Convenience vs. Pleasance: Matching on Horizontal and Vertical Dimensions and Platform Information Design *

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June 2019

Preliminary draft

*Authors are ordered alphabetically and have equal contributions to this paper. We thank participants at the 2019 Marketing Science conference, and seminar participants at Xiamen University, IESR at Jinan University, City University of Hong Kong, Peking University, Central University of Finance and Economics, and Shanghai University of Finance and Economics for their great comments. All errors are ours.

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Abstract

We consider a two-sided decentralized matching scenario on a peer-to-peer platform. Each platform user on one side, called a “sender”, sends a request to one user on the other side, called a “receiver”. A match is successful if the request is accepted. Individuals on both sides are differentiated in two dimensions: a vertical attribute, which reflects the individual’s quality of being high (H) or low (L), and a horizontal location, which captures the extent of mismatch between a sender and a receiver. A platform user’s utility from a successful match increases with the match’s vertical quality but decreases with their horizontal distance. We first derive the equilibrium when the platform fully discloses user information across sides. Interestingly, even when senders’ valuation for the horizontal closeness increases, an L-sender may instead choose a distant receiver to avoid competition. We then examine a platform’s three possible information designs: withholding the senders’ vertical information, withholding the receivers’ vertical information, and centralized matching . Surprisingly, withholding one side’s vertical information not only always hurts the H-type users on both sides, but may also hurt the L-type users on the opposite side. The centralized matching scheme can benefit both types of users. Lastly, we compare the transaction volume under all matching schemes. Among the decentralized matching schemes, withholding receiver’s vertical information results in the highest number of successful matches. Full information matching leads to the lowest number of transactions.

Keywords: Information Design, Peer-to-peer Platforms, Multi-dimensional Matching, Decentralization

1 Introduction

Peer-to-peer platforms, such as Airbnb and BlaBlaCar, provide online marketplaces for individuals with supply to meet those with demand: Airbnb matches house owners who have empty rooms to travelers who need lodging; BlaBlaCar connects drivers who travel between cities and have empty seats to passengers who need a ride. While each individual wants to match with "the best" option available, their perspectives regarding whom being the best do not necessarily agree. A house listed on Airbnb may be good in terms of offering better amenities, which can be viewed as an attribute on vertical dimension with an almost unanimously acceptable definition of "being pleasant", or it may be good in terms of its location convenience, which is a more horizontal dimension as travelers tend to have idiosyncratic definitions of "being convenient" depending on their different desired locations.

Platform users often trade off between the unanimous dimension and the idiosyncratic dimension in their match preferences: a traveler who uses Airbnb may choose between a more "convenient" house, which is close enough to her desired location but has very few essential amenities, and a more "pleasant" house, which is distant from the desired location but provides high quality amenities. The weights users put on the two dimensions can also significantly vary by the matching context. Travelers usually care a lot about the lodging quality, thus may prefer a more "pleasant" house to a more "convenient" house. In many other markets, the match's horizontal attributes are more important. To a part-time BlaBlaCar driver who shares his ride between cities, the passenger's overlapping trip route and desired departure time arguably matter more than the passenger's vertical attributes such as rating. How do platform users trade off the value from the match's horizontal attribute and that from the vertical attribute? How does users' perception on the relative importance of the two dimensions of attributes affect individual's strategy in seeking a match?

The answers to these questions are crucial for a platform which looks to improve matching efficiency. A decentralized peer-to-peer platform, which relies on users from two sides to proactively match with each other, are concerned about the coordination failure caused by same-side competition. For example, multiple passengers, without knowing each other's strategy, may end up sending their requests to the same driver, who eventually can only take one passenger. Centralizing the matching process to avoid the inefficiency, such as Uber which assigns a driver to each passenger, usually requires huge amount of computing and processing power, thus can be costly. Alternatively, a platform can influence user behavior and increase the number of matches through information design, i.e. control the user information to disclose across sides. OkCupid, an American-based online dating platform, conducted an experiment in 2013 by obscuring all profile photos. Interestingly, the platform users engaged in more meaningful conversations and responded to first messages more often[1]. How can a platform increase the number of successful matches through information design, especially when individuals value both horizontal attribute and vertical attribute of a match? To platform users, how will different designs affect their welfare?

In this paper, we distinguish between the vertical and horizontal types of individual attributes, and theoretically examine their effects on a decentralized platform's matching process. We describe each individual as the duplet of a vertical quality and a horizontal location. Each user on one side of the platform, which we call the "senders", send a matching request to a user on the opposite side, which we call the "receivers". Individual's utility from a successful match increases with the match's vertical quality but decreases with their horizontal distance. To reflect the fact that the relative importance of a match's horizontal and vertical attribute can vary by sides, we allow the two sides to put different weights on the match value from these two dimensions. For example, although a Blablacar driver highly values the passenger's horizontal attribute such as overlapping route and departure time, the passenger may care more about the driver's vertical attribute such as driver rating and car quality.

A key observation from our analysis is that users' valuation on the two dimensions exert distinct influences on the same-side competition: The more important the vertical attributes are, the more similar users' preferences towards a match are and the more intense the competition will be. Conversely, if the horizontal dimension is weighed more, the taste heterogeneity on match options is larger and the matching market becomes less competitive. We identify senders' incentive of sacrificing the match value from horizontal dimension to chase distant receivers with higher vertical quality, which leads to senders' requests concentrating to one receiver and results in coordination failure. Such incentive is even stronger among senders who themselves have higher vertical quality, as their competitive advantage lowers their chance of being rejected by receivers. One may expect that such incentive becomes weaker when senders put more weight on the match's horizontal value. Surprisingly, we find that this intuition does not hold for all senders, as those with lower vertical quality may be even more motivated to forgo a match's horizontal value and send

[1] `https://www.nytimes.com/2014/07/29/technology/okcupid-publishes-findings-of-user-experiments.html`

requests to distant receivers with higher vertical quality. By doing so, low-quality senders strategically avoid direct competition with high-quality senders and increase the their chance of requests being accepted.

Compared to traditional offline markets, today's peer-to-peer platforms obtain more user information and have better control on how information is displayed and how matches are formed. Given that individual's valuation for a match's vertical attribute causes coordination friction, a platform can reduce such inefficiency by withholding user's vertical information across sides. The idea is to make each individual send/accept a request based only on the match's horizontal attribute. Along this logic, we examine two information design schemes: withholding senders' vertical information, and withholding receivers' vertical information. Consistent with one's intuition, both design schemes increase the total number of matches. Interestingly, withholding receiver's vertical information increases the number of matches to a greater extent. To follow the intuition, note that if receivers cannot see senders' vertical attribute, high-quality senders lose their competitive advantage because receivers cannot distinguish them from low-quality senders. The incentive of chasing high-quality receivers thus become weaker for high-quality senders, but also becomes stronger for low-quality senders. The senders' overall incentive of chasing high-quality receivers is diminished, but is not fully eliminated. In contrast, if senders cannot see receivers' vertical quality, despite high-quality senders' competitive advantage, high-quality senders are not to utilize the advantage because as they not able to identify high-quality receivers. As a result, all senders choose receivers based on the horizontal attribute, which to a higher extent avoids requests crowding to one receiver.

It is then natural to ask whether the increase in number of matches also benefits each individual. We show that under both design schemes, senders and receivers with higher vertical attribute are all worse off. One may believe that low-quality users benefit from high-quality users' welfare loss. In contrast to this intuition, we find that withholding one side's vertical information can also hurt the low-quality users on the opposite side. This happens if users on one side highly value the match's horizontal attribute, thus their behavior won't dramatically change when their vertical attributes are hidden. As a result, the low-quality users on the opposite side benefit little from the change in their match's behavior, but are hurt as they cannot not make decision based on the match's vertical information. Lastly, we compare the two designs with centralized matching where no match failure happens. In contrast to the two decentralized design where high-quality users are always hurt, centralized matching can benefit high-quality users even though they cannot leverage their advantage to proactively seek a match.

The remainder of the paper is organized as follows: Section 2 provides a review of the literature. Section 3 sets the model. Section 4 derives the equilibrium in the baseline model where all user information are disclosed. Section 5 analyzes two information designs in which a platform displays less information, and derives the corresponding equilibrium and implications on user welfare. Section 6 studies a centralized matching scheme and the welfare implication. Section 7 compares the number of matches under four different matching schemes. Section 8 concludes the paper. The proofs for all the claims made in the paper can be seen in the appendix.

2 Literature

Our paper contributes to the literature of platform design (See Einav, Farronato, and Levin (2016) and Goldfarb and Tucker (2019) for a recent review). Researchers have examined various issues in this area, including search design (Hagiu and Jullien (2011), Chiou and Tucker (2012), Eliaz and Spiegler (2016),De los Santos and Koulayev (2017)), pricing mechanisms (Uetake (2018), Gomes and Pavan (2018), Guda and Subramanian (2019)), and reputation management (Bolton, Greiner, and Ockenfels (2013), Mayzlin, Dover, and Chevalier (2014),Nosko and Tadelis (2015)). The current paper focuses on a platform's information design. Ostrovsky and Schwarz (2010) show that schools can suppress student's information to prevent early contracting and to improve job placement. Hoppe, Moldovanu, and Ozdenoren (2011) show that when a platform cannot observe each individual's full information, implementing a coarse matching scheme (i.e. dividing each side to two categories) can be preferable to a finer matching scheme or a random matching scheme. Halaburda, Jan Piskorski, and Yıldırım (2017) find that by restricting agents' choice set, a platform can reduce same-side competition, charge agents higher participation fee, and earn higher profits. In contrast to these work, we study the information dimensions of user attributes (i.e. horizontal attribute and vertical vertical attribute) a platform can choose to disclose across sides in order to improve matching efficiency. Tadelis and Zettelmeyer (2015) find empirical evidence that in a auction context, sellers disclosing vertical information helps heterogeneous bidders to better sort into their most valuable market, and thus increases the transaction volume. By contrast, we examine a peer-to-peer matching context, and show that disclosing vertical information intensifies same-side competition and decreases the transaction volume. In a related paper, Romanyuk and Smolin (2018) study how limiting buyer's information to sellers can alleviate the congestion problem, and improve all buyers' and sellers' welfare. They consider a dynamic setting where buyers are short-lived, non-strategic, and homogeneous in their match value. This particular setup allows long-lived sellers to wait for high-value match, which leads to congestion. Our paper differs from theirs as we allow both sides to be heterogeneous in their preference for a certain match, and do not allow either side to wait for a later request because of high delay cost. The match failure from our model is not due to congestion, but is the result of multiple senders competing for the same receiver. We focus on sender's strategic behavior given the user information a platform reveals across sides. In contrast to Romanyuk and Smolin (2018) Pareto improvement result, we find that withholding vertical information improves the transaction volume, but lowers the welfare of certain groups of users.

This paper also contributes to the literature on multi-dimensional matching. The seminal work of Becker (1973) establishes the sorting result in a uni-dimensional matching context. Chiappori, Oreffice, and Quintana-Domeque (2012) study the marriage market, and find that if an agent's multi-dimensional attributes can be represented by some uni-dimensional index, then the index is ordinarily identifiable under certain assumptions. In a later paper, Chiappori, Oreffice, and Quintana-Domeque (2017) show that if the multi-dimensional attributes, such as education and smoking status, do not admit uni-dimensional representation, then the assortative matching is distorted, as a smoking individual may "marry down" to an individual with lower socioeconomic status. Lindenlaub (2017) generalizes the notion of assortative sorting from uni-dimensional matching to multi-dimensional matching, and apply the framework in labor market to explain how an increase in complementarity in worker's cognitive skills (relative to manual skills) leads to larger wage inequality. This strand of literature mainly uses assignment models and assumes complementarity between individual types in their production function. The focus is mostly on how the positive assortative matching (PAM) pattern changes if agents cannot be characterized by a uni-dimensional index. Our paper contrasts with the literature in three ways. First, instead of using an assignment model, we allow agents to proactively seek a match so that the matching process is decentralized. Second, although complementarity between agent types is a common assumption in marriage market and labor market, it is not quite applicable to peer-to-peer matching contexts such as ride sharing market or house sharing market. We drop the complementarity assumption, but allow the high-type agents on one side to differ in their horizontal match value to an agent on the opposite side. Third, the literature on multi-dimensional matching largely assumes that agents have the same preference ranking for each dimension of a match's attributes. By contrast, we look into the situation where individual's preference for a match only agrees on the match's vertical dimension but differs on the horizontal dimension. We show that individual's taste heterogeneity on the horizontal dimension is the key in reducing competition, which a platform can use in information design to increase number of matches[2].

3 The Model

Consider a market where two sides of individuals look to match with each other through an intermediary platform. Depending on which side initialize the matching request, we call one side of the individuals *senders* (he), and the other side *receivers* (she). In the case of Airbnb, travelers are usually senders who send the booking requests to the house owners whom we view as receivers. Each individual has two attributes: a horizontal attribute, denoted by the individual's location x, and a vertical attribute, taking value of either high (H) or low (L). To allow competition between peers on the same side, we have two senders and two receivers in our model. Without loss of generality, we normalize two receiver's location distance to 1 and let each sender's location to be a random variable taking value on the unit interval between the two receivers. Figure 1 visualizes the matching context we consider.

Figure 1: Matching Context

Theoretically, the two receivers' vertical types can have four possible combinations: $HH, HL, LH, \text{and} LL$. Given the interest in platform users' trade-off between the value from a match's vertical dimension and horizontal dimensions of attributes, we only study the asymmetric cases (HL and LH). In the other two symmetric case, since receivers are not vertically differentiated, senders' decision on whom to send matching requests does not depend on receivers' vertical attribute. The prior probability for the realization of the two states of the world (HL or LH) are equal to half, and is common knowledge to all individuals. Below we describe the details of the two sides' decisions in seeking a match, utilities derived from a match, and the timing of the game.

[2] Klumpp (2009) lets agents to be solely horizontally differentiated, and studies the structure of stable assignment under transferable and non-transferable utility. Our two-dimensional model allows one to examine individual's strategic trade-off for the value from a match's horizontal attribute and vertical attribute when matching is decentralized.

3.1 Senders

We allow randomness in a sender's both horizontal and vertical attributes: Ex-ante, a sender's horizontal location x follows a uniform distribution $U[0,1]$, while his vertical attribute has equal chance of being either H or L. Before sending any matching request, senders' two dimensions of attributes are independently realized according to the common prior. Each sender observes his own horizontal location and vertical type, as well as the two receiver's vertical types. However, he does not observe any information of the other sender, but can only use the common prior as his belief for the other sender's information. This setup is in line with many real world peer-to-peer matching contexts, where a platform user (such as a passenger on BlaBlaCar or a traveler on Airbnb) only observes the match options on the other side, but does not know whom he is competing with. Based on the information and the belief, each sender chooses one receiver to send an matching request to.

A sender gets utility $U_S(\delta, v_R^J)$ from a match with a type i-receiver, $i \in \{H, L\}$, at a distance of δ:

$$U_S(\delta, v_R^J) = \alpha \cdot (1-\delta) + (1-\alpha) \cdot v_R^J \tag{1}$$

where $\alpha \in (0,1)$ and v_R^j is the receiver's vertical attribute. Specifically, we assume $v_R^L = 0$ and $v_R^H = 1$.

The utility function reflects two things captured by this model: First, a sender values both dimensions of attributes of a match: Horizontally, the sender gets a higher utility if the receiver is closer to him; Vertically, the sender is better off if he is matched with an H type receiver. Second, along the horizontal dimension, a receiver is valued differently depending on the sender's location x and the resulting distance δ. However, along the vertical dimension, all senders' preferences are aligned in the sense that the H-receiver is preferred to the L-receiver. The parameter α measures how much a sender cares about the match's horizontal closeness relative to the the match's vertical quality.

If a sender is not matched with any receiver, the sender chooses his outside option, the utility of which is normalized to 0.

3.2 Receivers

One L-receiver and one H-receiver are located at two ends of a unit line. The prior probability of L-receiver being located at the left end is equal to half. Each receiver observes her own type and location, and the information of the senders who send her a request. The value $U_R(\delta, v_S^I)$ from matching with a type i-sender, $i \in \{H, L\}$, at a distance of δ, is

$$U_R(\delta, v_S^I) = \beta \cdot (1 - \delta) + (1 - \beta) \cdot v_S^I \tag{2}$$

where $\beta \in (0, 1)$ captures the weight a receiver puts on a match's horizontal value. Consistent with the sender side, we assume $v_S^L = 0$ and $v_S^H = 1$. Note that same as senders, each receiver also derives value from a match's both horizontal attribute and vertical attribute. However, the weight a receiver puts on these two dimensions does not necessarily agree with the sender side.

If a receiver does not receive any request, she chooses her outside option which, same as the sender side, yields 0 utility. If a receiver gets only one request, she will accept it as she is better off compared to choosing the outside option. If a receiver gets two requests, she chooses the one which has a higher match value to her.

3.3 Timing

The game unfolds in three stages. In the first stage, the locations and the types of the two senders and the two receivers are realized. In the second stage, each sender chooses one receiver to send a request to. In the last stage, each receiver chooses the best request she receives, if any. A match is formed if the sender's request is accepted, and utilities to each party are realized. If a sender gets rejected by a receiver, or a receiver does not get any request, then the individual chooses the outside option.

