A cash transportation vehicle routing problem with combinations of different cash denominations

Guoxun Xu[a], Yanfeng Li[a]*, W.Y. Szeto[b] and Jun Li[a]

[a]*School of Economics and Management, Southwest Jiaotong University, Chengdu, Sichuan, China*

[b]*Department of Civil Engineering, The University of Hong Kong, Hong Kong*

Email: xgxbing@126.com [Xu]; yanwaa@126.com [Li]; ceszeto@hku.hk [Szeto];

lijun@home.swjtu.edu.cn [Li]

Abstract

This paper investigates a new cash transportation problem, which is a variant of the capacitated vehicle routing problem. To better satisfy the demands of customers (e.g., banks, large retailers, shopping centers, automated teller machines (ATMs), etc.), the combination of different cash denominations (i.e., $100, $50, $20, etc.) is considered. The robbery risk, which is measured by both the amount of cash being carried and the distance covered by the vehicle carrying the cash, is limited by a risk threshold. The problem is formulated as mixed-integer linear programming to minimize the total cost, which consists of the total travel cost and the total penalties due to the unmet expected demand of customers. A combined hybrid tabu search metaheuristic is proposed to solve this problem. Tabu search is adopted for determining routing decisions while three methods, namely the exact method, the greedy method, and the mixed method, are proposed to be embedded within the tabu search to determine the denomination combination strategy. The numerical studies show that the proposed method makes a good tradeoff between the solution quality and the computation time. Experimental results also reveal the effects of the unit penalty and the risk threshold.

Keywords: vehicle routing problem; cash transportation; cash denominations; risk management; combined hybrid tabu search

1. Introduction and literature review

With the rapid development of the social economy, the amount of cash in circulation has played a quite important role in commerce and trade year by year, and it is expected to maintain its dominance in the near future, despite the fact that the use of non-traditional payments methods (e.g., credit, debit, prepaid cards and mobile payments) is continuously growing. In many developing countries, people prefer only cash as the payment method. In fact, more than 2.5 billion adult people do not have access to banks or to any other financial services. Specifically, in Brazil 57% of the adult population are unbanked, and in South Africa 54% of the adult population are unbanked (Geismar et al., 2016). In developed countries, in addition to small

* Corresponding author

value transactions, people prefer to use cash to preserve their anonymity. The cash demand is at slightly above 7% of GDP in the US and above10% of the GDP in Eurozone (Rogoff, 2017).

The volume of cash in circulation has been increasing drastically in many countries in the past ten years. In the U.S., the currency in circulation increased 82.1% from 828,938 million in 2007 to 1,509,440 million in 2016. In the developing countries, the currency in circulation increases much faster. For example, in China, the currency in circulation increased more than 124% in the past ten years.

With the circulation increasing, more and more currency issuance problems appear. For instance, the U.S. Federal Reserve reported that, in 2016, 660 million dollars (16.37% of its annual budget) were paid for costs associated with the production of nearly 7.3 billion Federal Reserve notes. In China, the total value of banknotes in the circulation market was 240 billion RMB in 2011. The average printing cost for each banknote is 0.5 RMB. By adding the transportation cost, inventory cost, disposal cost, and other handling fees, the total cost for each banknote is almost 1.2 RMB. Since the life cycle for each banknote is about three years, the monetary authority prints 80 billion pieces of banknotes every year, and the total cost is about 96 billion RMB. Some of the costs mentioned above (e.g., the printing cost) are unavoidable, while the other costs (e.g., the transportation cost) can be reduced by means of scientific management. In view of this, the cash transportation problem is a key factor worthy of further investigation.

The cash transportation problem is a real-life application of vehicle routing problem (VRP) (e.g., Partyka and Hall, 2000; Emir, 2002; Eksioglu et al., 2009; Michallet et al., 2014). In reality, to deal with the transfer of cash and valuables in this industry, logistics companies (self-operation or outsourcing) transport banknotes, coins, and other valuable items from the depot (the place where the cash and valuables are deposited) to customers (e.g., banks, large retailers, shopping centers, ATMs, jewelers, casinos, etc.). Nowadays, the cash transportation business is a huge market where the worldwide cost of handling cash exceeds $300 billion per year (Talarico, 2016).

However, the cash transportation problem has not received much attention so far. In the literature, few studies focus on this problem. Most researchers describe their studies as well-known variants of the classical vehicle routing problem. Boonsam et al. (2011) modeled the cash distribution problem as the vehicle routing problem with time windows. All branches of the bank were first clustered to different distribution centers, and then the routes for each distribution center were designed. Dai and Liu (2012) considered banking cash transportation as the single vehicle routing problem. The influence of the traffic jam was considered in their model. Anbuudayasankar et al. (2012) modeled the process of replenishing money in the ATMs as the bi-objective vehicle routing problem with forced backhauls to reduce the total routing cost and the span of travel tour. Michallet et al. (2014) presented the periodic vehicle routing problem with time windows to deal with a real-case cash transportation problem for a software company. In particular, for a customer needed to be visited a few times during a predefined planning horizon, the regularity of arrival times of the visits must be avoided. Talarico et al. (2015b) considered the cash transportation problem as a practical application of the k-dissimilar vehicle routing problem. In order to increase unpredictability, an index of similarity, on the basis of the number of identical edges common between alternative solutions, was proposed. Roel et al. (2016) considered the replenishment of ATMs as a rich multi-period inventory routing problem with pickup and delivery. The cash pickups and deliveries were introduced in inventory routing problem context. Larrain et al. (2017) modeled the management of the amount of cash for ATMs as the inventory routing problem with cassettes and stockouts. The stockouts were allowed but penalized.

In addition, few studies consider the robbery risk during transportation and propose different approaches to reduce the robbery risk. Tarantilis and Kiranoudis (2004) presented a decision support system to minimize the probability of having a successful vehicle robbery at any point in the road network. The risk of the robbery was associated with the minimum distance between the point of the road and the closest police department. Some researchers consider reducing the robbery risk by building peripatetic routes that are unpredictable for robbers. Ngueveu et al. (2010a, 2010b) proposed m-peripatetic vehicle routing problem (m-PVRP). In m-PVRP, each customer can be visited several times during the m periods, but the use of the same edge twice is explicitly forbidden. The m-peripatetic salesman problem (m-PSP) is a special case of the m-PVRP with single vehicle (e.g., Calvo and Cordone, 2003; Duchenne et al., 2007). Duchenne et al. (2012) proposed the m-capacitated peripatetic salesman problem (m-CPSP), which is an extended version of the m-PSP. In m-CPSP, the use of the same edge more than C_e times is explicitly forbidden. Some researchers propose models to formulate more flexible vehicle routing to reduce the robbery risk. Yan et al. (2012) utilized a time-space network technique to model the cash transportation. To reduce the risk of robbery, there must be variations in daily vehicle routes and schedules, including the daily vehicle arrival time at each demand point. Followed this study, Yan et al. (2014) incorporated stochastic travel times and added the unanticipated penalty cost for violating planned operation time windows. Bozkaya et al. (2017) considered two robbery risk components: (i) following the same or very similar routes arranged before and (ii) visiting neighborhoods with low socioeconomic status along the routes. A risk-based model was proposed to generate alternative routing solutions that make the routes unpredictable. Some researchers (e.g., Talarico et al., 2015a; Talarico et al., 2017a; Talarico et al., 2017b; Radojičić et al., 2018) considered the amount of cash being carried by vehicle and introduced a risk constraint when collecting the cash along the route and dealt with the robbery risk of a route that was increasing along the route.

