Pricing Risks across Currency Denominations *

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Abstract

We use principal component analysis on 55 bilateral exchange rates of 11 developed currencies to identify two important global risk sources in FX markets. The risk sources are related to Carry and Dollar but are not spanned by these factors. We estimate the market prices associated with the two risk sources in the cross-section of FX market returns and construct FX market implied country-specific SDFs. The SDF volatilities are related to interest rates and expected carry trade returns in the cross-section. The SDFs price international stock returns and are related to important financial stress indicators and macroeconomic fundamentals. The first principal risk is associated with the TED spread, quantities measuring volatility, tail and contagion risks and future economic growth. It earns a relatively small implied Sharpe ratio. The second principal risk is associated with the default and term spreads and quantities capturing volatility and illiquidity risks. It further correlates with future changes in the long term interest rate and earns a large implied Sharpe ratio.

JEL-Classification: F31, G15.

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1 Introduction

Understanding risks and their pricing implications in foreign exchange (FX) markets is important. We use principal component analysis (PCA) on 55 bilateral exchange rate growths of 11 developed currencies to identify the major risk sources. We focus on the first two principal components (PCs) as risk sources since they capture the most important common variation in all bilateral exchange rates according to the Eigenvalue and Growth Ratio criterions by Ahn and Horenstein (2013). We find that our identified risk sources have some overlap with the Carry and Dollar factors, but the relation to the Dollar is weaker.[1] Moreover, our risk sources are not fully spanned by the Carry and Dollar factors.

In a second step, we employ the fundamental economic identity that an exchange rate is equal to the ratio of its corresponding country-specific stochastic discount factors (SDFs) and take expectations to derive a cross-sectional relationship between expected FX market returns and market prices of our identified risk sources. This allows us to estimate market prices of risk and construct FX market implied country-specific SDFs. The theoretical identity between exchange rates and SDFs naturally arises in frictionless, fully integrated and arbitrage free international financial markets (e.g. Brandt et al. (2006), Maurer and Tran (2017a,b)). Moreover, a nice feature of this relationship is that every shock in FX markets must be a shock to (at least one) SDF and is priced. This is in stark contrast to other asset classes, such as stock markets for instance, where shocks can be priced or idiosyncratic.

Most FX market research focuses on risk pricing in USD. However, setting the USD as the base currency implicitly biases the analysis towards risks which are specifically important to a US investor but not necessarily to investors in other countries or from a global perspective. That is, these risks may be compensated by potentially insignificant market prices in a global context. For instance, Lustig et al. (2011) use PCA on exchange rates quoted against the USD and find that the market price of risk of the first PC (also known as the Dollar factor) is small. That is, while the first PC captures most of the time-series variation in exchange rates it does not explain the cross-section of expected returns, which confirms our concern.

We argue that global risks are better identified if we use all bilateral exchange rates (i.e., not only quoted against one base currency) in the PCA. Of course, the set of exchange rates quoted

[1] Lustig et al. (2011) define the Dollar as the strategy of borrowing in USD and equally lending in all other currencies, and the Carry as borrowing in low and lending in high interest rate currencies.

against the USD implies all bilateral exchange rates. However, the PCA strongly focuses on USD specific shocks when only exchange rates quoted against the USD are used, while the PCA on all bilateral exchange rates is impartial in weighting shocks across all exchange rates, which balances the impact of shocks specific to any one country and highlights global risks.[2]

Our estimated SDFs have several interseting implications. We find that the implied SDFs increase during historically bad times such as the Asian financial crisis, Russian sovereign default and the bailout of Long-Term Capital Management, the default of Lehman Brothers and the financial crisis, and the bailouts of Greece and the European sovereign debt crisis. Moreover, we show that currencies with lower interest rates have more volatile SDFs, and the carry trade of borrowing currencies associated with more volatile SDFs and lending currencies associated with less volatile SDFs is profitable.

We further use the non-parametric approach of Christensen (2017) to decompose our estimated SDFs into permanent and transitory components and show that these components satisfy the theoretical bounds derived by Alvarez and Jermann (2005). This approach also provides us with a non-parametric estimate of long term bond yields for each country. We find that these estimated yields are close to the data, which is interesting because our estimation did not use any information about long term bonds (but only exchange rates and short term bonds). Moreover, we estimate a theoretical relationship provided by Lustig et al. (2017) between long term bond excess returns and entropies of the permanent SDF components across countries and find that this relationship holds in our estimated model.

In additional out of sample tests, we show that our estimated SDFs price international stock returns. In particular, we show that our first two PCs from FX market data capture only about 10% of the time-series variation in international stock returns (when denominated in local currency) but explain about 30% of the historical equity premia across countries. Moreover, the cross-sectional correlation between the risk premia implied by our SDFs and historical premia is 67%. We further use Fama and MacBeth (1973) regressions to estimate the market prices of risk of our first two FX market PCs in the cross-section of international stock returns. The estimated market prices are large and highly significant even after controlling for popular pricing factors such as the world

[2] Note that for $N+1$ currencies only N out of all $\frac{N(N-1)}{2}$ bilateral exchange rates are linearly independent. Thus, PCA delivers only N PCs with non-zero eigenvalues. These N PCs span the same space as the N PCs of N exchange rates quoted against a single base currency (e.g. USD). But in general, the first $K < N$ PCs of all $\frac{N(N-1)}{2}$ bilateral exchange rates will not span the same space as the first K PCs of the N exchange rates quoted against a single base currency.

market portfolio, the five global Fama and French (2015) factors, global momentum and the Dollar and Carry factors. The market prices estimated in the cross-section of stock returns are comparable to the ones estimated in the cross-section of FX returns. We further find that the second PC is more important as a pricing factor than the first PC.

Furthermore, we document that our estimated SDF in the USA correlates with a broad set of US specific financial stress indicators. We also document that the first FX market PC is related to the TED spread and variables that quantify volatility, tail and contagion risk. In contrast, the second PC is associated with the default and term spreads and stress indicators that measure volatility and illiquidity.

Finally we test the relationship between our estimated SDFs and macroeconomic fundamentals. We confirm our economic intuition that an increase in the SDF (bad shock) has a negative effect on several measures of economic growth, a negative effect on short and long term interest rates, and a positive effect on unemployment. We further document that the first FX market PC is associated with a broad set of macroeconomic fundamentals which mostly capture economic growth. The second PC is weakly related to most macroeconomic quantities but has a significant association with changes in the long term interest rate.