We highlight two points of our model. First, the game we consider is a one-round matching game. This is suitable in modeling matching context where individuals have high delay cost, or when the market is thin. For example, the number of BlaBlaCar drivers who travel between certain cities on a certain day is usually limited. If a passenger is rejected in one round, it is very likely that he can only choose the outside option (e.g. taking the train). Second, we do not allow senders to request multiple receivers at the same time. Many platforms forbid a sender to simultaneously approach multiple receivers, or at least make it costly to senders. In the case of Airbnb, a booking request is required to add payment information, and once accepted the card will be charged in full for the reservation. Therefore, sending requests to multiple hosts at the same time is costly and will be warned of the risk of double booking by Airbnb.

For ease of exposition, in the following analysis we use H-request to refer to a request sent from an H-sender, and correspondingly L-request a request sent from an L-sender.

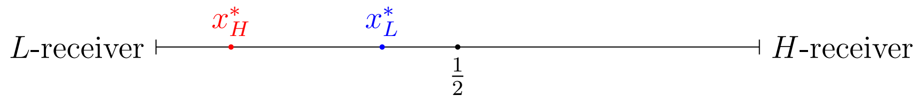

Figure 2: Thresholds of Sender Strategy

4 Full Information Matching

We first derive the Bayesian Nash Equilibrium of this two-stage game. We call this section "Full Information Matching" because each sender observes both receivers' full information before sending a request. Likewise, each receiver observes the full information of the senders who sent her a request. In later sections, we consider the possibility of platform withholding part of agents' information in its platform design. Given that the L-receiver has an equal chance of being located at either $x = 0$ or $x = 1$, and because of symmetry, we focus on the equilibrium where the L-receiver is located at $x = 0$. The H-receiver, as a result, is located at $x = 1$.

We only characterize the two senders' equilibrium strategies, as a receiver's strategy in the last stage is simply to choose the best request she has received (if she has received one or more). A sender's strategy depends on his belief on the other sender's type and strategy. In equilibrium, the belief needs to be consistent with the common prior of the sender's type and location, as well as the strategy each type of sender uses.

Proposition 1. *For a type $I \in \{L, H\}$ sender, there exists a threshold x^*_I, such that he chooses the L-receiver if he is located at $x \in [0, x^*_I)$, and the H-receiver otherwise. Furthermore,*

1. $x^*_H < x^*_L < \frac{1}{2}$
2. *It is always true that $x^*_L > 0$. However, $x^*_H = 0$ if α is below certain threshold.*
3. *As β increases, x^*_H weakly increases, while x^*_L always decreases.*
4. *As α increases, x^*_H always increases, while x^*_L decreases iff $\frac{1}{3} \le \alpha < \frac{1}{\sqrt{6}}$ and $\beta \le \frac{1+3\alpha}{3+3\alpha}$*

If receivers are not vertically differentiated, a sender's strategy is only driven by his horizontal distance with each receiver. Intuitively, he chooses the closer receiver, suggesting $x^*_I = \frac{1}{2}$, $I \in \{L, H\}$. However, in the current context where receivers are vertically differentiated and senders also gain utility from the receiver's vertical quality, we observe $x^*_I < \frac{1}{2}$, $I \in \{L, H\}$. This implies that both receivers are willing to chase the high-type receivers (remember that the H-receiver is located at $x = 1$), though doing so sacrifices some utility from a match's horizontal attribute. The inequality $x_H^* < x_L^*$ further suggests that the incentive of chasing high-quality receiver is even stronger for a sender who himself is of high-quality. The stronger incentive of chasing the H-receiver results from an H-sender's competitive advantage: compared to an L-sender,an H-sender has a higher chance of being accepted by the H-receiver, because receivers also value a sender's vertical quality.

We make a second observation that $x_L^* > 0$ always holds. That is, no matter how much an L-sender values a receiver's vertical quality, he chooses the low-quality receiver if the H-receiver is too far away. However, an H-sender, given high enough valuation for a receiver's quality (i.e., α is below a certain threshold), will always chase the H-receiver despite their distance in between. The divergence in senders' strategies again reflects H-sender's competitive advantage: an L-sender, because of a higher chance of getting rejected by the receiver, does not find it optimal to chase the H-receiver if the horizontal match is poor. In contrast, an H-sender can capitalize on his competitive advantage to always choose the H-receiver if he highly values the receiver's vertical quality.

An H-sender's competitive advantage comes from receivers' valuation for senders' vertical quality, which is mediated by β, the parameter which measures the weight receivers put on a match's horizontal dimension versus vertical dimension. Interestingly, if receivers care more about senders' horizontal distance (β increases), we observe a divergence in senders' strategies: an H-sender become less motivated to chase the H-receiver (x_H^* increases), while an L-sender becomes more motivated to chase the H-receiver (x_L^* decreases). To understand this, notice that the increase in receivers' valuation for a sender's horizontal attribute weakens an H-sender's advantage. As a result, an H-sender's incentive of chasing the H-receiver is attenuated. Anticipating less competition from H-senders, an L-sender becomes more incentivized to chase the H-receiver as the chance of getting rejected becomes smaller.

The last point of the proposition shows how a change in sender's own preference affects his matching strategy. If senders put more weight on receivers' horizontal attribute (i.e. α increases), one may expect that the incentive of chasing a distant H-receiver becomes weaker. Such incentive indeed is weakened for an H-sender (i.e. x_H^* always increases), but surprisingly, may become even stronger for an L-sender (i.e. x_L^* may decrease). In other words, even when an L-sender starts to care more about a match's horizontal closeness, he may switch from choosing the closer L-receiver to choosing the more distant H-receiver. To understand this, note that an increase in α has two opposing effects on an

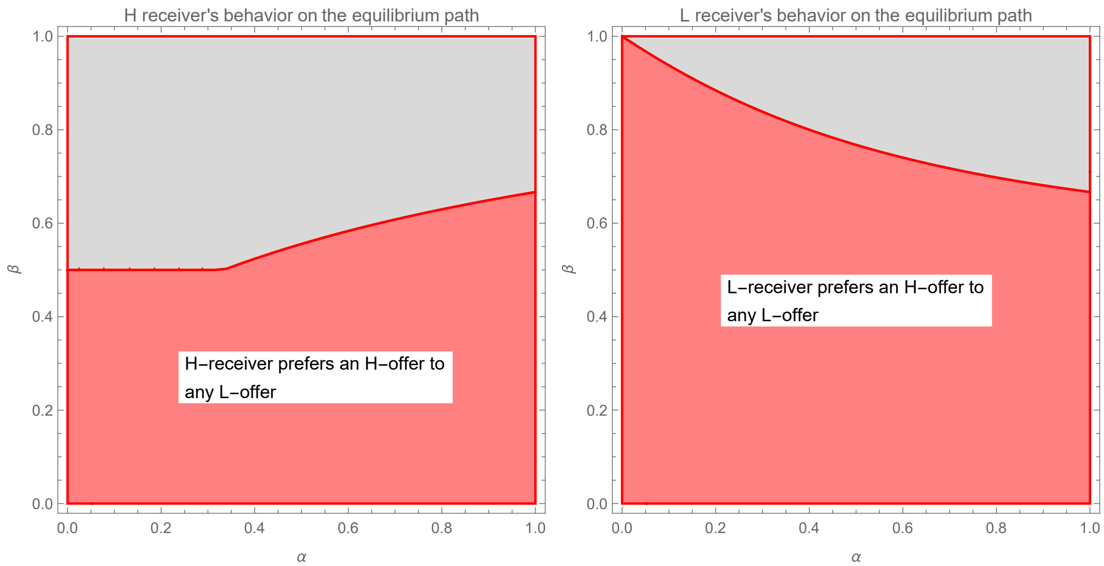

Figure 3: Receiver Strategies in Full Information Matching Scheme

L-sender's strategy. The direct effect is from the preference change: a higher valuation for receivers' horizontal closeness makes the distant H-receiver less attractive to an L-sender. However, there is an indirect effect: because H-senders are also less interested in chasing the H-receiver, the competition for H-receiver decreases, which leads to a higher chance of an L-sender's request being accepted by the H-receiver. The net outcome hinges on which effect dominates. We find that the indirect dominates when α is in a medium range, while β is small. The conditions imply that senders' valuation for both dimensions of a receiver's attribute is balanced, but receivers highly value senders' vertical quality so that the competition strongly favors the high-type sender.

Proposition 1 describes each sender's equilibrium strategy. Recall that a receiver's strategy is mechanical: she chooses the best request from what she received. The following proposition characterizes a receiver's behavior in the full information matching equilibrium, and Figure 3 provides a graphical illustration.

Proposition 2. *Upon receiving an H-request, a J-receiver ($J \in \{L, H\}$) never accepts an L-request if β is below certain threshold $\beta^J(\alpha)$. Furthermore, $\frac{\partial \beta^H(\alpha)}{\partial \alpha} \geq 0$ while $\frac{\partial \beta^L(\alpha)}{\partial \alpha} < 0$.*

Same as senders, receivers also value a match's vertical quality in addition to the horizontal closeness. Proposition 2 shows the condition for the extreme case: a receiver prefers an H-request, regardless of its horizontal value, to any L-request. Not surprisingly, this happens when β is below certain threshold, i.e. the receiver strongly values a sender's vertical quality. However, as α increases, the thresholds for the two types of receivers behave differently: the threshold for the H-receiver increases, but that for the L-receiver decreases. To see the reason, recall from Proposition 1, an increase in α makes an H-sender's strategy less distorted by the receiver's vertical quality, i.e. $\frac{\partial x_H^*}{\partial \alpha} > 0$. Note that the request from an H-sender located at x_H^* is the worst H-request for both types of receivers. As x_H^* increases, the worst H-request for the H-receiver becomes better in terms of horizontal value, but that for the L-receiver becomes worse in terms of horizontal value. To an H-receiver, because of the increase in the worst H-request's horizontal value, she still prefers an H-request to any L-request even if she starts to care more about the match's horizontal attribute ($\frac{\partial \beta^H(\alpha)}{\partial \alpha} \geq 0$). An L-receiver, because now can receive a worse H-request, needs a higher weight on a match's vertical value so that for her an H-request still dominates any L-request ($\frac{\partial \beta^L(\alpha)}{\partial \alpha} < 0$).

These two propositions depict a clear picture of sender's equilibrium strategy and receiver's behavior on the equilibrium path. In an one-dimensional matching context where individuals on both sides only care about the match's horizontal value, one can imagine that each sender chooses the closer receiver, and each receiver picks the request from the closer sender. The two propositions show that individual's valuation for a match's vertical value adds in some distortion. First, senders' valuation for a match's vertical quality creates the incentive of chasing a distant receiver. Second, receivers' valuation for a match's vertical quality makes such incentive differ for the two types of senders: the senders who themselves are of high quality have a stronger incentive to chase a distant receiver. From a platform's perspective, such distortion can be harmful as it makes senders' requests too concentrated to the H-receiver, but eventually the H-receiver can only be matched with one sender. The key of the distortion is that individuals' valuation for a match's vertical quality makes their preference more similar to each other. Understanding this, a platform can consider withholding users' vertical information to alleviate the distortion on individuals' behavior. In the next section, we follow this idea and examine two possible information designs.

5 Information Design

The analysis in the last section suggests that a platform can correct the matching distortion by withholding individual's vertical information. The question then is which side's vertical information to hide. We examine two possible designs. Under the first design, the platform withholds all senders' vertical information, so that receivers can only select sender's request based on their horizontal distance. Under the second design, the platform withholds all receivers' vertical information, so that a sender chooses whom to send a request to based their horizontal distance.

5.1 Design 1: Withholding The Senders' Vertical Information

If a receiver cannot observe a sender's vertical attribute, she is not able to tell the sender's vertical quality from a request [3]. Therefore, upon receiving two requests, a receiver accepts the one from the closer sender to maximize her expected utility. WLOG, we focus on the situation where the L-receiver is located at $x = 0$ and the H-receiver at $x = 1$. The following lemma summarizes the Bayesian Nash Equilibrium of this game.

Lemma 1. *When the platform withholds senders' vertical information from receivers, there exists a threshold $x^*_{D1} = \frac{\alpha}{1+\alpha}$, such that a sender, regardless of his vertical type, chooses the L-receiver is he is located at $x \in [0, x^*_{D1})$, and the H-receiver otherwise. Furthermore, $x^*_H < x^*_{D1} < x^*_L < \frac{1}{2}$.*

Intuitively, if receivers cannot distinguish an H-senders from an L-senders, an H-sender loses his competitive advantage and thus is less motivated to chase the H-receiver ($x^*_H < x^*_{D1}$). An L-sender, facing less competition from H-senders, is more incentivized to chase the H-receiver ($x^*_{D1} < x^*_L$). Moreover, the incentives of chasing the H-receivers are aligned for both types of senders.

One thing to notice is that even a platform withholds sender's information, the matching distortion is not fully eliminated, as $x^*_{D1} < \frac{1}{2}$, and requests still tend to concentrate to the H-receiver. This is because though senders are not vertically differentiated in receivers' eyes, senders are driven by their inherent preference to chase the H-receiver. Next we examine how the design affects the welfare of each types of individuals. The detailed discussion on the total number of matches are in Section 7.

Proposition 3. *Compared with full information matching, the platform withholding sender's vertical information*

- *always hurts an H-sender, but benefits an L-sender.*
- *always hurts an H-receiver, and hurts an L-receiver if and only if α is large.*

[3] In the appendix we show that a separating equilibrium where a receiver can infer the sender's vertical quality from the request's location cannot exist, and therefore $x^*_H = x^*_L$ in Design 1.

Consistent with one's intuition, when sender's vertical information is obscured, an H-sender is hurt due to the loss of competitive advantage, and an L-sender becomes better off as his weakness in the vertical dimension is not exposed to receivers.

The interesting welfare change happens to the opposite side. Not too surprisingly, the H-receiver becomes worse off if she cannot see the sender's vertical quality. One should understand that this is not only because the H-receiver cannot choose a request based on the sender's vertical information (direct effect), but also because the change in senders' strategies disfavors the H-receiver (indirect effect). Specifically, $x_H^* < x_{D1}^*$ implies that H-receiver's likelihood of receiving an H-request decreases, while $x_{D1}^* < x_L^*$ implies that her chance of receiving an L-request increases. These two effects, both being negative, reduce the H-receiver's welfare. One may expect that the L-receiver is better off because of the H-receiver's welfare loss. However, this intuition fails when α is large. The reason is that the L-receiver's welfare is also affected by two, but opposing effects. The direct effect still hurts the L-receiver as she also cannot judge a request's vertical quality. The indirect effect, however, favors the L-receiver, because to her the odds of receiving an H-request increases while that of receiving an L-request decreases. The direct effect only dominates the indirect effect when α is large. To see the reason, note when senders care a lot about the match's horizontal closeness, their strategies under full information matching is largely driven by the horizontal distance with each receiver. When their vertical information is obscured, their strategies won't significantly change, which leads to small indirect effect on the L-receiver's welfare. As a result, the L-receiver's welfare change is dominated by the negative direct effect. Therefore, despite H-receiver's welfare loss, the L-receiver can also be worse off when the platform withholds senders' vertical information.

5.2 Design 2: Withholding Receiver's Vertical Information

Now we consider the other possible information design: withholding the receiver's vertical information. The next lemma shows the Bayesian Nash Equilibrium of the game.

Lemma 2. *When the platform withholds receivers' vertical information, there exists a threshold $x_{D2}^* = \frac{1}{2}$, such that a sender chooses the receiver at 0 if his location $x \in [0, x_{D2}^*)$, and the receiver at 1 otherwise.*

If a sender cannot distinguish the vertical attribute of the two receivers, he will choose the receiver based only on the horizontal distance, i.e., choose the closer one. Interestingly, unlike in the first design where an H-sender loses his competitive advantage, under the current design an H-sender still has competitive advantage, as receivers can distinguish senders' quality and prefers an H-request, all else equal. However, the H-receiver is not able to leverage his competitive advantage to choose the receiver. Because of this, the thresholds for both senders move to $\frac{1}{2}$, which fully alleviate the distortion caused by senders' incentive of chasing an distant H-sender. We then analyze the design's influence on the each side's user welfare.

Proposition 4. *Compared with full information disclosure, the platform withholding receivers' vertical information*

- *always hurts an H-sender, but benefits an L-sender if β is small.*
- *always hurts an H-receiver, but benefits an L-receiver.*

It is easy to understand the welfare effect on receivers when their vertical information is hidden: the H-receiver's welfare decreases as he cannot be identified by the senders; the L-receiver is better off because now she is equally attractive as the H-receiver.

On the opposite side, one may believe that senders are worse off if they cannot make request decisions based on the receiver's vertical information. Interestingly, this intuition only applies to an H-sender, but is not always true to an L-sender. To see the reason, note that to an L-sender, in addition to the negative effect that he cannot chase the H-sender, there is also an positive indirect effect. This effect comes from L-sender's higher chance of being matched with the H-receiver, due to the H-receiver's inability of attracting H-requests. The net effect for the L-receiver's welfare thus depends on which effect dominates. If receivers highly values the sender's vertical quality (i.e. β is small), an H-sender's incentive of chasing the H-receiver is strong due to his big competitive advantage. As a result, when receivers' vertical information is withheld, the H-receiver's chance of getting an H-request significantly decreases, resulting in a large positive indirect effect on an L-sender's welfare. Therefore, even not able to observe receivers' vertical attribute, an L-receiver can still be better off.