The most important observation is that all the above studies do not consider the demand for different denomination cash and are restricted to a single commodity case. Actually, the demand of customers for different domination cash varies. For example, a great demand of large denomination cash (i.e., $100, $50) is always needed in banks and ATMs, while small denomination cash (i.e., $20, $10, etc.) is far more largely required in the supermarket and subway Ticket Vending Machine. When the demand for the required denomination cash deviates from the supply, the inconvenience caused to the customers may reduce customers' satisfaction. Therefore, the logistics company should design the distribution plan and vehicle routing by considering the demand of each denomination cash.

In this paper, we present a cash transportation problem with the consideration of the optimal mix of different denomination cash. Due to the special characteristic of cash to be transported, in addition to the robbery risk, there are some other important decisions to be made which include: routing of the vehicles for the cash distribution and the combination of different cash denominations to satisfy the requirement of the customers for their daily operations. The objective is to minimize the total transportation cost and the penalties associated with the supply amount differences compared with the expected amount of different cash denominations for all the customers. The penalty is incorporated in the model to consider the inconvenience caused to the customer if the supplied cash denominations are deviating from the expected demand. The proposed problem has some characteristics that make it different from the aforementioned studies. (1) The customer demands consist of the total cash amount and the amounts of different denomination cash. Different denomination cash for the same total cash amount results in different weights, which will affect the vehicle load and dispatching strategy. Therefore, this problem includes the decision of the mix of different denomination cash in addition to the routing decision. (2) For each customer, the demand is the amount of cash, instead of the weight of cash. However, there is a weight capacity restriction for each vehicle. Therefore, the conversion of the cash amount to the weight is needed. (3) The cash on board is decreasing when the vehicle delivers goods along the route. Therefore, the robbery risk index of a route is a decreasing measure.

The proposed problem is a multi-commodity capacitated vehicle routing problem with denomination and risk aspects, which is different from the capacitated vehicle routing problem in several ways. Firstly, the delivery quantity for each denomination cash at each customer is a decision variable and affects the objective function. Secondly, each customer has a demand for a total cash amount and has a minimum demand and maximum demand for each denomination cash. Each denomination cash transported is not independent but restricted to these demands. Thirdly, the robbery risk is considered in the course of cash transportation, and different risk threshold may affect the vehicle routing and the cash denomination combination transported.

To solve the proposed problem, we may consider different solution methods, including exact methods (e.g., Roel et al., 2016; Yan et al., 2012) and heuristics. The latter includes ant colony optimization with large neighborhood search (e.g., Talarico et al., 2017a), an adaptive and diversified vehicle routing approach (Bozkaya et al., 2017), a progressive multiobjective optimization with iterative local search(e.g., Talarico et al., 2017b), a variable mixed integer programming neighborhood search (e.g.,Larrain et al., 2017), the multi-start and perturb-and-improve metaheuristic (e.g., Talarico et al., 2015a), the multi-start iterated local search (e.g., Michallet, 2014), an improved ant colony algorithm (e.g., Dai and Liu, 2012), nearest neighbor algorithm (e.g., Boonsam et al., 2011), group sweep algorithm (e.g., Boonsam et al., 2011), and an adaptive memory-based metaheuristic (e.g., Tarantilis and Kiranoudis, 2004). The proposed problem is NP-hard since it is a variant of vehicle routing problem, exact methods may only obtain optimal solutions in quite small instances in a reasonable time. For large network applications, it is quite difficult to obtain optimal solutions efficiently. Therefore, heuristics are normally used for this problem. This paper adopts a tabu search method as the backbone of our solution method. However, the proposed optimization problem involves decision variables for the delivery quantity of different denomination cash, robbery risk, and cash amount on each vehicle on each arc in addition to vehicle route. Hence, we cannot apply the tabu search directly to solve the proposed problem. For this purpose, three methods namely the exact method, the greedy method, and the mixed method, are embedded within the tabu search.

The contributions of this study include the following.

(1) We propose a new cash transportation problem with the consideration of the optimal mix of different denominations.

(2) We develop an efficient metaheuristic to solve small, medium and large instances.

The remainder of this paper is organized as follows. Section 2 describes and formulates the problem. Section 3 presents the combined hybrid tabu search algorithm. Section 4 depicts the numerical examples. Section 5 gives our conclusions and directions for future research.

2. Problem description and formulation

The cash transportation problem is defined on a directed graph $G = (V, A)$ with vertex $V = \{0,\} \cup \{1, \dots, n\}$, where 0 is the depot, $\{1, \dots, n\}$ is the set of customers, and $A = \{(i,j): i, j \in V, i \neq j\}$ is the set of arcs. For the sake of simplicity, the depot 0 is replaced by two dummy nodes, s(start) from which all vehicle routes depart and e (end) where all routes end.

Each customer i has a non-negative demand q_i which represents the cash amount in value (say, 200 million dollars) to be delivered by the vehicle during its visit. Furthermore, there are $|M|$ kinds of denominations (e.g., \$100, \$50, \$20, etc.), where M is the set of the denominations. Although each customer has a demand for cash amount, they hope to get an expected allocation of different denomination cash for the service convenience, which is called an expected demand d_i^m for the m^{th} denomination cash at customer i. There is a unit penalty p_i^m which is associated with the deviation from the demand of the m^{th} denomination cash at customer i. Failing to meet the expected demand leads to a penalty cost for the logistics company because of the decline of customer satisfaction. Note that, for the purpose of reducing the total cost, delivering more denominations than the customer expected demand is allowed.

In addition, the robbery risk is integrated into this problem. Talarico et al. (2015) proposed two factors that are the cash amount being carried and the travel distance by the vehicle carrying the cash to measure the robbery risk. To determine the robbery risk incurred by the vehicle when it delivers cash along the route, we adopt the same factors and calculate the robbery risk R_i^k by using Eq. (1), where Q_i^k is the total amount of cash in vehicle k when it leaves customer i. c_{ij} is the length of the directed arc (i, j) and T is the risk threshold. Fig. 1 illustrate the calculation of risk.

$$\begin{cases} R_i^k = R_j^k + Q_i^k c_{ij} \\ R_i^k \leq T \end{cases} \tag{1}$$

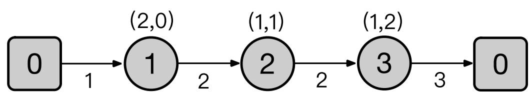


Fig. 1. Original delivery route.

