the remaining outer network keeps connected, given that p is not too small. Then a small random removal of links will not significant reduce the size of the giant subnetwork. Taking $p = 0.8$ as an example, after the removal of 20% of nodes from the centre, the remaining outer network still contains 80% of nodes. If we further disrupt 20% of links randomly, the size of the giant subnetwork almost keeps unchanged, which means no nodes are isolated. Next, if f_l decrease from 0.6 to 0.4, there will be a sharp decrease of the size.

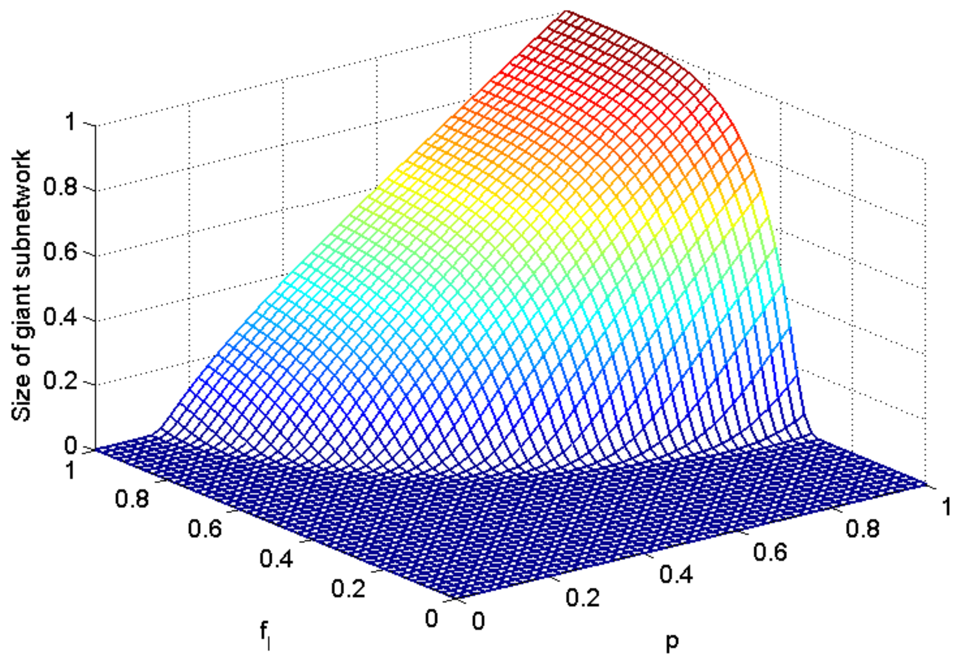

Figure 4. The size of the giant subnetwork with respect to different values of f_l and p in the earthquake-impacted road network.