6 Centralized Matching

Before discussing the influence of the two information designs on the number of successful matches, we consider the possibility of platform centralizing the matching process by assigning the closest sender to each receiver (Figure 4). It can be easily seen that under this assignment rule, (1) the number of matches is maximized and equals two, and (2) the (ex ante) total user welfare is maximized because the total horizontal mismatch (distance)

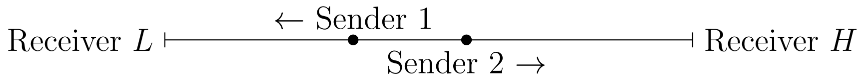

Figure 4: Assignment Rule in Centralized Matching

is minimized. Therefore, the centralized matching scheme here can be interpreted as the first best result for this matching model. In reality, centralized matching may need huge computing and processing power and thus is very costly to implement. However, it is still useful to theoretically understand the effect on the welfare of individuals from both sides.

Under this centralized matching scheme, a sender located at $x \in [0, 1]$ would expect to get utility $U_S(x) = [\alpha(1-x) + (1-\alpha) \cdot 0](1-x) + [\alpha x + (1-\alpha) \cdot 1]x$. The uncertainty comes from the unknown location of the other sender. Integrating with respect to x, we can get the *ex ante* expected sender utility on the platform before the location is realized:

$$EU_S^c = \int_0^1 U_S(x)f(x)dx = \frac{1}{2} + \frac{\alpha}{6}$$

Note that under centralized matching, the *ex ante* utility for both types of senders are the same. This is because the centralization rule we consider here does not discriminate senders by their vertical type.

In terms of receiver welfare, because of symmetry we can focus on the receiver located at $x = 0$. Under the centralization rule, the receiver has equal chance of matching with an H-sender or with an L-sender. Therefore, her utility from a match's vertical quality is $(1-\beta) \cdot \frac{1}{2}$. On the horizontal dimension, as the receiver is matched with the closer sender, the her expected utility from a match's horizontal closeness is $\int_0^1 \beta(1-t)g(t)dt$. Here $g(t) = 2(1-t)$ is the probability density function of $\min\{x_1, x_2\}$, where x_1 and x_2 are independent random variables which follow the uniform distribution $U[0, 1]$. We then obtain a receiver's *ex ante* expected utility, which also does not depend on the receiver's type

$$EU_R^c = \int_0^1 \beta(1-t)2(1-t)dt + (1-\beta) \cdot \frac{1}{2} = \frac{1}{2} + \frac{\beta}{6}$$

By comparing EU_R^c and EU_S^c, one can see that both senders' and receivers' ex-ante utility take the same expression: one half plus the weight for on a match horizontal's attribute adjusted by a factor $\frac{1}{6}$. This reflects the generality of our model: by fixing receivers' location and letting senders' location be random, the model is not creating any asymmetry between the senders and receivers. The next result is about the influence of centralized matching on user welfare, compared with when information is fully disclosed across sides. We use Figure 5 and Figure 6 to visualize the results.

Proposition 5. *Compared with full information matching, centralized matching:*

- *benefits an H-sender if and only if either α or β is large, and always benefits an L-sender.*
- *benefits an H-receiver if and only if either α or β is large, and always benefits an L-receiver.*

In contrast to the two previous designs which can hurt some L-type users, the centralized matching always benefit L-type users, regardless of which sides they are at. Centralized matching benefits an L-type user in two ways: first, it removes L-users' competitive disadvantage to H-type users. Second, it eliminates the possibility of getting unmatched.

Recall that both Design 1 and Design 2 hurt H-type users on both sides. The centralized matching, though does not favor H-type individuals in its assignment rule and thus makes an H-type individual's competitive advantage vanish, can in fact benefit an H-type individual on both sides. This is because centralized matching secures each user a match, and the benefit of this can outweigh an H-type user's loss of competitive advantage if either α or β is large. To see the reason, note that when at least one side cares a lot about the match's horizontal distance, then either a sender is less motivated to chase a H-receiver, or a receiver is less motivated to accept the request from a distant H-sender. Under both situations, an H-type individual's competitive advantage is weak, while the chance of getting unmatched is higher (compared to when both sides highly values a match's vertical quality). Therefore, the centralized matching, though eliminating an H-type user's already small competitive advantage, still benefits an H-type user by significantly increases his/her chance of getting matched. As a result, centralized matching, unlike the two decentralized designs, can benefit an H-type user from both sides.

7 Comparison Across Designs

We have studied the matching processes under three decentralized matching schemes (i.e. *full information matching, design 1*, and *design 2*) and the *centralized matching* scheme. In this section, we compare across these four matching schemes on two dimensions: the total number of matches and the sender/receiver welfare. A profit-driven platform that takes commissions out of matched transactions may care more about the total number of matches, while a growing platform that wishes to attract users by providing better matching service may care more about the welfare of senders or receivers (or both). Instead of restricting the objective of a platform, we provide comparisons in multiple dimensions for flexible considerations.

7.1 Number of Matches

Under decentralized matching schemes, match failure is caused by the sender-side competition when two requests are sent to the same receiver. Therefore, centralized matching achieves the maximum number of matches as the assignment rule ensures that no match failure occurs. In addition, a complete comparison in the number of matches across schemes is given in the following proposition

Proposition 6. *In terms of the number of successful matches,*
Centralized Matching $\geq$ Decentralized Design 2 $\geq$ Decentralized Design 1 $\geq$ Decentralized Full Information Matching.

Compared to withholding senders' vertical information (Design 1), withholding receivers' vertical information (Design 2) results in a higher number of matches. To see the reason, note that each match is initiated by a sender, thus the design which better corrects the distortion caused by a sender chasing the H-receiver further increases the total number of matches. In Design 1, H-senders lose their competitive advantage. Although it makes a H-sender less motivated to chase the H-receiver, it also further incentives each L-sender to chase the H-receiver. Consequently, the marginal sender of each type converges to $x_H^* = x_L^* = \frac{\alpha}{1+\alpha} < \frac{1}{2}$, implying that the matching distortion caused by a sender chasing the H-receiver is not fully eliminated, as senders' requests still crowd towards the H-receiver. In Design 2, H-senders cannot utilize their competitive advantage to chase the H-receiver. As a result, each sender's decision depends entirely on his horizontal distance to each receiver ($x_H^* = x_L^* = \frac{1}{2}$). The matching distortion from competition is fully eliminated, so the number of matches is higher than that in Design 1. Lastly, when user information is fully disclosed, sender-side competition is the most intense, because H-senders have full competitive advantage and can also fully utilize it to chase the H-receiver. Therefore, full information matching leads to the smallest transaction volume. This result suggest that a decentralized platform with the highest level of information transparency might end up with the lowest number of matches.

7.2 Welfare

We then compare user welfare across the four matching schemes. Given that each individual has equal chance of being H-type or L-type, we calculate the average user welfare on each side by averaging the welfare of each side's H-type individual and L-type individual derived from section 4, 5 and 6. Note that both senders' and receivers' welfare depend on the parameters α and β. However, for senders, α determines the preference weight on a match's two dimensions, thus affects senders' welfare directly, whereas β determines a receiver's probability of accepting a sender's request, thus affects senders' welfare indirectly. Similarly, for receivers, α has an indirect effect and β has a direct effect on their welfare. For ease of exposition, we fix the parameter that has direct welfare effect at different levels, and vary the parameter that has indirect welfare effect. This gives us Figure 5 for sender side welfare comparison, and Figure 6 for receiver side welfare comparison.

From both figures, one can observe that centralized matching maximizes the average user welfare on both sides. To see this, note that the centralized assignment rule eliminates match failure, so that the welfare gain from a match's vertical value is fully realized. Furthermore, it also assigns each receiver the closer sender, so that the welfare loss from the horizontal mismatch is minimized. A second observation is that each side's average user welfare is not affected by the parameter that has indirect welfare effect, but only varies by the parameter that has direct welfare effect. This is because the centralized assignment rule eliminates each user's strategic behavior, which makes the parameter that has indirect welfare effect irrelevant. With this knowledge, we use each side's average user welfare under centralized matching as the reference line, and examine the welfare loss under the three decentralized matching schemes caused by peer competition and individual strategic play.

Figure 5 first shows that among the three decentralized matching scheme, the average sender welfare is the highest under Design 1 (withholding senders' vertical information), and is the lowest under Design 2 (withholding receivers' vertical information). This suggests that on average, senders benefit if the platform makes receivers non-strategic (Design 1), but are hurt if the platform forces themselves to be non-strategic (Design 2). Second, when α approaches 0, though dominated by centralized matching, Design 1 can make sender's welfare as close as that under centralized matching. To understand this, note that when sender's utility is mainly derived from the match's vertical quality, then with probability almost equal to 1, a sender will chase the H-receiver. The H-receiver, because cannot tell sender's vertical quality, picks the closer sender, which largely aligns with the centralized assignment rule. Third, as α approaches 1, the average sender welfare

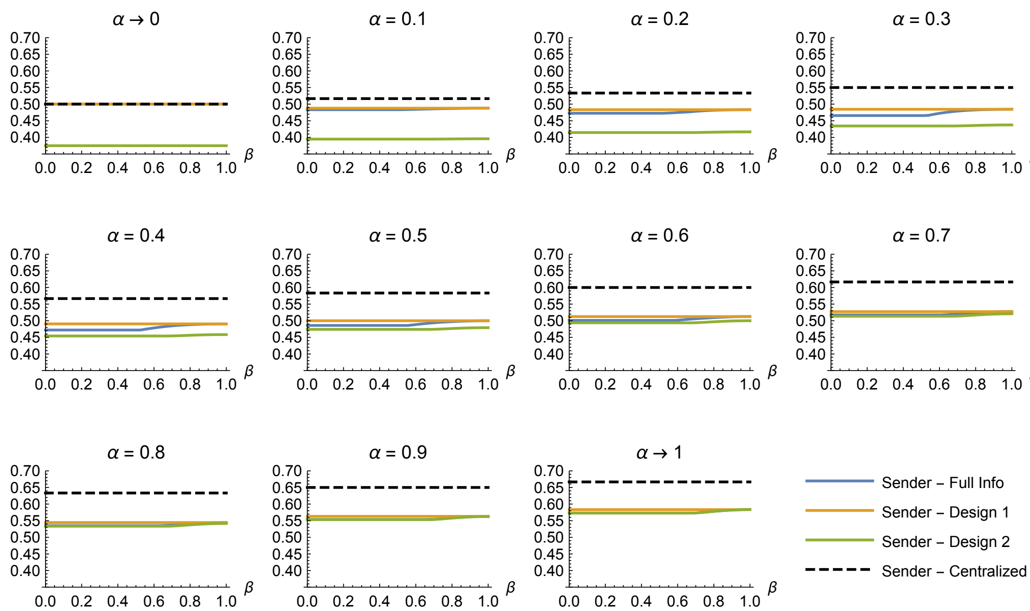

Figure 5: Average Sender Welfare in Four Matching Schemes

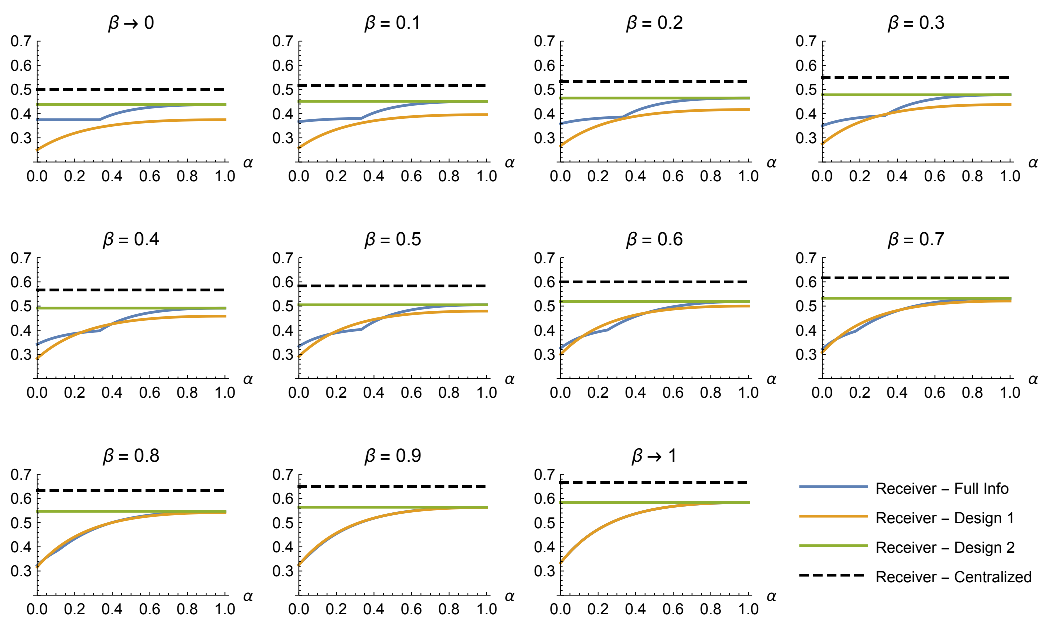

Figure 6: Average Receiver Welfare in Four Matching Schemes

converges under Full information Matching and Design 2. Intuitively, if senders mainly care about the receiver's horizontal distance, then concealing receiver's vertical quality has little influence on a sender's strategy, thus little effect on the average sender welfare.

Turning attention to Figure 6, we find that the average receiver's welfare is not well ordered under the three decentralized matching scheme. In particular, Design 2 (withholding receivers' vertical inforation) always dominates the other two, as it makes senders' behavior non-strategic. However, Design 1 (withholding senders' vertical information) and Full Information Matching is ambiguous on which one is better for the receivers. This is because Design 1 lowers H-sender's incentive of chasing the H-receiver, but in the meanwhile increases L-sender's incentive of chasing the H-receiver. As a result, the welfare effects on the H-receiver and the L-receiver can be opposite, and the magnitude of the two effects depend on α and β. The figure shows that Design 1 dominates Full Information design only when α and β are in middle ranges. However, as β approaches 1, the average receiver welfare under Design 1 and that under Full Information scheme converge: if receivers care little about the senders' quality, withholding this information has little effect on the average receiver welfare.

8 Conclusion

The purpose of this paper is to theoretically understand how platform users trade off the value from a match's horizontal attribute and vertical attribute, and how a decentralized platform can increase the number of matches through information design, as well as the influence of each design on user welfare. Toward this goal, we build a model where platform users, belonging to either sender side or receiver side depending on who initiate a matching request, proactively seek a match given the user information a platform discloses across sides. All users are differentiated in two dimensions of attributes, while the platform controls what user information to reveal across sides.

We start with a baseline model where the platform discloses full user information across sides. Though individuals gain match value from two dimensions, we find that their valuation for the vertical dimension creates distortion in a matching process. Specifically, all senders have incentive of chasing a distant high-quality receiver at the expense of horizontal value, making multiple sender requests crowding to one receiver. Such incentive, due to receivers' valuation for senders' vertical attribute, is even stronger for senders who themselves are of high quality. One may believe that the distortion will be attenuated if senders care more about a match's horizontal closeness. Contrary to this intuition, we find that even if senders put a higher weight on a match's horizontal closeness, those who are of low vertical quality may switch from choosing an closer receiver to choosing a distant receiver. The reason is that low-quality senders need to take into account the competition from high-quality senders when choosing a receiver. When a match's horizontal value becomes more important to senders, low-quality senders face stronger competition if choosing a closer low-quality receiver, but can have a higher chance of being accepted if choosing a distant high-quality receiver. Therefore, they may switch to the distant receiver to avoid competition. In terms of receiver behavior, we find that if the horizontal match value becomes more important to senders, a low-quality receiver needs to care more about senders' vertical quality, so that she prefers a high-quality sender to any low-quality sender, regardless of the senders' horizontal match value. A high-quality receiver, by contrast, still prefers a high-quality sender to any low-quality sender, even if she cares more about a match's horizontal value.

Understanding that individuals' preference for a match's vertical quality creates matching distortion, one can conjecture that platform can correct the distortion by withholding users' vertical information. Following this idea, we examine a platform's two possible information designs. We show that the first design, in which the platform withholds senders' vertical information to eliminate high-quality senders' advantage, alleviates but not fully removes the matching distortion, as senders still have incentives to chase a distant high-quality receiver. The second design, in which the platform hides receivers' vertical information to make high-quality senders unable to use their advantage, is more effective in correcting the distortion, as all senders can only make their request decisions based on a match's horizontal value. In terms of user welfare, one may be inclined to think that withholding users' vertical information hurts high type users while benefits low type users. Interestingly, we show that withholding one side's vertical information can also hurt the low-quality user on the opposite side. We also examine the centralized matching which can be viewed as the first-best outcome in the sense that it maximizes total number of matches and total user welfare . Surprisingly, unlike the two designs which always hurt high-quality users, the centralized matching scheme, though does not favor high-quality users in its assignment rule, can still benefit all high-quality users, given that at least one side of users care a lot about a match's horizontal value.