In Fig. 1, the weight of each arc represents the distance c_{ij} while the values above customer i denote the delivery amount of two cash denominations (\$10, \$20) for customer *i*. Vehicle k takes a delivery route $0 \to 1 \to 2 \to 3 \to 0$ with an initial cash amount 100 ($2 \cdot 10 + 0 \cdot 20 + 1 \cdot 10 + 1 \cdot 20 + 1 \cdot 10 + 2 \cdot 20 = 100$) on board. By using Eq. (1), We can compute the risk $R_3^k = 0(3 \to 0)$, $R_2^k = R_3^k + 2 \cdot (1 \cdot 10 + 2 \cdot 20) = 100(2 \to 3)$, $R_1^k = R_2^k + (1 \cdot 10 + 1 \cdot 20 + 1 \cdot 10 + 2 \cdot 20) \cdot 2 = 260(1 \to 2)$, $R_0^k = R_1^k + 1 \cdot (2 \cdot 10 + 0 \cdot 20 +$

$1 \cdot 10 + 1 \cdot 20 + 1 \cdot 10 + 2 \cdot 20) = 360(0 \to 1)$ sequentially. We can easily obtain that the risks along route are 360, 260, 100, 0, decreasingly.

In this problem, each vehicle starts from the depot and performs a single route to execute its dispatching task by visiting a sequence of customers before returning to the depot. Each customer is only allowed to be visited exactly once. To formulate the problem, the following notations are needed.

Set/Indices

M: Set of cash denominations, indexed by $m = 1, \ldots, |M|$;

N: Set of customers, indexed by $i = 1, \ldots, n$.

K: Set of vehicles, indexed by $k = 1, \ldots, |K|$.

Parameters

q_i: Demand/supply for total cash amount at customer i;

L_i^m: Minimum demand (in units) of the m^{th} denomination cash of customer i;

H_i^m Maximum demand (in units) of the m^{th} denomination cash of customer i;

d_i^m: Expected demand (in units) of the m^{th} denomination cash of customer i;

p_i^m: Unit penalty deviating from the expected demand for the m^{th} denomination cash of customer i;

a_m: Denomination coefficient of the m^{th} denomination cash;

w_m: Weight coefficient of the m^{th} denomination cash;

c_{ij}: The distance of arc (i, j);

T: The risk threshold;

C_k: Weight capacity of vehicle k;

M_0: A sufficiently large value.

Decision variables

$$x_{ij}^k = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed by vehicle } k; \\ 0 & \text{otherwise;} \end{cases}$$

y_{im}^k: Supply (in units) of the m^{th} denomination cash for customer i by vehicle k;

Q_i^k: Cash amount/value of vehicle k when leaving customer i;

R_i^k: Risk of vehicle k when leaving customer i;

B_i^k: Auxiliary continuous variable associated with vehicle k at customer i used by the sub-tour elimination constraint;

z_i^m: Auxiliary continuous variable used to linearize $|d_i^m - \sum_{k \in N} y_{im}^k|$ in the original objective function.

Formulation

$$min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ij}^k + \sum_{i \in N} \sum_{m \in M} p_i^m z_i^m \quad (2)$$

s.t.

Auxiliary constraints:

$$z_i^m \geq d_i^m - \sum_{k\in K} y_{im}^k \qquad \forall i \in N, m \in M \qquad (3)$$

$$z_i^m \geq -(d_i^m - \sum_{k\in K} y_{im}^k) \qquad \forall i \in N, m \in M \qquad (4)$$

Risk constraints and capacity constraints:

$$H_i^m \geq \sum_{k\in K} y_{im}^k \geq L_i^m \qquad \forall i \in N, m \in M \qquad (5)$$

$$y_{im}^k \leq M_0 \sum_{j\in N} x_{ij}^k \qquad \forall i \in N, m \in M, k \in K \qquad (6)$$

$$q_i = \sum_{k\in K} \sum_{m\in M} a_m y_{im}^k \qquad \forall i \in N \qquad (7)$$

$$Q_0^k = \sum_{i\in N} q_i \sum_{j\in N} x_{ij}^k \qquad \forall k \in K \qquad (8)$$

$$Q_j^k \geq Q_i^k - q_i - M_0(1 - x_{ij}^k) \qquad \forall (i,j) \in A,\ k \in K \qquad (9)$$

$$R_e^k = 0 \qquad \forall k \in K \qquad (10)$$

$$R_i^k \geq R_j^k + Q_i^k c_{ij} - M_0\left(1 - x_{ij}^k\right) \qquad \forall (i,j) \in A, k \in K \qquad (11)$$

$$0 \leq R_i^k \leq T \qquad \forall i \in N, \forall k \in K \qquad (12)$$

$$\sum_{i\in N} \sum_{m\in M} w_m y_{im}^k \leq C_k \qquad \forall k \in K \qquad (13)$$

Routing constraints:

$$\sum_{k\in K} \sum_{j:(i,j)\in A} x_{ij}^k = 1 \qquad \forall i \in N \qquad (14)$$

$$\sum_{i\in N} x_{i,e}^k = 1 \qquad \forall k \in K \qquad (15)$$

$$\sum_{j\in N} x_{sj}^k = 1 \qquad \forall k \in K \qquad (16)$$

$$\sum_{i\in V\setminus\{e\}} x_{ij}^k - \sum_{i\in V\setminus\{s\}} x_{ji}^k = 0 \qquad \forall j \in N, k \in K \qquad (17)$$

$$B_j^k \geq B_i^k + 1 - M_0\left(1 - x_{ij}^k\right) \qquad \forall (i,j) \in A, k \in K \qquad (18)$$

Variable constraints:

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j) \in A, k \in K \qquad (19)$$

$$y_{im}^k \geq 0 \qquad \forall i \in N, k \in K, m \in M \qquad (20)$$

$$B_i^k \geq 0 \qquad \forall k \in K, i \in V \qquad (21)$$

The objective function (2) is to minimize the total cost, including the total travel cost and the total penalty cost. As the two costs are monetary costs, they are combined directly in the objective of the model. Constraints (3) and (4) are used to linearize the absolute difference.

Constraints (5) ensure that customer's maximum and minimum demand for each denomination cash must be satisfied. Constraints (6) guarantee that if arc (i,j) is not traversed by vehicle k, the unloaded quantity for each cash denomination cash equals 0. Constraints (7) guarantee that the customer's demand for cash amount must be satisfied. Constraints (8) ensure that the vehicle starts with cash amount equal to the total amount to be delivered to all customers. It also ensures each vehicle must be empty when returning to the depot. Constraints (9) are the cash flow conservation conditions. They state that the quantity unloaded from a vehicle at a customer equals the difference between the quantity in the vehicle before and after visiting that customer. Constraints (10)-(12) are used to define the risk, which is mentioned in Section 2.1. Constraints (13) are used to restrict vehicle load not greater than the weight capacity of a vehicle throughout its tour. Constraints (14) state that every vertex has to be served exactly once. Constraints (15) and (16) guarantee that each vehicle starts at the depot and returns to the depot at the end of its route. Constraints (17) are the vehicle flow conservation conditions, which impose a requirement that vehicle k can leave customer j only if it has served i previously. Constraints (18) are the sub-tour elimination constraints. Constraints (19) are binary constraints for the routing decision variables. Constraints (20) are the non-negativity constraints for the unloading decision variables. Constraints (21) are the non-negativity constraints for auxiliary variables associated with the sub-tour elimination constraints.

