

# When Is Upstream Collusion Profitable?

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## Abstract

Motivated by the recent antitrust cases in which Japanese auto parts suppliers colluded to raise supply prices against their long-term collaborators, the Japanese carmakers, we study the conditions under which an upstream collusion is profitable even after compensating downstream direct purchasers. Oligopoly competition in successive industries is shown to give rise to a vertical externality and a horizontal externality. If a collusive price of intermediate goods better balances the two externalities, the collusion will raise the joint profit of all firms in the two industries and is therefore profitable for the upstream after compensation of downstream firms. We further link the profitability of such collusions to industry characteristics in terms of demand, cost, and competition intensity.

**Keywords:** upstream collusion, successive oligopoly, vertical externality, horizontal externality

**JEL Code:** L1, L2, L4, D43

## 1 Introduction

In 2008 the United States Department of Justice (DOJ) and Federal Bureau of Investigation (FBI) started investigating automobile parts suppliers for anticompetitive conduct in the US. The perpetrators were mostly Japanese firms including Furukawa, Yazaki, Denso, Mitsubishi, Sumitomo, Aisan, etc. They were accused of price fixing and bid rigging for more than a decade, from year 2000 to 2011, in the sales of automotive wire harnesses, bearing, air conditioning systems, windshield washer and wiper systems, compressors and condensers,

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radiators, seatbelts, etc.<sup>1</sup> By March 2017, 48 companies and 65 individuals had been charged; all had pleaded guilty and agreed to pay a combined fine of \$2.9 billion (DOJ, 2017). Alerted by the US investigation, antitrust authorities in Japan, the European Union, Canada, South Korea, Mexico, Australia, and China subsequently carried out their own investigations of Japanese suppliers operating in their countries. All companies pleaded guilty to the respective jurisdictions and paid various amounts of fines (DOJ, 2013; DG Competition, 2013; NDRC, 2014).

What's puzzling about this antitrust case is that the direct victims were mostly Japanese carmakers including Nissan, Toyota, Honda, Mazda, Mitsubishi, and Fuji Heavy Industries. Japanese companies, especially those in the automobile industry, are well known for maintaining long-term relationship and close collaboration with their suppliers. Many observers believe that such tight cooperation contributed greatly to the success of Japanese automobile industry, as close collaboration brings many benefits in the form of reduced time of new model development, shared cost of innovation, quick response to fluctuations in market demand, and so on (Ahmadjian and Lincoln, 2001). Why would the suppliers conspire against their long-term customers? Why did the Japanese carmakers not report or complain about the price fixing and bid rigging? Given that the victims were global heavyweights such as Toyota, Honda, and Nissan, it is hard to imagine that the carmakers were ignorant of the conspiracy or lacked the power to strike back. Then why had the practice persisted for so long?

We suggest one answer: that the downstream carmakers may have received compensations from these suppliers for the elevated supply prices. In fact, side payments between collusive suppliers and their direct purchasers are quite common, as discussed by Schinkel et al. (2008). The fundamental question then becomes: if downstream firms must be compensated for any damage arising from collusions among their suppliers, would these suppliers even want

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<sup>1</sup>Executives of the companies held regular meetings and telephone communications in the United States and Japan to rig bids, set prices, and allocate supplies. They used code names and met in remote locations to keep secrecy, and had measures to monitor and enforce the collusive agreements (DOJ, 2013). Most of the involved parts are standard products and were used by carmakers in a modular way. Although the costs of these parts accounted for only about 5-8% in the final prices of the automobiles, they were needed frequently in large quantities, which facilitated collusion and caused substantial damages to the carmakers.

to collude in the first place? This question is of general importance. In real life, many cartels exist in supply chains in which the buyers are intermediate firms who have the power, incentive, and information to report the cartel or sue its members, so it is reasonable to assume that at least some cartels must compensate their direct purchasers. Standard cartel theory, however, has focused almost exclusively on the incentive compatibility issue, i.e., a cartel must prevent its members from deviating from the collusive agreement. As the compensation and the incentive-compatibility constraints are orthogonal in nature, they are likely to lead to different cartel behavior and consequences. To better predict cartel incidence and inform antitrust policies, therefore, it is important to fully understand the implications of the compensation constraint in cartel practices.

In this article, we study the conditions under which upstream collusion is profitable even after compensating the downstream firms. Given the feasibility of many forms of side payments across vertical business partners, the task pins down to identifying the conditions under which an upstream collusion raises the joint profit of all firms in the two industries. Using a standard, general, two-stage model of successive oligopoly, we show that firms' equilibrium choices give rise to two externalities: a vertical externality by which a downstream firm ignores the benefits that its output quantity brings to upstream firms, and a horizontal externality by which a downstream firm ignores the damages that its output brings to other downstream firms. The vertical externality makes firms produce too little as compared to the monopoly level, whereas the horizontal externality makes them produce too much. If the horizontal externality dominates, firms over-produce in the oligopoly equilibrium. By raising the input price and therefore reducing the total output, then, an upstream collusion moves the outcome closer to the monopoly level and thereby raises the joint profit. We show that such collusions always reduce consumer surplus and social welfare.

The analytical framework helps us identify a number of parameters that affect the profitability of such upstream collusion. First, collusion is more likely to be profitable if either industry has more firms, or if the structure of the two industries is more balanced. Second, a smaller product differentiation or demand concavity for the final products are conducive to

upstream collusion. Third, upstream collusion is hindered by greater cost convexity in the downstream, but not affected by the cost structure in the upstream. These findings can inform antitrust authorities. Collusion in a supply chain is hard to uncover, as the real victims (i.e., consumers) and the conspirators (i.e., the upstream firms) are separated by legitimate and innocuous businesses (i.e., the downstream firms), which cannot be relied upon to report any wrongdoing.<sup>2</sup> As such, the above characterizations can help alert the antitrust authority. For example, a supply chain should be flagged if the downstream demand is inelastic, product differentiation is small, the vertical market structure is more or less balanced, and the downstream capacity constraint is relaxed.

Most studies of upstream collusion focus on the incentive issue, i.e., a cartel needs to ensure that no member wants to break away from the collusive agreement (Choe and Matsushima, 2013; Jullien and Rey, 2007; Piccolo and Reisinger, 2011; and Nocke and White, 2007; 2010). We, on the contrary, address the profitability issue of collusion with compensation consideration, setting aside the usual incentive constraint. The different angle that we take brings some new insights. For example, we find that a larger number of colluding firms (in the upstream) is conducive to profitable collusion. This is the opposite of the finding in traditional cartel theory, which suggests that a larger number of firms makes collusion more difficult. The two conclusions differ because they express different concerns (compensating downstream firms versus preventing deviations). If these two constraints are considered simultaneously, the relationship between cartel incidence and market concentration is likely to be non-monotonic, as suggested by the inverted U-shape in Symeonidis's (2003) empirical study.

Schinkel et al. (2008) also studied upstream collusion that must compensate downstream firms. They argue that the US Supreme Court's ruling that allowed only direct purchasers to claim antitrust damages would facilitate upstream collusion, as an upstream cartel can share the collusive benefits with its direct purchasers to avoid litigations. Although adequate compensation of direct purchasers is necessary for upstream cartel formation in Schinkel et al.'s

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<sup>2</sup>Citing *Illinois Brick* and similar practices, Schinkel et al. (2008, p.695) have concluded: "One of the lessons drawn by the U.S. Department of Justice's Antitrust Division from these and other big price-fixing cases was that large sophisticated buyers could not be relied upon to 'defeat cartel activity.'"