Lastly, we compare the transaction volume (i.e. successful matches) and average user welfare across different matching schemes. Among the three decentralized matching schemes, we show that withholding receivers' vertical information is the best matching scheme to maximize transaction volume. A decentralized peer-to-peer platform may think that more information to users can facilitate cross-side matching. Our result shows that full information matching leads to the lowest number of matches, because the distortion which causes requests concentrating to one receiver is the most severe. In terms of user welfare, we show that a decentralized platform can improve the average user welfare on one side by withholding the focal side's vertical information.

Managerial Implications. Considering that centralized matching for large scale platforms can be very costly and algorithm-demanding, analyses from this paper provide managerial implications on how a platform with cost concern or being algorithm-immature may choose a decentralized matching scheme to save the cost while getting as close as possible to the centralized outcome.

For platforms that focus more on maximizing transaction volume, such us ride-sharing platforms that take commissions out of matched rides or online dating platforms that charge service fee from matched interactions, withholding or reduce the amount of vertical dimension information (like exact car model on ride-sharing platforms or a person's income on online-dating platforms) is an optimal information design for decentralized matching. In particular, our analysis show that withholding vertical information of receivers (e.g. drivers on BlaBlaCar) yields more matches than withholding vertical information of senders.

Some other platforms focus on offering superior user experience from matches. For example, home-sharing platforms value the variety of houses listed on the platform as well as user experience from each stay. For these platforms, senders (e.g. the guests for home-sharing platforms) can benefit from the information design of withholding sender vertical information, and receivers (e.g. the hosts for home-sharing platforms) benefit from the information design of withholding receiver vertical information. If a home-sharing platform is at the growing stage and wants to attract more travelers/guests, our analysis suggests that Design 1 is a better choice. If the platform wants to grow the receiver side (home-owners/hosts), then Design 2 can better achieve the goal.

Directions for further research. Decentralized platforms which rely on two sides of users to proactively form matches is an evolving phenomenon. In this paper, we focus on a platform's information design problem when both sides of users value a match's horizontal attribute and vertical attribute. Future research can extend this model to explore the competition between decentralized platforms. In particular, the intensity of competition between platforms may vary by the two sides, which is another force that can shape a competing platform's information design strategy. In addition, competing platforms may differentiate by using different information design schemes, so that each platform can focus on one side and avoid head-to-head competition. The current research also provides theoretical foundation for empirical vitrifaction and measurement with field data.

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Main Appendix

In this main appendix, we present the detailed proof of each lemma and proposition in the main paper. For the ease of exposition, we suppress some lengthy expressions in this main appendix, and make them available in the supplementary appendix.

Proof for Proposition 1.

We first prove the following claim, and then use it to prove the proposition.

Claim 1. *If a type-I sender located at x prefers the L-receiver, then any type-I sender located at $x' < x$ prefers the L-receiver; If a type-I sender located at x prefers the H-receiver, then any type-I sender located at $x'' > x$ prefers the H-receiver.*

Proof. A type-I sender located at x prefers the L-receiver if and only if the expected utility he derives from choosing the L-receiver is (weakly) larger than that from choosing the H-receiver, i.e. $EU_S(v_R^L|x) \geq EU_S(v_R^H|x)$, where $EU_S(v_R^L|x) = U_S(v_R^L|x) \cdot \Pr(x \text{ accepted by L-receiver})$ and $EU_S(v_R^H|x) = U_S(v_R^H|x) \cdot \Pr(x \text{ accepted by H-receiver})$.

Now consider a type-I sender located at $x' < x$. Note that $U_S(v_R^L|x') = \alpha(1-x') > \alpha(1-x) = U_S(v_R^L|x)$, and $\Pr(x' \text{ accepted by L-receiver}) \geq \Pr(x \text{ accepted by L-receiver})$ as the L-receiver strictly prefers the closer sender who is located at x'. As a result, $EU_S(v_R^L|x') \geq EU_S(v_R^L|x)$. Similarly, $U_S(v_R^H|x') = \alpha x' + (1-\alpha) < \alpha x + (1-\alpha) = U_S(v_R^H|x)$, and $\Pr(x' \text{ accepted by H-receiver}) \leq \Pr(x \text{ accepted by H-receiver})$ as the H-receiver strictly prefers the closer sender who is located at x. Therefore, $EU_S(v_R^L|x') \leq EU_S(v_R^L|x)$. Taken together, we have $EU_S(v_R^L|x') \geq EU_S(v_R^L|x) \geq EU_S(v_R^L|x) \geq EU_S(v_R^L|x')$, which implies that any type-I sender located at $x' < x$ prefers L-receiver. This completes the proof of the first part of the claim.

The second part of the claim can be proved by taking the exact same approach. ■

Claim 1, together with the continuity of each individual's utility function, imply that senders' strategy can be summarized by the equilibrium thresholds (x_L^*, x_H^*), such that a type I-sender $I \in \{L, H\}$ chooses the L-receiver if he is located at $x < x_I^*$, , and the H-receiver if $x > x_I^*$. If $x_I^* \in (0, 1)$, then the type-I sender located at $x = x_I^*$ is indifferent between choosing the L-receiver and the H-receiver. Otherwise, the type-I sender located at $x = x_I^*$ prefers the L-receiver (if $x_I^* = 1$) or the H-receiver (if $x_I^* = 1$). It is easy to see that $x_I^* \neq 1$: Suppose type-I sender at $x = 1$ chooses the L-receiver. However, by doing so, he gets zero utility, and can be strictly better off by choosing the H-receiver. So see this, note that the H-receiver prefers the type-I sender at $x = 1$ to any type-I sender at $x < 1$. This means the possibility of type-I's request being accepted by the H-receiver is positive. Furthermore, type-I sender also derives positive utility if matched with the H-receiver. Taken together, type-I sender's expected utility of choosing the H-receiver is positive. As a result $x_I^* = 1$, $I \in \{L, H\}$, cannot hold.

Now consider the receiver's strategy. The receiver chooses the outside option if she does not receive any request. If receiving only one request, the receiver accepts it. If receiving two requests from the same type senders, the receiver accepts the closer one. If receiving an H-request at a distance δ_H and an L-request at a distance δ_L, accepting the H-request yields a utility $U_R(\delta_H, v_S^H) = \beta(1-\delta_H) + (1-\beta)\cdot 1$, while accepting the L-request yields a utility of $U_R(x_L, v_S^L) = \beta(1-\delta_L)+(1-\beta)\cdot 0$. The receiver chooses the L-request if $U_R(\delta_L, v_S^L) > U_R(\delta_H, v_S^H)$, i.e. $\delta_H - \delta_L > \frac{1-\beta}{\beta}$, and the H-request otherwise. We use $\delta_0 \equiv \frac{1-\beta}{\beta}$ to denote this threshold.

Next, depending on the magnitude of x_L^* and x_H^*, we have the following three cases.

Case 1: $0 < x_I^* < 1$**,** $I \in \{L, H\}$**,**

In this situation, a type-I sender, $I \in \{L, H\}$, located at x_I^*, is indifferent between choosing the L-receiver and choosing the H-receiver. With a slight abuse of notations, we also use x_I^* to denote the type-I sender located at $x = x_I^*$. Thus we have $EU_S(v_R^L|x_L^*) = EU_S(v_R^H|x_L^*)$ and $EU_S(v_R^L|x_H^*) = EU_S(v_R^H|x_H^*)$, where

$$EU_S(v_R^L|x_L^*) = \big(\alpha(1-x_L^*) + (1-\alpha)\cdot 0\big)\cdot \Pr(x_L^* \text{ accepted by } L\text{-receiver}) \tag{A1}$$
$$EU_S(v_R^H|x_L^*) = \big(\alpha\cdot x_L^* + (1-\alpha)\cdot 0\big)\cdot \Pr(x_L^* \text{ accepted by } H\text{-receiver}) \tag{A2}$$
$$EU_S(v_R^L|x_H^*) = \big(\alpha(1-x_H^*) + (1-\alpha)\cdot 0\big)\cdot \Pr(x_H^* \text{ accepted by } L\text{-receiver}) \tag{A3}$$
$$EU_S(v_R^H|x_H^*) = \big(\alpha\cdot x_H^* + (1-\alpha)\cdot 0\big)\cdot \Pr(x_H^* \text{ accepted by } H\text{-receiver}) \tag{A4}$$

Note that there are four probabilities in these four expected utility expressions. To calculate them, we need to consider the following 7 possible sub cases depending on x_L^*, x_H^*, and δ_0:

	Conditions
Case 1.1:	$\{x_H^* \le \delta_0, 1 - x_H^* \le \delta_0\}$
Case 1.2:	$\{x_H^* \le \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* \le \delta_0\}$
Case 1.3:	$\{x_H^* > \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* \le \delta_0\}$
Case 1.4:	$\{x_H^* > \delta_0, 1 - x_H^* \le \delta_0, x_L^* - x_H^* \le \delta_0\}$
Case 1.5:	$\{x_H^* \le \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* > \delta_0\}$
Case 1.6:	$\{x_H^* > \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* > \delta_0\}$
Case 1.7:	$\{x_H^* > \delta_0, 1 - x_H^* \le \delta_0, x_L^* - x_H^* > \delta_0\}$

- *Case 1.1:* $\{x_H^* \le \delta_0, 1 - x_H^* \le \delta_0\}$

  In this situation, both L-receiver and H-receiver prefer an H-request to any L-request. To see this, note that the best L-request the L-receiver can receive is from $x = 0$, which is (weakly) dominated by the worst H-request she can receive, which is from $x = x_H^*$, because $x_H^* \le \delta_0$. The same reasoning applies to the H-receiver because $1 - x_H^* \le \delta_0$.

  Now consider the sender x_L^*. If choosing the L-receiver, his request is accepted if and only if the other sender is an H-sender and is located at $x \in [x_H^*, 1]$, or the other sender is an L-sender and is located at $x \in (x_L^*, 1]$. Therefore, $\Pr(x_L^*$ accepted by L-receiver$) = \frac{1}{2} \cdot (1 - x_H^*) + \frac{1}{2} \cdot (1 - x_L^*)$. Similarly, $\Pr(x_L^*$ accepted by H-receiver$) = \frac{1}{2} \cdot x_H^* + \frac{1}{2} \cdot x_L^*$.

Next consider sender x_H^*. Note that he is preferred by both receivers to any L-sender. If choosing the L-receiver, his request is accepted if and only if the other sender is an H-sender and is located at $x \in [x_H^*, 1]$, or the other sender is an L-sender. Thus we have $\Pr(x_H^*$ accepted by L-receiver$) = \frac{1}{2} \cdot (1 - x_H^*) + \frac{1}{2}$. Similarly, $\Pr(x_H^*$ accepted by H-receiver$) = \frac{1}{2} \cdot x_H^* + \frac{1}{2}$.

We then substitute these four probability expressions into the expected utility functions, and solve the equations $EU_S(v_R^L | x_L^*) = EU_S(v_R^H | x_L^*)$ and $EU_S(v_R^L | x_H^*) = EU_S(v_R^H | x_H^*)$. This gives us

$$x_L^* = \frac{6\alpha^2 - \alpha + 1}{(1 + 2\alpha)(1 + 3\alpha)} \tag{A5}$$

$$x_H^* = \frac{3\alpha - 1}{3\alpha + 1} \tag{A6}$$

Then we need to ensure that $x_I^* \in (0, 1)$, $I \in \{L, H\}$, as well as the conditions $\{x_H^* \leq \delta_0, 1 - x_H^* \leq 0\}$ are satisfied. The solution set for the these inequalities, which we name as Rn1, are

$$Rn1 = \{0 < \beta \leq \frac{1}{2}, \frac{1}{3} < \alpha < 1\} \cup \{\frac{1}{2} < \beta < \frac{2}{3}, \frac{3\beta - 1}{3 - 3\beta} \leq \alpha < 1\} \tag{A7}$$

- *Case 1.2:* $\{x_H^* \leq \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* \leq \delta_0\}$

  In this situation, the L-receiver still prefers an H-request to any L-request. The H-sender, when receiving both an H-request and an L-request, will accept the L-request if $(1 - x_H) - (1 - x_L) = x_L - x_H > \delta_0$.

  For sender x_L^*, same as in Case 1.1, we have $\Pr(x_L^*$ accepted by L-receiver$) = \frac{1}{2} \cdot (1 - x_H^*) + \frac{1}{2} \cdot (1 - x_L^*)$. Also note the inequality $x_L^* - x_H^* \leq \delta_0$ implies that x_L^* will not be chosen by the H-receiver if she has received an H-request. Thus we have $\Pr(x_L^*$ accepted by H-receiver$) = \frac{1}{2} \cdot x_H^* + \frac{1}{2} \cdot x_L^*$, same as in Case 1.1.

  For sender x_H^*, $\Pr(x_H^*$ accepted by L-receiver$) = \frac{1}{2} \cdot (1 - x_H^*) + \frac{1}{2}$ is the same as in Case 1.1. However, $\Pr(x_H^*$ accepted by H-receiver$)$ is different. If sender x_H^* chooses the H-receiver, his request is accepted if the other sender is H-sender and is located at $x \in [0, x_H^*)$, or the other sender is L-sender and is located at $x \in [0, x_H^* + \beta_0]$. Therefore, $\Pr(x_H^*$ accepted by H-receiver$) = \frac{1}{2} \cdot x_H^* + \frac{1}{2} \cdot (x_H^* + \delta_0)$.

  Substituting these probability expressions into the expected utility functions and solving the equations $EU_S(v_R^L | x_L^*) = EU_S(v_R^H | x_L^*)$ and $EU_S(v_R^L | x_H^*) = EU_S(v_R^H | x_H^*)$, we have

$$x_L^* = \frac{\alpha + 2\beta + 4\alpha^2\beta - \sqrt{4\beta^2 + 4\alpha\beta^2 + (\alpha + 2\alpha\beta)^2}}{2\alpha(1+2\alpha)\beta} \tag{A8}$$

$$x_H^* = \frac{-\alpha - 2\beta + \sqrt{4\beta^2 + 4\alpha\beta^2 + (\alpha + 2\alpha\beta)^2}}{2\alpha\beta} \tag{A9}$$

Then ensuring the conditions $x_I^* \in (0,1)$, $I \in \{L, H\}$ and $\{x_H^* \leq \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* < \delta_0\}$ are satisfied, we get the solution set, which we name as Rn2, as

$$Rn2 = \{\frac{1}{2} < \beta \leq \frac{2}{3}, \frac{1-\beta}{1+\beta} < \alpha < \frac{3\beta - 1}{3 - 3\beta}\} \cup \{\frac{2}{3} < \beta < 1, \frac{1-\beta}{1+\beta} < \alpha \leq \frac{3\beta(1-\beta)}{4\beta - 2}\} \tag{A10}$$

- *Case 1.3:* $\{x_H^* > \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* \leq \delta_0\}$

  In this situation, when receiving both an H-request and an L-request, the L-receiver accepts the L-request if $x_H - x_L > \delta_0$, while the H-receiver accepts the L-request if $x_L - x_H > \delta_0$.

  For sender x_L^*, same as in Case 1.2, we can derive $\Pr(x_L^*$ accepted by L-receiver$) = \frac{1}{2} \cdot (1 - x_H^*) + \frac{1}{2} \cdot (1 - x_L^*)$ and $\Pr(x_L^*$ accepted by H-receiver$) = \frac{1}{2} \cdot x_H^* + \frac{1}{2} \cdot x_L^*$.

  For sender x_H^*, if choosing the L-receiver, his request is accepted if the other sender is H-sender and is located at $x \in (x_H^*, 1]$, or the other sender is L-sender and is located at $[x_H^* - \delta_0, 1]$, thus $\Pr(x_H^*$ accepted by L-receiver$) = \frac{1}{2} \cdot (1 - x_H^*) + \frac{1}{2} \cdot (1 - (x_H^* - \delta_0))$. Same as in Case 1.2, $\Pr(x_H^*$ accepted by H-receiver$) = \frac{1}{2} \cdot x_H^* + \frac{1}{2} \cdot (x_H^* + \delta_0)$.

  Substituting these expressions into the expected utility functions and solving $EU_S(v_R^L|x_L^*) = EU_S(v_R^H|x_L^*)$ and $EU_S(v_R^L|x_H^*) = EU_S(v_R^H|x_H^*)$, we have

$$x_L^* = \frac{2\alpha(2\alpha + 2\beta - 1) - \beta + 1}{2(1+2\alpha)(\alpha+\beta)} \tag{A11}$$

$$x_H^* = \frac{2\alpha + \beta - 1}{2(\alpha + \beta)} \tag{A12}$$

  Then checking the conditions $x_I^* \in (0,1)$, $I \in \{L, H\}$ and$\{x_H^* > \delta_0, 1 - x_H^* > \delta_0, x_L^* - x_H^* \leq \delta_0\}$, we have the solution set, named Rn3, as

$$Rn3 = \{\frac{2}{3} < \beta < 1, \frac{3\beta(1-\beta)}{4\beta - 2} < \alpha < 1\} \tag{A13}$$

- *Case 1.4 - Case 1.7*

For all these cases, we follow the exact same steps in Case 1.1 - 1.3 to first derive the four probability expressions, second solve the equations $EU_S(v_R^L|x_L^*) = EU_S(v_R^H|x_L^*)$ and $EU_S(v_R^L|x_H^*) = EU_S(v_R^H|x_H^*)$, and last check the conditions in each case. There are no solutions of x_I^*, $I \in \{L, H\}$, that satisfy all the constraints in each sub cases.