Related Literature

The framework connecting moments of SDF growths to exchange rates is for instance suggested by Bekaert and Hodrick (1992), Bekaert (1996) and Backus et al. (2001). Lustig and Verdelhan (2007, 2011) and Burnside (2011, 2012) discuss the connection between carry trade returns and aggregate consumption growth (CCAPM) and other popular asset pricing factors, which are known to explain the cross-section of stock returns. Recently, a large literature has emerged introducing new currency risk factors: carry factor (Lustig et al., 2011), global volatility factor (Menkhoff et al., 2012a,b), global currency skewness factor (Rafferty, 2012), FX correlation risk factor (Mueller et al., 2013), Dollar factor (Lustig et al., 2014; Verdelhan, 2015), Euro factor (Greenaway-McGrevy et al., 2016), downside beta risk factor (Dobrynskaya, 2014; Lettau et al., 2014; Galsband and Nitschka, 2013), FX liquidity risk factor (Mancini et al., 2013), economic size factor (Hassan, 2013), surplus-consumption risk factor (Riddiough, 2014). Some recent papers link some of these factors to macroeconomic conditions and explore what conditions are associated with "safe haven" properties of currencies (e.g., Habib and Stracca (2012), Cenedese (2012), Dobrynskaya (2015), Berg and Mark

(2016) and Dahlquis and Hasseltoft (2017) to name a few). Daniel et al. (2014) shows that Dollar-neutral carry trades and strategies with a Dollar exposure are different and the aforementioned factors appear to explain only Dollar-neutral returns. Bekaert and Panayotov (2016) show that excluding the Australian dollar, Japanese Yen, and Norwegian Krone from the asset universe substantially improves the Sharpe ratio and lowers the downside risk of carry trade strategies.

Another literature employs and examines statistical approaches to build factors. Meese and Rogoff (1983) challenge structural models for exchange rates and show that these models are unable to outperform a simple random walk model. Bakshi and Panayotov (2013) show that time-series predictability of carry trades is significant for dynamic currency portfolios (while being absent in fixed currency pairs). Koedij and Schotman (1989) use PCA to build groups of currencies with similar characteristics and single out four leading currencies: US dollar (USD), Yen (JPY), Deutsche Mark (DM), British Pound (GBP). Similarly, Greenaway-McGrevy et al. (2012) show that the JPY/USD, Euro/USD and GBP/USD exchange rates capture most of the variation in 23 exchange rates. Engel et al. (2007) estimate a factor model which is able to predict exchange rates at long horizons in the sample after 1999 but not in earlier samples. Sarno et al. (2012) estimate an affine multi-currency model with four latent variables which explains exchange rate fluctuations. Dong (2006) estimates a VAR model and finds that inflation and output gap are important to exchange rate dynamics. Rapach and Wohar (2006) and Maasoumi and Bulut (2012) test several exchange rate factor models and conclude that it is hard to consistently outperform a simple random walk model.[3,4]

We use PCA on all bilateral exchange rates to identify major risk sources and a cross-sectional regression of FX market returns to construct country-specific SDFs. An advantage of our approach over other empirical factor models is that we are able to provide a clear theoretical set-up to identify risk sources using the theoretical relationship between exchange rates and SDFs. As a comparison we focus on the well-known and dominant Dollar and Carry factors as a benchmark. We show that our factors and estimated SDFs capture important risks not spanned by the Dollar-Carry two factor model. Moreover, we related our PCs to financial stress indicators and macroeconomic fundamentals. We show that the first PC is related to the TED spread and quantities which

[3]See Maasoumi and Bulut (2012) for additional references on structural exchange rate models.

[4]Yet another literature uses option prices to quantify risks of currency crashes and peso events and explain carry trade returns (e.g. Brunnermeier et al. 2008; Burnside et al. 2011; Farhi et al. 2014; Chernov et al. 2013 and Jurek 2014; see Chernov et al. 2013 for a comprehensive literature review on exchange rate crash risks). We focus on diffusion risks in our analysis.

measure volatility and contagion risk and economic growth, while the second PC is related to the default and term spreads and variables which measure volatility and illiquidity and to changes in the long term interest rate.

Our paper is also related to the literature which links FX markets and stock returns. Solnik (1974) was arguably the first to theoretically show that a FX market factor is important in an international CAPM. Dumas and Solnik (1995) estimate market prices of a four factor model (world stock market portfolio and three exchange rates). Bekaert and Hodrick (1992) analyze predictable components in FX and stock returns and estimate a VAR model. Patro et al. (2002) introduce a two factor model (world stock index and a currency basket) to explain stock market returns across developed countries. Fama and French (2015) test an international five factor model (based on size and valuation ratios). Brusa et al. (2015) introduce an international CAPM model with one global stock market factor and two currency factors (Dollar and Carry), which does a better job pricing a broad set of international assets than other international factor models. We show that our first two PCs of 55 bilateral exchange rates are important to price stocks and earn large market prices in the cross-section of stock returns, even after controlling for the world market, global Fama-French, global momentum, Dollar and Carry factors.

Finally, based on our estimation approach Maurer et al. (2017) construct a dynamic trading strategy and find that the strategy earns a large Sharpe ratio out-of-sample and outperforms many popular currency trading strategies across various performance measures and sub-samples.

Our paper is structured as follows. Section 2 presents our estimation approach to construct SDFs from priced risks in FX markets. Section 3 implements the approach in the data and investigates model implications and in-sample evidence. Section 4 investigates out-of-sample evidence supporting the validity of our estimation. Section 5 concludes. The appendices provide additional results, list details on data sources and provide derivations for theoretical results in the paper.

2 SDF Estimation from FX Market Data

In this section we present key steps to estimate country-specific SDFs from FX data and the principal component analysis. We then relate our estimation procedure to the standard Fama and MacBeth (1973) regression of factor pricing models.

2.1 Setup

We model $N+1$ countries (or currencies) indexed by $I \in \{1, \ldots, N+1\}$. We focus on diffusion risks. We employ the standard filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P}\}$, wherein $\{\mathcal{F}_t\}_{t\geq 0}$ is the natural filtration associated with the n-dimensional standard Brownian motion Z_t as diffusion risks in the market. Our specification assumptions for the diffusion model of FX market risks are: (A1) no-arbitrage, (A2) complete and frictionless financial markets, (A3) diffusion processes of exchange rates (A4) sufficient stationarity in the exchange rate processes (for the time windows of our study).[5] The market completeness and the continuous-time setting (i.e., the diffusion risk specification) are convenient assumptions, and can be relaxed by replacing SDFs by their respective projectors.[6]

The risk pricing in country I's currency is characterized by the country-specific SDF M_I,

$$\frac{dM_{t,I}}{M_{t,I}} = -r_I dt - \eta_I^T dZ_t, \quad \forall I, t. \tag{1}$$

