Regret aversion and asymmetric price distribution

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Abstract This paper examines the economic asymmetries between a regret-averse firm and a risk-averse firm under price uncertainty. We show that the global and marginal effects of price uncertainty on production are both positive (negative) when regret aversion prevails if the random output price is asymmetrically distributed with positive (negative) skewness. In this case, high (low) output prices are much more likely to be seen than low (high) output prices. To minimize regret, the firm is induced to raise (lower) its output optimal level. The skewness of the price distribution as such plays a pivotal role in determining the regret-averse firm's production decision under price uncertainty.

JEL classification: D21; D24; D81

Keywords: Production; Regret aversion; Risk aversion; Skewness

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Abstract This paper examines the economic asymmetries between a regret-averse firm and a risk-averse firm under price uncertainty. We show that the global and marginal effects of price uncertainty on production are both positive (negative) when regret aversion prevails if the random output price is asymmetrically distributed with positive (negative) skewness. In this case, high (low) output prices are much more likely to be seen than low (high) output prices. To minimize regret, the firm is induced to raise (lower) its output optimal level. The skewness of the price distribution as such plays a pivotal role in determining the regret-averse firm's production decision price uncertainty.

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1. Introduction

There is ample evidence that individuals and firms have desires to avoid adverse consequences arising from their ex-ante optimal decisions that turn out to be ex-post suboptimal (Loomes, 1988; Loomes et al., 1992; Loomes and Sugden, 1987; Starmer and Sugden, 1993). To reconcile these pervasive regret-averse preferences, Bell (1982, 1983) and Loomes and Sugden (1982) develop regret theory that defines regret as the disutility arising from not having chosen the ex-post optimal alternative. An axiomatic foundation of regret theory is later offered by Quiggin (1994) and Sugden (1993).

In a recent article, Broll et al. (2016) have incorporated regret theory into Sandmo's (1971) model of the competitive firm under price uncertainty.[1] The firm's regret-averse preferences are characterized by a bivariate utility function that includes additive separable disutility from having chosen ex-post suboptimal alternatives.[2] The extent of regret is

[1]Broll and Wong (2015) introduce ambiguity and ambiguity aversion to exporting firms to study their incentives to export to foreign countries.

[2]Other applications of regret theory include Braun and Muermann (2004), Guo et al. (2015), Muermann et al. (2006), and Wong (2012, 2015).

gauged by the difference between the actual profit and the maximum profit attained by making the optimal production decision had the firm observed the true realized output price. Broll et al. (2016) derive a sufficient condition under which the regret-averse firm optimally produces less when the output price becomes uncertain. As an extension, Niu et al. (2014) derive an alternative sufficient condition for such a negative global effect of price uncertainty on production.

In this paper, we revisit the results of Broll et al. (2016) by examining not only the global effect but also the marginal effect of changes in price uncertainty on production. We show that both effects are positive (negative) if the random output price is asymmetrically distributed with positive (negative) skewness. In this case, high (low) output prices are much more likely to be seen than low (high) output prices. To minimize regret, the firm is induced to raise (lower) its optimal output level. We as such show that the skewness of the price distribution plays a pivotal role in determining the regret-averse firm's production decision. These are novel results when contrasting with those of the literature on the competitive firm under price uncertainty, thereby highlighting the economic asymmetries between regret aversion and risk aversion in shaping decision making processes under uncertainty.

The rest of this paper is organized as follows. Section 2 incorporates regret aversion into the model of the competitive firm under price uncertainty. Section 3 examines the global effect of price uncertainty on production. Section 4 examines the marginal effect of changes in price uncertainty on production. The final section concludes.

2. The model

Consider the competitive firm under price uncertainty *à la* Sandmo (1971). There is one period with two dates, 0 and 1. To begin, the firm produces a single commodity according to a deterministic cost function, $C(Q)$, where $Q \geq 0$ is the output level, and $C(Q)$ is compounded to date 1 with the properties that $C(0) = C'(0) = 0$, and $C'(Q) > 0$ and

$C''(Q) > 0$ for all $Q > 0$. The strict convexity of $C(Q)$ reflects the fact that the firm's production technology exhibits decreasing returns to scale.

At date 1, the firm sells the entirety of its output, Q, at the then prevailing per-unit price, $\tilde{P}$, which is not known ex ante.[3] The uncertain per-unit price, $\tilde{P}$, is distributed according to a known cumulative distribution function (CDF), $F(P)$, over support $[\underline{P}, \overline{P}]$, where $0 < \underline{P} < \overline{P}$. The firm's profit at date 1 as a function of P is, therefore, given by

$$\Pi(P) = PQ - C(Q), \tag{1}$$

for all $P \in [\underline{P}, \overline{P}]$.