Case 2: $x_H^* = 0$

In this situation, an H-sender always prefers the H-receiver, regardless of their horizontal distance. If $x_H^* = 0$, then it must be $x_L^* > 0$: suppose $x_L^* = 0$, then the L-sender at $x = 0$ derives 0 utility by choosing the the H-receiver, because the probability of getting accepted by H-receiver is 0. He can be strictly better off by switching to the L-receiver, as he will be accepted by L-receiver with probability 1, and derives positive utility α. By continuity, there exists $\epsilon > 0$, such that an L-sender, if located at $x \in [0, \epsilon)$, can be strictly better off by switching to the L-receiver, which leads to a contradiction.

If $x_H^* = 0$ and $x_L^* > 0$, then we have $EU_S(v_R^L|x_L^*) = EU_S(v_R^H|x_L^*)$ and $EU_S(v_R^L|x_H^* = 0) \leq EU_S(v_R^H|x_H^* = 0)$. Note that $EU_S(v_R^L|x_L^*)$ and $EU_S(v_R^H|x_L^*)$ are given in (A1) and (A2). $EU_S(v_R^L|x_H^* = 0)$ and $EU_S(v_R^H|x_H^* = 0)$ can be written as

$$EU_S(v_R^L|x_H^* = 0) = \big(\alpha \cdot (1 - 0) + (1 - \alpha) \cdot 0\big) \cdot \Pr(x_H^* = 0 \text{ accepted by } L\text{-receiver}) \tag{A14}$$

$$EU_S(v_R^H|x_H^* = 0) = \big(\alpha \cdot (1 - 1) + (1 - \alpha) \cdot 1\big) \cdot \Pr(x_H^* = 0 \text{ accepted by } H\text{-receiver}) \tag{A15}$$

Again, to calculate the four probabilities, we need to consider the following 3 sub cases, depending on x_L^* and δ_0

	Conditions
Case 2.1:	$\{x_L^* \leq \delta_0\}$
Case 2.2:	$\{x_L^* > \delta_0\}$

- *Case 2.1* $x_L^* \leq \delta_0$

  Consider the sender x_L^*. If he chooses the L-receiver, he is accepted if the other sender is an H-sender, because all an H-sender never chooses the L-receiver, or the other sender is an L-sender and is located at $x \in (x_L^*, 1]$. Therefore, $\Pr(x_L^* \text{ accepted by } L\text{-receiver}) = \frac{1}{2} + \frac{1}{2} \cdot (1 - x_L^*)$. If x_L^* chooses the H-receiver, he will be accepted only if the other sender is an L-sender and is located at $x \in [0, x_L^*)$, as the H-receiver prefers the H-sender at $x = 0$ to x_L^* because $x_L^* - 0 = x_L^* < \delta_0$, thus prefers any H-sender to x_L^*. Therefore, $\Pr(x_L^* \text{ accepted by } H\text{-receiver}) = \frac{1}{2} \cdot x_L^*$. Substituting these two probability expressions to (A1) and (A2) and solving $EU_S(v_R^L|x_L^*) = EU_S(v_R^H|x_L^*)$, we have

$$x_L^* = \frac{2\alpha}{2\alpha + 1} \tag{A16}$$

Next consider the sender $x_H^* = 0$. If he chooses the L-receiver, his request is accepted for sure, thus $\Pr(x_H^* = 0 \text{ accepted by } L\text{-receiver }) = 1$. If he chooses the H-receiver, $\Pr(x_H^* = 0 \text{ accepted by } H\text{-receiver}) = \frac{1}{2}$, if $\delta_0 \geq 1$, as in this case $x_H^* = 0$ is accepted by the H-receiver if and only if the other sender is an L-sender ; Or $\Pr(x_H^* = 0 \text{ accepted by } H\text{-receiver}) = \frac{1}{2} \cdot \delta_0$, if $\delta_0 < 1$, as $x_H^* = 0$ is accepted by the H-receiver if and only if the other sender is an L-sender and is located at $x \in [0, \delta_0)$. These two situations can be written together as $\Pr(x_H^* = 0 \text{ accepted by } H\text{-receiver}) = \frac{1}{2} \cdot \min\{1, \delta_0\}$.

Substituting $\Pr(x_H^* = 0 \text{ accepted by } H\text{-receiver}) = \frac{1}{2}$ and $\Pr(x_H^* = 0 \text{ accepted by } H\text{-receiver}) = \frac{1}{2} \cdot \min\{1, \delta_0\}$ in (A14) and (A15), and solve the inequality $x_L^* \leq \delta_0$ and $EU_R(v_R^L | x_H^* = 0) \leq EU_R(v_R^H | x_H^* = 0)$, we derive the solution set, named Rn4, as

$$Rn4 = \{0 < \beta \leq \frac{1}{2}, 0 < \alpha \leq \frac{1}{3}\} \cup \{\frac{1}{2} < \beta < 1, 0 < \alpha \leq \frac{1-\beta}{1+\beta}\} \tag{A17}$$

- *Case 2.2* $x_L^* > \delta_0$

  Following the exact same step in Case 2.1 to first derive x_L^*, and check the conditions $x_L^* > \delta_0$ and $EU_R(v_R^L | x_H^* = 0) \leq EU_R(v_R^H | x_H^* = 0)$ we find no solution x_L^* that satisfy all the constraints.

Case 3: $x_L^* = 0$

Following the same approach in Case 2, it can be shown that $x_L^* = 0$ is never an equilibrium.

Combine the above analysis, the equilibrium $\{x_L^*, x_H^*\}$ are given as follows.

$$x_L^* = \begin{cases} \frac{6\alpha^2-\alpha+1}{(1+2\alpha)(1+3\alpha)} & \text{if } \{\alpha,\beta\} \in Rn1 \\ \frac{\alpha+2\beta+4\alpha^2\beta-\sqrt{4\beta^2(1+\alpha)+(\alpha+2\alpha\beta)^2}}{2\alpha(1+2\alpha)\beta} & \text{if } \{\alpha,\beta\} \in Rn2 \\ \frac{2\alpha(2\alpha+2\beta-1)-\beta+1}{2(1+2\alpha)(\alpha+\beta)} & \text{if } \{\alpha,\beta\} \in Rn3 \\ \frac{2\alpha}{2\alpha+1} & \text{if } \{\alpha,\beta\} \in Rn4 \end{cases} \tag{A18}$$

$$x_H^* = \begin{cases} \frac{3\alpha-1}{3\alpha+1} & \text{if } \{\alpha,\beta\} \in Rn1 \\ \frac{-\alpha-2\beta+\sqrt{4\beta^2(1+\alpha)+(\alpha+2\alpha\beta)^2}}{2\alpha\beta} & \text{if } \{\alpha,\beta\} \in Rn2 \\ \frac{2\alpha+\beta-1}{2(\alpha+\beta)} & \text{if } \{\alpha,\beta\} \in Rn3 \\ 0 & \text{if } \{\alpha,\beta\} \in Rn4 \end{cases} \tag{A19}$$

where Rn1, Rn2, Rn3, and Rn4 are given in (A7), (A10),(A13), and (A17). One can also verify that Rn1, Rn2, Rn3, and Rn4 are disjoint sets, whose union is $\{0 < \alpha < 1, 0 < \beta < 1\}$. Figure

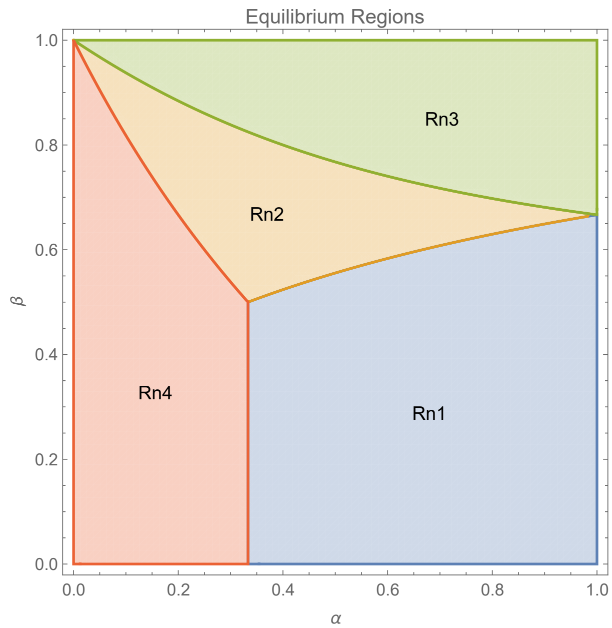

Figure 1: Equilibrium Regions in The Parameter Space

1 visualizes this fact. This means that the analysis is complete and the equilibrium is always unique.

With (A18) and (A19), one can easily verify the statements in Proposition 1. This completes the proof. $\square$

Proof for Proposition 2.

We first derive $\beta^H(\alpha)$. According to the analysis in Preposition 1, the H-receiver prefers an H-request to any L-request, when $\{\alpha,\beta\} \in Rn1$, or when $\{\alpha,\beta\} \in Rn9$ and meanwhile $\delta_0 \geq 1$. Note that $Rn1$ can be rewritten as $\{\frac{1}{3} \leq \alpha < 1, 0 < \beta \leq \frac{1+3\alpha}{3+3\alpha}\}$, while $Rn9 \cap \{\delta_0 \geq 1\} = \{0 < \alpha < \frac{1}{3}, 0 < \beta \leq \frac{1}{2}\}$. Therefore, $\beta^H(\alpha)$ is given as

$$\beta^H(\alpha) = \begin{cases} \frac{1}{2} & \text{if } 0 < \alpha < \frac{1}{3} \\ \frac{1+3\alpha}{3+3\alpha} & \text{if } \frac{1}{3} \leq \alpha < 1 \end{cases} \tag{A20}$$

It can be easily seen that $\frac{\partial \beta^H(\alpha)}{\partial \alpha} \geq 0$, $\forall \alpha \in (0,1)$.

We then derive $\beta^L(\alpha)$. From the the analysis in Preposition 1, we know that upon receiving an H-request, the L-receiver never accepts an L-request when $\{\alpha,\beta\} \in Rn1 \cup Rn2 \cup Rn4$. Note that $Rn1 \cup Rn2 \cup Rn4$ can be written as $\{\alpha \in (0,1), 0 < \beta \leq \frac{3-4\alpha+\sqrt{9+16\alpha^2}}{6}\}$. Therefore, we have $\beta^L(\alpha)$ as

$$\beta^L(\alpha) = \frac{3 - 4\alpha + \sqrt{9+16\alpha^2}}{6} \tag{A21}$$

and $\frac{\partial \beta^L(\alpha)}{\partial \alpha} = \frac{8\alpha - 2\sqrt{9+16\alpha^2}}{3\sqrt{9+16\alpha^2}} < 0$, $\forall \alpha \in (0, 1)$. This completes the proof.

$\square$

Proof for Lemma 1.

If the platform withholds senders' vertical information, then receivers can only make decisions based on the senders' horizontal distance. We consider the equilibrium that is characterized by thresholds for each type of senders, x_L^* and x_H^*. This proof consists of two parts. In the first part, we prove that $x_L^* = x_H^*$. In the second part, we derive the expression of this threshold.

Part 1: Prove that $x_L^* = x_H^*$

First, it is easy to see that $x_I^* \neq 1$, $I \in \{L, H\}$. Otherwise, the type-I sender located at $(1-\epsilon, 1]$ can strictly be better off by switching from the L-receiver to the H-receiver.

Suppose that $x_L^* > x_H^*$ in equilibrium. This means that type I-sender, $I \in \{L, H\}$, strictly prefers the L-receiver if he is located at $x \in [0, x_I^*)$, while strictly prefers the H-receiver if he is located at $x \in (x_I^*, 1]$. Furthermore, if the L-receiver gets a request from $x \in [0, x_H^*)$, she does not know the sender's type but can only use the common prior that the sender has equal chance of being L or H. However, if the request is from $x \in (x_H^*, 1]$, then the L-receiver infers that it is from an L-sender Similarly, the H-receiver cannot tell the sender's type if the request is from $x \in (x_L^*, 1]$, but can infer that the sender is H-type if the request is from $x \in [0, x_L^*)$

If a receiver gets a request at a distance δ', but does not know the sender's type, the expected utility from accepting the request is $\beta(1-\delta') + (1-\beta)\cdot(\frac{1}{2}\cdot v_S^L + \frac{1}{2}\cdot v_S^H) = \beta(1-\delta') + \frac{1-\beta}{2}$. On the other hand, if he gets an H-request at a distance δ'', the expected utility from accepting the request is $\beta(1-\delta'') + (1-\beta)$. Comparing these two expressions, it can be seen that upon receiving an ambiguous request and an H-request, the receiver accepts the H-request if and only if $\delta'' - \delta' \leq \frac{1-\beta}{2\beta}$. We use $\delta_{00} \equiv \frac{1-\beta}{2\beta}$ to denote this threshold.

Now consider $x_{L1} = x_L^* - \epsilon_1$ and $x_{L2} = x_L^* + \epsilon_2$. Both ϵ_1 and ϵ_2 are sufficiently small, so that $x_{L1} \in (x_H^*, x_L^*)$, $x_{L2} \in (x_L^*, 1]$, and both x_{L1} and x_{L2} are enough close to x_L^*. With a slight abuse of notation, we also use x_{L1} to denote an L-sender located at $x = x_{L1}$, and x_{L2} an L-sender located at $x = x_{L2}$.

Sender x_{L1} strictly prefers choosing the L-receiver, which means $EU_S(v_R^L|x_{L1}) > EU_S(v_R^H|x_{L1})$. If x_{L1} chooses the L-receiver, he is accepted if the other sender is L-type and is located at $(x_{L1}, 1]$, or if the other sender is H-type and is located at $[x_H^*, 1]$. If x_{L1} deviates to the H-receiver, he will be inferred as an H-sender. Therefore, he is accepted if the other sender is L-type and is located at $[0, \min\{x_{L1} + \delta_{00}, 1\}]$, or if the other is H-type, and is located at $[0, x_{L1}) \cup [x_L^*, \min\{x_{L1} + \delta_{00}, 1\}]$.

Therefore, we have the expressions for $EU_S(v_R^L|x_{L1})$ and $EU_S(v_R^H|x_{L1})$,

$$
\begin{aligned}
EU_S(v_R^L|x_{L1}) &= \alpha(1-x_{L1})\cdot \Pr(x_{L1}\text{accepted by } L\text{-receiver}) \\
&= \alpha(1-x_{L1})\cdot[\frac{1}{2}(1-x_{L1})+\frac{1}{2}(1-x_H^*)] \\
EU_S(v_R^H|x_{L1}) &= \Big(\alpha\cdot x_{L1}+(1-\alpha)\Big)\cdot \Pr(x_{L1}\text{ accepted by } H\text{-receiver}) \\
&= \Big(\alpha\cdot x_{L1}+(1-\alpha)\Big)[\frac{1}{2}\cdot(\min\{x_{L1}+\delta_{00},1\})+\frac{1}{2}\cdot(x_{L1}+(\min\{x_{L1}+\delta_{00},1\}-x_L^*))]
\end{aligned}
$$

Note that $\lim_{\epsilon_1\to 0} EU_S(v_R^L|x_{L1}) = \alpha(1-x_L^*)\cdot(1-\frac{x_L^*}{2}-\frac{x_H^*}{2})$ and $\lim_{\epsilon_1\to 0} EU_S(v_R^H|x_{L1}) = \Big(\alpha\cdot x_L^*+(1-\alpha)\Big)\cdot\min\{x_L^*+\delta_{00},1\}$. Because $EU_S(v_R^L|x_{L1}) > EU_S(v_R^H|x_{L1})$, and by continuity, we have $\lim_{\epsilon_1\to 0} EU_S(v_R^L|x_{L1}) \geq \lim_{\epsilon_1\to 0} EU_S(v_R^H|x_{L1})$.

Now consider sender x_{L2}. He strictly prefers choosing the H-receiver, which means $EU_S(v_R^L|x_{L2}) < EU_S(v_R^H|x_{L2})$. Similarly, we can derive

$$
\begin{aligned}
EU_S(v_R^L|x_{L2}) &= \alpha(1-x_{L2})\cdot \Pr(x_{L2}\text{ accepted by } L\text{-receiver}) \\
&= \alpha(1-x_{L2})\cdot[\frac{1}{2}\cdot(1-x_L^*)+\frac{1}{2}\cdot(1-x_H^*)] \\
EU_S(v_R^H|x_{L2}) &= \Big(\alpha\cdot x_{L2}+(1-\alpha)\Big)\cdot \Pr(x_{L2}\text{ accepted by } H\text{-receiver}) \\
&= \Big(\alpha\cdot x_{L2}+(1-\alpha)\Big)\cdot[\frac{1}{2}\cdot x_{L2}+\frac{1}{2}\cdot(\max\{x_{L2}-\delta_{00},x_H^*\})]
\end{aligned}
$$

Note that $\lim_{\epsilon_2\to 0} EU_S(v_R^L|x_{L2}) = \alpha(1-x_L^*)\cdot(1-\frac{x_L^*}{2}-\frac{x_H^*}{2})$ and $\lim_{\epsilon_2\to 0} EU_S(v_R^H|x_{L2}) = \Big(\alpha\cdot x_L^*+(1-\alpha)\Big)\cdot\frac{1}{2}(x_L^*+\max\{x_L^*-\delta_{00},x_H^*\})$. Because $EU_S(v_R^L|x_{L2}) < EU_S(v_R^R|x_{L2})$, and by continuity, we have $\lim_{\epsilon_2\to 0} EU_S(v_R^L|x_{L2}) \leq \lim_{\epsilon_2\to 0} EU_S(v_R^H|x_{L2})$.