3. The solution method

We develop a combined hybrid tabu search to solve the proposed problem, by adopting a hybrid tabu search as the backbone algorithm to determine the routes and embedding three methods, namely the exact method, the greedy method and the mixed method, to determine supply quantity of each denomination cash to each customer, cash amount on each vehicle on each arc, and the risk of each arc on the given route.

3.1. The hybrid tabu search

The hybrid tabu search involves solution representation, initial solution, evaluation of solutions, neighborhoods, repairing, tabu list and aspiration criterion, diversification. The procedure of the algorithm is given below.

Algorithm. Combined hybrid tabu search

```
Generate an initial solution p
p_best ← p
Elite set S ← ∅
While stopping condition is not satisfied do
    Generate a selected neighborhood of p by six moves
    Repair all infeasible solutions in the neighborhood, then find the best feasible p'
    if p' is better than p_best then
        p_best ← p'
        Add p_best to the elite set S, update S
    end if
```

11: $p \leftarrow p'$

12: **if** p_{best} is not improved for IT iterations **then**

13: Select a solution p_r randomly and remove it from the elite set S

14: $p \leftarrow p_r$

15: Reset the tabu list

16: **end if**

17: **end While**

18: Return p_{best}

The initial setup is shown in lines 1-2; an initial solution p is generated using nearest neighbor together with greedy randomized selection mechanism. At each iteration, a selected neighborhood is generated (line 5). However, they may exceed the vehicle capacity or the risk threshold. Therefore, infeasible solutions must undergo the repair operation (Line 6). Note that according to our aspiration criterion, if a forbidden move is able to improve the current best solution p_{best}, it is revoked the tabu status. If p_{best} is improved, the algorithm adds the new solution to the elite set S (line 7-10). If the best solution is not improved for IT iterations (line 12), the algorithm selects a solution p_r randomly from the elite set S randomly and updates current solution p (line 13). Then, the tabu list is reset to empty (line 15).

3.1.1. Solution representation

In our algorithm, a solution p is represented by a permutation of the visited customers. Suppose n customers visited by k vehicle routes, the representation is formed by a vector of length $(n + k)$. For instance, if seven customers are visited by three trucks, the solution can be represented as $0 \rightarrow 1 \rightarrow 2 \rightarrow 0 \rightarrow 5 \rightarrow 7 \rightarrow 4 \rightarrow 0 \rightarrow 6 \rightarrow 3 \rightarrow 0$. The three zeros mean there are three routes: $0 \rightarrow 1 \rightarrow 2 \rightarrow 0$, $0 \rightarrow 5 \rightarrow 7 \rightarrow 4 \rightarrow 0$ and $0 \rightarrow 6 \rightarrow 3 \rightarrow 0$.

3.1.2. Initial solution

The initial solution for the algorithm is created with the nearest neighbor heuristic combined with a greedy randomized selection mechanism. The standard nearest neighbor heuristic constructs a solution by selecting the closest unvisited customer at each iteration. In this method, a greedy randomized selection mechanism is used instead of a simple greedy procedure. It means that the next customer is selected randomly from the restricted candidate set containing the first μ closest unvisited customers. If it is not possible to add any customer to the current route that satisfies the risk constraint and vehicle capacity constraint, a new route is started.

3.1.3. Evaluation of solutions

A solution p gives an objective function value $f(p)$, which is the sum of total transportation costs and the penalty costs. In our algorithm, infeasible solutions are allowed to enhance the performance of the search. The infeasible solution can be yielded by relaxing the limit on the vehicle capacity and risk threshold. We set $W(p)$ and $D(p)$ as the penalties for exceeding the capacity and risk threshold, respectively. Therefore, $f(p)$ is modified to a fitness function value $F(p)$, defined as

$$F(p) = f(p) + \alpha \cdot W(p) + \beta \cdot D(p) \tag{22}$$

$$W(p) = max\{0,\ max(\sum_{i\in N}\sum_{m\in M} w_m y_{im}^k - C_k)_{k\in K}\} \tag{23}$$

$$D(p) = max\{0, max(R_i^k)_{i\in N,k\in K} - T\} \tag{24}$$

Where α, β represents the penalty parameter associated with $W(p)$, $D(p)$, respectively. Initially, $\alpha = \beta = 1$. The penalties are updated when repairing the solutions.

3.1.4. Neighborhood

The neighborhood is a set of solutions obtained by applying some moves in all possible ways to the current solution. In each iteration, the solutions in the neighborhood may be feasible or infeasible in terms of the vehicle capacity and the risk threshold. Infeasible solutions must undergo the repair operation. After repairing all infeasible ones, the best feasible solution in the neighborhood is chosen. In the tabu search framework, six moves are adopted, including random insertions, random insertions of subsequences, random insertions of reversed subsequences, random swaps of subsequences, reversing a subsequence, random swaps of reversed subsequences. A detailed description of the six moves is given below.

(1) Node relocation. Randomly select and relocate a node into another random position.

(2) Subsequence relocation. Randomly select and relocate a subsequence with random length into another random position.

(3) Reversed subsequence relocation. Randomly select and relocate a subsequence with random length into another random position. Then, reverse the selected subsequence.

(4) Subsequence reversing. Randomly select and reverse a subsequence with random length.

(5) Subsequences swapping. Random select and swap two independent subsequences with random length.

(6) Reversed subsequences swapping. Random select and swap two independent subsequences with random length. Then, reverse the two selected subsequences.

3.1.5. Repairing

The solutions obtained from the neighborhood operator may be feasible and infeasible, because they may exceed the vehicle capacity or the risk threshold. Infeasible solutions must undergo the repair operation. Repair operation includes temporarily multiplying the penalty parameters α, β by $1+\gamma$. If the capacity (or risk threshold) is exceeded, α (or β) is temporarily multiplied by $1+\gamma$. If the capacity and risk threshold are exceeded at the same time, both α and β are temporarily multiplied by $1+\gamma$. The value of γ is chosen uniformly randomly in (0,1] at each iteration. Then, the neighborhood operator is restarted. The increase in the penalty parameters aims to redirect the search to feasible solutions.

3.1.5. Tabu list and aspiration criterion

When a move improves the current best solution p_{best}, the move is declared forbidden for τ iterations. The tabu tenure τ is randomly chosen in a uniform interval $[1,\sqrt{|N|}]$, where N is the set of customers. If all the six moves are tabu, the one with the shortest tabu tenure is revoked the tabu status. If the current best solution p_{best} is not improved for IT iterations, the current solution p is updated by p_r randomly selected from the elite set S, and the tabu list is reset to empty. We use a single aspiration criterion: if a move can improve the solution better than other moves, it is performed even when it is tabu.