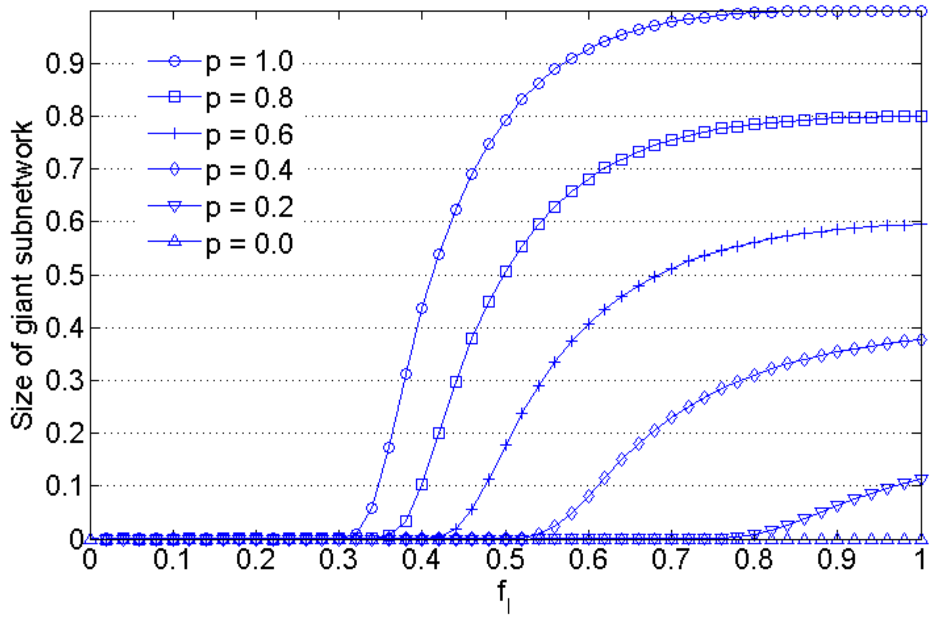

Figure 5. The size of the giant subnetwork with respect to different f_l in the earthquake-impacted road network.

5.1.3 Average size of small subnetworks

After excluding the giant subnetwork, the average size of the small subnetworks in the remaining road network impacted by different magnitudes of earthquakes are illustrated in Figure 6-7. Note that the size of a small subnetwork is represented by the number of nodes contained in it, while the size of the giant subnetwork above is represented by the percentage of nodes in it compared with that in the pre-disaster network. We find that, for a fixed p, the average size of small subnetworks firstly increases with f_l, and then decreases after reaching a maximum value. The maximum value of average size for each fixed p exactly occurs at the percolation threshold. We explain this from the perspective of building the network. Firstly when $f_l = 0$, all the nodes are separated, the average size is 1. Then, with the increase of f_l, several nodes are connected, and the average size increase. This trend continues until the giant subnetwork appears. This giant subnetwork is formed by many small subnetworks and after that, more and more small subnetworks will be included into the giant one. Furthermore, the probability that a small subnetwork will be included into the giant one increases with its size. Therefore, only those very small subnetworks keep isolated. That is why the average size decreases with f_l after the percolation threshold. Finally, when all the nodes are contained in the giant subnetwork, the average size of small subnetworks decrease to 1 when $p < 1$, and decrease to 0 when $p = 1$, where there is no small subnetworks.

We also find that the maximal value of the average size of small subnetworks always happens when f_l takes a medium value. If f_l is too large, the network is well connected. If f_l is too small, the network is mostly divided into separate nodes. Differently, p does not have an significant impact on the average size of small subnetworks. For a fixed f_l, the value of p only affects the proportion of nodes that are removed from the centre, and also the size of the remaining outer network. That is why the maximal value of the average size of small subnetworks increases linearly with p in Figure 6. In Figure 7, the maximal value of the average size of small subnetworks decreases nonlinearly with f_l. When p is small, most nodes are removed from centre, and the remaining outer network is small. These small subnetworks caused by random failures of links will be smaller.