Following Paroush and Venezia (1979), we assume that the firm's preferences are represented by the following bivariate utility function:

$$V(\Pi, R) = U(\Pi) - \beta G(R), \tag{2}$$

where $U(\Pi)$ is a von Neumann-Morgenstern utility function with $U'(\Pi) > 0$ and $U''(\Pi) < 0$ for all $\Pi > 0$, β is a positive constant, and $G(R)$ is a regret function defined over the magnitude of regret, R, such that $G(0) = 0$, and $G'(R) > 0$ and $G''(R) > 0$ for all $R > 0$.[4] The magnitude of regret, $R = \Pi^{\max} - \Pi$, is gauged by the difference between the actual profit, Π, and the maximum profit, $\Pi^{\max}$, that the firm could have earned at date 1 should the firm have made the optimal production decision based on knowing the true per-unit price, P. Since Π cannot exceed $\Pi^{\max}$, the firm experiences disutility from forgoing the possibility of undertaking the ex-post optimal production decision. The parameter, β, is a constant regret coefficient that reflects the increasing importance of regret aversion in representing the firm's preferences as β increases.

To characterize the regret-averse firm's optimal production decision, we have to first determine the maximum profit, $\Pi^{\max}$, at date 1. If the firm could have observed the true

[3]Throughout the paper, random variables have a tilde (~) while their realizations do not.
[4]Bleichrodt et al. (2010) provide empirical evidence that regret functions are indeed convex.

per-unit price, P, the maximum profit at date 1 would be achieved if the firm had chosen $Q(P)$, which is the solution to $C'[Q(P)] = P$. The maximum profit at date 1 as a function of P is, therefore, given by

$$\Pi^{\max}(P) = PQ(P) - C[Q(P)], \tag{3}$$

for all $P \in [\underline{P}, \overline{P}]$. Using Eqs. (1) and (3), we can write the magnitude of regret, $R(P)$, as

$$R(P) = \Pi^{\max}(P) - \Pi(P) = PQ(P) - C[Q(P)] - [PQ - C(Q)], \tag{4}$$

for all $P \in [\underline{P}, \overline{P}]$.

We can now state the regret-averse firm's ex-ante decision problem. At date 0, the firm chooses an output level, Q, so as to maximize the expected value of the bivariate utility function defined in Eq. (2):

$$\max_{Q \geq 0} \; \mathrm{E}\{U[\Pi(\tilde{P})] - \beta G[R(\tilde{P})]\}, \tag{5}$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the CDF of $\tilde{P}$, and $\Pi(P)$ and $R(P)$ are given by Eqs. (1) and (4), respectively. The first-order condition for program (5) is given by

$$\mathrm{E}\Big\{\{U'[\Pi^*(\tilde{P})] + \beta G'[R^*(\tilde{P})]\}[\tilde{P} - C'(Q^*)]\Big\} = 0, \tag{6}$$

where an asterisk (*) signifies an optimal level.

Differentiating the objective function of program (5) twice with respect to Q yields

$$\mathrm{E}\Big\{\{U''[\Pi(\tilde{P})] - \beta G''[R(\tilde{P})]\}[\tilde{P} - C'(Q)]^2 - \{U'[\Pi(\tilde{P})] + \beta G'[R(\tilde{P})]\}C''(Q)\Big\} < 0, \tag{7}$$

for all $Q > 0$, where the inequality follows from the properties of $U(\Pi)$, $G(R)$, and $C(Q)$. Eq. (7) implies that Eq. (6) is both necessary and sufficient for Q^* to be the unique optimal solution to program (5).

3. The global effect of price uncertainty

As a benchmark, we consider the case wherein the uncertain per-unit price, $\tilde{P}$, is fixed at its expected value, $\mathrm{E}(\tilde{P})$. In this benchmark case of certainty, Eq. (6) reduces to $C'(Q^\circ) = \mathrm{E}(\tilde{P})$, where Q° is the optimal output level under certainty. This is the usual optimality condition under which the marginal cost of production is equated to the expected per-unit price.

To examine the global effect of price uncertainty on the firm's production decision, we compare Q^* with Q°. To this end, we first consider the case that the firm is risk neutral and regret averse, i.e., $U(\Pi) = \Pi$ and $G''(R) > 0$. Differentiating the objective function of program (5) with $U(\Pi) = \Pi$ with respect to Q, and evaluating the resulting the derivative at $Q = Q^\circ$ yields

$$\beta \mathrm{E}\{G'[R^\circ(\tilde{P})][\tilde{P} - \mathrm{E}(\tilde{P})]\} = \beta \mathrm{Cov}\{G'[R^\circ(\tilde{P})], \tilde{P}\}, \tag{8}$$

where we have used the fact that $C'(Q^\circ) = \mathrm{E}(\tilde{P})$, and $\mathrm{Cov}(\cdot, \cdot)$ is the covariance operator with respect to the CDF of $\tilde{P}$.[5] It then follows from Eqs. (6) and (7) that $Q^* > (<)\ Q^\circ$ if, and only if, the covariance term on the right-hand side of Eq. (8) is positive (negative). We state and prove our first proposition.