3.1.6. Diversification

In this paper, the diversification strategy is based on an elite set. Nguyen et al. (2013) have introduced an elite set to direct the search to potential unexplored promising regions when the search begins to stagnate. An elite set is designed as a diversified pool of high-quality solutions found during the tabu search.

The elite set starts empty and is limited in size. The diversity of the elite set is controlled by inserting new best solutions produced by the tabu search and the elimination of the existing solutions in the elite set. Different from Nguyen et al. (2013), in our algorithm, the elimination is based on the similarity of solutions. The similarity $\Delta(p_i,p_j)$ is defined as the sum of the number of common arcs between solution p_i and solution p_j. Note that, $\Delta(p_i,p_j)\le n-1$, where n is the number of nodes of a solution p, and $\Delta(p_i,p_j)=n-1$ means that p_i is the same as p_j. For instance, we suppose $p_1=$ (0,1,2,3,4,0,5,6,7,8), $p_2=$ (0,1,2,5,4,0,6,8,3,7). The common arcs between p_1 and p_2 are (0,1) and (1,2). Hence, the similarity $\Delta(p_1,p_2)$ is equal to 2. The elimination of a solution from the elite set is considered when a new best solution p_{best} is inserted. There are two cases. While the elite set is not yet full, we delete the solutions which are very similar to p_{best}. In our algorithm, a solution p is deleted when the similarity $\Delta(p,p_{best})\le n-2$. When the elite set is full, p_{best} replaces the solution p that is the most similar to it.

3.2. The embedded methods

The tabu search handles the search space of vehicle routes and determines x_{ij}^k. For a given solution generated by tabu search, the embedded methods determine the supply quantity (in units) y_{im}^{k}. After x_{ij}^{k} and y_{im}^{k} are obtained, the value of the objective function can be determined. This subsection describes three embedded methods, namely the exact method, the greedy method and the mixed method.

3.2.1. Exact method

The optimal solutions of y_{im}^{k} can be obtained by solving a simplified version of the model formulated in section 2.2. As the routing problem is solved by the tabu search, x_{ij}^{k} becomes a known parameter, and therefore the model can be solved by a common linear programing (LP) solver by omitting routing constraints (14) - (18), the 0-1 constraints (19) and auxiliary constraints (21). We use Gurobi 7.0 to solve the LP problem. Although this method can obtain an optimal solution, it is restricted to small size networks because the computation time drastically increases with increasing network sizes. For larger networks, heuristics become more practical solution methods.

3.2.2. Greedy method

The basic idea of this method is to give a higher priority to the denomination who has a larger unit penalty of the unmet demand. The procedure is depicted as follow:

$Step1$: Determine x_{ij}^{k}. We generate the vehicle routes by tabu search. We can obtain the value of x_{ij}^{k}, as well as the route set K, indexed by $k = 1, \dots, |K|$.

$Step2$: Determine y_{im}^{k}.

$Step2.1$: Set $k = 1$.

$Step2.2$: Set the customer list $V_k = \emptyset$.

$Step2.3$: Let i be the i'^{th} customer to be visited in route k. Set $i' = 1$.

$Step2.4$: Initialize y_{im}^{k}. Set $y_{im}^{k} = L_i^m,\ \forall\ m \in M$ to satisfy constraints (5). Update the demand in value $q_i \leftarrow q_i - \sum_{m \epsilon M} L_i^m$.

$Step2.5$: Select the denomination with the largest penalty coefficient from the denomination set M, and mark it as max. Then, set $y_{imax}^{k} = \min\ \{d_i^{max}, \frac{q_i}{a_{max}}\}$, update $q_i \leftarrow q_i - a_{max} y_{imax}^{k}$ and remove max from M. Repeat this step until $q_i = 0$.

$Step2.6$: Put i into V_k. If V_k includes all the customers visited by vehicle k, go to $Step2.7$. Otherwise, update $i' \leftarrow i' + 1$, and go to $Step2.4$.

$Step2.7$: If $k < |K|$, update $k \leftarrow k + 1$ and go to Step 2.2. Otherwise, go to Step 3.

$Step3$: Determine Q_i^k and R_i^k. Q_i^k and R_i^k are determined by Eq. (25).

$$\begin{cases} Q_0^k = \sum_{i\in N} q_i \sum_{j\in N} x_{ij}^k \\ Q_j^k = Q_i^k - q_i \\ R_0^k = 0 \\ R_i^k = R_j^k + Q_i^k c_{ij} \end{cases} \tag{25}$$

3.2.3. Mixed method

Adopting a greedy method is the simplest and fastest way to obtain a near-optimal solution but the optimality is not guaranteed, while the exact method can obtain an optimal solution with huge computation time. To balance the solution quality and the computation time, we propose a hybrid solution method by mixing these two methods. For the route candidates in the neighborhood generated in each iteration when adopting the tabu search, we divide these route candidates into two parts randomly with the proportion $\eta: 1-\eta$. One part (η) are solved optimally by the exact method in Section 3.2.1, while another part $(1-\eta)$ is solved heuristically by the greedy method in Section 3.2.2. Thus, we can get an acceptable tradeoff between the solution quality and computation time.

4. Numerical studies

In this section, numerical examples are set up to illustrate the problem properties and the performance of the combined hybrid tabu search algorithm. The solution method is coded in C# on a 3.4GHz Intel i7-2600 processor with 16GB of RAM.

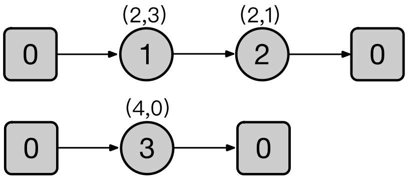
4.1. Effect of the unit penalty on the delivery strategy

In this section, we consider a small network with 2 trucks, 3 nodes ($N = \{1,2,3\}$) and 2 denominations ($M = \{\$10, \$20\}$) to show the effects of the unit penalty on the delivery strategy. The data are shown in Table 1.

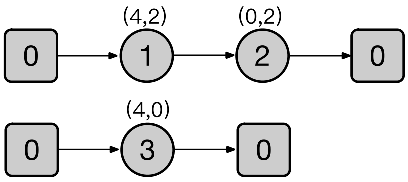
Table 1

The data of an instance with 3 nodes

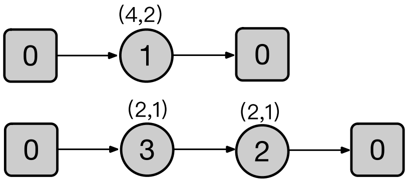
Parameters	Values
c_{ij}, w_m, T	1, 1, 200
C_k	8, 6
q_i	80, 40, 40
L_i^m, H_i^m	0, 0
d_i^m	4, 2; 2, 1; 4, 0
p_i^m	1, 1; 10, 10; 100, 1



(a) The optimal solution with original parameters



(b) The optimal solution when changing p_1^1 from 1 to 10



(c) The optimal solution when changing p_1^1 from 1 to 10, p_3^1 from 100 to 1

Fig. 2. Effect of the unit penalty p_i^m

Fig. 2a represents the initial optimal solution consisting of route $0 \to 1 \to 2 \to 0$ and route $0 \to 3 \to 0$. The values of y_{im}^k are shown in the parentheses near a node.