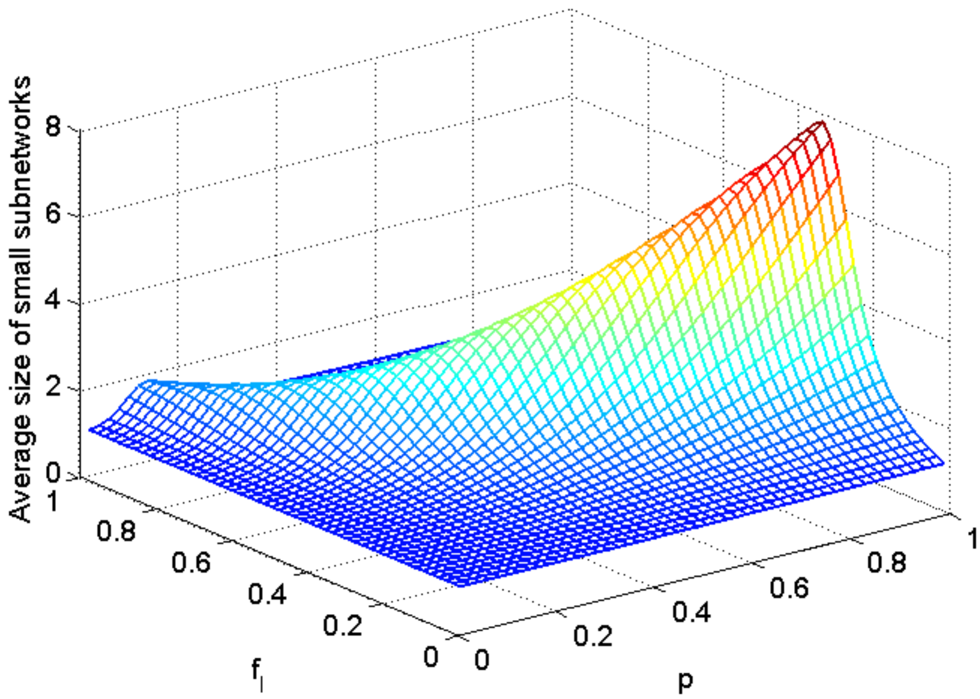

Figure 6. The average size of small subnetworks with respect to different values of f_l and p in the earthquake-impacted road network.

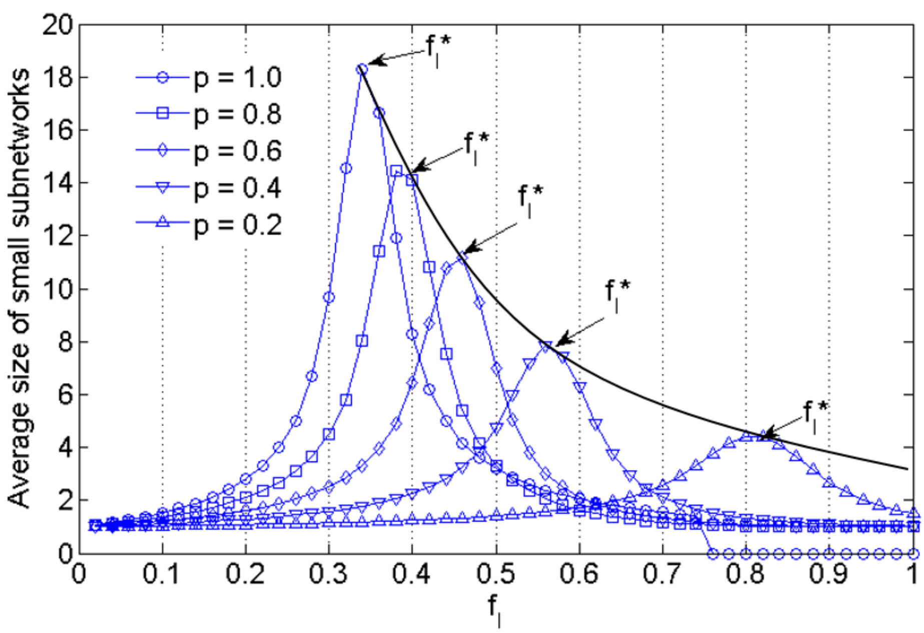

Figure 7. The average size of small subnetworks with respect to different value of f_l in the earthquake-impacted road network.

5.2 Local connectivity

5.2.1 Number of neighbors

In the last section, we compared the impact of different magnitudes of disasters on global connectivity of road networks. Specifically, we find that when a small fraction of links are disrupted, i.e. $0.8 \leq f_l \leq 1$, the global connectivity almost keeps unchanged. However, the distances between node pairs may increase due to the disruption of certain links on their shortest path. We quantify local connectivity by the number of neighbors with different distances.

The probability distribution of the second-nearest neighbors in the original network are provided in Table 3. For example, 0.94% nodes in the network have 6 second-nearest neighbors. No nodes have less than 6 or more than 23 second-nearest neighbors. The probability distributions of the second-nearest neighbors in the remaining network impacted by different magnitudes of natural disasters are shown in Table 3-6. It is obvious that the numbers of neighbors reduce a lot. Taking the case in Table 4 as an example ($f_l = 0.5, p = 1$), over 90% nodes have less than 6 second-nearest neighbors, even though we find that over 80% nodes keep connected as a giant subnetwork from Figure 5. Local connectivity can provide more information that cannot be revealed by global connectivity.

Table 3. Probability distribution of the number of second-nearest neighbors in the original network ($f_l = 1, p = 1$).