Proposition 1. *The regret-averse, but risk-neutral, competitive firm increases or decreases its optimal output level, i.e., Q^* is greater or smaller than Q°, when the per-unit price becomes uncertain, depending on whether $\mathrm{E}\{G'[R^\circ(\tilde{P})]\}$ is no less than $G'[R^\circ(\underline{P})]$ or $G'[R^\circ(\overline{P})]$, respectively.*

Proof. Note that

$$\frac{\partial G'[R^\circ(P)]}{\partial P} = G''[R^\circ(P)][Q(P) - Q^\circ]. \tag{9}$$

[5] For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

Since $Q'(P) = 1/C''[Q(P)] > 0$ and $Q[\mathrm{E}(\tilde{P})] = Q^\circ$, it follows from Eq. (9) and $G''(R) > 0$ that $G'[R^\circ(P)]$ is decreasing (increasing) in P for all $P < (>)\ \mathrm{E}(\tilde{P})$. Hence, $G'[R^\circ(P)]$ is U-shaped and reaches a unique minimum at $P = \mathrm{E}(\tilde{P})$. If $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\underline{P})]$, there must exist a unique per-unit price, $P^\circ \in (\mathrm{E}(\tilde{P}), \overline{P})$, such that $G'[R^\circ(P^\circ)] = \mathrm{E}\{G'[R^\circ(\tilde{P})]\}$. Since $G'[R^\circ(P)]$ is U-shaped and $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\underline{P})]$, it follows that $G'[R^\circ(P)] < \mathrm{E}\{G'[R^\circ(\tilde{P})]\}$ for all $P \in (\underline{P}, P^\circ)$ and $G'[R^\circ(P)] > \mathrm{E}\{G'[R^\circ(\tilde{P})]\}$ for all $P \in (P^\circ, \overline{P}]$. Hence, we have

$$\mathrm{Cov}\{G'[R^\circ(\tilde{P})], \tilde{P}\} = \mathrm{E}\bigg\{\bigg\{G'[R^\circ(\tilde{P})] - \mathrm{E}\{G'[R^\circ(\tilde{P})]\}\bigg\}(\tilde{P} - P^\circ)\bigg\} > 0. \tag{10}$$

Eq. (10) then implies that $Q^* > Q^\circ$. The proof that $Q^* < Q^\circ$ if $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\overline{P})]$ can be done analogously and thus is omitted. □

To see the validity of the condition that $\mathrm{E}\{G'[R^\circ(\tilde{P})]\}$ can be no less than $G'[R^\circ(\underline{P})]$ or $G'[R^\circ(\overline{P})]$, we consider the case that $\tilde{P}$ is a binary random variable such that $\tilde{P}$ is equal to either $\underline{P}$ or $\overline{P}$ with probability p or $1 - p$, respectively. In this binary example, $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} > (<)\ G'[R^\circ(\underline{P})]$ if, and only if, $R^\circ(\underline{P}) < (>)\ R^\circ(\overline{P})$. To be more concrete, we assume that the cost function is quadratic, $C(Q) = cQ^2$, where c is a positive constant. Then, we have $Q(P) = P/2c$ and $R^\circ(P) = [P - \mathrm{E}(\tilde{P})]^2/4c$. In this case, we have $R^\circ(\underline{P}) < (>)\ R^\circ(\overline{P})$, thereby $Q^* > (<)\ Q^\circ$, if, and only if, $p > (<)\ 1/2$.

The intuition for Proposition 1 is as follows. If $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\underline{P})]$, it follows from Eq. (10) that $G'[R^\circ(\tilde{P})]$ is positively correlated with $\tilde{P}$. Introducing regret aversion to the firm makes the firm raise more concerns about the disutility from the discrepancy of its output level, $Q(P) - Q^\circ$, when high realizations of $\tilde{P}$ are revealed. To minimize regret, the regret-averse firm optimally adjusts its output level upward from Q° so that $Q^* > Q^\circ$. On the other hand, if $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\overline{P})]$, then $G'[R^\circ(\tilde{P})]$ is negatively correlated with $\tilde{P}$. Introducing regret aversion to the firm makes the firm raise more concerns about the disutility from the discrepancy of its output level, $Q^\circ - Q(P)$, when low realizations of $\tilde{P}$ are revealed. To minimize regret, the regret-averse firm optimally adjusts its output level downward from Q° so that $Q^* < Q^\circ$.