When p_1^1 is increased to 10 (shown in Fig. 2b), a new optimal solution is obtained and the values of y_{im}^k are changed. At customer 1, the delivery quantities with \$10 cash denomination increases to 4 to satisfy the expected demand. But the delivery quantity with \$20 cash denomination decreases 2, because the total demand of the customer must be satisfied. Meanwhile, the delivery amounts at customer 2 change to (0, 2) due to the vehicle capacity restriction. When p_1^1 is increased to 10 and p_3^1 is decreased to 1(shown in Fig. 2c), both x_{ij}^k and y_{im}^k are changed. At customer 1, the total weight of delivered cash increases to 6. Meanwhile, customer 2 is visited by the second truck due to the vehicle capacity restriction.

It is concluded that different unit penalties may lead to different cash denomination combination and different vehicle routing due to the vehicle capacity restriction.

4.2. Effect of the risk threshold on the delivery strategy

In this section, we consider a network with 2 trucks, 4 customers ($N = \{1, 2, 3, 4\}$) and 2 denominations ($M = \{\$10, \$20\}$) to investigate the relationship between the risk threshold and the delivery strategy. The data are shown in Table 2.

Table 2

The data of an instance with 4 nodes

Parameters	Values
$w_m,\ T$	1, 250
C_k	7, 5
q_i	30, 40, 20, 50
L_i^m, H_i^m	0, 10
d_i^m	1, 1; 2, 1; 2, 0; 5, 0
p_i^m	10, 10; 10, 10; 1, 1; 5, 5

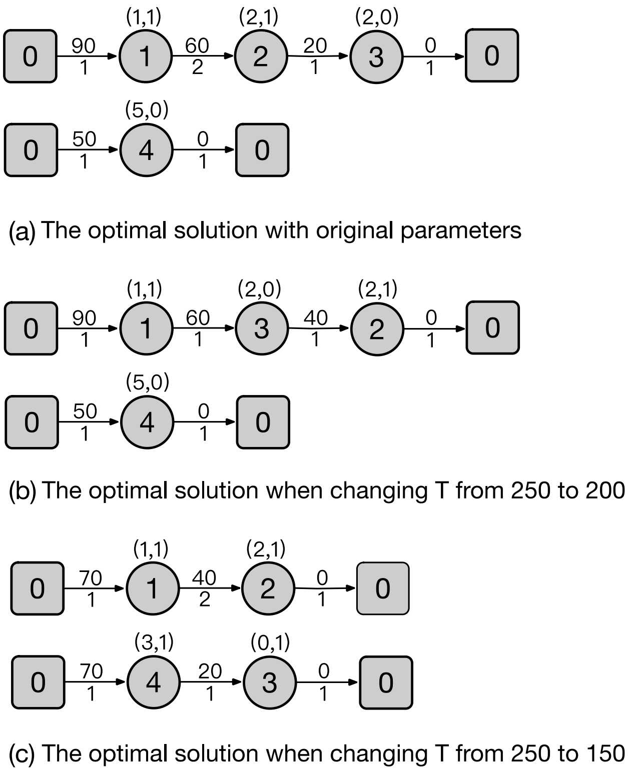


Fig. 3. Effect of the risk threshold T.

The initial optimal solution is shown in Fig. 3a. The values in the parentheses near a node are the values of y_{im}^k. Q_i^k and c_{ij} are above and below the arc respectively. We can easily obtain that the risk along route 1 are 230, 140, 20, 0, and along route 2 are 50, 0.

When risk threshold T is decreased from 250 to 200, a new optimal solution is obtained by reordering the customers of route 1 (shown in Fig. 3b). Furtherly, when risk threshold T is decreased to 150, the customers visited by vehicles are changed (shown in Fig. 3c). Another observation is that the risk threshold can also affect the delivery quantity of dominations cash.

When T=150, customer 3 is changed to be visited in route 2. Meanwhile, the cash mixes of customer 3 and customer 4 change due to the capacity restriction of truck 2.

It is concluded that different risk threshold may lead to different vehicle routing and different cash denomination combination due to the vehicle capacity restriction.

4.3. Performance analysis of the combined hybrid tabu search

This section investigates the performance of the combined hybrid tabu search, including the hybrid tabu search embedded exact method (TS-EM), the hybrid tabu search embedded mixed method (TS-MM) and the hybrid tabu search embedded greedy method (TS-GM). Since the proposed problem has not been studied so far, no test instances are available in the literature. We generated 10 instances with $|K| \times |N| = 2 \times 10, 2 \times 13, 3 \times 21, 4 \times 25, 5 \times 28, 5 \times 31, 7 \times 49, 7 \times 69, 10 \times 81, 10 \times 103$. The data are shown in Table 3.

Table 3

All instances data

Parameter	Values
d_i^m, L_i^m, H_i^m	[1, 99]
p_i^m	[0, 9]
q_i	[1, 5]
a_m	0.001, 0.005, 0.01, 0.02, 0.05, 0.1
w_m	0.8, 0.9, 1, 1, 1, 1.1
C_k	1000
T	100, 100, 300, 500, 1600, 1000,
	1500, 18000, 6000, 39000
$\|M\|$	6
node coordinates	[-10, 10]

4.3.1. Parameter settings

Talarico et al. (2015a) performed a meta-calibration experiment to generate good values of parameter μ mentioned in Section 3.1.2. We adopt their parameter setting and set μ to 3. In the study of Nguyen et al. (2013), increasing the size of the elite set improves only slightly the solution quality. They found that setting the size of the elite set to 5 achieves a better balance between solution quality and computation. We also set $|S|$ to 5. For the sake of computation efficiency, IT is set as 20 and the termination criterion is set to IT_{max}= 500.

To test the parameter η proposed in Section 3.2.3, an experimental analysis is performed. Fig. 4. shows the computation results for different η (in steps of 0.1) in instance 3×21. With the increase of η, the objective value decreases, while the computing time increases. It suggests that the increase of the proportion of the exact method can lead to improving the solution quality at the cost of an increase in computing time. When $\eta \geq 0.4$, The objective value decreases more slowly. Thus, $\eta = 0.4$ can be considered as a good tradeoff between the solution quality and computation time. The 10 generated instances are tested in the same way. A good tradeoff (good solution in a short time) can be obtained with different η in the range [0.3, 0.5]. Thus, for the sake of simplicity, we adopt $\eta = 0.4$ in the next section.