Neighbors	**≤5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Probability %	0.00	0.94	5.31	11.36	13.43	14.49	16.50	13.88	9.43	5.93
Neighbors	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**≥24**
Probability %	3.43	2.13	1.36	0.86	0.50	0.26	0.12	0.05	0.02	0.00

Table 4. Probability distribution of the number of second-nearest neighbors ($f_l = 0.5, p = 1$).

Neighbors	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**≥9**
Probability %	13.36	15.89	20.78	18.13	13.66	8.88	4.97	2.50	1.12	0.72

Table 5. Probability distribution of the number of second-nearest neighbors ($f_l = 1, p = 0.5$).

Neighbors	0	1	2	3	4	5	6	7	8	≥9
Probability %	50.99	1.77	4.25	5.99	7.57	8.08	7.15	5.65	3.87	4.68

Table 6. Probability distribution of the number of second-nearest neighbors ($f_l = 0.7, p = 0.7$).

Neighbors	0	1	2	3	4	5	6	7	8	≥9
Probability %	34.81	6.94	11.56	12.37	11.52	9.18	6.21	3.75	2.01	1.64

Next, we further investigate the impact of disasters on local connectivity. The average number of first-, second-, third-, and fourth- nearest neighbors in the remaining road networks impacted by different magnitudes of earthquakes are illustrated in Figure 8. Both localized attacks in the inner network and random failures in the outer network have significant effects on the average number of neighbors. Furthermore, this impact increases with the distances. The number of the first-nearest neighbors approximately changes linearly with f_l for a fixed p. While for the fourth-nearest neighbors, if f_l for a fixed p decreases from 100% to 50%, the number of neighbors almost decreases to 10%.

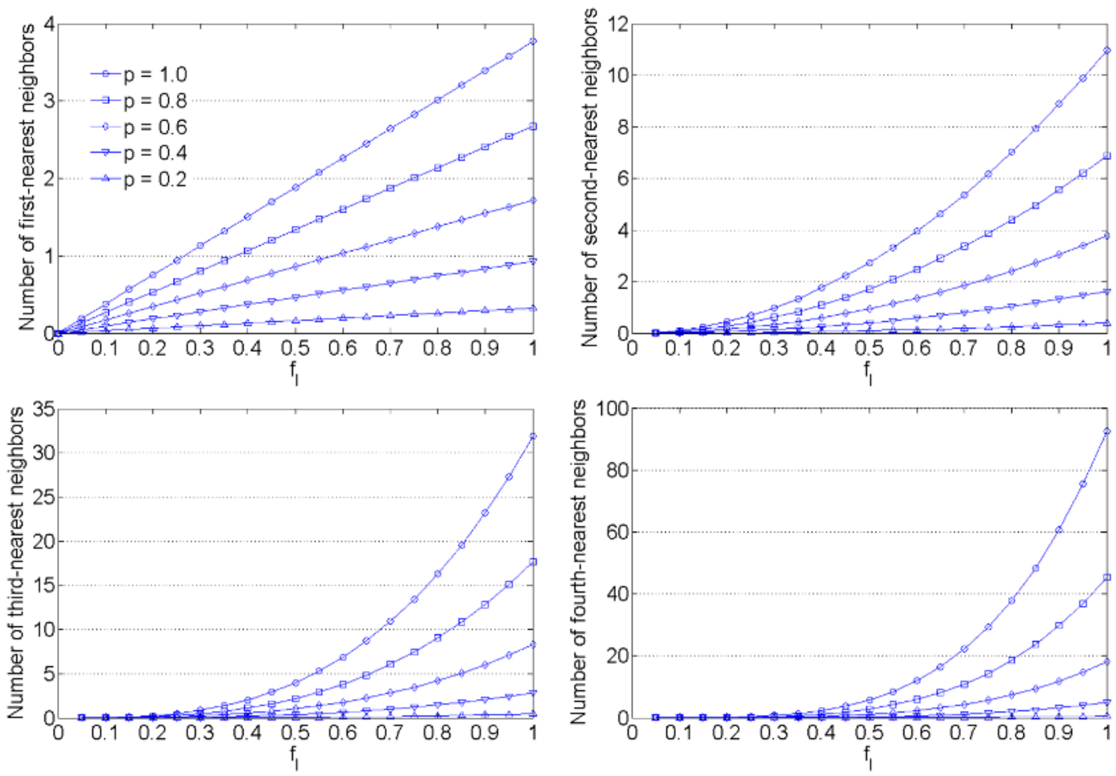

Figure 8. Average number of first-, second-, third-, and fourth- nearest neighbors with respect to

different f_l in localized natural disasters.