(2008) model, these authors simply assume that this necessary condition is satisfied. They therefore focus on the traditional condition of cartel stability among upstream members. In contrast, we assume that the cartel is stable (imagine a standard dynamic model in which detection is immediate and the cartel breaks down permanently whenever cheating is detected), and focus instead on the profitability problem. As our analysis demonstrates, higher prices do not always raise the joint profit of all upstream and downstream firms; a cartel may well be unprofitable after the necessary compensation of downstream firms. The profitability conditions must be taken seriously, as they are likely to affect conclusions established elsewhere when cartel members' incentive-compatibility was the only concern.

The analytical framework of vertical and horizontal externalities is very general and can be applied to various vertical conducts.<sup>3</sup> Toward the end of the article we illustrate the usefulness of such approach by applying it to the incentive of vertical disintegration (Bonanno and Vickers, 1988; Lin, 1988). In a series of studies, Winter and his coauthors (Mathewson and Winter, 1984; Winter, 1993; Krishnan and Winter, 2007) have used vertical and horizontal externalities to analyze vertical control. Their models typically assume a monopoly supplier who can extract all of the surplus, so the major purpose of these studies is to explain whether a particular vertical conduct can achieve the monopoly profit. By contrast, our upstream industry is oligopoly, and we focus on the conditions under which upstream collusion is profitable given that the direct downstream has to be compensated.

The rest of the article is organized as follows. After setting up the model in section 2, we analyze the equilibrium choices in section 3 and identify the condition under which an upstream collusion is profitable after paying out to downstream firms. Section 4 extends the model to different settings to show that the major finding and its driving force are general and robust. Section 5 concludes.

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<sup>3</sup>For example, it can shed light on retail price maintenance (Rey and Tirole, 1986), quantity constraint (Schinkel et al., 2008), exclusive dealing (Lin, 1990; O'Brien and Shaffer, 1993), exclusive territories (Rey and Stiglitz, 1988), etc.

## 2 Model

Consider two vertically related industries with  $m$  ( $m \geq 2$ ) identical upstream firms and  $n$  ( $n \geq 1$ ) identical downstream firm(s). A homogeneous input is produced by upstream firms at constant marginal cost,  $f$ , and sold at a uniform price to downstream firms, which then transform the input into a homogeneous final product at constant marginal cost,  $c$ , on a one-for-one basis. Let  $p = p(Q)$  be the inverse demand for the final product where  $p$  is the price and  $Q$  is the total quantity of sales.

Firms in both industries compete à la Cournot, and the two industries interact through an endogenous demand for input, which is determined in the following way. For any given input price,  $t$ , downstream Cournot competition results in an equilibrium output quantity for each downstream firm and hence a total quantity,  $Q$ . The relationship between  $t$  and  $Q$  thus derived,  $t(Q)$ , is regarded by upstream firms as the demand function for the input. More specifically, the successive oligopoly game is played as follows (Lewis et al., 1986; Salinger, 1988). All upstream firms simultaneously choose their quantities with the understanding that the resulting input price is determined from  $t = t(\sum_{j=1}^m q_j^u)$ , where  $q_j^u$  is the quantity produced by upstream firm  $j$ ,  $j \in \{1, 2, \dots, m\}$ . The profit of firm  $j$  is therefore  $\pi_j^u = q_j^u [t(\sum_{k=1}^m q_k^u) - f]$ . Given any input price,  $t$ , downstream firms simultaneously choose their quantities,  $q_i^d$  for downstream firm  $i$ ,  $i \in \{1, 2, \dots, n\}$ , and the profit of firm  $i$  is given by  $\pi_i^d = q_i^d [p(\sum_{k=1}^n q_k^d) - c - t]$ . In equilibrium,  $\sum_{j=1}^m q_j^u = \sum_{i=1}^n q_i^d = Q$ .

The demand function  $p(Q)$  is assumed to be such that  $p(Q) > 0$  and  $p'(Q) < 0$  for any  $Q > 0$ , and  $\lim_{Q \rightarrow \infty} p(Q) = 0$ . In addition, to ensure that the successive oligopoly equilibrium behaves properly, we assume that  $p(Q)$  satisfies the following two properties for any  $Q > 0$ :

**A1.**  $p''(Q)Q + p'(Q) < 0$ ;

**A2.**  $p'''(Q)Q^2 + (n+3)p''(Q)Q + (n+1)p'(Q) < 0$ .

A1 implies that output quantities of downstream firms are strategic substitutes, and each firm's profit is strictly concave in its quantity choice, whereas A2 implies similar properties for the upstream industry given that the input demand is derived from downstream competition.

A1 is common in Cournot competition (Novshek, 1985; Nocke and Whinston, 2009), and A2 is a counterpart of A1 and is implicitly assumed in the literature of successive oligopoly (Greenhut and Ohta, 1979; Salinger, 1988). Under these two assumptions, there exists a unique Nash equilibrium in quantities in each industry, and the equilibrium is stable.

Suppose that the successive oligopoly is originally at equilibrium with total output  $Q^*$  and input price  $t^* = t(Q^*)$ , and consider the following collusion game. All the upstream firms decide whether or not to collude. A collusion is a collective, binding decision by the upstream firms to charge a different input price  $\tilde{t} \neq t^*$ .<sup>4</sup> If they do so, the collusive input price  $\tilde{t}$  will result in a corresponding  $\tilde{Q}$  that still satisfies the input demand function, i.e.,  $\tilde{t} = t(\tilde{Q})$ . Alternatively and equivalently, we may conceptualize collusion as producing the input at total quantity  $\tilde{Q}$  rather than the equilibrium  $Q^*$  (so each upstream firm produces  $\frac{\tilde{Q}}{m}$  rather than the equilibrium  $\frac{Q^*}{m}$ ). An upstream firm's profit in the collusive outcome is then  $\tilde{\pi}^u = (\tilde{t} - f)\frac{\tilde{Q}}{m}$ , whereas a downstream firm  $i$ 's profit is  $\tilde{\pi}_i^d = q_i^d [p(\sum_{k=1}^n q_k^d) - c - \tilde{t}]$  when it produces  $q_i^d$ .

Unlike most studies of upstream collusions where the major concern is individual members' deviation from the collusive agreement, here the constraint is that the collusion cannot hurt downstream firms. In other words, the downstream firms must be compensated for any losses arising from the upstream collusion. The question is, would an upstream collusion still be profitable after compensating the downstream? We will focus on this question in the analysis, and ignore a related question of exactly how the compensation is done, as it is straightforward and has been studied in the literature in various forms, such as passive ownership (Hunold and Stahl, 2016) or transfer pricing with rationing (Schinkel et al., 2008). Consequently, our task is to identify conditions under which an upstream collusion raises the joint profits of the upstream and downstream industries.

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<sup>4</sup>The collusion can be implemented in a non-cooperative, repeated game such that no firm has any incentive to deviate as long as the collusion raises each colluding firm's profit and the discount factor is sufficiently large.

### 3 Analysis

#### 3.1 Demand for input

For any given input price  $t$ , a downstream firm  $i$  chooses its output quantity  $q_i^d$  to maximize its profit  $\pi_i^d$ . In equilibrium, all downstream firms choose the same quantity  $q_i^d = \frac{Q}{n}$ , so the first-order condition,  $\frac{\partial \pi_i^d}{\partial q_i^d} = 0$ , leads to:

$$t(Q) = p'(Q) \frac{Q}{n} + p(Q) - c, \quad (1)$$

which defines the endogenous demand function for input.<sup>5</sup> Note that  $t'(Q) = p''(Q) \frac{Q}{n} + p'(Q) \frac{n+1}{n}$ , then A1 implies that  $t'(Q) < 0$ , or equivalently  $Q'(t) < 0$ , meaning that there is a one-to-one mapping between  $t$  and  $Q$ , and that a higher input price always results in a smaller total production quantity.