The drift and volatility of SDF growths are country I's instantaneously risk-free rate $r_I \in \mathbf{R}$ and the prices of n diffusion risks $\eta_I \in \mathbf{R}^n$ respectively. Let the exchange rate $EX_{t,J/I}$ be the number of units of currency J that buys one unit of currency I at time t. Market completeness implies that the exchange rate equals the ratio of SDFs, $EX_{t,J/I} = \frac{M_{t,I}}{M_{t,J}}$, $\forall I, J$. From this follows the exchange rate growths,

$$\frac{dEX_{t,J/I}}{EX_{t,J/I}} = \left[r_J - r_I + \eta_J^T \Delta\eta_{J/I}\right] dt + \Delta\eta_{J/I}^T dZ_t, \quad \text{where} \quad \Delta\eta_{J/I} \equiv \eta_J - \eta_I. \tag{2}$$

To see how exchange rate risks are priced in asset markets, we consider a typical net-zero carry trade strategy from the perspective of currency denomination I, which we take as USD in this paper. At time t, the strategy borrows currency B (paying interest rate r_B) and lends currency L (paying interest rate r_L). At $t+dt$, liquidating all positions and converting the payoff to the denomination currency I yields the realized excess return $CT^I_{t+dt,-B/+L}$ and the expected value

[5] We provide empirical evidence to justify this stationarity assumption in Appendix A.

[6] Such a replacement is fully adequate as long as risks are not entangled in FX markets, see Maurer and Tran (2017a).

$ECT^I_{-B/+L}$,

$$CT^I_{t+dt,-B/+L} = \eta_I^T \Delta\eta_{B/L} dt + \Delta\eta_{B/L}^T dZ_t,$$
$$ECT^I_{-B/+L} = \eta_I^T \Delta\eta_{B/L} dt, \qquad \Delta\eta_{B/L} \equiv \eta_B - \eta_L. \tag{3}$$

We observe that the innovation structures in exchange rates (2) and realized carry trade returns (3) are identical, as both are driven by the differential prices of risks of the form $\Delta\eta_{t,C/D} \equiv \eta_{t,C} - \eta_{t,D}$. Motivated by this oservation, we apply the PCA directly on the denomination-free exchange rate covariance matrix (as opposed to the covariance matrix of carry trade returns) to identify important risk factors in FX markets in our construction of SDFs below.

2.2 SDF Estimation Approach

Our procedure to estimate country-specific SDFs M_I (1) has two stages. The first stage employs a principal component analysis (PCA) to extract important and identifiable risk factors in FX markets. The second stage employs a cross-sectional regression of mean carry trade returns on factor loadings (obtained in the first stage) to reconstruct SDFs in FX markets. In essence, PCA organizes exchange rate risks into identifiable components. Because carry trade strategies load on these risks, their expected returns shed light on the pricing of these principal risks, which then help us to estimate SDFs as the pricing kernels. By construction, our estimated SDF is the SDF projected onto the FX market risk space.

First Stage: Identifying Principal FX Risk Factors

To identify and organize the risk structure in FX markets, we apply a principal component analysis on the exchange rate growths of currency pairs, which share identical risks with carry trade returns (2), (3). We briefly describe the main analysis here and relegate technical details and notations to Appendix F.2.

Let $\mathcal{P}$ denote the set of P currency pairs in the analysis, $P \equiv dim(\mathcal{P})$, and X the matrix of innovations in exchange rate growths (2). Specifically, each column of matrix X denotes the demeaned exchange rate growth time series of a currency pair in $\mathcal{P}$ (see (26)). The PCA starts with the diagonalization of the exchange rate sample covariance matrix $X^T X$,

$$W^T \left[X^T X\right] W = \text{Diag}[\lambda_1; \ldots; \lambda_P],$$

where λ's are eigenvalues, and W is a $P \times P$ orthogonal matrix whose elements are referred to as loadings in the PCA. For convenience, we work with rescaled and standardized quantities,

$$\Delta\overline{\eta} \equiv \Delta\eta\, W \,\mathrm{Diag}\left[\tfrac{1}{\sqrt{\lambda_1}};\ldots;\tfrac{1}{\sqrt{\lambda_P}}\right], \qquad \Delta\overline{\eta}^T \Delta\overline{\eta} = \mathbf{1}_{P\times P},$$
$$\overline{\Pi} \equiv XW\mathrm{Diag}\left[\tfrac{1}{\sqrt{\lambda_1}};\ldots;\tfrac{1}{\sqrt{\lambda_P}}\right], \qquad \overline{\Pi}^T \overline{\Pi} = \mathbf{1}_{P\times P}, \tag{4}$$
$$\overline{W} \equiv W\mathrm{Diag}\left[\sqrt{\lambda_1};\ldots;\sqrt{\lambda_P}\right], \qquad \overline{W}^T \overline{W} = \mathrm{Diag}\left[\lambda_1;\ldots;\lambda_P\right],$$

where each of the P columns of matrix $\Delta\eta$ denotes a differential price-of-risk vector $\Delta\eta_{C/D}$, and each of the P columns of matrix $\overline{\Pi}$ denotes a (rescaled) principal component (see (3) and (26)). When eigenvalues $\{\lambda_1;\ldots;\lambda_P\}$ are sorted in descending order, the K-th column of matrix $\overline{\Pi}$ represents the K-th observable (rescaled) principal component (as a time series),

$$\overline{\Pi}_{t,K} = \frac{1}{\sqrt{\lambda_K}} \sum_{C/D\in\mathcal{P}} X_{t,C/D} W_{C/D,K} = \frac{1}{\sqrt{\lambda_K}} \sum_{C/D\in\mathcal{P}} W_{C/D,K} \Delta\eta^T_{C/D} dZ_t = \Delta\overline{\eta}^T_K dZ_t, \tag{5}$$

where the sum runs over all currency pairs $C/D \in \mathcal{P}$ in the analysis, and K denotes any such pair.[7] The last equality has employed (4), with $\Delta\overline{\eta}_K$ denoting the K-th column of matrix $\Delta\overline{\eta}$ (4), (26). Note that while we neither observe differential prices of risks $\Delta\eta$ (nor $\Delta\overline{\eta}$) nor the original diffusion dZ_t, the PCA in this first stage identifies the observable loadings W, principal components $\overline{\Pi}$ and eigenvalues λ.