We now resume the original case that the firm is both risk averse and regret averse, i.e., $U''(\Pi) < 0$ and $G''(R) > 0$. Differentiating the objective function of program (5) with respect to Q, and evaluating the resulting derivative at $Q = Q^\circ$ yields

$$\mathrm{E}\{\Psi(\tilde{P})[\tilde{P} - \mathrm{E}(\tilde{P})]\} = \mathrm{Cov}[\Psi(\tilde{P}), \tilde{P}], \tag{11}$$

where $\Psi(P) = U'[\Pi^\circ(P)] + \beta G'[R^\circ(P)]$ and we have used the fact that $C'(Q^\circ) = \mathrm{E}(\tilde{P})$. It then follows from Eqs. (6) and (7) that $Q^* > (<)\ Q^\circ$ if, and only if, the covariance term on the right-hand side of Eq. (11) is positive (negative). We state and prove the following proposition.

Proposition 2. *If $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, the regret-averse competitive firm increases or decreases its optimal output level, i.e., Q^* is greater or smaller than Q°, when the per-unit price becomes uncertain, depending on whether $\mathrm{E}[\Psi(\tilde{P})]$ is no less than $\Psi(\underline{P})$ or $\Psi(\overline{P})$, respectively.*

Proof. Note that

$$\Psi'(P) = U''[\Pi^\circ(P)]Q^\circ + \beta G''[R^\circ(P)][Q(P) - Q^\circ], \tag{12}$$

and

$$\Psi''(P) = U'''[\Pi^\circ(P)]Q^{\circ 2} + \beta G'''[R^\circ(P)][Q(P) - Q^\circ]^2 + \beta G''[R^\circ(P)]Q'(P). \tag{13}$$

Since $Q(P) < (>)\ Q^\circ$ for all $P < (>)\ \mathrm{E}(\tilde{P})$, Eq. (12) implies that $\Psi'(P) < 0$ for all $P \leq \mathrm{E}(\tilde{P})$. Since $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, Eq. (13) implies that $\Psi''(P) > 0$ for all $P \in [\underline{P}, \overline{P}]$. By Jensen's inequality, we have $\mathrm{E}[\Psi(\tilde{P})] > \Psi[\mathrm{E}(\tilde{P})]$. If $\mathrm{E}[\Psi(\tilde{P})] \geq \Psi(\underline{P})$, there must exist a unique per-unit price, $P^\circ \in (\mathrm{E}(\tilde{P}), \overline{P})$, such that $\Psi(P^\circ) = \mathrm{E}[\Psi(\tilde{P})]$. Since

$\Psi(P)$ is convex and $\mathrm{E}[\Psi(\tilde{P})] \geq \Psi(\underline{P})$, it follows that $\Psi(P) < \mathrm{E}[\Psi(\tilde{P})]$ for all $P \in (\underline{P}, P^\circ)$ and $\Psi(P) > \mathrm{E}[\Psi(\tilde{P})]$ for all $P \in (P^\circ, \overline{P}]$. Hence, we have

$$\mathrm{Cov}[\Psi(\tilde{P}), \tilde{P}] = \mathrm{E}\Big\{\{\Psi(\tilde{P}) - \mathrm{E}[\Psi(\tilde{P})]\}(\tilde{P} - P^\circ)\Big\} > 0. \tag{14}$$

Eq. (14) then implies that $Q^* > Q^\circ$. The proof that $Q^* < Q^\circ$ if $\mathrm{E}[\Psi(\tilde{P})] \geq \Psi(\overline{P})$ can be done analogously and thus is omitted. □

The sufficient condition for $Q^* > Q^\circ$, i.e., $\mathrm{E}[\Psi(\tilde{P})] \geq \Psi(\underline{P})$, can be written as

$$\beta\Big\{\mathrm{E}\{G'[R^\circ(\tilde{P})] - G'[R^\circ(\underline{P})]\}\Big\} \geq U'[\Pi^\circ(\underline{P})] - \mathrm{E}\{U'[\Pi^\circ(\tilde{P})]\}. \tag{15}$$

The right-hand side of condition (15) is positive since $U''(\Pi) < 0$. Condition (15) never holds for all $\beta > 0$ if $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \leq G'[R^\circ(\underline{P})]$. In this case, we cannot unambiguously compare Q^* with Q°. For condition (15) to hold, we need $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} > G'[R^\circ(\underline{P})]$ and

$$\beta \geq \frac{U'[\Pi^\circ(\underline{P})] - \mathrm{E}\{U'[\Pi^\circ(\tilde{P})]\}}{\mathrm{E}\{G'[R^\circ(\tilde{P})]\} - G'[R^\circ(\underline{P})]} > 0. \tag{16}$$

Given that $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} > G'[R^\circ(\underline{P})]$, Proposition 1 implies that the regret-averse firm has an incentive to produce beyond Q° should the firm be risk neutral. Since the firm is in fact risk averse, there is a countervailing incentive that induces the firm to reduce its output level when the per-unit price becomes uncertain. Condition (16) simply says that the firm is sufficiently regret averse, i.e., β is sufficiently large, in that the incentive driven by regret aversion dominates the opposing incentive driven by risk aversion. The firm as such optimally produces more upon introducing the price uncertainty.