The input demand function,  $t(Q)$ , is valid in both oligopoly competition and collusion. In what follows, it will be convenient to treat  $Q$  as the instrument of collusion rather than  $t$ . It is also useful to define the downstream markup as  $\sigma(Q) \equiv p(Q) - t(Q) - c$ . Given (1),  $\sigma(Q) = -p'(Q) \frac{Q}{n} > 0$ . Then  $\sigma'(Q) = -\frac{1}{n}[p''(Q)Q + p'(Q)] > 0$ . Given the inverse relationship between  $t$  and  $Q$ ,  $\sigma'(Q) > 0$  is equivalent to  $\sigma'(t) < 0$ . That is, when the input price is higher, the downstream markup becomes smaller. This is because A1 and A2 of the demand function imply cost absorption, i.e., any increase in the input price will be absorbed by downstream firms, albeit only partially.

**Lemma 1** *When the input price increases, the industrial total output ( $Q$ ) and the downstream markup ( $\sigma$ ) both decrease.*

#### 3.2 Successive oligopoly

The successive oligopoly equilibrium is solved by backward induction. The downstream competition has been derived above and captured by equation (1). Facing such demand

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<sup>5</sup>The second derivative is  $\frac{\partial^2 \pi_i^d}{\partial (q_i^d)^2} = p''(Q)q_i^d + 2p'(Q)$ . If  $p''(Q) < 0$ , then  $p''(Q)q_i^d + 2p'(Q) < 0$ ; if  $p''(Q) \geq 0$ , then  $p''(Q)q_i^d + 2p'(Q) \leq p''(Q)Q + 2p'(Q) < p''(Q)Q + p'(Q) < 0$ , where the last inequality is due to A1. In subsequent analysis, the second order conditions are always satisfied given A1 and A2, and are therefore omitted to save space.



for input, upstream firm  $j$  chooses quantity  $q_j^u$  to maximize its profit  $\pi_j^u$ . The first-order condition,  $\frac{\partial \pi_j^u}{\partial q_j^u} = 0$ , then leads to:

$$\left[ p'(Q) \frac{Q}{n} + p(Q) - c - f \right] + \left[ \frac{n+1}{n} p'(Q) + \frac{Q}{n} p''(Q) \right] q_j^u = 0.$$

In equilibrium, each upstream firm produces the same amount:  $q_j^u = \frac{Q}{m}$ , so we have

$$\left[ (m+n+1-nm)p'(Q^*) + p''(Q^*)Q^* \right] \frac{Q^*}{nm} + [p'(Q^*)Q^* + p(Q^*)] = c + f. \quad (2)$$

By A1 and A2, the equilibrium quantity,  $Q^*$ , is determined uniquely by (2). Given  $Q^*$ , the equilibrium values of all the other variables can then be uniquely determined.

### 3.3 Joint-profit maximization

Consider the two industries' total profits as a function of the total quantity,  $Q$ :

$$\Pi(Q) = [p(Q) - c - f] Q,$$

which is maximized at  $\frac{\partial \Pi(Q)}{\partial Q} = 0$ , or

$$p'(Q^{**})Q^{**} + p(Q^{**}) = c + f. \quad (3)$$

(3) defines a unique quantity,  $Q^{**}$ , referred to hereafter as the *monopoly quantity*, which maximizes the two industries' joint profits. The left hand side of (3) is the marginal revenue from all firms in the two industries, and the right hand side is the (joint) marginal cost.

### 3.4 Vertical and horizontal externalities

Compare (2) with (3). The two equations differ in that (2) has an extra term:

$$NE \equiv \left[ (m+n+1-nm)p'(Q^*) + p''(Q^*)Q^* \right] \frac{Q^*}{nm}. \quad (4)$$

This term captures the extra effect due to competition and is referred to as the *net externality* (hence the notation  $NE$ ). If  $NE > 0$ , we will have  $p'(Q^*)Q^* + p(Q^*) < c + f = p'(Q^{**})Q^{**} + p(Q^{**})$ , which in turns implies  $Q^* > Q^{**}$  given that  $\Pi(Q)$  is concave in  $Q$  by A1. That is a

situation in which firms over-produce in successive oligopoly as compared to the monopoly quantity. If  $NE < 0$ , firms under-produce, leading to  $Q^* < Q^{**}$ .

**Lemma 2** *The equilibrium total output in successive oligopoly ( $Q^*$ ) is greater than the monopoly output ( $Q^{**}$ ) if and only if  $NE > 0$ .*

To understand how the competitive outcome differs from the monopoly outcome, we show below that  $NE$  captures the discrepancy between the individual and the collective incentives, and can be further decomposed into two externalities:<sup>6</sup>

$$NE = \frac{\partial \pi_i^d}{\partial q_i^d} - \frac{\partial \Pi(Q)}{\partial q_i^d} = \underbrace{-(t^* - f)}_{\substack{\text{Vertical externality} \\ (-)}} + \underbrace{\left[ -\frac{n-1}{n} p'(Q^*) Q^* \right]}_{\substack{\text{Horizontal externality} \\ (+)}}. \quad (5)$$

When choosing its output quantity, an individual downstream firm ignores two effects on the profits of other firms in the two industries, as captured by the two terms on the right hand side of equation (5). The first term,  $-(t^* - f)$ , comes from  $-\frac{\partial(\sum_{j=1}^m \pi_j^u)}{\partial q_i^d} = -\frac{\partial[(t^* - f)Q]}{\partial Q} \frac{\partial Q}{\partial q_i^d} = -(t^* - f)$ . It measures how a downstream firm's output affects the total upstream profits, and is therefore referred to as a *vertical externality*. As upstream firms enjoy a positive markup (i.e.,  $t^* > f$ ) in equilibrium, an additional unit of downstream sales would have benefited upstream firms, but downstream firms ignore such benefits, so they produce too little from the viewpoint of all firms' collective interest. The second term in (5),  $-\frac{n-1}{n} p'(Q^*) Q^*$ , comes from  $-\frac{\partial(\sum_{k \neq i}^n \pi_k^d)}{\partial q_i^d} = -\frac{n-1}{n} Q^* \frac{\partial[p(Q) - t^* - c]}{\partial Q} \frac{\partial Q}{\partial q_i^d} = -\frac{n-1}{n} p'(Q^*) Q^*$ . It measures how an individual downstream firm's output affects the profits of all other downstream firms, and is thereafter referred to as a *horizontal externality*. Because a downstream firm's output hurts

<sup>6</sup>In successive oligopoly, a downstream firm's individual incentive is captured by:

$$\begin{aligned} \frac{\partial \pi_i^d}{\partial q_i^d} &= \frac{\partial \pi_i^d(Q)}{\partial Q} \frac{\partial Q}{\partial q_i^d} = p'(Q) \frac{Q}{n} + p(Q) - c - t \\ &= [p'(Q)Q + p(Q) - c - f] + (f - t) - \frac{n-1}{n} p'(Q)Q \\ &= \frac{\partial \Pi(Q)}{\partial q_i^d} + (f - t) + \left[ -\frac{n-1}{n} p'(Q)Q \right], \end{aligned}$$

where  $\frac{\partial \Pi(Q)}{\partial q_i^d} = \frac{\partial \Pi(Q)}{\partial Q} \frac{\partial Q}{\partial q_i^d} \equiv p'(Q)Q + p(Q) - c - f$  represents the collective incentive, i.e., how this downstream firm's output affects the upstream and downstream firms' joint profit.

its competitors but it ignores such damage, it tends to produce too much from the viewpoint of collective interest.