Second Stage: Cross-Sectional Regression

We aim to construct an estimate $\widehat{M}_{t,I}$ of SDF $M_{t,I}$ (1) by projecting country I's prices of risk in the space spanned by the PCA rescaled prices of risks (4) as follows,

$$\widehat{\eta}_I = \sum_{C/D\in\mathcal{P}} \gamma^I_{C/D} \Delta\overline{\eta}_{C/D}, \tag{6}$$

where $n \times 1$ (rescaled) differential price of risk vector $\Delta\overline{\eta}_{C/D}$ is defined in (4) and (26), and $\widehat{\eta}_I$ is also a $n \times 1$ column vector. Coefficients γ in the above projection are factor prices (associated with (rescaled) principal factors $\overline{\Pi}_{t,K}$, $\forall K \in \mathcal{P}$) and can be estimated via a cross-sectional regression on

[7] In matrix (discrete) notation (26) of Appendix F.2, $n \times 1$ diffusion innovation vector dZ_t is the t-th row of matrix dZ, and $n \times 1$ differential price of risk vector $\Delta\eta_{C/D} \equiv \eta_C - \eta_D$ is the C/D-th column of matrix $\Delta\eta$.

carry trade returns as we explain next.

First, observe that as a result of the definitions in (4), the element $\overline{W}_{B/L,K}$ of matrix $\overline{W}$ is the loading of the carry trade return $CT^{I}_{t+dt,-B/+L}$ (3) on the K-th (rescaled) principal components $\overline{\Pi}_{t,K}$, $\forall K \in \mathcal{P}$.[8] Second, under the linear specification (6), expected carry trade returns (3) become,

$$\frac{1}{dt} ECT^{I}_{-B/+L} = \sum_{C/D \in \mathcal{P}} \gamma^{I}_{C/D} \Delta \overline{\eta}^{T}_{C/D} \Delta \eta_{B/L} = \sum_{C/D \in \mathcal{P}} \gamma^{I}_{C/D} \overline{W}_{B/L,C/D}, \qquad \forall B/L \in \mathcal{P}, \tag{7}$$

where in the last equality we have used rescaling and orthogonality relationships (4). Combining the two observations above indeed implies that the coefficient γ^{I}_{K} in (6) is the factor price (in currency I) of the K-th principal risk factor $\overline{\Pi}_{t,K}$, for each $K \in \mathcal{P}$.

Furthermore, because we observe the loadings W and eigenvalues λ's from PCA, equation (7) suggests that coefficients γ in (6) can be estimated from a cross-sectional regression of the mean carry trade returns (varying currency pairs B/L, while fixing denomination currency I) on the rescaled scores $\overline{W}$ (4). As a result, we obtain the estimates (stacked in $P \times 1$ column vector $\widehat{\gamma}^{I}$, see (27), Appendix F.2 for notation),

$$\widehat{\gamma}^{I} = \frac{1}{dt} \left(\overline{W}^{T} \overline{W} \right)^{-1} \overline{W}^{T} ECT^{I}. \tag{8}$$

These coefficients then generate an estimate for country I's prices of risks (6), and in turn, for country I's SDF,

$$\frac{d\widehat{M}_{t,I}}{\widehat{M}_{t,I}} = -r_I dt - \widehat{\eta}^{T}_{I} dZ_t = -r_I dt - \sum_{K \in \mathcal{P}} \overline{\Pi}_{t,K} \, \widehat{\gamma}^{I}_{K}, \qquad \forall I, \tag{9}$$

where the last equality is derived using (5). Clearly, $\widehat{\gamma}^{I}$ are factor prices (in currency I) associated with principal factors $\overline{\Pi}_{t,K}$. Furthermore, our estimated SDF is fully identified because it is expressed in observable principal components $\overline{\Pi}$ and estimated $\widehat{\gamma}^{I}$ determined in (8).

2.3 Discussion

Several important observations concerning the estimation of SDFs from FX data are in order. First, all risks in FX markets must be priced by at least one country's SDF. This is because an

[8] To see this, note that relationships in (4) imply $\overline{\Pi}\, \overline{W}^{T} = \overline{\Pi}\, \text{Diag}\left[\sqrt{\lambda_1}; \ldots; \sqrt{\lambda_P}\right] W^{T} = \Pi\, W^{T} = X$. As noted below (3), because innovations in exchange rate growths X (2), (26) equal innovations in realized carry trade returns (3), the previous identity $\overline{\Pi}\, \overline{W}^{T} = X$ implies $CT^{I}_{t+dt,-B/+L} = \sum \overline{\Pi}_{t,K} \overline{W}_{B/L,K}$ for all currency pairs $B/L \in \mathcal{P}$. Then indeed, $\overline{W}_{B/L,K}$ is the loading of the carry trade return $CT^{I}_{t+dt,-B/+L}$ on the K-th principal components $\overline{\Pi}_{t,K}$.

exchange rate equals the ratio of the involved SDFs, hence any shock to an exchange rate must be a shock to at least one SDF. This feature makes FX markets a desirable setting to estimate SDFs as opposed to other asset markets, parts of which are idiosyncratic and not priced. Second, any residual risk inherent in η_I but not priced in the carry trade returns (3) must both (i) carry same prices in all currencies, and (ii) be orthogonal to the risks revealed by exchange rate fluctuations.[9] Our estimated SDF from FX data do not price these residual risks. It is an empirical question as to how important these residual risks are and we address this quesiton in subsequent sections on empirical tests. Third, the PCA in the first stage organizes FX risks in descending order of co-variations. It therefore systematically informs us on selecting and retaining only principal risks while dropping risks of minor statistical significance. Such a selection is highly desirable, e.g., to eliminate portfolio strategies of spuriously high Sharpe ratios (Ross, 1976; Kozak et al., 2015).

Finally, we observe that formally, our two-stage estimate of the SDF may also be cast as a Fama and MacBeth (1973) two-stage regression. Practically, however, our estimate differs from Fama-MacBeth regressions in the implementation of the first stage. Therein, we exploit the fact that all exchange rate risks are necessarily priced by SDFs to implement the PCA directly on the exchange rate covariance matrix (as opposed to running time-series regressions as in the Fama-MacBeth first stage). To see this connection, we consider principal components as risk factors and carry trades as test assets. The Fama-MacBeth first stage is the (time-series) regression of realized carry trade returns (3) on rescaled principal components (4). For a specific strategy (of borrowing B and lending L, from the perspective of denomination currency I), this first-stage regression is the following linear decomposition,

$$CT^I_{t+dt,-B/+L} = \sum_{K\in\mathcal{P}} b^I_{K,B/L}\overline{\Pi}_{t,K} + \epsilon^I_{t,B/L}.$$

We can stack these regressions for all strategies $B/L \in \mathcal{P}$, yielding matrix equation (28), from which the OLS estimate follows,

$$\widehat{b}^I = \left(\overline{\Pi}^T\overline{\Pi}\right)^{-1}\overline{\Pi}^T CT^I = \overline{\Pi}^T X = \overline{W}^T, \tag{10}$$

where the last equality follows from relationships (4). Clearly, these Fama-MacBeth first-stage estimates are the transpose of (rescaled) loadings from the PCA. The Fama-MacBeth second stage

[9] Condition (i) implies that the residual risks are canceled and do not affect exchange rate fluctuations. Condition (ii) implies that expected carry trade returns have no information to estimate the residual risks.

is the (cross-sectional) regression of the mean carry trade returns on the first-stage factor estimates $\widehat{b}$ (28). Then indeed the Fama-MacBeth regression approach yields price of risk estimates (29) identical to those obtained from our second-stage regression (7) (since the loadings $\left(\widehat{b}^I\right)^T = \overline{W}$ (10) in the first stage are the same in the two approaches).