The sufficient condition for $Q^* < Q^\circ$, i.e., $\mathrm{E}[\Psi(\tilde{P})] \geq \Psi(\overline{P})$, can be written as

$$\beta\Big\{G'[R^\circ(\overline{P})] - \mathrm{E}\{G'[R^\circ(\tilde{P})]\}\Big\} \leq \mathrm{E}\{U'[\Pi^\circ(\tilde{P})]\} - U'[\Pi^\circ(\overline{P})]. \tag{17}$$

The right-hand side of condition (17) is positive since $U''(\Pi) < 0$. Condition (17) holds for all $\beta > 0$ if $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\overline{P})]$. Given that $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\overline{P})]$, Proposition

1 implies that the regret-averse firm has an incentive to produce below Q° should the firm be risk neutral. Since the firm is in fact risk averse, there is a reinforcing incentive that induces the firm to reduce its output level when the price uncertainty prevails. For all $\beta > 0$, the firm optimally produces less when the per-unit price becomes uncertain.

If $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} < G'[R^\circ(\overline{P})]$, condition (17) reduces to $\beta \leq \beta_1$, where

$$\beta_1 = \frac{\mathrm{E}\{U'[\Pi^\circ(\tilde{P})]\} - U'[\Pi^\circ(\overline{P})]}{G'[R^\circ(\overline{P})] - \mathrm{E}\{G'[R^\circ(\tilde{P})]\}} > 0. \tag{18}$$

Given that $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} < G'[R^\circ(\overline{P})]$, the results of Proposition 1 are not applicable so that the incentive driven by regret aversion is ambiguous. However, if the firm is not too regret averse in that $\beta \leq \beta_1$, the incentive driven by risk aversion to reduce output in response to the presence of price uncertainty becomes the dominant factor, thereby rendering $Q^* < Q^\circ$.

Wong (2014) derives a sufficient condition for $Q^* < Q^\circ$, which requires $\beta \leq \beta_2$ if $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, where

$$\beta_2 = \frac{U'\{\Pi^\circ[\mathrm{E}(\tilde{P})]\} - U'[\Pi^\circ(\overline{P})]}{G'[R^\circ(\overline{P})] - G'(0)} > 0, \tag{19}$$

and $G'(0) = G'\{R^\circ[\mathrm{E}(\tilde{P})]\}$. Niu et al. (2014) derive an alternative sufficient condition for $Q^* < Q^\circ$, which requires $\beta \leq \beta_3$ if $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, where

$$\beta_3 = -\frac{U''[\Pi^\circ(\overline{P})]Q^\circ}{G''[R^\circ(\overline{P})][Q(\overline{P}) - Q^\circ]}. \tag{20}$$

Since $U'''(\Pi) \geq 0$, we have

$$U''[\Pi^\circ(\overline{P})] \geq \frac{U'\{\Pi^\circ[\mathrm{E}(\tilde{P})]\} - U'[\Pi^\circ(\overline{P})]}{\Pi^\circ[\mathrm{E}(\tilde{P})]\} - \Pi^\circ(\overline{P})}. \tag{21}$$

Since $\Pi^\circ(\overline{P}) - \Pi^\circ[\mathrm{E}(\tilde{P})] = [\overline{P} - \mathrm{E}(\tilde{P})]Q^\circ$, we can write inequality (21) as

$$-U''[\Pi^\circ(\overline{P})]Q^\circ \leq \frac{U'\{\Pi^\circ[\mathrm{E}(\tilde{P})]\} - U'[\Pi^\circ(\overline{P})]}{\overline{P} - \mathrm{E}(\tilde{P})}. \tag{22}$$

Since $G'''(R) \geq 0$ and $G'(0) = G'\{R^\circ[\mathrm{E}(\tilde{P})]\}$, we have

$$\frac{G'[R^\circ(\overline{P})] - G'(0)}{\overline{P} - \mathrm{E}(\tilde{P})} \leq \frac{\partial G'[R^\circ(P)]}{\partial P}\bigg|_{P=\overline{P}} = G''[R^\circ(\overline{P})][Q(\overline{P}) - Q^\circ]. \tag{23}$$

It then follows from inequalities (22) and (23) that $\beta_3 \leq \beta_2$. Hence, the alternative sufficient condition of Niu et al. (2014) implies that of Wong (2014), but not vice versa. In other words, the sufficient condition of Wong (2014) is less restrictive than that of Niu et al. (2014).