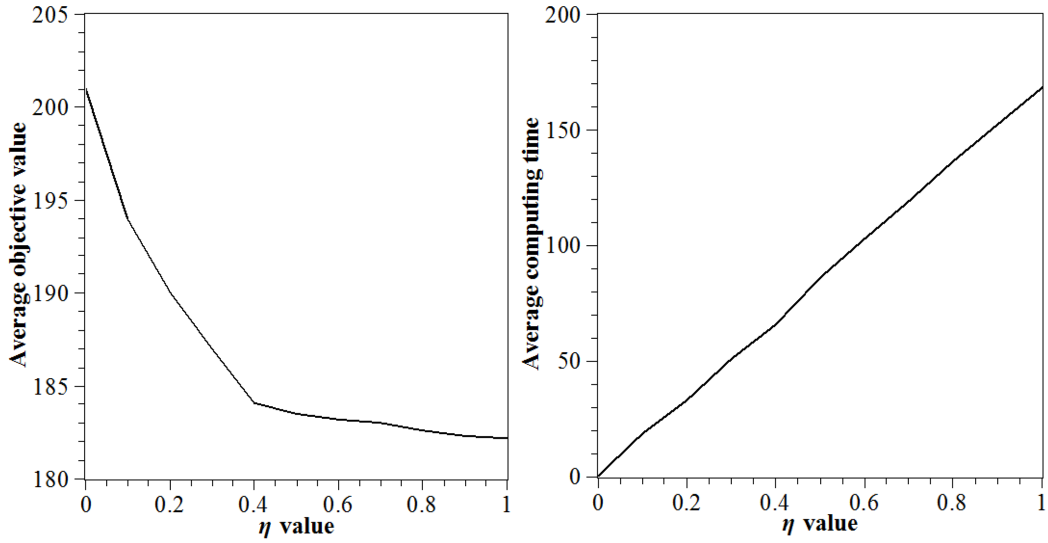


Fig. 4. Computation results of instance 3×21

4.3.2. Results and comparison

To illustrates the performance of the combined hybrid tabu search, the 10 instances solved with combined hybrid tabu search were also solved with Gurobi 7.0. A running time of 2 h was imposed on Gurobi. Table 4 reports the running times (CPU), the upper bound s(UB), the lower bounds (LB) and the corresponding gaps obtained from Gurobi, including. Table 4 also reports the average objective values (Avg.obj), the standard deviation of the objective values (Std) and the average running times obtained with the combined hybrid tabu search in 20 runs, as well as the gaps representing the deviation of the Avg.obj from the LB.

For small-size instances ($|N| = 10, 13$), Gurobi obtained the optimal solutions. However, the computation time increased exponentially with the problem size. Whereas the combined hybrid tabu search could obtain optimal solutions in a very short time compared with Gurobi.

Table 4

Comparison of the performance of Gurobi and the combined hybrid tabu search.

Instance	Gurobi				Combined hybrid tabu search											
					TS-EM				TS-MM				TS-GM			
	CPU/*s*	UB	LB	Gap/%	Avg.obj	CPU/*s*	Std	Gap/%	Avg.obj	CPU/*s*	Std	Gap/%	Avg.obj	CPU/*s*	Std	Gap/%
2×10	51.3	90.0*	90.0	0	90.0	25.1	0	0	90.0	11.0	0	0	90.0	0.1	0.21	0
2×13	6402	247.1*	247.1	0	247.1	28.8	0	0	247.1	12.5	0	0	255.0	0.2	2.6	3.2
3×21	7200	179.9	176.6	1.86	168.2	91.1	1.3	3.1	184.5	65.8	4.5	7.6	200.9	0.4	8.7	13.7
4×25	7200	388.2	380.3	2.05	396.0	265.6	0.9	4.1	403.9	134.2	5.7	6.2	429.6	1.2	13.2	13.0
5×28	7200	333.3	308.1	7.55	328.8	328.9	2.5	6.7	338.7	166.3	10.9	9.3	361.7	1.4	19.5	17.4
5×31	7200	686.8	525.9	23.4	567.3	652.5	1.6	7.8	580.0	373.3	9.1	10.3	612.7	2.2	17.6	16.5
7×49	7200	649.0	425.0	34.5	466.5	740.1	2.9	9.7	475.8	439.6	11.7	12.0	499	3.1	22.4	17.4
7×69	7200	1771.5	1223.3	44.8	1429.6	1542.2	6.7	16.9	1458.2	848.3	12.5	19.2	1558.2	3.5	28.7	27.3
10×81	7200	–	–	–	740.9	1993.3	7.1	–	750.5	1140.2	10.0	–	792.8	5.8	15.1	–
10×103	7200	–	–	–	2112.9	3648.1	17.9	–	2155.2	2247.3	32.7	–	2303	7.7	50.4	–

For medium-size instances (21≤ $|N| \leq 69$), Gurobi could not find the optimal solutions within the 2-h limit. When $|N|$ ≥28, the upper bounds were not too good compared to the results obtained from the combined hybrid tabu search and the average gap was over 27.5%. Whereas the combined hybrid tabu search could produce high-quality solutions within short computing times. As can be seen in table 4, the average gaps obtained with the combined hybrid tabu search are 10.3% (TS-EM), 12.7% (TS-MM) and 19.7% (TS-GM).

For large-size instance ($|N| \geq 81$), Gurobi could not find feasible solutions within 2-h limit. Unlike Gurobi, the combined hybrid tabu search always found feasible solutions quickly. From table 4, it can be observed that TS-GM only takes 7.7s when $|N| = 103$.

Table 4 also shows the tradeoff between computation time and solution quality when solving larger networks. Among the three schemes of combined hybrid tabu search, TS-EM always got the best solutions compared with others at the cost of requiring the longest computation times. TS-GM always got the worst solutions among the three schemes. However, it only took less than 2% computation time of other two methods. TS-MM could always illustrate a tradeoff between the solution quality and the computation time.

4.4. Effect of unit penalty and risk threshold towards the computing speed

In this section, several thresholds and penalty levels are generated to test the model. For each instance, the base penalty level P and the base risk level T are defined as the initial unit penalty and the initial risk threshold, respectively. Additional levels are generated by using several multiplicative factors. Table 5 reports the computation results. As can be seen in table 5, an increase for the unit penalty or the risk threshold does not make a significant change for the combined hybrid tabu search in computing time.

Table 5

Average running times at different penalty levels or risk levels.