5.2.2 Integrated local connectivity: efficiency

To bring number of neighbors with different distances into one single index for the whole network, the efficiency of the road network impacted by different magnitudes of earthquakes are calculate and illustrate in Figure 9. Different from the situation in global connectivity, localized attacks in the inner network and random failures in the outer network have similar impacts on local connectivity. Figure 10 provides a comparison between global and local connectivity with respect to different value of f_l under several representative value of p. We can find that when f_l is large, local connectivity is more sensitive than global connectivity. Even though the disruption of a small proportion of nodes will not affect global connectivity, it truly reduces local connectivity. Global connectivity and local connectivity decrease to 0 at the same point when the giant component disappears.

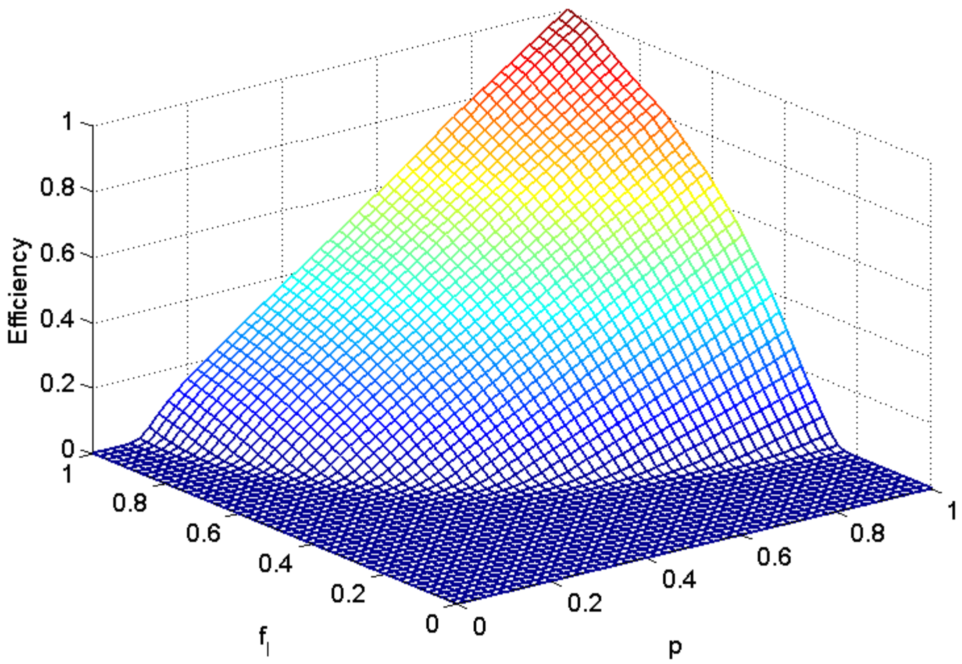

Figure 9. Efficiency of the earthquake-impacted road network with respect to different values of f_l and p.