If $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} \geq G'[R^\circ(\overline{P})]$, we show that $Q^* < Q^\circ$ for all $\beta > 0$. On the other hand, if $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} < G'[R^\circ(\overline{P})]$, we show that $Q^* < Q^\circ$ for all $\beta \leq \beta_1$. Since $U'''(\Pi) \geq 0$, we have $\mathrm{E}\{U'[\Pi^\circ(\tilde{P})]\} \geq U'\{\Pi^\circ[\mathrm{E}(\tilde{P})]\}$. Since $G''(R) > 0$, we have $\mathrm{E}\{G'[R^\circ(\tilde{P})]\} > G'(0)$. It then follows from Eqs. (18) and (19) that $\beta_1 > \beta_2$. Hence, our sufficient conditions are implied by those of Wong (2014) and Broll et al. (2016), but not vice versa. In other words, our sufficient conditions are more general.

4. The marginal effect of price uncertainty

In this section, we examine the marginal effect of changes in price uncertainty on production when regret aversion prevails. To this end, we let $\hat{F}(P)$ be a new CDF of $\tilde{P}$. When the original CDF, $F(P)$, is replaced by the new CDF, $\hat{F}(P)$, the first-order condition for program (5) becomes

$$\int_{\underline{P}}^{\overline{P}} \{U'[\Pi^\dagger(P)] + \beta G'[R^\dagger(P)]\}[P - C'(Q^\dagger)]\mathrm{d}\hat{F}(P) = 0, \tag{24}$$

where a dagger (†) signifies an optimal level. To compare $Q^\dagger$ with Q^*, we evaluate the left-hand side of Eq. (24) at Q^* to yield

$$\int_{\underline{P}}^{\overline{P}} \{U'[\Pi^*(P)] + \beta G'[R^*(P)]\}[P - C'(Q^*)]\mathrm{d}\hat{F}(P)$$

$$= \int_{\underline{P}}^{\overline{P}} \{U'[\Pi^*(P)] + \beta G'[R^*(P)]\}[P - C'(Q^*)]\mathrm{d}[\hat{F}(P) - F(P)], \tag{25}$$

where the equality follows from Eq. (6). It then follows from Eqs. (7) and (24) that $Q^\dagger > (<)\ Q^*$ if, and only if, the right-hand side of Eq. (25) is positive (negative).

We first consider the case wherein $\hat{F}(P)$ is riskier than $F(P)$ in the sense of first-order stochastic dominance (FSD), i.e., $\hat{F}(P) \geq F(P)$ for all $P \in [\underline{P}, \overline{P}]$, with strict inequality at some P. We state and prove the following proposition.

Proposition 3. *When the uncertain per-unit price, $\tilde{P}$, experiences an increase in risk via a FSD shift from $F(P)$ to $\hat{F}(P)$, the regret-averse competitive firm reduces its optimal output level, i.e., $Q^\dagger < Q^*$, if the firm's coefficient of relative risk aversion does not exceed unity, i.e., $-\Pi U''(\Pi)/U'(\Pi) \leq 1$ for all $\Pi > 0$.*

Proof. Let $\Phi(P) = \{U'[\Pi^*(P)] + \beta G'[R^*(P)]\}[P - C'(Q^*)]$. Then, we have

$$\Phi'(P) = U''[\Pi^*(P)][C(Q^*) - C'(Q^*)Q^*] + U'[\Pi^*(P)]\left\{1 + \Pi^*(P)\frac{U''[\Pi^*(P)]}{U'[\Pi^*(P)]}\right\}$$
$$+\beta G''[R^*(P)][Q(P) - Q^*][P - C'(Q^*)] + \beta G'[R^*(P)]. \tag{26}$$

Since $C''(Q) > 0$ and $U''(\Pi) < 0$, the first term on the right-hand side of Eq. (26) is positive. Given that $-\Pi U''(\Pi)/U'(\Pi) \leq 1$ for all $\Pi > 0$, the second term is also positive. Since $Q'(P) > 0$, we have $Q(P) < (>)\ Q^*$ whenever $P < (>)\ C'(Q^*)$. The third term on the right-hand side of Eq. (26) is positive so that $\Phi'(P) > 0$. Using $\Phi(P)$, we can write the right-hand side of Eq. (25) as

$$\int_{\underline{P}}^{\overline{P}} \Phi(P)\mathrm{d}[\hat{F}(P) - F(P)] = -\int_{\underline{P}}^{\overline{P}} \Phi'(P)[\hat{F}(P) - F(P)]\mathrm{d}P < 0. \tag{27}$$

where the equality follows from integration by parts, and the inequality follows from the fact that $\hat{F}(P) \geq F(P)$ for all $P \in [\underline{P}, \overline{P}]$, with strict inequality at some P. Hence, we conclude from Eq. (27) that $Q^\dagger < Q^*$. $\square$