In sum, competition in successive oligopoly results in a vertical externality and a horizontal externality.<sup>7</sup> From the perspective of joint profits, vertical externality tends to reduce the total output whereas horizontal externality tends to increase it. The firms' joint profit is maximized if the two externalities exactly cancel out. If the horizontal externality dominates, i.e., if  $NE > 0$ , firms produce too much to the detriment of their collective interest.

### 3.5 The upstream, downstream, and joint profits

We now look at how a change in  $Q$  affects the profits in the two industries. Because  $t(Q)$  decreases in  $Q$  (Lemma 1), there exists a  $Q^{max}$  such that  $t(Q^{max}) = f$ , at which point the upstream profit is zero, so the range of permissible  $Q$  is between 0 and  $Q^{max}$ .

The downstream total profit is  $\Pi^d(Q) = \sigma(Q)Q$ , where  $\sigma(Q)$  is the downstream markup as defined earlier. Then  $\frac{\partial \Pi^d(Q)}{\partial Q} = \sigma'(Q)Q + \sigma(Q)$ . By Lemma 1,  $\sigma'(Q) > 0$ , so  $\frac{\partial \Pi^d(Q)}{\partial Q} > 0$ , meaning that the downstream total profit increases with industry total output. This is because a larger total output indicates a smaller input price, which in turn implies a greater downstream markup (Lemma 1). Greater total output and greater markup reinforce each other to lead to a greater total downstream profit. Therefore,  $\Pi^d$  is a monotonically increasing function of  $Q$ , reaching the largest level at  $Q^{max}$ .

The upstream firms' total profit,  $\Pi^u(Q) = (t - f)Q$ , is maximized when

$$\frac{Q}{n} [p''(Q)Q + 2p'(Q)] + [p'(Q)Q + p(Q)] = c + f. \quad (6)$$

Given A2,  $\Pi^u(Q)$  is concave, so (6) has a unique solution, which is denoted as  $Q^u$ . Because

<sup>7</sup>The decomposition is from the perspective of downstream firms. A similar and equivalent decomposition can be carried out from the angle of upstream firms:

$$NE = \frac{\partial \pi_j^u}{\partial q_j^u} - \frac{\partial \Pi(Q)}{\partial q_j^u} = \underbrace{-\frac{m-1}{mn} Q^* [p''(Q^*)Q^* + (n+1)p'(Q^*)]}_{\substack{\text{Horizontal externality} \\ (+)}} + \underbrace{\frac{Q^*}{n} [p''(Q^*)Q^* + 2p'(Q^*)]}_{\substack{\text{Vertical externality} \\ (-)}}$$

$p''(Q)Q + 2p'(Q) < 0$ , a comparison between (6) and (3) reveals:

$$Q^u < Q^{**}. \quad (7)$$

That is, the upstream total profit is maximized at an output level smaller than the monopoly quantity. This is because a larger  $Q$  benefits downstream firms, and the downstream gain outweighs the upstream loss if the current output is  $Q^u$ , at which the upstream profit changes by a secondary order. For a similar reason, after comparing (6) with (2), we have

$$Q^u < Q^*. \quad (8)$$

That is, the upstream firms would collectively prefer an output level that is smaller than the equilibrium level.

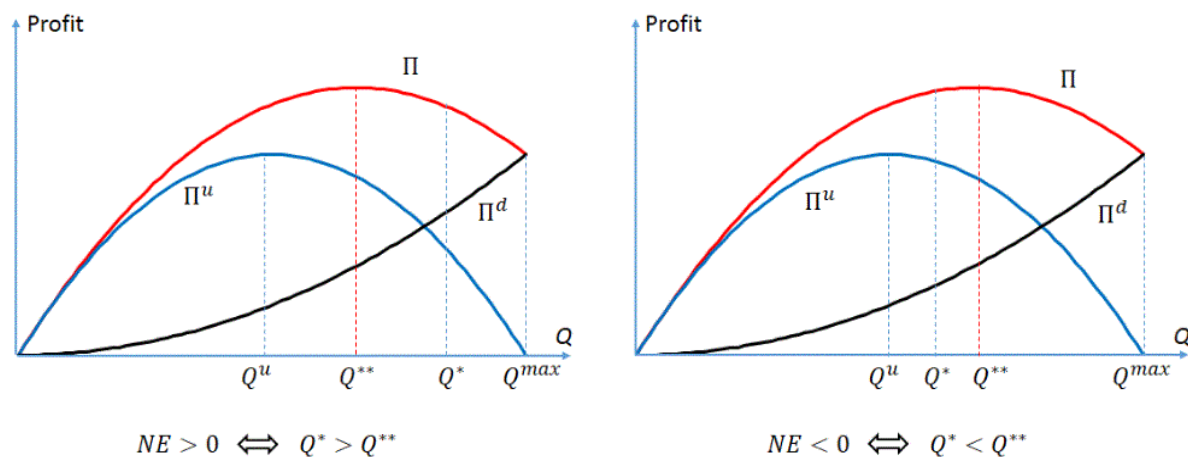


Figure 1: Profit curves as functions of  $Q$

In sum, among the three output levels (the equilibrium quantity,  $Q^*$ ; the monopoly quantity,  $Q^{**}$ ; and the one that maximizes the total upstream profit,  $Q^u$ ),  $Q^u < Q^{**}$  and  $Q^u < Q^*$  both holds unconditionally, whereas the comparison between  $Q^{**}$  and  $Q^*$  depends on the sign of  $NE$ . Figure 1 draws the total profits of the upstream ( $\Pi^u$ ), the downstream ( $\Pi^d$ ), and all firms combined ( $\Pi$ ) as functions of  $Q$ . In the left panel,  $NE > 0$  so  $Q^* > Q^{**}$ , and we have  $Q^u < Q^{**} < Q^*$ . In the right panel,  $NE < 0$  so  $Q^* < Q^{**}$ , and we have  $Q^u < Q^* < Q^{**}$ . The comparison between  $Q^{**}$  and  $Q^*$  is crucial because, starting from the

equilibrium  $Q^*$ , a smaller  $Q$  (resulting from a higher collusive input price) raises the joint profit if and only if  $Q^* > Q^{**}$  to begin with.

The results are summarized in the following proposition:

**Proposition 1** *Consider the equilibrium total output  $Q^*$  in successive oligopoly.*

- (i) *When  $NE > 0$ , we have  $Q^u < Q^{**} < Q^*$ . Then a small reduction of total output from  $Q^*$  raises the upstream and joint profits but reduces the downstream profits.*
- (ii) *When  $NE < 0$ , we have  $Q^u < Q^* < Q^{**}$ . Then a small reduction of total output from  $Q^*$  raises the upstream profits but reduces the downstream and joint profits. Conversely, a small increase of total output reduces the upstream profits but raises the downstream and joint profits.*
- (iii) *When  $NE = 0$ , we have  $Q^u < Q^* = Q^{**}$ . Then a different  $Q$  (either larger or smaller than  $Q^*$ ) can only reduce the total profit.*

### 3.6 The profitability of price-raising upstream collusion

Most real-life cases of upstream collusion, including the Japanese automobile parts litigations, involved colluding to raise the input price. In our setting, this means that starting from the successive oligopoly equilibrium (i.e.,  $t^* = t(Q^*)$ ), the upstream firms raise the input price ( $\tilde{t} > t^*$ ) by a small amount and correspondingly reduce the total production quantity ( $\tilde{Q} < Q^*$ ). Most of our analysis below will focus on such kind of collusion, hereafter referred to as *price-raising upstream collusion* (or simply upstream collusion when there is no ambiguity).

In this article, we look for profitable upstream collusion which must compensate the downstream. Apparently, it requires the collusion to raise the two industries' joint profits. According to Proposition 1, the joint profit is raised if and only if  $NE > 0$ .