3 Estimation and Model Implied Results

We apply the methodology introduced in the previous section to the data to estimate the proposed diffusion model and country-specific SDFs in FX markets and present in- and out-of-sample evidence to examine the validity of our approach. We show that our estimated SDFs are consistent with important empirical patterns in the data.

3.1 FX Market Data

We use daily exchange rates between 11 developed countries: Australia, Canada, Denmark, Eurozone[10], Japan, New Zealand, Norway, Sweden, Switzerland, UK and USA. FX markets in these developed currencies are typically more liquid, feature a higher trading volume, lower transaction costs, less capital controls and markets are more likely to be fully integrated, frictionless and free of arbitrage in comparison to emerging countries.[11] Since our theoretical model assumes fully integrated, frictionless and arbitrage free markets with completely disentangled risks[12], our set of developed countries fits our theoretical model better than a larger set of developed and emerging countries.

Spot and forward exchange rates against the US dollar are provided by Barclays Capital and WM/Reuters (WMR). In cases where data for one currency is available from both sources, the longer series is used. We check the discrepancies between the two sources and they are negligible. We use data from 1984 to 2014. Exchange rates of all currencies except for the Euro are available for the entire sample period. The inception of the Euro was in 1999 when 15 developed countries in Europe formed the Eurozone. Germany is one of the largest economies in the Eurozone and we

[10] For simplicity we refer to the Eurozone simply as Euro or EU, although not all countries in the EU use the Euro.

[11] There are several recent papers that discuss the possibility of arbitrage due to a failure in the covered interest rate parity (CIP) in the last decade (Borio et al., 2016; Cenedese et al., 2016; Du et al., 2017; Rime et al., 2016). Overall, these papers suggest that possible (if any) arbitrage opportunities are small and only accessible by very few large financial institutions.

[12] Complete risk disentanglement is a sufficient and necessary condition for the equality between exchange rates and ratios of (projected) country-specific SDFs to hold (Maurer and Tran, 2017a,b).

use the German Mark to extend the data of the Euro from 1999 back to 1984. This helps us to keep our panel of data balanced.

Data for the US short term interest rate is from the Center for Research in Security Prices (CRSP) US Treasury Databases, series "CRSP Monthly Treasury - Fama Risk Free Rates". This series contains 1-month risk free rates. We use the midpoint between bid and ask rates. We use the forward and spot exchange rates to construct interest rate differentials of short term bonds between currencies (based on the covered interest rate parity).

3.2 Principal Component Analysis

We use demeaned daily exchange rate growths of all $P = 55$ bilateral exchange rates between our 11 currencies for the PCA. To determine the number of common factors we use the Eigenvalue Ratio and Growth Ratio estimators proposed by Ahn and Horenstein (2013). They show that these two estimators perform better in small samples and are more robust than alternative estimators. The Eigenvalue Ratio is defined as $ER(k) = \frac{\lambda_k}{\lambda_{k+1}}$, where λ_j is the eigenvalue associated with the jth PCs. The Growth Ratio is $GR(k) = \frac{\ln(1+\lambda_k/V(k))}{\ln(1+\lambda_{k+1}/V(k+1))}$ with $V(j) = \sum_{i=j+1}^{P} \lambda_i$. The Eigenvalue Ratio and Growth Ratio estimators choose k^*_{ER} and k^*_{GR} to maximize $ER(k)$ and $GR(k)$, i.e., $k^*_{ER} = \arg\max_{1 \leq k \leq k_{max}} \{ER(k)\}$ and $k^*_{GR} = \arg\max_{1 \leq k \leq k_{max}} \{GR(k)\}$ where $k_{max} = \frac{P}{10}$. We find $k^*_{ER} = k^*_{GR} = 2$, i.e., the Eigenvalue and Growth Ratio estimators of Ahn and Horenstein (2013) both suggest that the first two PCs capture the common variation of the 55 bilateral exchange rates of our 11 currencies (see Table B.1 for the values of $ER(k)$ and $GR(k)$ $\forall k \in \{1, \ldots, k_{max}\}$). The first (rescaled) PC $\overline{\Pi}_{t,1}$ captures 33% and the second $\overline{\Pi}_{t,2}$ 21% of the total variation of all exchange rate growths. In the following we construct country-specific SDFs M_J as described in (9) based on only the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$.

Lustig et al. (2011) work with exchange rates quoted against the USD, sort currencies according to interest rates into quintiles and construct five equally weighted currency portfolios. From the return time-series of these five portfolios, they then construct PCs. They find that the first two PCs explain almost all the variation in returns of the five portfolios. Moreover, the first component has a correlation of 99% with the Dollar factor, which borrows USD and equally lends in all other currencies. Similarly, the second PC has a correlation of 94% with the Carry factor, which sells the bottom and buys the top interest rate quintile portfolios.

An important difference between Lustig et al. (2011) and our analysis is the set of exchange rates, i.e., using only exchange rates against the USD versus all bilateral exchange rates. Of course, the set of exchange rates quoted against the USD implies all bilateral exchange rates. However, the PCA strongly focuses on USD specific shocks when only exchange rates quoted against the USD are used, while the PCA on all bilateral exchange rates puts more balanced weights on shocks across all currencies and emphasizes shocks common to multiple currencies. Intuitively, if every country is exposed to i.i.d. country-specific shocks, then the US specific-shock affects every exchange rate in the set of exchange rates quoted against the USD, while other country-specific shocks only affect one exchange rate in that set. Thus, one of the first few PCs is likely to load on the US specific shock even though it may not necessarily be an important global risk or may not be important from the perspective of investors outside the US. In contrast, using all bilateral exchange rates reduces the emphasis on any country-specific shock (including the US). Thus, the use of all bilateral exchange rates is better suited to capture dominant global risks in international FX markets without focusing on a particular investor or currency denomination.