Now we have two inequalities: $\lim_{\epsilon_1\to 0} EU_S(v_R^L|x_{L1}) \geq \lim_{\epsilon_1\to 0} EU_S(v_R^H|x_{L1})$ and $\lim_{\epsilon_2\to 0} EU_S(v_R^L|x_{L2}) \leq \lim_{\epsilon_2\to 0} EU_S(v_R^H|x_{L2})$. Notice that $\lim_{\epsilon_1\to 0} EU_S(v_R^L|x_{L1}) = \lim_{\epsilon_2\to 0} EU_S(v_R^L|x_{L2}) = \alpha(1-x_L^*)\cdot(1-\frac{x_L^*}{2}-\frac{x_H^*}{2})$. Therefore, $\lim_{\epsilon_1\to 0} EU_S(v_R^H|x_{L1}) \leq \lim_{\epsilon_1\to 0} EU_S(v_R^H|x_{L2})$, which is

$$
\Big(\alpha\cdot x_L^*+(1-\alpha)\Big)\cdot\min\{x_L^*+\delta_{00},1\} \leq \Big(\alpha\cdot x_L^*+(1-\alpha)\Big)\cdot\frac{1}{2}(\max\{x_L^*-\delta_{00},x_H^*\}+x_L^*) \tag{A22}
$$

This inequality holds if and only if $\min\{x_L^*+\delta_{00},1\} \leq \frac{1}{2}(\max\{x_L^*-\delta_{00},x_H^*\}+x_L^*)$. However, because $x_H^* < x_L^*$, we have $\frac{1}{2}(\max\{x_L^*-\delta_{00},x_H^*\}+x_L^*) < \frac{1}{2}(x_L^*+x_L^*) = x_L^* < \min\{x_L^*+\delta_{00},1\}$, thus a contradiction. Therefore, $x_L^* > x_H^*$ cannot hold in equilibrium.

Next we need to show that $x_H^* > x_L^*$ cannot hold in equilibrium. First notice that $x_L^* \neq 0$, otherwise, an L-receiver located at $[0,\epsilon)$ can be better off by switching to the L-receiver. Then we can use the exact same approach as in the case $x_L^* > x_H^*$ to show contradiction. Specifically, we use the decisions of one L-receiver located at $x = x_L^* - \epsilon_1$, and another L-receiver located at $x = x_L^* + \epsilon_2$, to derive an inequality, which cannot not hold if $x_H^* > x_L^*$. Because the proof follows exact the same steps in the case $x_L^* > x_H^*$, we omit the details.

Part 2: Derive the threshold $x_{D1}^* \equiv x_L^* = x_H^*$

We use x_{D1}^* to denote the common threshold for both type of senders. From Part 1, we know that $x_{D1}^* = 1$ cannot hold in equilibrium. Is is obvious that $x_{D1}^* = 0$ cannot hold in equilibrium, because otherwise a sender located at $x \in [0, \epsilon)$, regardless of his type, can be better off by switching to the L-receiver. Therefore, we have $x_{D1}^* \in (0, 1)$.

Because senders of both types use the same strategy, receivers cannot infer the sender's type from the request's horizontal location. As a result, receivers' optimal strategy is to choose the closer request to maximize the expected utility.

Consider the sender located at $x = x_{D1}^*$. He is indifferent between choosing the L-receiver and the H-receiver. If choosing the L-receiver, his request is accepted if the other sender, regardless of the type, is located at $x > x_{D1}^*$. Therefore, the expected utility of choosing the L-receiver is

$$EU_S(v_R^L|x_{D1}^*) = \alpha(1 - x_{D1}^*) \cdot [\frac{1}{2}(1 - x_{D1}^*) + \frac{1}{2}(1 - x_{D1}^*)] \tag{A23}$$

If choosing the H-receiver, his request is accepted if the other sender, regardless of the type, is located at $x < x_{D1}^*$. The expected utility of choosing he H-receiver is

$$EU_S(v_R^H|x_{D1}^*) = \Big(\alpha \cdot x_{D1}^* + (1 - \alpha)\Big) \cdot [\frac{1}{2} \cdot x_{D1}^* + \frac{1}{2} \cdot x_{D1}^*] \tag{A24}$$

By solving $EU_S(v_R^L|x_{D1}^*) = EU_S(v_R^H|x_{D1}^*)$, we obtain $x_{D1}^* = \frac{\alpha}{1+\alpha}$. Comparing x_{D1}^* with x_L^* and x_H^* as given in (A18) and (A19) , we have $x_H^* < x_{D1}^* < x_L^* < \frac{1}{2}$.

This completes the proof. □

Proof for Proposition 3.

We first derive the expected utility of each type of individuals under full information matching and under Design 1, and then make the comparison.

User welfare under full information matching:

We first calculate the senders' welfare, and then derive the receivers' welfare

- **Senders' welfare:** We separately calculate the welfare of an L-sender and an H-sender:

  1. L-sender's welfare:

  If an L-sender is located at $x_L \in [0, x_L^*)$, he chooses the L-receiver, and is accepted if and only if one of the following situations happens: (1) The other sender is L-type and is located at $x \in (x_L, 1]$; (2) The other sender is H-type and is located at $x \in (\min\{x_L + \delta_0, x_H^*\}, x_H^*)$, where $\delta_0 \equiv \frac{1-\beta}{\beta}$ as defined in Proposition 1 [1]; Or (3) the other sender is H-type and is

[1] As shown in Proposition 1, δ_0 is the minimum horizontal closeness advantage that an L-sender needs to have, so that he will be favored by a receiver over an H-sender.

located at $x \in [x_H^*, 1]$. One should notice that depending on the value of α, β, and x_L, the second situation not necessarily exists as $(\min\{x_L + \delta_0, x_H^*\}, x_H^*)$ may be a null set. Regardless of $(\min\{x_L + \delta_0, x_H^*\}, x_H^*)$ being a null set or not, the expected utility of an L-sender's with $x_L \in [0, x_L^*)$ can be written as

$$EU_{S_L}(x_L | x_L < x_L^*) = \alpha \cdot (1 - x_L) \cdot \left[\frac{1}{2}(1 - x_L) + \frac{1}{2}(1 - \min\{x_L + \delta_0, x_H^*\})\right] \tag{A25}$$

If the L-sender is located at $x_L \in [x_L^*, 1]$, he chooses the H-receiver, and is accepted if and only if :(1) The other sender is L-type and is located at $[0, x_L)$, or (2) the other sender is H-type and is located at $x \in [x_H^*, \max\{x_L - \delta_0), x_H^*\})$, or (3) the other sender is H-type and is located at $x \in [0, x_H^*)$. Depending on α, β, and x_L, the second situation may not exist. But regardless, the expected utility of an L-sender's with $x_L \in [x_L^*, 1]$ can be written as

$$EU_{S_L}(x_L | x_L \geq x_L^*) = \Big(\alpha \cdot x_L + (1 - \alpha)\Big) \cdot \left[\frac{1}{2} \cdot x_L + \frac{1}{2} \cdot \max\{x_L - \delta_0, x_H^*\})\right] \tag{A26}$$

Therefore, before the L-sender's location is realized, his ex-ante expected utility under full information matching is

$$EU_{S_L}^{Full} = \int_0^{x_L^*} EU_{S_L}(x_L | x_L < x_L^*) dx_L + \int_0^{x_L^*} EU_{S_L}(x_L | x_L \geq x_L^*) dx_L \tag{A27}$$

The thresholds x_L^* and x_H^*, as given in (A18) and (A19), vary across parameter regions. Hence the integrals vary across different parameter regions and have very complicated expressions. For the compactness of proof, the expression of $EU_{S_L}^{Full}$ is suppressed here and are left in the supplementary appendix.

2. H-sender's welfare:

Similarly, if an H-sender is located at $x_H \in [0, x_H^*)$, he chooses the L-receiver, and is accepted if (1) the other sender is H-type and is located at $x \in (x_H, 1]$, or (2) the other sender is L-type and is located at $x \in (\max\{x_H - \delta_0, 0\}, x_L^*)$, or (3) the other sender is L-type and is located at $x \in [x_L^*, 1]$. Therefore, the expected utility of an H-sender located at $x_H \in [0, x_H^*)$ is

$$EU_{S_H}(x_H | x_H < x_H^*) = \alpha \cdot (1 - x_H) \cdot \left[\frac{1}{2}(1 - x_H) + \frac{1}{2}(1 - \max\{x_H - \delta_0, 0\})\right] \tag{A28}$$

If the H-sender is located at $x_H \in [x_H^*, 1]$, he chooses the H-receiver, and is accepted if (1) the other sender is H-type and is located at $x \in [0, x_H)$, or (2) the other sender is L-type and is located at $x \in [x_L^*, \min\{x_H + \delta_0, 1\}]$, or (3) the other sender is H-type and is located at $x \in [0, x_L^*)$. Although the second situation may not exist, the expected utility of an

H-sender located at $x_H \in [x_H^*, 1]$ can be written as

$$EU_{S_H}(x_H|x_H \geq x_H^*) = \Big(\alpha \cdot x_H + (1-\alpha)\Big) \cdot \Big[\frac{1}{2} \cdot x_H + \frac{1}{2} \cdot \min\{x_H + \delta_0, 1\}\Big)\Big] \tag{A29}$$

Before the H-sender's location is realized, his ex-ante expected utility under full information matching is

$$EU_{S_H}^{Full} = \int_0^{x_H^*} EU_{S_H}(x_H|x_H < x_H^*)dx_H + \int_0^{x_H^*} EU_{S_L}(x_H|x_L \geq x_H^*)dx_H \tag{A30}$$

We leave the expression of $EU_{S_H}^{Full}$ in the supplementary appendix.

- **Receivers' welfare:**

We first calculate the welfare of an L-receiver and then the welfare of an H-receiver. Ex-ante, the probability of both senders being L-type is $\frac{1}{4}$, both senders being H-type is $\frac{1}{4}$, and one sender being H-type while the other being L-type is $\frac{1}{2}$. We use *LL*, *HH*, and *HL* to denote these three cases.

1. L-receiver's welfare:

In the case of *LL*, let x_{L1} and x_{L2} denote the two L-senders' locations, and define $x'_L \equiv \min\{x_{L1}, x_{L2}\}$. The L-receiver accepts the request from the sender at x'_L, if and only if $x'_L \in [0, x_L^*)$. Because x_{L1} and x_{L2} are independent and identically distributed as $U[0, 1]$, the probability density function (pdf) of x'_L can be derived as $g(x'_L) = 2(1-x'_L)$. Therefore, the L-receiver's expected utility given the *LL* case is

$$EU_{R_L}^{Full}(LL) = \int_0^{x_L^*} \beta(1-x'_L) \cdot 2(1-x'_L)dx'_L \tag{A31}$$

Similarly, given the HH case, the L-receiver's expected utility is

$$EU_{R_L}^{Full}(HH) = \int_0^{x_H^*} \Big(\beta(1-x'_H) + (1-\beta)\Big) \cdot 2(1-x'_H)dx'_H \tag{A32}$$

In the case of *HL*, let x_L and x_H denote the locations of the L-sender and H-sender respectfully. The L-receiver accepts the L-request, if and only if $x_L \in [0, x_L^*)$ and $x_H \in [\min\{x_L + \delta_0, x_H^*\}, 1]$, and he accepts the H-request if and only if $x_H \in [0, x_H^*]$ and $x_L \in [\min\{x_H - \delta_0, 0\}, 1]$. As a result, the L-receiver's expected utility given the *HL* case is

$$\begin{aligned} EU_{R_L}^{Full}(HL) = &\int_0^{x_L^*} \beta(1-x_L) \cdot (1 - \min\{x_L + \frac{1-\beta}{\beta}, x_H^*\})dx_L + \\ &\int_0^{x_H^*} [\beta(1-x_H) + (1-\beta)] \cdot (1 - \max\{x_H - \frac{1-\beta}{\beta}, 0\})dx_H \end{aligned} \tag{A33}$$

Therefore, the L-receiver's ex-ante expected utility under full information matching is

$$EU_{R_L}^{Full} = \frac{1}{4}EU_{R_L}^{Full}(LL) + \frac{1}{4}EU_{R_L}^{Full}(HH) + \frac{1}{2}EU_{R_L}^{Full}(HL) \tag{A34}$$

We leave the expression of $EU_{R_L}^{Full}$ in the supplementary appendix.

<u>2. H-receiver's welfare:</u>

Following the same steps as those in the L-receiver's case, one can derive that the H-receiver's welfare, given the LL, HH, and HL cases are

$$EU_{R_H}^{Full}(LL) = \int_{x_L^*}^{1} \beta x_L'' \cdot 2x_L'' dx_L'' \tag{A35}$$

$$EU_{R_H}^{Full}(HH) = \int_{x_H^*}^{1} [\beta x_H'' + (1-\beta)] \cdot 2x_H'' dx_H'' \tag{A36}$$

$$\begin{aligned} EU_{R_H}^{Full}(HL) = &\int_{x_L^*}^{1} \beta x_L \cdot \max\{x_L - \delta_0, x_H^*\} dx_L + \\ &\int_{x_H^*}^{1} [\beta x_H + (1-\beta)] \cdot \min\{x_H + \delta_0, 1\} dx_H \end{aligned} \tag{A37}$$

Here $x_L'' = \max\{x_{L1}, x_{L2}\}$ denotes the closer L-sender to the H-receiver when LL is the case, while $x_H'' = \max\{x_{H1}, x_{H2}\}$ denotes the closer H-sender to the $H-$-receiver when HH is the case. Then we have the H-receiver's ex-ante expected utility under full information matching

$$EU_{R_H}^{Full} = \frac{1}{4}EU_{R_H}^{Full}(LL) + \frac{1}{4}EU_{R_H}^{Full}(HH) + \frac{1}{2}EU_{R_H}^{Full}(HL) \tag{A38}$$

We leave the expression of $EU_{R_H}^{Full}$ in the supplementary appendix.

User welfare under Design 1

Same as the analysis of the full information matching, we first calculate the senders' welfare and then the receivers' welfare.

- **Senders' welfare:**

  From Lemma 1, we know that both type of senders use the same strategy. As a result, receivers cannot choose request based on the senders' vertical quality. This means both an H-sender and an L-sender have the same expected utility. We use EU_S^{D1} to denote both an H-sender and an L-sender's expected utility. If a sender is located at $x \in [0, x_{D1}^*)$, he chooses the L-receiver, and is accepted if the other sender is located at $x' \in (x, 1]$. If a sender is located at $x \in [x_{D1}^*, 1]$, he chooses the H-receiver, and is accepted if the other sender is located at $x' \in [0, x)$. Therefore,

$$
\begin{aligned}
EU_S^{D1} &= \int_0^{x^*_{D1}} \alpha(1-x)\cdot(1-x)dx + \int_{x^*_{D1}}^1 \Big(\alpha x + (1-\alpha)\Big)\cdot x dx \\
&= \frac{3+2\alpha+2\alpha^2}{6(1+\alpha)} \qquad \text{(A39)}
\end{aligned}
$$

- **Receivers' welfare:** We follow the same steps as those in the case of full information matching, and derive the expected utility for each type of receivers.