**Proposition 2** *Price-raising upstream collusion is profitable if and only if  $NE > 0$ . When such collusion takes place, the price for the final product is higher than in successive oligopoly, whereas social welfare and consumer surplus are both lower.*

Because the collusion raises the input price, the total output quantity drops and thus the final price must rise. Because the final price is higher, consumers are worse off. Because the final price moves further away from the social marginal cost (i.e.,  $c + f$ ), social welfare also drops. These unambiguous negative effects of profitable upstream collusion justifies the antitrust lawsuits mentioned in Introduction.

Re-arrangement of the terms in (4) reveals that  $NE > 0$  is equivalent to:

$$(m - 1)(n - 1) > \rho(Q^*) + 2, \quad (9)$$

where

$$\rho(Q) \equiv \frac{p''(Q)Q}{p'(Q)}$$

is the degree of demand concavity (Fauli-Oller, 1997; Mrazova and Neary, 2017; Ziss, 2001).<sup>8</sup> Given A1, we have  $\rho(Q) > -1$ . Immediately we conclude that upstream collusion is never profitable if the downstream has only one firm ( $n = 1$ ), or each industry has exactly two firms ( $m = n = 2$ ).

Clearly, (9) is easier to satisfy when  $m$  or  $n$  is larger or  $\rho(Q^*)$  is smaller. The following proposition summarizes the factors that affect the profitability of upstream collusion.

**Proposition 3** *A price-raising upstream collusion is more likely to be profitable if:*

- (i) *demand is less concave (i.e.,  $\rho(Q^*)$  is smaller); or*
- (ii) *the upstream has more firms (i.e.,  $m$  is larger); or*
- (iii) *the downstream has more firms (i.e.,  $n$  is larger); or*
- (iv) *the vertical market structure is more balanced (i.e.,  $|m - n|$  is smaller for fixed  $m + n$ ), but is unaffected if  $m$  and  $n$  are switched.*

To understand (i), note that a smaller  $\rho(Q^*)$  indicates a less elastic demand for the final product,<sup>9</sup> which allows firms to charge a higher price. This is true for both monopoly and

<sup>8</sup>Mrazova and Neary (2017) used the term “demand convexity” for  $-\frac{p''(Q)Q}{p'(Q)}$ , whereas Fauli-Oller (1997) and Ziss (2001) used the term “demand concavity”.

<sup>9</sup>For example, when the demand is  $p = a - bQ^x$  (for  $x > 0$ ),  $\rho(Q^*) = x - 1$ , and a smaller  $\rho(Q^*)$  corresponds to a smaller  $x$ , which means smaller elasticity.

oligopoly.<sup>10</sup> However, monopoly price tends to rise more rapidly than oligopoly price, as the latter is constrained by horizontal competition. This makes it more likely for the oligopoly price to be smaller than the monopoly price.

As for (ii), (iii) and (iv), note that collusion profitability is solely determined by a comparison between the competitive output, which depends on  $m$  and  $n$ , and the monopoly output, which does not. When  $m$  or  $n$  increases, apparently the oligopoly output increases. In the extreme case of either  $m = \infty$  or  $n = \infty$  (so that either industry becomes perfectly competitive), a slight increase of the input price always raises the joint profit. If the two industries become more imbalanced, the stronger industry will take advantage of the weaker industry by having a larger markup, thereby reducing the total output. To see this, compare the market structure of  $5 \times 1$  versus  $3 \times 3$ . In  $5 \times 1$ , the upstream is quite competitive whereas the downstream is a monopoly. Although the input price is low, the downstream will be able to charge a large markup so that the equilibrium output tends to be small. By contrast, in  $3 \times 3$ , neither industry is dominant, and the equilibrium output will be greater than in  $5 \times 1$ .

Conclusion (i) is consistent with the findings by Pindyck (1979), who suggests that inelastic demands facilitate collusion. Demand concavity in our two-layer setting is similar to the demand elasticity in Pindyck's one-layer industry. The driving force is similar, too. In our setting, inelastic demand is conducive to collusion profitability because the demand condition raises monopoly price faster than it does oligopoly price, making it more likely for the latter to fall below the former. In conventional cartel setting of Pindyck (1979), an inelastic demand means greater collusive profits, which are needed to cover the cost of organizing and maintaining a cartel.

Conclusion (ii) links cartel success to the number of firms in the industry where collusion is taking place. It is the opposite of findings in conventional cartel theory, which typically considers a single industry without any downstream oligopoly buyers. In the traditional setting, collusion is constrained by individual members' incentive to deviate. When there are more firms in that industry, each member's market share in the collusive arrangement

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<sup>10</sup>That's why Pindyck (1979) argues that a smaller demand elasticity allows cartels to obtain larger profits.

becomes smaller. This means breaking away from the agreement (and enjoy the entire market for a short time) is more tempting, making collusion more difficult. In our setting, by contrast, collusion is constrained by the compensation requirement. When there are more upstream firms, it is more likely for firms to over-produce in equilibrium, in which case a higher input raises the total profits. We can imagine a more general setting in which upstream collusions face both the incentive constraint and the compensation constraint. The interaction between the two forces in a repeated game will likely lead to a non-monotonic relationship between upstream collusion and the number of firms. Indeed, Symeonidis's (2003) empirical study has uncovered an inverted U-relationship between market concentration and collusion incidence.

Because a greater number of firms in either industry is conducive to profitable upstream collusion, entry of new competitors into the upstream or downstream industry may precipitate upstream collusion. Exit has the opposite effect: collusion is less likely to be profitable. Finally, horizontal merger in either industry reduces the intensity of horizontal competition, making collusion less likely to be profitable.

## 4 Extensions

In what follows we will consider four extensions: collusion when firms compete in price, collusion under increasing marginal costs, price-reducing collusions, and vertical disintegration. These extensions establish the robustness of the major insights, reveal additional factors that affect collusion profitability, and demonstrate the usefulness of the horizontal-vertical externality framework in understanding vertical conducts well beyond upstream collusions.

### 4.1 Bertrand competition

The main analysis has assumed that both industries compete in quantities. Now suppose that downstream firms carry out Bertrand competition (the nature of upstream competition will be specified later). If the final products continue to be perfect substitutes, then each downstream firm's profit is zero, and the equilibrium total production quantity is such that  $Q^* > Q^u = Q^{**}$ . In that case, upstream collusion always increases upstream profit without hurting downstream firms.



Now suppose that the final products are differentiated, with downstream firm  $i$  facing demand  $q_i(p_i, \mathbf{p}_{-i})$  with  $\frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} < 0$  and  $\frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_{-i}} > 0$  for  $i \in \{1, \dots, n\}$ , where  $\mathbf{p}_{-i}$  is the price vector of all other downstream firms except  $i$ .<sup>11</sup> The equilibrium price  $p_i^*$  must satisfy the individual FOC:

$$q_i(p_i, \mathbf{p}_{-i}) + (p_i - t - c) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0. \quad (10)$$

On the other hand, the joint profit of all firms,  $\Pi = \sum_{i=1}^n (p_i - c - f) q_i(p_i, \mathbf{p}_{-i})$ , is maximized by firm  $i$ 's price,  $p_i^{**}$ , if the following FOC holds:

$$q_i(p_i, \mathbf{p}_{-i}) + (p_i - c - f) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} + \sum_{j \neq i} (p_j - c - f) \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0. \quad (11)$$