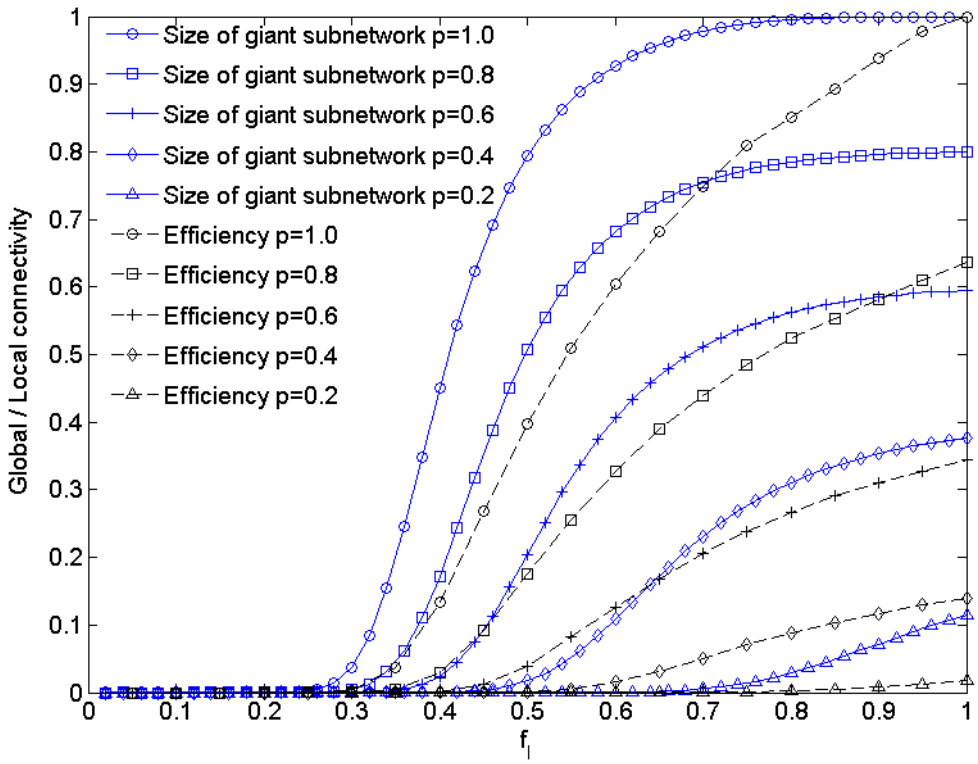

Figure 10. Comparison between global connectivity and local connectivity of the earthquake-impacted road network with respect to different value of f_l.

5.3 Simulation

To validate the proposed method, we conduct simulations on the same road network. The road network is built in Matlab and nodes and links are randomly broken according to the values of f_l and p. For each case, the simulation is repeated for 100 times and the mean value of the size of the giant subnetwork is obtained, which is further compared with the results from the proposed percolation method (Figure 11). We first look at the comparison of global connectivity, and find that the value of global connectivity by percolation and that by simulation fits well when $f_l \geq 0.6$. When $0.4 < f_l < 0.6$, the theoretical value of global connectivity is a little larger than the practical value in the first two subfigures. The reason is that percolation process is based on infinite networks, while the examined road network is too small. The existence of the boundary of the network will make the size of the giant subnetwork smaller than the theoretical value. When $f_l \leq 0.4$, the theoretical value of global connectivity is a little smaller than the practical value. In an infinite network, if the giant subnetwork contains finite number of nodes, than the global connectivity of this network will be regarded as 0. However, in practical networks, even if a large quantity of links are removed, the giant subnetwork still contains several nodes, and that is why the value of global connectivity by simulation will not decrease to 0 as we see from the results by percolation.