Lustig et al. (2011) use PCA on exchange rates quoted against the USD and find that the market price of risk of the first PC (or also known as the Dollar factor) is small. The price of risk of the Dollar factor is also found to be statistically insignificant in other studies (for instance Menkhoff et al. (2012a) or Maurer et al. (2017) among many others). That is, while the first PC (or Dollar) captures most of the time-series variation in exchange rates (quoted against the USD) it does not explain the cross-section of expected returns. Hence, this empirical finding confirms our concern of using only exchange rates quoted against one base currency in the PCA. In contrast, we show below that the PCs which we construct from all bilateral exchange rates all have substantial market prices and are thus important to capture both the time-series variation in changes in exchange rates and explain the cross-section of expected FX returns.

Empirically, if we use only exchange rates quoted against the USD in the PCA, we confirm the result of Lustig et al. (2011) that the first two PCs contain the same information as the Dollar and Carry factors. In particular, the correlation between the first PC and the Dollar is 99.6%, and the one between the second PC and the Carry is 96.6%. Moreover, regressing the first (second) PC on the Dollar and Carry factors yields an R-squared of 99.3% (93.5%). In contrast, we find that the relation between the first two PCs $\overline{\Pi}_{t,1}$, $\overline{\Pi}_{t,2}$ and the Dollar and Carry factors is weaker when we use all bilateral exchange rates. $\overline{\Pi}_{t,1}$ has correlations of -30.3% and 88.9% with the Dollar and Carry. The regression R-squared when regressing $\overline{\Pi}_{t,1}$ on the two factors is 88.1%. The corresponding correlations for $\overline{\Pi}_{t,2}$ are -66.5% and -40.4%, and the regression R-squared is 60.8%.

It is not surprising that there is some overlap between $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ of all bilateral exchange rates and the first two PCs of exchange rates defined against the USD (or the Dollar and Carry factors), but clearly significant differences remain. To conclude, we emphasize that these differences arise due to the strong USD focus of the PCA which only uses exchange rates quoted against the USD, while the PCA which uses all bilateral exchange rates attempts to focus less on country-specific and more on global risks.

We also investigate and visualize the decomposition of the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$. By construction, each PC loads on all 55 bilateral exchange rates. However, any exchange rate J/I can be expressed in terms of the two exchange rates J/USD and I/USD against the USD. Thus, we can re-write the original loadings of each PC on the 55 bilateral exchange rates, as linear combinations of only 10 exchange rates against the USD. These loadings of $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ on the 10 exchange rates J/USD are reported in the first two columns in Table 1 (first to second-to-last rows). In the last row indicated by USA we report 1 minus the sum of all loadings on the 10 exchange rates J/USD. Thus, the sum of the entire column adds up to 1 and can be interpreted as a portfolio of short term bonds in the 11 countries.[13] Columns 6 in Table 1 provides information on the average interest rate in each country relative to the US. The discussion of all other columns is deferred until later.

$\overline{\Pi}_{t,1}$ invests in AUD, NZD, USD and CAD. The weights on AUD and NZD are almost identical, 1.727 and 1.792, and the investments in USD and CAD are slightly lower with weights 1.428 and 0.876. It borrows in all other currencies, predominantly in CHF, EUR and DKK with weights -1.161, -0.884 and -0.828. The exposure to JPY is somewhat lower with a weight of -0.682. In comparison Carry borrows equally in CHF and JPY (currencies with lowest interest rates) and lends equally in AUD and NZD (currencies with highest interest rates). While there is some overlap with Carry (i.e., CHF and JPY are still funding and AUD and NZD are investment currencies), the investments of $\overline{\Pi}_{t,1}$ are clearly different. Particularly interesting is that $\overline{\Pi}_{t,1}$ assigns large negative weights to EUR and DKK and a large positive weight to USD although their interest rates are almost identical. Moreover, the JPY does not have an important role as a funding currency as CHF, EUR and DKK, but it is the most important funding currency in Carry. The weights of $\overline{\Pi}_{t,1}$ are very different from the Dollar factor, which borrows 100% in USD and lends 10% in each of the other 10 currencies.

[13]Note that the two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ are denominated in USD. However, for the initial construction we have used all bilateral exchange rates in the PCA which we argue shifts the focus away from the USD and more to globally important risks.

Table 1: SDF Estimations and Country-Specific Characteristics

Country J	(1) First PC $\overline{\Pi}_{t,1}$ loading on Currency J	(2) Second PC $\overline{\Pi}_{t,2}$ loading on Currency J	(3) Market Price $\hat{\gamma}_1^J$ of $\overline{\Pi}_{t,1}$	(4) Market Price $\hat{\gamma}_2^J$ of $\overline{\Pi}_{t,2}$	(5) Volatility of SDF $\widehat{M}_{t,J}$	(6) Average Interest Rate Differential J minus US	(7) Sharpe Ratio of borrowing USD and lending J
Australia	1.727	-0.468	-0.087	0.307	0.319	0.030	0.041
Canada	0.876	0.598	-0.124	0.343	0.364	0.007	0.016
Denmark	-0.828	-0.608	-0.196	0.302	0.360	0.008	0.038
Euro	-0.884	-0.469	-0.198	0.307	0.365	-0.004	0.027
Japan	-0.682	2.365	-0.190	0.402	0.445	-0.024	0.010
New Zealand	1.792	-0.525	-0.085	0.305	0.317	0.041	0.068
Norway	-0.541	-0.984	-0.184	0.290	0.343	0.022	0.041
Sweden	-0.460	-0.994	-0.180	0.289	0.341	0.016	0.032
Switzerland	-1.161	-0.285	-0.210	0.313	0.377	-0.016	0.026
United Kingdom	-0.266	-0.018	-0.172	0.322	0.365	0.019	0.036
USA	1.428	2.388	-0.143	0.369	0.396	N/A	N/A

Notes: Columns 1 & 2: decomposition of first and second PC into linear combination of exchange rates J/USD; last row (denoted USA) reports 1 minus the sum of all weights in the above rows. Columns 3 & 4: estimated market prices of risk of first two PCs across countries according to (8). Column 5: volatilities of estimated SDFs across countries according to (11). Column 6: time series average of difference between interest rates in country J and US. Column 7: Sharpe ratio of carry trade return of borrowing USD and lending in currency J from the perspective of a US investor.

$\overline{\Pi}_{t,2}$ lends in JPY, USD and CAD, and borrows predominantly in NOK and SEK. Interest rates are on average larger in Norway and Sweden than in Japan, the US and Canada. Thus, $\overline{\Pi}_{t,2}$ has some exposure to a long-short strategy based on interest rate differentials but the relation to Carry is relatively weak. The weights of $\overline{\Pi}_{t,2}$ also do not appear to align with the composition of the Dollar factor.