  For the L-receiver, let x'_L denote the closer sender in the case of LL, and x'_H the closer sender in the case of HH. In each of the three cases, we have

$$EU_{R_L}^{D1}(LL) = \int_0^{x^*_{D1}} \beta(1-x'_L)\cdot 2(1-x'_L)dx'_L \qquad \text{(A40)}$$

$$EU_{R_L}^{D1}(HH) = \int_0^{x^*_{D1}} [\beta(1-x'_H)+(1-\beta)]\cdot 2(1-x'_H)dx'_H \qquad \text{(A41)}$$

$$EU_{R_L}^{D1}(HL) = \int_0^{x^*_{D1}} \beta(1-x_L)\cdot(1-x_L)dx_L + \int_0^{x^*_{D1}} [\beta(1-x_H)+(1-\beta)]\cdot(1-x_H)dx_H \qquad \text{(A42)}$$

Therefore, the L-receiver's ex-ante expected utility is

$$
\begin{aligned}
EU_{R_L}^{D1} &= \frac{1}{4}EU_{R_L}^{D1}(LL) + \frac{1}{4}EU_{R_L}^{D1}(HH) + \frac{1}{2}EU_{R_L}^{D1}(HL) \\
&= \frac{6\alpha(1+\beta)+\alpha^2(3+\alpha)(3+\beta)}{6(1+\alpha)^3} \qquad \text{(A43)}
\end{aligned}
$$

For the H-receiver, let x''_L denote the closer sender in the case of LL, and x''_H the closer sender in the case of HH. Then we have

$$
\begin{aligned}
EU_{R_H}^{D1}(LL) &= \int_{x^*_{D1}}^1 \beta x''_L \cdot 2x''_L dx''_L \\
EU_{R_H}^{D1}(HH) &= \int_{x^*_{D1}}^1 [\beta x''_H + (1-\beta)]\cdot 2x''_H dx''_H \\
EU_{R_H}^{D1}(HL) &= \int_{x^*_{D1}}^1 \beta x_L \cdot x_L dx_L + \int_{x^*_{D1}}^1 [\beta x_H + (1-\beta)]\cdot x_H dx_H
\end{aligned}
$$

Therefore, the H-receiver's ex-ante expected utility is

$$\begin{aligned} EU_{R_H}^{D1} &= \frac{1}{4}EU_{R_H}^{D1}(LL) + \frac{1}{4}EU_{R_H}^{D1}(HH) + \frac{1}{2}EU_{R_H}^{D1}(HL) \\ &= \frac{6\alpha^2(1+\beta) + (1+3\alpha)(3+\beta)}{6(1+\alpha)^3} \end{aligned} \tag{A44}$$

Welfare Comparison: Full Information Matching vs. Design 1

Now that we have derived $EU_{S_L}^{Full}$, $EU_{S_H}^{Full}$, $EU_{R_L}^{Full}$, and $EU_{R_H}^{Full}$ under the full information matching, and $EU_{S_L}^{D1}$, $EU_{S_H}^{D1}$, $EU_{R_L}^{D1}$, and $EU_{R_H}^{D1}$ under Design 1, we can compare the user welfare under these two matching schemes.

On the sender side, through direct comparison, we have $EU_{S_H}^{D1} < EU_{S_H}^{Full}$, and $EU_{S_L}^{D1} > EU_{S_L}^{Full}$. Therefore, Design 1 hurts an H-sender, but benefits an L-sender.

On the receiver side, direct comparison shows that $EU_{R_H}^{D1} < EU_{R_H}^{Full}$ holds always. However, $EU_{R_L}^{D1} < EU_{R_L}^{Full}$ holds if and only if $\alpha > \alpha(\beta)$, where $\alpha(\beta)$ takes the following form

- If $\beta \in (0, 0.627685]$, $\alpha(\beta)$ is the largest real root of the equation

$$\begin{aligned} &15 + (174-78\beta)\alpha + (801-516\beta)\alpha^2 + (1770-1168\beta)\alpha^3 + (1473-1260\beta)\alpha^4 + (-1458+558\beta)\alpha^5 + \\ &(-4773+4336\beta)\alpha^6 + (-5022+5904\beta)\alpha^7 + (-2700+3456\beta])\alpha^8 + (-648+864\beta)\alpha^9 = 0 \end{aligned}$$

- If $\beta \in (0.627685, 725766]$, $\alpha(\beta)$ is the largest real root of the equation

$$\begin{aligned} &27\beta + 135\beta^2 - 351\beta^3 + 189\beta^4 + (18 + 612\beta + 1998\beta^2 - 5616\beta^3 + 2988\beta^4)\alpha + \\ &(360 + 5796\beta + 11988\beta^2 - 38592\beta^3 + 20448\beta^4)\alpha^2 + (3204 + 32196\beta + 37224\beta^2 - 160236\beta^3 + 87348\beta^4)\alpha^3 + \\ &(16824 + 120840\beta + 45522\beta^2 - 455016\beta^3 + 268062\beta^4)\alpha^4 + (58200 + 325212\beta - 107034\beta^2 - 926082\beta^3 + 628716\beta^4)\alpha^5 + \\ &(139968 + 638380\beta - 652656\beta^2 - 1350264\beta^3 + 1160380\beta^4)\alpha^6 + \\ &(239784 + 903708\beta - 1652688\beta^2 - 1316640\beta^3 + 1706880\beta^4)\alpha^7 + \\ &(292920 + 878553\beta - 2699697\beta^2 - 591525\beta^3 + 2002197\beta^4)\alpha^8 + \\ &(247614 + 496040\beta - 3087780\beta^2 + 506070\beta^3 + 1855644\beta^4)\alpha^9 + \\ &(130200 + 6996\beta - 2504388\beta^2 + 1275396\beta^3 + 1333740\beta^4)\alpha^{10} + \\ &(23340 - 265848\beta - 1411152\beta^2 + 1302840\beta^3 + 720192\beta^4)\alpha^{11} + \\ &(-21456 - 255312\beta - 522864\beta^2 + 821520\beta^3 + 277488\beta^4)\alpha^{12} + (-19104 - 128064\beta - 111936\beta^2 + 332064\beta^3 + 70080\beta^4)\alpha^{13} + \\ &(-6720 - 35904\beta - 8640\beta^2 + 80064\beta^3 + 9792\beta^4)\alpha^{14} + (-960 - 4480\beta + 768\beta^2 + 8832\beta^3 + 512\beta^4)\alpha^{15} = 0 \end{aligned}$$

- If $\beta \in (0.725766, 1)$, $\alpha(\beta)$ is the largest root of the equation

$$\begin{aligned} &9\beta^3 - 9\beta^5 + (15\beta^2 + 57\beta^3 - 51\beta^4 + 3\beta^5)\alpha + (9\beta + 105\beta^2 + 39\beta^3 - 147\beta^4 + 114\beta^5)\alpha^2 + \\ &(2 + 71\beta + 221\beta^2 - 331\beta^3 - 79\beta^4 + 284\beta^5)\alpha^3 + (18 + 207\beta - 57\beta^2 - 888\beta^3 + 327\beta^4 + 369\beta^5)\alpha^4 + \\ &(66 + 237\beta - 852\beta^2 - 714\beta^3 + 810\beta^4 + 261\beta^5)\alpha^5 + (126 - 36\beta - 1176\beta^2 + 232\beta^3 + 700\beta^4 + 58\beta^5)\alpha^6 + \\ &(132 - 336\beta - 528\beta^2 + 564\beta^3 + 168\beta^4)\alpha^7 + (72 - 288\beta + 48\beta^2 + 168\beta^3)\alpha^8 + (16 - 80\beta + 64\beta^2)\alpha^9 = 0 \end{aligned}$$

This completes the proof. $\square$

Proof for Lemma 2.

When the platform withholds receivers' vertical information, senders cannot observe the vertical types of either receiver. Two receivers appears the same in the vertical dimension to senders, and each sender sends request to the closer receiver to maximize his expected utility. Thus the threshold for both types of senders become $x^*_{D2} = \frac{1}{2}$. It is also easily seen that given the other sender chooses the threshold $\frac{1}{2}$, choosing $\frac{1}{2}$ is also the best response. Thus, $x^*_{D2} = \frac{1}{2}$ constitutes the equilibrium.

□

Proof for Proposition 4.

We have derived the welfare of each type of individuals on each side under the full information matching. In this proof, we first calculate user welfare under design 2, and then make the welfare comparison.

User welfare under Design 2

We first calculate the senders' welfare, and then the receivers' welfare.

- **Senders' welfare:**

  In what follows, we calculate the welfare of an L-sender and an H-sender separately.

  1. L-sender's welfare

  From Lemma 2, we know that $x^*_{D2} = \frac{1}{2}$ for both type of senders. If an L-sender is located at $x_L \in [0, \frac{1}{2})$, he sends his request to the receiver located at $x = 0$. The sender's request is accepted if the other sender is an L-sender and is located at $x \in (x_L, 1]$, or if the other sender is an H-sender and is located at $x \in [\min\{x_L + \delta_0 + \epsilon, \frac{1}{2}\}, 1]$, where $\epsilon \to 0$. Here $\delta_0 \equiv \frac{1-\beta}{\beta}$ as defined in the proof of proposition 1. If he is located at $x_L = \frac{1}{2}$, he is indifferent between the two receivers. If he is located at $x_L \in (\frac{1}{2}, 1]$, he chooses the receiver at $x = 1$, and his request is accepted if the other sender is an L-sender and is located at $x \in [0, x_L)$, or if the other sender is an H-sender and is located at $x \in [0, \max\{x_L - \delta_0 - \epsilon, \frac{1}{2}\}]$, where $\epsilon \to 0$. Because the chosen receiver has equal chance of being either H-type or L-type, we have an L-receiver's ex-ante expected utility as

$$\begin{aligned} EU^{D2}_{S_L} &= \int_0^{\frac{1}{2}} \Big(\frac{\alpha(1-x_L)}{2} + \frac{(\alpha(1-x_L) + (1-\alpha)}{2}\Big)\Big(\frac{(1-x_L)}{2} + \frac{(1-\min\{x_L+\delta_0, \frac{1}{2}\})}{2}\Big)dx_L + \\ &\int_{\frac{1}{2}}^1 \Big(\frac{\alpha x_L}{2} + \frac{(\alpha x_L + (1-\alpha)}{2}\Big)\Big(\frac{x_L}{2} + \frac{\max\{x_L - \delta_0, \frac{1}{2}\}}{2}\Big)dx_L \\ &= \begin{cases} \frac{5}{16} + \frac{\alpha}{6} & \text{if } \beta \in (0, \frac{2}{3}] \\ \frac{3\beta(2-6\beta+7\beta^2)+\alpha(4-12\beta+9\beta^2+4\beta^3)}{24\beta^3} & \text{if } \beta \in (\frac{2}{3}, 1) \end{cases} \end{aligned} \tag{A45}$$

2. H-sender's welfare

If an H-sender is located at $x_H \in [0, \frac{1}{2})$, he sends his request to the receiver located at $x = 0$. He is accepted by the receiver, if the other sender is an H-sender and is located at $x \in (x_H, 1]$, or if the other sender is an L-sender and is located at $x \in [\max\{x_H - \delta_0, 0\}, 1]$. If he is located at $x_H = \frac{1}{2}$, he is indifferent between choosing either receiver. If he is located at $x_H \in (\frac{1}{2}, 1]$, he chooses the receiver at $x = 1$, and his request is accepted if the other sender is an H-sender and is located at $x \in [0, x_H)$, or if the other sender is an L-sender and is located at $x \in [0, \min\{x_H + \delta_0, 1\}]$. As the selected receiver has equal chance of being either type, we have an H-receiver's ex-ante expected utility as

$$\begin{aligned} EU_{S_H}^{D2} = &\int_0^{\frac{1}{2}} \Big(\frac{\alpha(1-x_H)}{2} + \frac{(\alpha(1-x_H)+(1-\alpha)}{2}\Big)\Big(\frac{(1-x_H)}{2} + \frac{(1-\max\{x_H-\delta_0, 0\})}{2}\Big)dx_H + \\ &\int_{\frac{1}{2}}^1 \Big(\frac{\alpha x_H}{2} + \frac{(\alpha x_H + (1-\alpha)}{2}\Big)\Big(\frac{x_H}{2} + \frac{\min\{x_H+\delta_0, 1\}}{2}\Big)dx_H \\ = &\begin{cases} \frac{21+11\alpha}{48} & \text{if } \beta \in (0, \frac{2}{3}] \\ \frac{-3\beta(2-6\beta+\beta^2)+\alpha(4-18\beta+27\beta^2-8\beta^3)}{24\beta^3} & \text{if } \beta \in (\frac{2}{3}, 1) \end{cases} \end{aligned} \tag{A46}$$

- **Receivers' welfare:**

Because senders cannot observe receivers' vertical quality, thus only make decisions based on the horizontal distance, both the H-receiver and the L-receiver have the same expected utility. We use EU_R^{D2} to denote the expected utility of both types of receivers. Because of symmetry, we only need to calculate the welfare of the sender located at $x = 0$. Following the same steps as in the case of Design 1, we have the receiver's utility given the case LL, HH, and HL

$$EU_R^{D2}(LL) = \int_0^{\frac{1}{2}} \beta(1-x'_L)\cdot 2(1-x'_L)dx'_L \tag{A47}$$

$$EU_R^{D2}(HH) = \int_0^{\frac{1}{2}} \beta(1-x'_H)\cdot 2(1-x'_H)dx'_H \tag{A48}$$

$$\begin{aligned} EU_R^{D2}(HL) = &\int_0^{\frac{1}{2}} \beta(1-x_L)\cdot\Big(1-\min\{x+\delta_0, \frac{1}{2}\Big)dx_L + \\ &\int_0^{\frac{1}{2}} [\beta(1-x_H)+(1-\beta)]\cdot\Big(1-\min\{x_H-\delta_0, 0\}\Big)dx_H \end{aligned} \tag{A49}$$

And the receiver's ex-ante expected utility is

$$
\begin{aligned}
EU_R^{D2} &= \frac{1}{4}EU_R^{D2}(LL) + \frac{1}{4}EU_R^{D2}(HH) + \frac{1}{2}EU_R^{D2}(HL) \\
&= \begin{cases} \frac{42+13\beta}{96} & \text{if } \beta \in (0, \frac{2}{3}] \\ \frac{10\beta^3-3\beta^2+9\beta-2}{24\beta^2} & \text{if } \beta \in (\frac{2}{3}, 1) \end{cases}
\end{aligned} \tag{A50}
$$

Welfare Comparison: Full Information Matching vs. Design 2

On the receiver side, through direct comparison, we have $EU_{R_H}^{D2} < EU_{R_H}^{Full}$ and $EU_{R_L}^{D2} > EU_{R_L}^{Full}$. Therefore, Design 2 hurts an H-receiver but benefits an L-receiver.

On the sender side, direct comparison shows that $EU_{S_H}^{D2} < EU_{S_H}^{Full}$ always holds. When comparing $EU_{S_L}^{D2}$ and $EU_{S_L}^{Full}$, we can first verify that $\frac{\partial(EU_{S_L}^{D2} - EU_{S_L}^{Full})}{\partial\beta} \leq 0$. Next,

$$
\begin{aligned}
\lim_{\beta\to 0^+}\left(EU_{S_L}^{D2} - EU_{S_L}^{Full}\right) &= \begin{cases} \frac{1+6\alpha-8\alpha^2}{16+32\alpha} & \text{if } \alpha \in (0, \frac{1}{3}] \\ \frac{(1-\alpha)^2(5+42\alpha)}{16(1+2\alpha)(1+3\alpha)^2} & \text{if } \alpha \in (\frac{1}{3}, 1) \end{cases} > 0 \\
\lim_{\beta\to 1^-}\left(EU_{S_L}^{D2} - EU_{S_L}^{Full}\right) &= -\frac{(1-\alpha)^2}{8(1+\alpha)} < 0
\end{aligned}
$$

The derivative and the two inequalities imply that $\forall\alpha \in (0,1)$, $\exists\beta(\alpha)$ such that $EU_{S_L}^{D2} > EU_{S_L}^{Full}$ if and only if $\beta < \beta(\alpha)$. This completes the proof. $\square$

Proof for Proposition 5.

In section 6 of the main paper we have derived that sender's and receiver's expected utility under centralized matching as $EU_S^c = \frac{1}{2} + \frac{\alpha}{6}$ and $EU_R^c = \frac{1}{2} + \frac{\beta}{6}$. Then we can make the welfare comparison.

On the sender side, through direct comparison, we have $EU_S^c > EU_{S_L}^{Full}$ for all $\{\alpha \in (0,1), \beta \in (0,1)\}$. However, $EU_S^c > EU_{S_H}^{Full}$ if and only if $\alpha > \frac{2}{3}$, or $\alpha \leq \frac{2}{3}$ and $\beta > \beta_0$, where β_0 is the largest real root of of the equation

$$
\begin{aligned}
&3\alpha^3 + 9\alpha^4 + (-6\alpha^2 - 36\alpha^3 - 18\alpha^4)\beta + (4\alpha + 48\alpha^2 + 57\alpha^3 + 6\alpha^4)\beta^2 + (-12 - 60\alpha - 87\alpha^2 - 102\alpha^3 + 27\alpha^4)\beta^3 + \\
&(72 + 108\alpha + 198\alpha^2 + 18\alpha^3 + 36\alpha^4)\beta^4 + (-60 - 84\alpha - 153\alpha^2 - 12\alpha^3 + 12\alpha^4)\beta^5 = 0
\end{aligned}
$$

Therefore, centralized matching always benefits an L-sender, and benefits an H-sender if and only if either α or β is large.