The intuition for Proposition 3 is as follows. Since there is a FSD shift from the original CDF, $F(P)$, to the new CDF, $\hat{F}(P)$, the realizations of $\tilde{P}$ close to $\underline{P}$ are now much more likely to be seen than those close to $\overline{P}$. The regret-averse firm as such raises more concerns about the disutility from the discrepancy of its output level, $Q^* - Q(P)$, when low realizations of $\tilde{P}$ are revealed given the FSD shift from $F(P)$ to $\hat{F}(P)$. To minimize regret, the firm optimally adjusts its output level downward from Q^*. The FSD shift from $F(P)$ to $\hat{F}(P)$, albeit reducing the expected per-unit price, has a side effect that induces the risk-averse firm to raise its output level in order to better stabilize its marginal utility, $U'[\Pi^*(\tilde{P})]$. Since the elasticity of the firm's marginal utility is gauged by the coefficient of relative risk aversion, $-\Pi U''(\Pi)/U'(\Pi)$, the firm's marginal utility would be insensitive to changes in profit when $-\Pi U''(\Pi)/U'(\Pi) \leq 1$ for all $\Pi > 0$. In this case, risk aversion also induces the firm to produce less so that $Q^\dagger < Q^*$.

Since $Q'(P) > 0$, there must exist a unique per-unit price, $P^* \in (\underline{P}, \overline{P})$, at which $Q(P^*) = Q^*$, i.e., $P^* = C'(Q^*)$. The following definition is adopted from the definition of downside risk *à la* Menezes et al. (1980).[6]

Definition 1. *The CDF, $\hat{F}(P)$, is said to have more simple positive (negative) skewness than the CDF, $F(P)$, if, and only if,*

$$\int_{\underline{P}}^{\overline{P}} [\hat{F}(P) - F(P)]\mathrm{d}P = 0, \tag{28}$$

$$\int_{\underline{P}}^{\overline{P}} \left\{ \int_{\underline{P}}^{P} [\hat{F}(x) - F(x)]\mathrm{d}x \right\} \mathrm{d}P = 0, \tag{29}$$

$$\int_{\underline{P}}^{P} [\hat{F}(x) - F(x)]\mathrm{d}x \leq (\geq)\ 0 \ \ \textit{for all } P \leq P^*, \tag{30}$$

$$\int_{\underline{P}}^{P} [\hat{F}(x) - F(x)]\mathrm{d}x \geq (\leq)\ 0 \ \ \textit{for all } P \geq P^*, \tag{31}$$

and

$$\int_{\underline{P}}^{P} \left\{ \int_{\underline{P}}^{x} [\hat{F}(y) - F(y)]\mathrm{d}y \right\} \mathrm{d}x \leq (\geq)\ 0 \ \ \textit{for all } P \in [\underline{P}, \overline{P}]. \tag{32}$$

[6] An increase in downside risk in the sense of Menezes et al. (1980) is simply a third-degree increase in risk in the sense of Ekern (1980).

Eq. (28) ensures that $\tilde{P}$ has the same mean under $F(P)$ and $\hat{F}(P)$. Eq. (29) ensures that $\tilde{P}$ has the same variance, denoted by σ^2, under $F(P)$ and $\hat{F}(P)$. Eq. (32) ensures that $\tilde{P}$ has more positive (negative) skewness under $\hat{F}(P)$ than under $F(P)$, while Eqs. (30) and (31) ensure a single-crossing property. To see this, we compare the central third moment under $\hat{F}(P)$ and that under $F(P)$:

$$\begin{aligned}
&\int_{\underline{P}}^{\overline{P}} \left[\frac{P-\mathrm{E}(\tilde{P})}{\sigma}\right]^3 \mathrm{d}[\hat{F}(P)-F(P)] \\
&= \frac{6}{\sigma^3}\int_{\underline{P}}^{\overline{P}} [P-\mathrm{E}(\tilde{P})]\left\{\int_{\underline{P}}^{P}[\hat{F}(x)-F(x)]\mathrm{d}x\right\}\mathrm{d}P \\
&= \frac{6}{\sigma^3}\int_{\underline{P}}^{\overline{P}} (P-P^*)\left\{\int_{\underline{P}}^{P}[\hat{F}(x)-F(x)]\mathrm{d}x\right\}\mathrm{d}P \\
&= -\frac{6}{\sigma^3}\int_{\underline{P}}^{\overline{P}} \left\{\int_{\underline{P}}^{P}\left\{\int_{\underline{P}}^{x}[\hat{F}(y)-F(y)]\mathrm{d}y\right\}\mathrm{d}x\right\}\mathrm{d}P, \qquad (33)
\end{aligned}$$

where the first equality follows from integration by parts and Eq. (28), the second equality follows from Eq. (29), and the last equality follows from integration by parts and Eq. (29). If $\hat{F}(P)$ has more simple positive (negative) skewness than $F(P)$, the right-hand side of Eq. (33) is positive (negative) so that the third central moment under $\hat{F}(P)$ is indeed larger (smaller) than that under $F(P)$.

We state and prove our final proposition.