To sum up, the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ constructed from the set of all 55 bilateral exchange rates display some overlap with the Carry (or the second PC of the 10 exchange rates quoted against the USD) and the Dollar factor (or the first PC of the 10 exchange rates quoted against the USD) but there are significant differences. Most notably, the Dollar factor is less prevalent in our analysis than in Lustig et al. (2011) because by construction country-specific risks in our PCA on all bilateral exchange rates get less attention and the focus is directed towards global risks (i.e., independent of a base currency) compared to an analysis based on exchange rates only quoted against the USD. In the following we provide additional estimation results and tests to demonstrate that our risk factors are distinct from Dollar and Carry in several other important dimensions.

3.3 Estimation of Country-Specific SDFs

Given the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ as new risk sources, we use the regression proposed in equation (7) to estimate the corresponding market prices of risk $\widehat{\gamma}_1^J$ and $\widehat{\gamma}_2^J$ specified in (8), and construct country J's SDF $\widehat{M}_{t,J}$ according to (9).

Columns 3 and 4 in Table 1 show market prices of risk (or risk loadings of SDFs) $\widehat{\gamma}_1^J$ and $\widehat{\gamma}_2^J$ on the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ according to (8). Column 5 reports the estimated annual volatilities of country-specific SDFs,

$$Vol\left(\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}\right) = \|\widehat{\gamma}^J\| = \sqrt{\widehat{\gamma}^{JT}\widehat{\gamma}^J}. \tag{11}$$

Columns 6 and 7 further report for each country J the average annual interest rate differential between country J and the US and the annual Sharpe ratio of the bilateral carry trade of borrowing USD and lending currency J.

The risk loadings in columns 3 and 4 do not differ a lot across countries which is consistent with the strong cross-country correlation of SDFs. For every country $\widehat{\gamma}_1^J$ is between -0.21 and -0.085, and $\widehat{\gamma}_2^J$ is between 0.289 and 0.402. Negative (positive) market prices $\widehat{\gamma}_1^J$ ($\widehat{\gamma}_2^J$) imply that $\overline{\Pi}_{t,1}$

($\overline{\Pi}_{t,2}$) is positively (negatively) related to the SDF growth $\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}$ and a positive realization in $\overline{\Pi}_{t,1}$ ($\overline{\Pi}_{t,2}$) is bad (good) news for marginal investors (see equation (9)). Market prices $\widehat{\gamma}_2^J$ for the second PC $\overline{\Pi}_{t,2}$ are larger in magnitude than $\widehat{\gamma}_1^J$ for the first PC $\overline{\Pi}_{t,1}$, which is interesting because $\overline{\Pi}_{t,2}$ is less correlated to the Carry factor (correlation of -40%) than $\overline{\Pi}_{t,1}$ (correlation of 88.9%). Thus, our estimation suggests that the Carry factor may not capture the most important priced risks in FX markets. This is an important contribution because identifying and quantifying the dominant priced risk sources is the first step to understand FX markets. To emphasize the importance of the first two PCs in our analysis we demonstrate in the tests in Section 4 that they are also essential risk sources in the context of equity markets and are related to financial stress indicators and macroeconomic fundamentals.

The variation in SDF volatilities across countries is economically large: SDF volatilities range from 31.7% and 31.9% in Australia and New Zealand to 40.2% in Japan. Moreover, the cross-country variation in SDF volatilities is strongly associated with average interest rates. Figure 1 plots average interest rate differentials in column 6 in Table 1 against SDF volatilities in column 5 and documents a striking negative relationship with a correlation of -89%. Column 1 in Table 2 provides statistical properties and confirms that the negative relationship is highly statistically significant with a t-statistic of 6.10 (Panel A) or 2.05 after controlling for inflation (Panel B). A common economic intuition is that volatility in the SDF is positively associated with precautionary savings. Based on this perception a large (small) SDF volatility indeed implies much (little) precautionary savings and a relatively low (high) interest rate in equilibrium. Though, such an argument requires additional assumptions on preferences and the risk sources in the economy than what we are assuming in the current paper.

Our finding differs from Gavazzoni et al. (2013) who show, in an affine diffusion model, that interest rates and market prices of risk are positively associated. In particular, they show that under certain parametric assumptions the volatility of the SDF is proportional to the volatility of the interest rate. They further document empirically that high interest rates tend to be more volatile, and therefore, are associated with more volatile SDFs under their modeling assumptions. In contrast, our estimates imply a negative relation between interest rates and SDF volatilities. The difference arises because our estimation is non-parametric and does not make any assumptions (such as an affine structure) on the relationship between interest rates and market prices of risks.[14]

[14] In light of Gavazzoni et al. (2013), we can conclude that our estimated SDFs do not fit into the parametric restrictions imposed on their affine risk setting. For instance, it is important in Gavazzoni et al. (2013) that interest

Interest Rates and SDF Volatilities

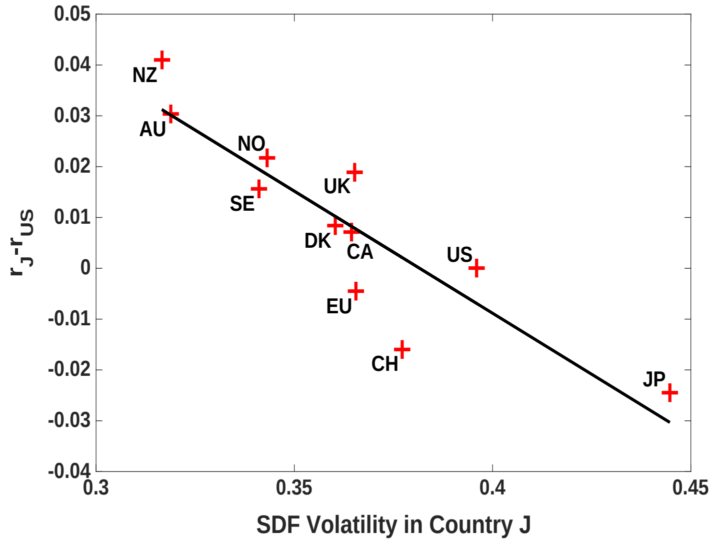

Figure 1: Cross-Sectional relationship between (time-series average of) interest rates and volatilities of country-specific SDFs as defined in equation (11).