On the receiver side, we also have $EU_R^c > EU_{R_L}^{Full}$ for all $\{\alpha \in (0,1), \beta \in (0,1)\}$. On the other hand, $EU_R^c > EU_{R_H}^{Full}$ holds if and only if one of the following conditions hold: (1) $\alpha > 0.804738$, or (2) $0.603418 < \alpha \leq 0.804738$ and $\beta > \frac{3+45\alpha+225\alpha^2+429\alpha^3+54\alpha^4-756\alpha^5-648\alpha^6}{5+75\alpha+303\alpha^2+391\alpha^3+90\alpha^4-756\alpha^5+216\alpha^6}$, or (3) $0.16846 < \alpha \leq 0.603418$ and $\beta > \beta_{00}$, or (4) $\alpha \leq 0.603418$ and $\beta > \beta_{000}$, where β_{00} is the second largest real root of the equation

$$
\begin{aligned}
&-12\alpha^5-80\alpha^6-192\alpha^7-192\alpha^8-64\alpha^9+(3\alpha^2+45\alpha^3+288\alpha^4+1044\alpha^5+2304\alpha^6+3024\alpha^7+2112\alpha^8+576\alpha^9)\beta+\\
&(-15\alpha^2-273\alpha^3-1512\alpha^4-4068\alpha^5-6528\alpha^6-7056\alpha^7-4800\alpha^8-1344\alpha^9)\beta^2+\\
&(16+216\alpha+1206\alpha^2+4008\alpha^3+8736\alpha^4+13656\alpha^5+13328\alpha^6+13152\alpha^7+10560\alpha^8+3328\alpha^9)\beta^3+\\
&(-144-1944\alpha-11166\alpha^2-36384\alpha^3-72552\alpha^4-95052\alpha^5-70944\alpha^6-34272\alpha^7-8832\alpha^8+576\alpha^9)\beta^4+\\
&(240+3240\alpha+18951\alpha^2+63435\alpha^3+131328\alpha^4+183876\alpha^5+163920\alpha^6+105552\alpha^7+45312\alpha^8+10176\alpha^9)\beta^5+\\
&(-112-1512\alpha-8979\alpha^2-30575\alpha^3-63600\alpha^4-88020\alpha^5-74288\alpha^6-39504\alpha^7-8832\alpha^8+1088\alpha^9)\beta^6=0
\end{aligned}
$$

and β_{000} is the second largest real root of the equation

$$
\begin{aligned}
&-2\alpha^3-12\alpha^4-24\alpha^5-16\alpha^6+(-3\alpha^2-12\alpha^3+48\alpha^5+48\alpha^6)\beta+(-3\alpha+9\alpha^2+108\alpha^3+192\alpha^4-144\alpha^6)\beta^2+\\
&(-1+21\alpha+102\alpha^2+80\alpha^3-372\alpha^4-192\alpha^5-16\alpha^6)\beta^3+(9+9\alpha-120\alpha^2-468\alpha^3+108\alpha^4-24\alpha^5)\beta^4+\\
&(-15-69\alpha-99\alpha^2+276\alpha^3-12\alpha^4)\beta^5+(7+42\alpha+111\alpha^2+2\alpha^3)\beta^6=0
\end{aligned}
$$

Therefore, qualitatively same as the sender side, centralized matching always benefits an L-receiver, and benefits an H-receiver if and only if either α or β is large. This completes the proof. □

Proof for Proposition 6.

We use M to denote the number of successful matches. Let x^*_L and x^*_H denote the threshold for each types of senders in equilibrium. If both types of senders are of the same type, then $M = 2$ if the two senders are located at the two sides of the threshold, and $M = 1$ if the two senders are located at the same side of the threshold. If the two senders are of different types, then $M = 2$ if $x_L < x^*_L$ and $x_H > x^*_H$, or $x_L \geq x^*_L$ and $x_H < x^*_H$, and $M = 1$ if $x_L < x^*_L$ and $x_H < x^*_H$, or $x_L \geq x^*_L$ and $x_H \geq x^*_H$. Therefore, ex ante, we have

$$
\begin{aligned}
M =&\frac{1}{4}\cdot\Big(2\cdot(x^*_L(1-x^*_L)+(1-x^*_L)x^*_L)+1\cdot({x^*_L}^2+(1-x_L)^2)\Big)+\\
&\frac{1}{4}\cdot\Big(2\cdot(x^*_H(1-x^*_H)+(1-x^*_H)x^*_H)+1\cdot({x^*_H}^2+(1-x^*_H)^2)\Big)+\\
&\frac{1}{2}\cdot\Big(2\cdot(x^*_L(1-x^*_H)+(1-x^*_L)x^*_H)+1\cdot(x^*_Lx^*_H+(1-x^*_L)(1-x^*_H)\Big)
\end{aligned}
\tag{A51}
$$

Under full information matching, x^*_H and x^*_L are given in (A19) and (A18). Under Design 1, $x^*_H = x^*_L = x^*_{D1} = \frac{\alpha}{1+\alpha}$. Under Design 2, $x^*_H = x^*_L = x^*_{D2} = \frac{1}{2}$. Let M_{Full}, M_{D1}, M_{D2}, and M_C denote the expected number of matches under the full information matching, design 1 matching, design 2 matching, and centralized matching, respectively. Apparently, $M_C = 2$. Substituting different expressions of x_H^* and x_L^* in (A51) given each matching scheme, we have

$$M_{Full} = \begin{cases} \frac{1+10\alpha+49\alpha^2+120\alpha^3+36\alpha^4}{(1+5\alpha+6\alpha^2)^2} & \text{if } \{\alpha,\beta\} \in Rn1 \\ \frac{\alpha^2(-1-2\beta+4\beta^2)+\alpha(-3\beta+4\beta^2+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\beta(-5\beta+3\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})}{(\beta+2\alpha\beta)^2} & \text{if } \{\alpha,\beta\} \in Rn2 \\ \frac{8\alpha^4+2\beta^2+20\alpha^3(1+\beta)+2\alpha\beta(1+7\beta)+\alpha^2(-1+32\beta+11\beta^2)}{2(1+2\alpha)^2(\alpha+\beta)^2} & \text{if } \{\alpha,\beta\} \in Rn3 \\ \frac{1+6\alpha+6\alpha^2}{(1+2\alpha)^2} & \text{if } \{\alpha,\beta\} \in Rn4 \end{cases} \tag{A52}$$

$$M_{D1} = \frac{1+4\alpha+\alpha^2}{(1+\alpha)^2} \tag{A53}$$

$$M_{D2} = \frac{3}{2} \tag{A54}$$

where Rn1, Rn2, Rn3, and Rn4 are given in (A7), (A10),(A13), and (A17). It then can be verified that $M_C > M_{D2} > M_{D1} > M_{Full}$. This completes the proof. □

Supplementary Appendix

In this supplementary appendix, we present the expressions of $EU_{S_L}^{Full}$, $EU_{S_H}^{Full}$, $EU_{R_L}^{Full}$, and $EU_{R_H}^{Full}$ derived in the proof of proposition 3 in the main appendix.

From the main appendix, we know that the parameter space $\{\alpha, \beta\} \in (0,1)^2$ can be partitioned into $Rn1 \cup Rn2 \cup Rn3 \cup Rn4$, where

$$
\begin{aligned}
Rn1 &= \{0 < \beta \leq \frac{1}{2}, \frac{1}{3} < \alpha < 1\} \cup \{\frac{1}{2} < \beta < \frac{2}{3}, \frac{3\beta - 1}{3 - 3\beta} \leq \alpha < 1\} \\
Rn2 &= \{\frac{1}{2} < \beta \leq \frac{2}{3}, \frac{1-\beta}{1+\beta} < \alpha < \frac{3\beta - 1}{3 - 3\beta}\} \cup \{\frac{2}{3} < \beta < 1, \frac{1-\beta}{1+\beta} < \alpha \leq \frac{3\beta(1-\beta)}{4\beta - 2}\} \\
Rn3 &= \{\frac{2}{3} < \beta < 1, \frac{3\beta(1-\beta)}{4\beta - 2} < \alpha < 1\} \\
Rn4 &= \{0 < \beta \leq \frac{1}{2}, 0 < \alpha \leq \frac{1}{3}\} \cup \{\frac{1}{2} < \beta < 1, 0 < \alpha \leq \frac{1-\beta}{1+\beta}\}
\end{aligned}
$$

We further make the partition $Rn4 = Rn4a \cup Rn4b$, where

$$
\begin{aligned}
Rn4a &= \{\frac{1}{2} < \beta < 1, 0 < \alpha \leq \frac{1-\beta}{1+\beta}\} \\
Rn4b &= \{0 < \beta \leq \frac{1}{2}, 0 < \alpha \leq \frac{1}{3}\}
\end{aligned}
$$

Then we have the parameter space $\alpha, \beta \in (0,1)^2 = Rn1 \cup Rn2 \cup Rn3 \cup Rn4a \cup Rn4b$. With this partition, the expressions of $EU_{S_L}^{Full}$, $EU_{S_H}^{Full}$, $EU_{R_L}^{Full}$, and $EU_{R_H}^{Full}$ are given as follows.

1. The expression of $EU_{S_L}^{Full}$:

$$
EU_{S_L}^{Full} = \begin{cases}
\frac{\alpha(4+77\alpha+39\alpha^2+18\alpha^3)}{6(1+2\alpha)(1+3\alpha)^2} & \text{if } \{\alpha, \beta\} \in Rn1 \\
\frac{1}{24\alpha^2(1+2\alpha)\beta^3} \cdot F_1(\alpha, \beta) & \text{if } \{\alpha, \beta\} \in Rn2 \\
\frac{1}{24(1+2\alpha)\beta^3(\alpha+\beta)^2} \cdot F_2(\alpha, \beta) & \text{if } \{\alpha, \beta\} \in Rn3 \\
\frac{3\beta(1-4\beta+5\beta^2)+\alpha^2(2-12\beta+24\beta^2-6\beta^3)+\alpha(1-12\beta^2+21\beta^3)}{12(1+2\alpha)\beta^3} & \text{if } \{\alpha, \beta\} \in Rn4a \\
\frac{3+5\alpha+10\alpha^2}{12+24\alpha} & \text{if } \{\alpha, \beta\} \in Rn4b
\end{cases} \tag{B1}
$$

where

$$\begin{aligned}F_1(\alpha,\beta)=&\alpha^4(2-6\beta+12\beta^2-36\beta^3)+8\beta^2(2\beta-\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\\&\alpha\beta\Big(2\beta^2-3\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+3\beta(2+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)+\\&\alpha^2\Big(6\beta^3+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\beta(3-5\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\\&3\beta^2(1+4\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)+\\&\alpha^3\Big(1-30\beta^3+2\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+4\beta^2(3+5\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})-\\&\beta(3+10\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)\\F_2(\alpha,\beta)=&3(1-3\beta)^2\beta^3+4\alpha^4(2-6\beta+3\beta^2+5\beta^3)+\alpha\beta^2(13-42\beta+39\beta^2+34\beta^3)+\\&2\alpha^2\beta(7-5\beta-12\beta^2+31\beta^3+7\beta^4)+2\alpha^3(2+8\beta-33\beta^2+23\beta^3+20\beta^4)\end{aligned}$$

2. The expression of $EU_{S_H}^{Full}$:

$$EU_{S_H}^{Full}=\begin{cases}\frac{2+5\alpha+15\alpha^2}{2+36\alpha} & \text{if } \{\alpha,\beta\}\in Rn1\\ \frac{1}{24\alpha^2\beta^3}\cdot F_3(\alpha,\beta) & \text{if } \{\alpha,\beta\}\in Rn2\\ \frac{-3\beta^2(1-6\beta+\beta^2)+\alpha^2(4-18\beta+30\beta^2-8\beta^3)-2\alpha\beta(1+3\beta-12\beta^2+4\beta^3)}{24\beta^3(\alpha+\beta)} & \text{if } \{\alpha,\beta\}\in Rn3\\ \frac{\alpha-3\alpha\beta-3\beta(1-4\beta+\beta^2)}{12\beta^3} & \text{if } \{\alpha,\beta\}\in Rn4a\\ \frac{3}{4}-\frac{\alpha}{3} & \text{if } \{\alpha,\beta\}\in Rn4b\end{cases}\tag{B2}$$

where

$$\begin{aligned}F_3(\alpha,\beta)=&\alpha^3(1-12\beta-6\beta^2)+4\alpha\beta^2(-3\beta+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\\&4\beta^2(-2\beta+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\alpha^2\Big(-18\beta^3+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\\&\beta(-6+4\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\beta^2(6+4\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)\end{aligned}$$

3. The expression of $EU_{R_L}^{Full}$:

$$EU_{R_L}^{Full}=\begin{cases}\frac{(1+2\alpha)^3(-1+3\alpha)(1+3\alpha)(5+9\alpha)+6\alpha(5+\alpha(24+\alpha(45+2\alpha(47+6(1-2\alpha)\alpha))))\beta}{4(1+\alpha(5+6\alpha))^3} & \text{if } \{\alpha,\beta\}\in Rn1\\ \frac{1}{24\alpha^3(1+2\alpha)^3\beta^2}\cdot G_1(\alpha,\beta) & \text{if } \{\alpha,\beta\}\in Rn2\\ \frac{1}{24(1+2\alpha)^3\beta^2(\alpha+\beta)^3}\cdot G_2(\alpha,\beta) & \text{if } \{\alpha,\beta\}\in Rn3\\ \frac{\alpha(6+5\alpha(3+2\alpha))\beta}{3(1+2\alpha)^3} & \text{if } \{\alpha,\beta\}\in Rn4\end{cases}\tag{B3}$$

where

$$
\begin{aligned}
G_1(\alpha,\beta) =& -8\alpha^6\Big(5+\beta(30+\beta(21+2\beta))\Big)+12\beta^2\Big(-2\beta+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}\Big)+\\
& 3\alpha\beta\Big(5\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\beta(-10-66\beta+31\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)-\\
& 4\alpha^5\Big(15-10\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\beta(114-40\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\\
& \beta(183+42\beta+2\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}))\Big)+\\
& 2\alpha^4\Big(6(-3+5\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\beta(168(-1+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\\
& \beta(-519-258\beta+50\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}))\Big)+\\
& 3\alpha^2\Big(2\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\beta(-5+38\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\\
& \beta(-85-144\beta+55\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}))\Big)+\\
& \alpha^3\Big(-6+36\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\beta(6(-21+44\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\\
& \beta(-711-698\beta+226\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}))\Big)\\
G_2(\alpha,\beta) =& -2\alpha^3(1+2\alpha)^3+3\alpha^2(1+2\alpha)^3(-3+4\alpha)\beta-3\alpha(1+2\alpha)^3(5+\alpha(-13+2\alpha))\beta^2+\\
& 3\Big(-3+\alpha(-5+2\alpha(23+2\alpha(22+\alpha(17+2\alpha+8\alpha^2)))))\Big)\beta^3+\\
& 3\Big(3+\alpha(27+4\alpha(-1+\alpha(12+\alpha(27+22\alpha))))\Big)\beta^4+3\Big(3+\alpha(-19+\alpha(3+4\alpha)(15+22\alpha))\Big)\beta^5+\\
& 9\Big(-1+\alpha(6+5\alpha(3+2\alpha))\Big)\beta^6
\end{aligned}
$$

4. The expression of $EU_{R_H}^{Full}$:

$$
EU_{R_H}^{Full} = \begin{cases}
\frac{3(1+2\alpha)^3(1+3\alpha)(1+6\alpha)+(-2+\alpha(-30+\alpha(-105+\alpha\ (-43+18\alpha(13+36\alpha)))))\beta}{3(1+\alpha(5+6\alpha))^3} & \text{if } \{\alpha,\beta\}\in Rn1\\
\frac{1}{24\alpha^3(1+2\alpha)^3\beta^2}\cdot G_3(\alpha,\beta) & \text{if } \{\alpha,\beta\}\in Rn2\\
\frac{1}{24(1+2\alpha)^3\beta^2(\alpha+\beta)^3}\cdot G_4(\alpha,\beta) & \text{if } \{\alpha,\beta\}\in Rn3\\
\frac{1}{12}\Big(-3-\frac{1}{\beta^2}+\frac{6}{\beta}+(6-\frac{16\alpha^3}{(1+2\alpha)^3})\beta\Big) & \text{if } \{\alpha,\beta\}\in Rn4a\\
\frac{3}{4}+\Big(-\frac{1}{6}-\frac{4\alpha^3}{3(1+2\alpha)^3}\Big)\beta & \text{if } \{\alpha,\beta\}\in Rn4b
\end{cases}
\tag{B4}
$$

where

$$
\begin{aligned}
G_3(\alpha,\beta) =& 8\alpha^6(-2+9\beta+9\beta^2+20\beta^3)+16\beta^2(2\beta-\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})- \\
& 16\alpha\beta^2(-15\beta+7\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+12\alpha^5\Big(-2+56\beta^3+ \\
& \beta^2(19-6\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+\beta(13-2\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)+ \\
& 6\alpha^4\Big(-1+156\beta^3+\beta^2(39-30\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})- \\
& 6\beta(-3+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)+\alpha^3\Big(-1+1068\beta^3-6\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+ \\
& \beta^2(81-366\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+12\beta(4+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big)+ \\
& \alpha^2\Big(-618\beta^3+\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2}+\beta(-6+4\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})+ \\
& \beta^2(-24+247\sqrt{4\beta^2+4\alpha\beta^2+(\alpha+2\alpha\beta)^2})\Big) \\
G_4(\alpha,\beta) =& 16\alpha^6(-1+\beta)^3+\beta^3(-1+9\beta-3\beta^2+11\beta^3)+24\alpha^5(-1+2\beta+6\beta^2+6\beta^3+3\beta^4)+ \\
& 3\alpha\beta^2(-1+7\beta+15\beta^2+5\beta^3+22\beta^4)+12\alpha^4(-1+22\beta^2+7\beta^3+45\beta^4+7\beta^5)+ \\
& 3\alpha^2\beta(-1+3\beta+46\beta^2+36\beta^3+39\beta^4+53\beta^5)+ \\
& 2\alpha^3(-1-6\beta+60\beta^2+150\beta^3+18\beta^4+258\beta^5+17\beta^6)
\end{aligned}
$$