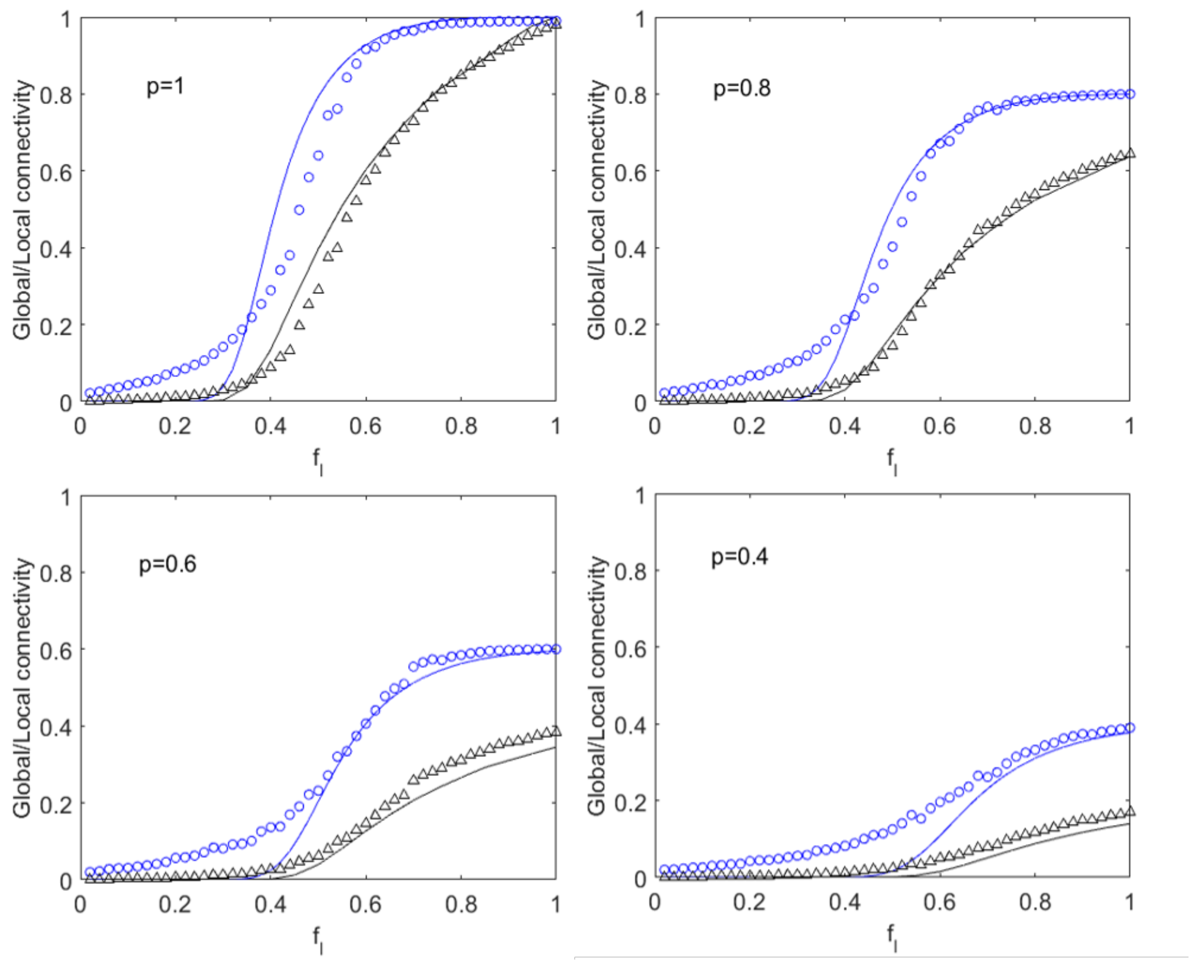

Figure 11. Comparison of theoretical and practical results with four different values of *p*. The solid lines are the values obtained using our method, and the points are results from simulations. The blue lines/points represent global connectivity, and the black lines/points represent integrated local connectivity.

Figure 11 also shows the comparison of theoretical and practical results of local connectivity. We know that the existence of cycles in the road network will result into some overlap when calculating the number of neighbors. It means that the value of local connectivity obtained by our method will be larger than the practical value. However, we find in Figure 11 that after normalizing local connectivity, the practical value and theoretical value fit well. It indicates that our propose method cannot provide the accurate value of local connectivity, but can precisely evaluate the changing trend of local connectivity when facing different magnitude of earthquake.

5.4 Computational complexity and the impact of network sizes

In this method, the structure of road network is described by the generating function of node degree distribution. As shown in Table 1, all of the metrics have closed-form expressions except subnetwork sizes distribution, which can only be obtained by iterative computing. Theoretically, to get the probability distribution of subnetwork sizes distribution of a network with n nodes, n+1 iterations are needed. That is to say, for a network with 1000 nodes, equation (10) has to be iteratively calculated by 1000 times. This can be completed within 1 second in Matlab R2013a on an Intel 2.8GHz PC with 4 GB RAM running Windows 7. For a network consisting of 10,000 nodes and 40,000 links, it take 68 seconds to calculate the subnetwork sizes distribution. Since percolation theory is developed on infinite networks, the accuracy of this method increases with the size of networks.