Rearranging (10), we have

$$\begin{aligned} \frac{\partial \pi_i^d}{\partial p_i} - \frac{\partial \Pi}{\partial p_i} &= -\frac{\partial \Pi^u}{\partial p_i} + \left( \frac{\partial \pi_i^d}{\partial p_i} - \frac{\partial \Pi^d}{\partial p_i} \right) \\ &= \underbrace{\left[ (f - t) \sum_j \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} \right]}_{\text{Vertical externality}} + \underbrace{\left[ -\sum_{j \neq i} (p_j - c - t) \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} \right]}_{\text{Horizontal externality}}, \\ &\quad (+) \qquad \qquad \qquad (-) \end{aligned}$$

To analyze the magnitude of vertical and horizontal externalities, we need to explicitly specify the vertical relation. If upstream firms sell the input through a “market interface” (Inderst, 2010; Ghosh et al., 2014), then the equilibrium input price mainly depends on upstream competition (given the input demand arising from downstream competition). For example, if upstream firms carry out Bertrand competition (with homogeneous input), then we have  $t = f$ . The vertical externality vanishes completely, and the horizontal externality always dominates. By increasing the vertical externality, an upstream collusion will better balance the two externalities and is therefore always profitable. If upstream firms carry out

<sup>11</sup>In addition, we assume that an increase of any individual downstream firm's final price reduces the total output, i.e.,  $\sum_j \frac{\partial q_j(p_i, \mathbf{p}_{-i})}{\partial p_i} < 0$ . If upstream firms sell the input at price  $t$  ( $t \geq f$ ), then downstream firm  $i$  chooses price,  $p_i$ , to maximize  $\pi_i^d = (p_i - t - c) q_i(p_i, \mathbf{p}_{-i})$ , treating  $t$  as given. We assume  $\frac{\partial^2 \pi_i^d}{\partial (p_i)^2} < 0$  to guarantee that the FOCs are sufficient to characterize the equilibrium,  $\frac{\partial^2 \pi_i^d}{\partial p_i \partial p_{-i}} > 0$  to ensure that downstream firms' prices are strategic complements, and  $\frac{\partial^2 \pi_i^d}{\partial (p_i)^2} + \sum_{j \neq i} \frac{\partial^2 \pi_i^d}{\partial p_i \partial p_j} < 0$  to ensure stability of the equilibrium.

Cournot competition,  $t(Q)$  is characterized by the downstream FOCs, i.e.,  $q_i(p_i, \mathbf{p}_{-i}) + (p_i - t - c) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0$ ,  $i = 1, \dots, n$ . And the equilibrium is solved from upstream FOCs,  $\frac{\partial \pi_j^u}{\partial q_j^u} = 0$ ,  $j = 1, \dots, m$ , where  $\pi_j^u = [t(Q) - f]q_j^u$ .

**Example 1** Consider linear demand functions  $q_i = a - p_i + \lambda \sum_{j=1}^n (p_j - p_i)$  for  $i = 1, \dots, n$  and  $\lambda > 0$ . In successive oligopoly in which upstream competition is Cournot and downstream competition is Bertrand,  $t^* = \frac{1}{m+1}(a - c - f) + f$  and  $p_i^* = a - \frac{[1+(n-1)\lambda]m}{[2+(n-1)\lambda](m+1)}(a - c - f)$ . Consequently the vertical externality is  $\frac{1}{m+1}(a - c - f)$  and the horizontal externality is  $-\frac{m}{(m+1)} \frac{(n-1)\lambda}{2+(n-1)\lambda}(a - c - f)$ . Horizontal externality dominates if and only if

$$(m-1)(n-1) \geq \frac{2}{\lambda}.$$

Just like in the case of Cournot competition, a larger number of firms in either industry makes it more likely for firms to over-produce in equilibrium and hence for a price-raising upstream collusion to increase the joint profit. Unlike in Cournot competition, a relatively small  $m$  and  $n$  (such as  $m = n = 2$ ) can still support a profitable collusion, as  $\lambda \leq 1$  is not required.<sup>12</sup> Finally a smaller degree of product differentiation (i.e., a greater  $\lambda$ ) increases the profitability of upstream collusions. Greater similarity among final products will reduce the total output in both monopoly and oligopoly, but the output drops slower in the latter due to horizontal competition. This makes it more likely for the equilibrium oligopoly total output to exceed the monopoly quantity.

If every upstream firm supplies exclusively to one downstream firm under the constraint that the number of upstream and downstream firms are equal (Lin, 1988, 1990; O'Brien and Shaffer, 1993), then we only need to replace  $t$  with  $t_i$  in the above expressions, where  $t_i$  maximize  $\pi_i^u = (t_i - f)q_i(\mathbf{p}(\mathbf{t}))$ , and  $q_i(\mathbf{p}(\mathbf{t}))$  is characterized by downstream firms' FOCs, i.e.,  $q_i(p_i, \mathbf{p}_{-i}) + (p_i - t_i - c) \frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0$ ,  $i = 1, \dots, n$ . Again, a larger number of firms in either

<sup>12</sup>Note that equilibrium stability requires self-price effect to be stronger than cross-price effect, which always holds:  $\left| \frac{\partial q_i}{\partial p_i} \right| > \left| \sum_{j \neq i} \frac{\partial q_i}{\partial p_j} \right|$  given that  $\frac{\partial q_i}{\partial p_i} = -1 - (n-1)\lambda$  and  $\frac{\partial q_i}{\partial p_j} = \lambda$ . So there is no need to restrict  $\lambda$  to be smaller than one. Since  $\frac{\left| \sum_{j \neq i} \frac{\partial q_i}{\partial p_j} \right|}{\left| \frac{\partial q_i}{\partial p_i} \right|} = \frac{(n-1)\lambda}{1+(n-1)\lambda}$  increases with  $\lambda$ , a greater  $\lambda$  indicates more homogeneous products.

industry or a smaller degree of product differentiation intensifies downstream competition, and therefore makes it more likely for the horizontal externality to dominate the vertical externality.

**Example 2** Let the final demand be  $q_i = a - p_i + \lambda \sum_{j=1}^n (p_j - p_i)$  for  $i \in \{1, \dots, n\}$  and  $\lambda > 0$ . Let  $m = n \geq 2$  and consider exclusive dealing. Then in equilibrium we have  $t_i^* = \frac{(2n-1)\lambda+2}{(n-1)^2\lambda^2+(5n-3)\lambda+4}(a-c-f) + f$  and  $p_i^* = \frac{(n-1)(3n-2)\lambda^2+3(3n-2)\lambda+6}{[(n-1)\lambda+2][(n-1)^2\lambda^2+(5n-3)\lambda+4]}(a-c-f) + c + f$ . Hence, the vertical externality is  $\frac{(2n-1)\lambda+2}{(n-1)^2\lambda^2+(5n-3)\lambda+4}(a-c-f)$  and the horizontal externality is  $\frac{(n-1)\lambda[(n-1)^2\lambda^2+(3n-2)\lambda+2]}{[(n-1)\lambda+2][(n-1)^2\lambda^2+(5n-3)\lambda+4]}(a-c-f)$ . Horizontal externality dominates vertical externality if and only if

$$\left[ (n-1) - \frac{2}{\lambda} \right] [(n-1)\lambda + 1] [(n-1)\lambda + 2] > 2. \quad (12)$$

It is straightforward to verify that (12) is easier to satisfy if the final products are more similar (i.e.,  $\lambda$  is larger) or there are more upstream-downstream pairs (i.e.,  $n$  is larger). For example, upstream collusion is profitable for  $\lambda > 2.32$  if  $n = 2$ , and for  $n \geq 4$  if  $\lambda = 1$ .

The above results are summarized in the following proposition:<sup>13</sup>

**Proposition 4** Suppose that downstream firms carry out Bertrand competition with differentiated products. Then upstream collusion is more likely to be profitable if there are more firms in either industry or the final products are less differentiated.