Instance	Penalty level	Risk level	CPU time (s)		
			TS-EM	TS-MM	TS-GM
2×10	0.1P	0.5T	25.12	10.96	0.12
	P	T	25.16	11.0	0.11
	10P	2T	25.45	11.23	0.15
2×13	0.1P	0.5T	28.72	12.56	0.25
	P	T	28.81	12.50	0.24
	10P	2T	28.91	12.54	0.25
3×21	0.1P	0.5T	91.26	65.93	0.42
	P	T	91.15	65.81	0.43

	10P	2T	91.80	65.30	0.41
4×25	0.1P	0.5T	396.29	134.81	1.28
	P	T	396.02	134.84	1.26
	10P	2T	396.27	134.66	1.21
5×28	0.1P	0.5T	328.81	166.19	1.40
	P	T	328.80	166.30	1.43
	10P	2T	328.74	166.74	1.41
5×31	0.1P	0.5T	567.62	373.37	2.25
	P	T	567.32	373.30	2.21
	10P	2T	567.19	373.25	2.26
7×49	0.1P	0.5T	741.08	438.77	3.16
	P	T	740.14	439.67	3.18
	10P	2T	739.50	440.31	3.18
7×69	0.1P	0.5T	1428.43	847.74	3.54
	P	T	1429.63	848.34	3.59
	10P	2T	1427.96	846.56	3.56
10×81	0.1P	0.5T	1993.98	1139.47	5.84
	P	T	1993.34	1140.25	5.86
	10P	2T	1990.83	1130.75	5.86
10×103	0.1P	0.5T	3646.75	2246.15	7.67
	P	T	3648.10	2247.31	7.70
	10P	2T	3648.33	2245.22	7.73

5. Conclusions

In this paper, we introduce a new cash-in-transit problem in order to optimize the mix of different denomination cash and build safe routes. We develop a mixed integer programming model and propose a combined hybrid tabu search metaheuristic to solve the model. The results show that the proposed method yields high-quality solutions and makes a good tradeoff between the solution quality and the computation time. The experimental results also show that the unit penalty determines the mix of different denomination cash and has an indirect effect on the vehicle routes. The risk threshold determines vehicle routes and has an indirect impact on the mix of different denomination cash. Future research can focus on addressing transporting multiples type of cash (i.e., new, used, damaged banknotes).

Acknowledgments

This work was jointly supported by the Humanities and Social Science Foundation of The Ministry of Education (14YJA630026), the National Natural Science Foundation of China (71571150). The authors are grateful to the reviewer for his/her constructive comments.

References

Anbuudayasankar, S. P., Ganesh, K., Koh, S. L., Ducq, Y., 2012. Modified savings heuristics and genetic algorithm for bi-objective vehicle routing problem with forced backhauls. *Expert Systems with Applications* 39, 3, 2296-2305.

Bozkaya, B., Salman, F. S., Telciler, K., 2017. An adaptive and diversified vehicle routing approach to reducing the security risk of cash-in-transit operations. *Networks* 69, 3, 256-269.

Boonsam, P., Suthikarnnarunai, N., Chitphaiboon, W., 2011. Assignment problem and vehicle routing problem for an improvement of cash distribution. *In Proceedings of the World Congress on Engineering and Computer Science* 2, 1160-1164.

Calvo, R. W., & Cordone, R., 2003. A heuristic approach to the overnight security service problem. *Computers & Operations Research*, 30, 9, 1269-1287.

Dai, M., Liu, X.C., 2012. An improved ant colony algorithm for single vehicle route optimization. *Journal of Computational Information Systems* 10, 8, 3963-3969.

Duchenne, É., Laporte, G., Semet, F., 2007. The undirected m-peripatetic salesman problem: polyhedral results and new algorithms. *Operations Research*, 55, 5, 949-965.

Duchenne, É., Laporte, G., Semet, F., 2012. The undirected m-capacitated peripatetic salesman problem. *European Journal of Operational Research*, 223, 3, 637-643.

Eksioglu, B., Vural, A. V., Reisman, A., 2009. Survey: the vehicle routing problem: a taxonomic review. *Computers & Industrial Engineering* 57, 4, 1472-1483.

Emir, A., 2002. Delivery pricing for different demand price elasticity functions. *Doctoral dissertation*, University of Florida, United States.

Geismar, H. N., Sriskandarajah, C., Zhu, Y., 2017. A review of operational issues in managing physical currency supply chains. *Production and Operations Management* 26, 6, 976-996.

Larrain, H., Coelho, L. C., Cataldo, A., 2017. A variable MIP neighborhood descent algorithm for managing inventory and distribution of cash in automated teller machines. *Computers & Operations Research* 85, 22-31.

Michallet, J., Prins, C., Amodeo, L., Yalaoui, F., Vitry, G., 2014. Multi-start iterated local search for the periodic vehicle routing problem with time windows and time spread constraints on services. *Computers & Operations Research* 41, 1, 196-207.

Ngueveu, S. U., Prins, C., Wolfler Calvo, R., 2010a. A hybrid tabu search for the m-peripatetic vehicle routing problem. *Matheuristics*, 10, 253–266.

Ngueveu, S. U., Prins, C., Calvo, R. W., 2010b. Lower and upper bounds for the m-peripatetic vehicle routing problem. *4OR*, 8, 4, 387-406.

Nguyen, P. K., Crainic, T. G., Toulouse, M., 2013. A tabu search for time-dependent multi-zone multi-trip vehicle routing problem with time windows. *European Journal of Operational Research* 231, 1, 43-56.

Partyka, J. G., Hall, R. W., 2000. On the road to service. *OR/MS Today* 215, 3, 572-580.

Radojičić, N., Marić, M., Takači, A., 2018. A New Fuzzy Version of the Risk-constrained Cash-in-Transit Vehicle Routing Problem. *Information Technology and Contro*l, 47, 2, 321-337.

Rogoff, K. S., 2017. The curse of cash: How large-denomination bills aid crime and tax evasion and constrain monetary policy. Princeton University Press.

Roel G. Van Anholt, Leandro C. Coelho, Gilbert Laporte, Iris F. A. Vis., 2016. An inventory-routing problem with pickups and deliveries arising in the replenishment of automated teller machines. *Transportation Science* 50, 3, 1077-1091.

Talarico, L., Sörensen, K., Springael, J., 2015a. Metaheuristics for the risk-constrained cash-in-transit vehicle routing problem. *European Journal of Operational Research* 244, 2, 457-470.

Talarico, L., Sörensen, K., Springael, J., 2015b. The k-dissimilar vehicle routing problem. *European Journal of Operational Research* 244, 1, 129-140.

Talarico, L., (2016). Secure vehicle routing: models and algorithms to increase security and reduce costs in the cash-in-transit sector. *4OR* 146, 1, 105-105.

Talarico, L., Springael, J., Sörensen, K., Talarico, F., 2017a. A large neighbourhood metaheuristic for the risk-constrained cash-in-transit vehicle routing problem. *Computers & Operations Research* 78, 547-556.

Talarico, L., Sörensen, K., Springael, J., 2017b. A biobjective decision model to increase security and reduce transportation costs in the cash-in-transit sector. *International Transactions in Operational Research* 24, 1-2, 59-76.

Tarantilis, C. D., Kiranoudis, C. T., 2004. An adaptive memory programming method for risk logistics operations. *International journal of systems science* 35, 10, 579-590.

Yan, S., Wang, S. S., Wu, M. W., 2012. A model with a solution algorithm for the cash transportation vehicle routing and scheduling problem. *Computers & Industrial Engineering* 63, 2, 464-473.

Yan, S., Wang, S. S., Chang, Y. H., 2014. Cash transportation vehicle routing and scheduling under stochastic travel times. *Engineering Optimization* 46, 3, 289-307.