6 Conclusions

Connectivity of road networks in post-earthquake stage is vital for evacuation and relief distribution. This paper first proposed an analytical method for analyzing the connectivity of road networks when being impacted by different magnitudes of earthquakes. Previous methods in the literature can only assess the connectivity of road networks with determined structures, and cannot analytically study the change of network connectivity with respect to different magnitudes of earthquakes. Therefore, we introduced percolation theory to study the connectivity of post-earthquake road networks.

In post-earthquake road networks, the probability of component disruption decreases with its distance from the epicenter. With consideration of this feature, we propose a new percolation process which integrates the traditional localized attack and random failure. The post-earthquake network is divided into two parts: inner network and outer network. The inner network is affected by a localized attack, and the outer network is affected by a random failure.

We proposed the concepts of global connectivity and local connectivity for post-earthquake road networks and chose from the literature the most appropriate metrics to measure them. Global connectivity measures the extent to which the whole network is connected, quantified by the size of giant subnetwork and average size of small subnetworks, and local connectivity measures the distances between each node to the other nodes, quantified by number of neighbors. Furthermore, to bring the number of neighbors with different distances together into one single index for the whole network, we employed the concept of efficiency to combine them into one metric for the local connectivity of the whole network.

The application to a real-world road network shows that this method can assess the connectivity of road networks impacted by different magnitudes of earthquakes from both global and local perspectives, in a systematic and efficient way.

There are several limitations of this method. The first one is that cycles in the network are neglected when using generating function. Although the simulation in section 5.3 shows that cycles do not have large impact on the calculation of global connectivity, they truly affect the assessment of local connectivity. Due to the existence of cycles, there would be some overlapping when calculating the number of neighbors. We also find that the impact of cycles reduces with the increase of the magnitude of earthquake, as the road network becomes more and more sparse. One of the future work will focus on eliminating the impact of cycles on the assessment of network connectivity. The second limitation is that since percolation theory is developed on infinite networks, there could be some differences between the theoretical and practical results when the road network is too small. For the network used in this study which consists of 109 nodes, the method works well. In spite of these limitations, we still think the proposed framework is an alternative way to study the connectivity of post-earthquake road networks and it sheds light on the application of percolation to more practical networks with consideration of their features.

More characteristics of road networks, such as the dynamic traffic demand, and continuously degradable road capacities, could be considered in the model to make it applicable for analyzing more practical issues.

Appendix I

Lemma A1 Given the generating function of the original network is $G_0(x) = \sum_{k=0}^{\infty} P(k) \cdot x^k$, and the probability of being disrupted for each link in the network is $1 - f_l$, the generating function of the remaining network is $G_0^{f_l}(x) = G_0(1 - f_l + f_l x)$.

Proof Because the probability that a randomly chosen link is disrupted is $1 - f_l$, the probability that a randomly chosen node has degree k' will be

$$P(k') = \sum_{k=k'}^{\infty} P(k) \binom{k}{k'} {f_l}^{k'} (1 - f_l)^{k-k'}.$$

Thus, the generating function will be

$$\begin{aligned} G_0^{f_l}(x) = \sum_{k'=0}^{\infty} P(k') x^k &= \sum_{k'=0}^{\infty} \sum_{k=k'}^{\infty} P(k) \binom{k}{k'} {f_l}^{k'} (1 - f_l)^{k-k'} x^{k'} \\ &= \sum_{k=k'}^{\infty} P(k) \sum_{k'=0}^{\infty} \binom{k}{k'} {f_l}^{k'} (1 - f_l)^{k-k'} x^{k'} \\ &= \sum_{k=k'}^{\infty} P(k) \sum_{k'=0}^{\infty} \binom{k}{k'} (x f_l)^{k'} (1 - f_l)^{k-k'} \\ &= \sum_{k=k'}^{\infty} P(k) (1 - f_l + f_l x)^k \\ &= G_0(1 - f_l + f_l x). \end{aligned}$$

QED.

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