## 4.2 Increasing marginal costs

In the main model, we have assumed that the marginal costs in both industries are constant. Now suppose marginal costs are increasing such that an upstream firm  $j$ 's cost is  $F(q_j^u)$ ,  $j \in \{1, \dots, m\}$ , and a downstream firm  $i$ 's cost is  $C(q_i^d)$ ,  $i \in \{1, \dots, n\}$ , with  $F'(q_j^u) \geq 0$ ,  $F''(q_j^u) \geq 0$ ,  $C'(q_i^d) \geq 0$ , and  $C''(q_i^d) \geq 0$ . The other settings remain the same.

In successive oligopoly, given input price,  $t$ , each downstream firm maximizes its profit:  $\max_{q_i^d \geq 0} \pi_i^d \equiv q_i^d [p(\sum_{k=1}^n q_k^d) - t] - C(q_i^d)$ , which gives rise to the inverse demand for input:

$$t(Q) = p'(Q) \frac{Q}{n} + p(Q) - C'(\frac{Q}{n}).$$

<sup>13</sup>For product differentiation, the proposition also holds if the downstream competition is Cournot.

Following the same procedure as in the main model (see the appendix for details), we can show that a slightly higher input price increases joint profit if and only if:

$$(m-1)(n-1) \geq \frac{1 + \gamma(Q^*)}{\gamma(Q^*)} [\rho(Q^*) + 1], \quad (13)$$

where  $\gamma(Q) \equiv \frac{p''(Q)Q + p'(Q)}{p'(Q) - C''(\frac{Q}{n})}$  is a variable that reflects the curvatures of the demand and marginal cost functions.<sup>14</sup> Noting that  $\gamma(Q) \in (0, \rho(Q) + 1]$  (with  $\gamma(Q) = \rho(Q) + 1$  when  $C''(\cdot) = 0$ ) and that  $\gamma(Q)$  increases with  $C''(\frac{Q}{n})$  (as the numerator and denominator are both negative), we have:

**Proposition 5** *Fixing the equilibrium output  $Q^*$ , upstream collusions are more likely to be profitable if the downstream cost is less convex (i.e., (13) is easier to satisfy when  $C''(\cdot)$  is smaller), but are unaffected by the convexity of the upstream cost.*

The proposition indicates that an increasing marginal cost in the downstream makes it more difficult for upstream collusion to be profitable, more so when the downstream cost is more convex. When  $C''(\cdot)$  is smaller, the oligopoly and monopoly output both expand, but the oligopoly output expands more due to downstream competition, just as the effect of a smaller  $\rho(Q^*)$  discussed earlier. This makes it more likely for the oligopoly output to be greater than the monopoly output.

### 4.3 Price-reducing upstream collusion

We have established that a price-raising upstream collusions is profitable if and only if  $NE > 0$ . What about  $NE < 0$ ? In that case, the equilibrium total output  $Q^*$  falls below the monopoly level (i.e.,  $Q^* \in (Q^u, Q^{**})$ ) and thus  $\frac{\partial \Pi(Q^*)}{\partial Q} > 0$  (Proposition 1, see the right panel of Figure 1). If upstream firms collude to raise the input price, they would still gain, but the two industries' joint profits decrease. Then there is no way for the upstream to compensate the downstream to make every firm better off. However, if upstream firms collude to *reduce* the input price, the total profit would increase. A lower input price benefits all downstream firms but hurts upstream firms (Proposition 1). In that case, if there is a mechanism for

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<sup>14</sup> Farrell and Shapiro (1990) and Gaudet and Salant (1991) use a similar expression  $\frac{p''(Q)q_i + p'(Q)}{p'(Q) - C''(q_i)}$ .

the downstream to properly compensate the upstream (through, for example, franchise fees), then all firms can again be better off.

Because consumer surplus and total welfare increase if and only if the final price decreases or, equivalently, total output increases, price-reducing upstream collusion improves consumer surplus and total welfare.

**Proposition 6** *If  $NE < 0$ , then a price-reducing upstream collusion*

*(i) raises the joint profit of the two industries;*

*(ii) is profitable for the upstream if they are properly compensated by the downstream firms;*

*(iii) raises consumer surplus and social welfare.*

If the demand for final product is linear,  $\rho(Q) = 0$ , then  $NE < 0$  (i.e., the opposite of (9)) boils down to  $(m - 1)(n - 1) < 2$ , which holds when  $m = n = 2$ . In that case, upstream firms would have an incentive to collude to reduce the input price, which raises the profit of all four firms as well as the consumer surplus and social welfare. It is a win-win-win.<sup>15</sup>

#### 4.4 Vertical disintegration

In upstream collusion, the imbalance between vertical and horizontal externalities is mitigated by a marginal change of the input price. There is, however, a more dramatic adjustments to the two externalities in the form of vertical disintegration. When firms are vertically integrated, each downstream unit will obtain the input at cost. In that case there is no vertical externality, and the remaining horizontal externality clearly hurts total profits. Vertical disintegration would then create a vertical externality, which can offset the horizontal externality and may therefore increase the total profits (unless the vertical externality is too large).

**Example 3** *Consider a  $2 \times 2$  market structure (i.e., vertical separation) with downstream demands  $q_i = a - p_i + \lambda(p_{-i} - p_i)$  for  $i \in \{1, 2\}$  and  $\lambda > 0$ . Normalize  $a - c - f = 1$ .*

<sup>15</sup>To be consistent with the main analysis, here we imply that upstream firms reach a binding agreement to charge a price that is lower than the equilibrium level. This is similar, although not identical, to the price caps that Rey and Tirole (forthcoming) have analyzed.

Suppose upstream carry out Cournot competition. The joint profit of an upstream-downstream pair is  $\pi^{VS} = \frac{2(\lambda+1)(\lambda+4)}{9(\lambda+2)^2}$ . If both pairs integrate vertically, the profit of each integrated firm is  $\pi^{VI} = \frac{\lambda+1}{(\lambda+2)^2}$ . Then  $\pi^{VS} > \pi^{VI}$  if and only if  $\lambda > \frac{1}{2}$ .

Suppose each upstream firm exclusively supplies one downstream firm, then we have  $\pi^{VS} = \frac{2(2\lambda^5+16\lambda^4+45\lambda^3+55\lambda^2+30\lambda+6)}{(\lambda+2)^2(\lambda^2+7\lambda+4)^2}$  and  $\pi^{VI} = \frac{\lambda+1}{(\lambda+2)^2}$ . Again,  $\pi^{VS} > \pi^{VI}$  if and only if  $2 > (4 + \lambda)(1 - \lambda^2)$ , i.e.,  $\lambda > 0.76$ . Therefore, when the final products are sufficiently similar, vertical separation can increase the four firms' joint profit.

In the above example, the benefit of vertical separation is conditional—the final products must be sufficiently similar. If products are very different (i.e.,  $\lambda$  is very small), vertical disintegration generates a vertical externality that is too large, which hurts the total profit. This result contrasts with Lin (1988), who finds that vertical separation always increases the joint profit regardless of the degree of product differentiation. That is because Lin (1988) considers downstream competition in a Hotelling setting, which means the total demand for final products is perfectly inelastic. A higher final price (due to vertical disintegration) unambiguously increases the joint profit. By contrast, here we have adopted a more general demand function such that the total quantity sold decreases with the final price.

Bonanno and Vickers (1988) and Vickers (1985) have shown that the advantage of vertical separation depends on the nature of downstream competition. In their models, upstream firms in vertical separation utilize exclusive dealing and franchise fees. If the downstream competition is Bertrand, each upstream firm sets a wholesale price above marginal cost, thereby generating a vertical externality that can partially offset the horizontal externality. This is optimal both collectively and individually (Bonanno and Vickers, 1988). If the downstream competition is Cournot, however, vertically separated upstream firms will set wholesale prices below cost, generating a negative vertical externality that will exacerbate the horizontal externality. This is optimal individually but not collectively (Vickers, 1985). In our setting, the vertical relation is determined by arm's length transactions. The input price is always greater than cost, and therefore there is always a positive vertical externality under vertical separation. That is why in our setting, vertical separation can be optimal for

both Bertrand and Cournot competition in the downstream.