Proposition 4. *If $U'''(\Pi) \geq 0$ and $G'''(R) \geq 0$, the regret-averse competitive firm increases (decreases) its optimal output level, i.e., $Q^\dagger > (<)\ Q^*$, when the CDF of the uncertain per-unit price, $\tilde{P}$, shifts from $F(P)$ to $\hat{F}(P)$, where $\hat{F}(P)$ has more simple positive (negative) skewness than $F(P)$.*

Proof. Let $\Phi(P) = \{U'[\Pi^*(P)] + \beta G'[R^*(P)]\}(P-P^*)$. Then, we have $\Phi''(P) = H(P) + +2U''[\Pi^*(P)]Q^*$, where

$$H(P) = U'''[\Pi^*(P)]Q^{*2}(P-P^*) + \beta G'''[R^*(P)][Q(P)-Q^*]^2(P-P^*)$$

$$+\beta G''[R^*(P)](P-P^*)Q'(P)+2\beta G''[R^*(P)][Q(P)-Q^*]. \tag{34}$$

Since $Q(P) < (>)\ Q^*$ whenever $P < (>)\ P^*$ and $Q'(P) > 0$, Eq. (34) implies that $H(P) < (>)\ 0$ whenever $P < (>)\ P^*$. Using $\Phi(P)$, we can write the right-hand side of Eq. (25) as

$$\int_{\underline{P}}^{\overline{P}} \Phi(P)\mathrm{d}[F(P)-F^\circ(P)]$$

$$= \int_{\underline{P}}^{\overline{P}} \{H(P)+2U''[\Pi^*(P)]Q^*\}\bigg\{\int_{\underline{P}}^{P}[F(x)-F^\circ(x)]\mathrm{d}x\bigg\}\mathrm{d}P$$

$$= \int_{\underline{P}}^{\overline{P}} H(P)\bigg\{\int_{\underline{P}}^{P}[F(x)-F^\circ(x)]\mathrm{d}x\bigg\}\mathrm{d}P$$

$$-2Q^{*2}\int_{\underline{P}}^{\overline{P}} U'''[\Pi^*(P)]\bigg\{\int_{\underline{P}}^{P}\bigg\{\int_{\underline{P}}^{x}[\hat{F}(y)-F(y)]\mathrm{d}y\bigg\}\mathrm{d}x\bigg\}\mathrm{d}P, \tag{35}$$

where the first equality follows from integration by parts and Eq. (28), and the second equality follows from integration by parts and Eq. (29). Since $H(P) < (>)\ 0$ whenever $P < (>)\ P^*$, Eqs. (30) and (31) imply that the first term on the right-hand side of Eq. (35) is positive (negative). Eq. (32) implies that the second term on the right-hand side of Eq. (35) is also positive (negative). Hence, we conclude that $Q^\dagger > (<)\ Q^*$ if $\hat{F}(P)$ has more simple positive (negative) skewness than $F(P)$. □

The intuition for Proposition 4 is as follows. When $\hat{F}(P)$ has more simple positive skewness than $F(P)$, realizations of $\tilde{P}$ close to $\underline{P}$ are much less likely to be seen than those close to $\overline{P}$. The regret-averse firm as such raises more concerns about the disutility from the discrepancy of its output level, $Q(P)-Q^*$, when high realizations of $\tilde{P}$ are revealed. To minimize regret, the regret-averse firm optimally adjusts its output level upward from Q^*. Prudence, i.e., $U'''(\Pi) \geq 0$, further reinforces the firm's preferences for positive skewness and thus $Q^\dagger > Q^*$. On the other hand, when $\hat{F}(P)$ has more simple negative skewness than $F(P)$, realizations of $\tilde{P}$ close to $\underline{P}$ are much more likely to be seen than those close to

$\overline{P}$. The regret-averse firm as such optimally adjusts its output level downward from Q^* to reduce the discrepancy of its output level, $Q^* - Q(P)$, when low output prices are revealed. Prudence implies that the firm would like to minimize its exposure to negative skewness and thus $Q^\dagger < Q^*$.

5. Conclusion

In this paper, we revisit the model of a regret-averse competitive firm under price uncertainty *à la* Broll et al. (2016). The firm's regret-averse preferences are characterized by a bivariate utility function that includes additive separable disutility from having chosen ex-post suboptimal alternatives. The extent of regret is gauged by the difference between the actual profit and the maximum profit attained by making the optimal production decision had the firm observed the true realized output price. We show that the global and marginal effects of price uncertainty on production are both positive (negative) when regret aversion prevails if the random output price is asymmetrically distributed with positive (negative) skewness. In this case, high (low) output prices are much more likely to be seen than low (high) output prices. To minimize regret, the firm is induced to raise (lower) its optimal output level. As such, the economic asymmetries between regret aversion and risk aversion in shaping decision making processes under uncertainty are significant and deserve more scrutiny from future research.

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