## 5 Conclusion

In this article, we have studied the profitability of upstream collusions when compensating direct purchasers is necessary. In vertically related industries characterized by oligopoly competition, we demonstrate that there always exist a vertical externality and a horizontal externality. From the viewpoint of the firms' collective interest, horizontal competition can be too fierce or too mild depending on which externality dominates. If cutting back on quantity and raising prices mitigates the imbalance between the two externalities, a collusion can move the total output toward the monopoly outcome and therefore benefit all firms in the two industries. In fact, a higher joint profit is not only necessary for an upstream collusion to be profitable, but also tends to be sufficient, as many forms of side payments across vertical business partners are readily available, such as transfer pricing, quantity forcing, passive ownership, or sophisticated long-term contracting.<sup>16</sup>

By focusing on the profitability issue that has been largely ignored by the literature, we have set aside the incentive issues in this research. A more realistic setting, of course, is to consider both constraints simultaneously. For example, upstream firms may engage in a repeated game in which every firm decides in each period whether or not to deviate and, if no firm deviates, the cartel must compensate the downstream firms for any losses caused by the increased price of the intermediate goods. Collusion is sustained only if the colluding firms gain collectively and the discount factor is sufficiently large. The implications can then be studied by investigating how the cutoff discount factor depends on the underlying parameters. It is a fertile and yet challenging research, which we leave for future work.

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<sup>16</sup>Schinkel et al. (2008) have mentioned many forms by which collusive suppliers compensate their direct customers. In the case of Japanese automobile industry, partial ownership of suppliers by major carmakers seems particularly relevant and widespread. Most Japanese auto parts suppliers are partially owned by one or more major Japanese carmakers. For example, Toyota holds 41% of Denso and 33% of Aisan. In turn, Denso and Aisan sell 40% of their products to Toyota. In China, Japanese car manufacturing operates mostly through joint ventures, with Toyota holding 5.3% of Nachi Bearing and 22% of JTEKT, while Mitsubishi holds 6.3% of NTN Bearing.

## 6 Appendix

### 6.1 Proof of Proposition 3

(i) is straightforward even though  $Q^*$  is endogenous. As for (ii), we have

$$\frac{\partial Q^*}{\partial m} = \frac{Q^* [p''(Q^*)Q^* + (n+1)p'(Q^*)]}{m \left[ p'''(Q^*) (Q^*)^2 + (m+n+3)p''(Q^*)Q^* + (m+1)(n+1)p'(Q^*) \right]} > 0.$$

The numerator is negative given A1, and the denominator is also negative given A1 and A2. Therefore, the equilibrium total output increases when the upstream has more firms. Now re-write (2) as  $NE + [p'(Q^*)Q^* + p(Q^*)] = c + f$ , and take derivative with respect to  $m$ :  $\frac{\partial(NE)}{\partial m} + [p''(Q^*)Q^* + 2p'(Q^*)] \frac{\partial Q^*}{\partial m} = 0$ . As  $p''(Q^*)Q^* + 2p'(Q^*) < 0$  and  $\frac{\partial Q^*}{\partial m} > 0$ , we conclude  $\frac{\partial(NE)}{\partial m} > 0$ . Similarly for (iii) linking  $NE$  to  $n$ .

Finally for (iv), the equilibrium condition (2) is symmetric between  $m$  and  $n$ , so swapping the two parameters will not change the equilibrium  $Q^*$  and hence will not affect  $NE$ . Now fix  $m+n = X$  and substitute  $n = X - m$  into (2). Taking derivative with respect to  $m$ , we have

$$\left. \frac{\partial Q^*}{\partial m} \right|_{m+n=X} = \frac{(2n-X)[p(Q^*) - c - f]}{p'''(Q^*) (Q^*)^2 + (X+3)p''(Q^*)Q^* + [(X+1) + n(X-n)]p'(Q^*)}.$$

The denominator is negative given A1 and A2, and the joint markup  $p(Q^*) - c - f > 0$ . Then  $\frac{\partial Q^*}{\partial m} > 0$  if and only if  $2n - X < 0$ , i.e.,  $m < n$ . That is, fixing  $m+n$ , increasing  $m$  will increase  $Q^*$  when  $m < n$ , but will decrease  $Q^*$  when  $m > n$ . Using the same method as for (ii), we also know that  $NE$  increases if and only if  $Q^*$  increases.

### 6.2 Increasing marginal costs

In successive oligopoly, given input price  $t$ , each downstream firm maximizes its profit:  $\max_{q_i^d \geq 0} \pi_i^d \equiv q_i^d [p(\sum_{k=1}^n q_k^d) - t] - C(q_i^d)$ , which gives rise to the inverse demand for input:

$$t(Q) = p'(Q) \frac{Q}{n} + p(Q) - C'(\frac{Q}{n}).$$

Substitute this input demand function into an upstream firm's profit function to obtain  $\pi_j^u \equiv t(\sum_{k=1}^m q_k^u) q_j^u - F(q_j^u)$ , the FOCs of which determine the unique equilibrium in successive



oligopoly. Therefore, the total output  $Q^*$  is characterized by:

$$\frac{Q^*}{nm} \left[ p''(Q^*)Q^* + (m+n+1-nm)p'(Q^*) - C''\left(\frac{Q^*}{n}\right) \right] + p'(Q^*)Q^* + p(Q^*) = C'\left(\frac{Q^*}{n}\right) + F'\left(\frac{Q^*}{m}\right).$$

The joint profit is  $\Pi(Q) = p(Q)Q - nC\left(\frac{Q}{n}\right) - mF\left(\frac{Q}{m}\right)$ . It is maximized at  $Q^{**}$ , which is characterized by:

$$p'(Q^{**})Q^{**} + p(Q^{**}) = C'\left(\frac{Q^{**}}{n}\right) + F'\left(\frac{Q^{**}}{m}\right).$$

Similarly, the firms over-produce (i.e.,  $Q^* > Q^{**}$ ) if and only if

$$\frac{Q^*}{nm} \left[ p''(Q^*)Q^* + (m+n+1-nm)p'(Q^*) - C''\left(\frac{Q^*}{n}\right) \right] > 0.$$

A downstream firm's individual incentive can be decomposed as:

$$\frac{\partial \pi^d}{\partial q^d} = \frac{\partial \Pi(Q)}{\partial q^d} + \underbrace{\left[ F'\left(\frac{Q}{m}\right) - t \right]}_{\substack{\text{Vertical externality} \\ (-)}} + \underbrace{\left[ -\frac{n-1}{n}p'(Q)Q \right]}_{\substack{\text{Horizontal externality} \\ (+)}}.$$

On the other hand, upstream firms' joint profit is  $\Pi^u(Q) = t(Q)Q - mF\left(\frac{Q}{m}\right)$ , and its optimization leads to a solution of  $Q^u$ , which satisfies:

$$\frac{Q^u}{n} \left[ p''(Q^u)Q^u + 2p'(Q^u) - C''\left(\frac{Q^u}{n}\right) \right] + \left[ p'(Q^u)Q^u + p(Q^u) - C'\left(\frac{Q^u}{n}\right) - F'\left(\frac{Q^u}{m}\right) \right] = 0.$$

It is easy to show that this optimal output is smaller than the monopoly level (i.e.,  $Q^u < Q^{**}$ ).

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