We further investigate the relationship between the SDF volatility in country J and carry trade returns of borrowing USD and lending currency J, $CT^{US}_{t+dt,-US/+J}$. Expected carry trade returns $ECT^{US}_{-US/+J}$ (i.e., the time-series average of $CT^{US}_{t+dt,-US/+J}$) vary substantially across countries J while the variances of $CT^{US}_{t+dt,-US/+J}$ hardly change as illustrated in Figure 2. Given this empirical pattern we can show in the context of a diffusion model that there must be a strong relationship between the expected carry trade return $ECT^{US}_{-US/+J}$ and the volatility of country J's SDF. Indeed,

$$\begin{aligned}\|\widehat{\gamma}^J\|^2 &= \|\widehat{\gamma}^{US}\|^2 - 2\left(\widehat{\gamma}^{US} - \widehat{\gamma}^J\right)^T \widehat{\gamma}^{US} + \|\widehat{\gamma}^{US} - \widehat{\gamma}^J\|^2 \\ &= \|\widehat{\gamma}^{US}\|^2 - \frac{2}{dt} ECT^{US}_{-US/+J} + \frac{1}{dt} Var\left[CT^{US}_{t+dt,-US/+J}\right].\end{aligned}$$

It is apparent from Figure 2 that the cross-country variation of $Var\left[CT^{US}_{t+dt,-US/+I}\right]$ is almost zero. We get the approximate empirical relationship,

$$\|\widehat{\gamma}^I\|^2 - \|\widehat{\gamma}^J\|^2 \approx \frac{2}{dt}\left[ECT^{US}_{-US/+J} - ECT^{US}_{-US/+I}\right] = \frac{2}{dt} ECT^{US}_{-I/+J}. \qquad (12)$$

rate volatilities sort monotonically with SDF volatilities in the cross section – which relies on the affine setting and parametric assumptions in their paper. Our procedure aims to estimate SDF volatilities from asset prices, and makes no assumption on the pattern of the cross-sectional variation of interest rate volatilities a priori.

Table 2: Cross-Sectional Regressions on SDF Volatilities and Interest Rate Differentials

	(1) Ave Interest Rate Diff $r_J - r_{US}$	(2) Average CT -US/+J to US investor	(3) Sharpe Ratio CT -US/+J to US investor	(4) Average CT -US/+J to US investor	(5) Sharpe Ratio CT -US/+J to US investor
Panel A:					
$Vol\left(\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}\right)$	-0.48^{***}	-0.40^{***}	-3.31^{***}		
	(-6.10)	(-4.14)	(-4.21)		
$r_J - r_{US}$				0.72^{***}	5.95^{***}
				(3.91)	(3.93)
R^2	79%	63%	64%	61%	61%
Panel B:					
$Vol\left(\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}\right)$	-0.20^{*}	-0.55^{***}	-4.59^{***}		
	(-2.05)	(-3.33)	(-3.41)		
$r_J - r_{US}$				1.77^{***}	14.47^{***}
				(5.09)	(5.06)
$i_J - i_{US}$	1.39^{***}	-0.76	-6.37	-2.51^{***}	-20.52^{***}
	(3.44)	(-1.11)	(-1.14)	(-3.24)	(-3.21)
R^2	90%	67%	68%	81%	81%
Panel C:					
$Vol\left(\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}\right)$		-0.29^{**}	-2.42^{**}		
		(-2.34)	(-2.44)		
$r_J - r_{US}$		1.34^{***}	10.89^{***}		
		(4.04)	(4.03)		
$i_J - i_{US}$		-2.62^{***}	-21.45^{***}		
		(-4.19)	(-4.22)		
R^2		88%	88%		

Notes: Cross-country OLS regressions $Y_J = \alpha + \sum \beta_h X_h + \epsilon_J$ with explanatory variables X: estimated SDF volatility $Vol\left(\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}\right)$ in country J (11), average interest rate differential $r_J - r_{US}$, average inflation differential $i_J - i_{US}$. Dependent variable Y: average interest rate differential $r_J - r_{US}$ (Column 1), average carry trade return $CT^{US}_{-US/+J}$ (Column 2 & 4), Sharpe ratio of $CT^{US}_{-US/+J}$ (Column 3 & 5). Panel A and B are separate regression results. Values in parentheses below each regression coefficient are t-statistics. We have 11 observations. 10%, 5%, 1% significance levels of two sided t-statistics are indicated by *, ** and ***, respectively.

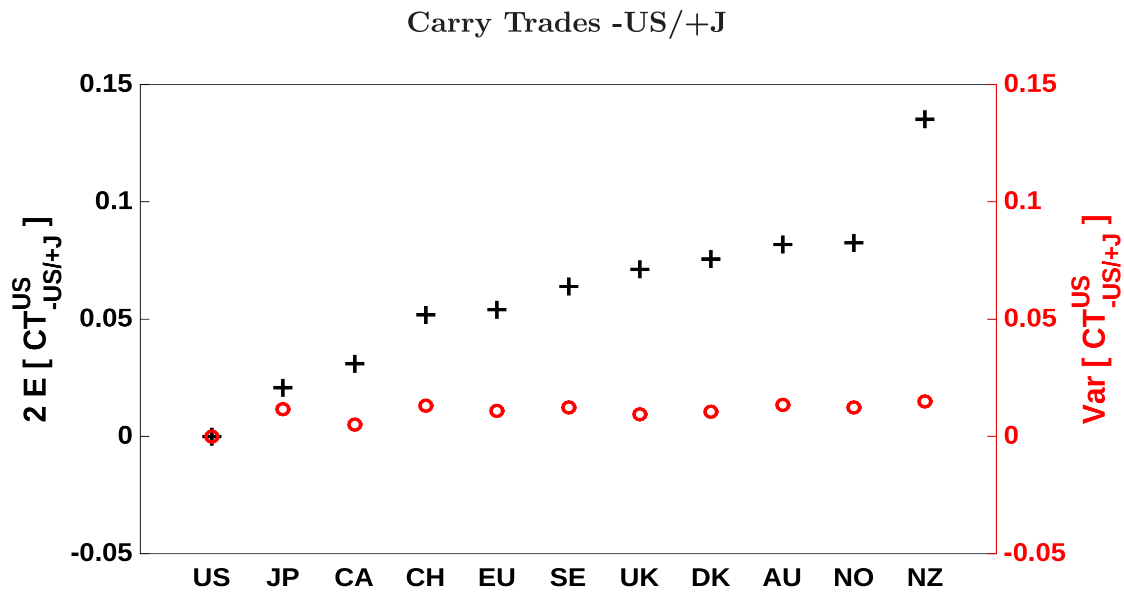

Figure 2: Carry trade strategies of borrowing USD and lending currency J from the perspective of a US investor. Left vertical axis–black crosses: Cross-sectional variation in $2\times$ average carry trade returns. Right axis–red circles: Cross-sectional variation in the variance of carry trade returns.

While relationship (12) appears similar to equation (4) in Verdelhan (2010)[15], there are some key differences. Verdelhan (2010) derives his equation (4) for the expected log-return instead of the expected (continuously compounded) return an investor earns. While co-variations between SDFs across countries are not the focus in his analysis, they are a conceptually important piece when modeling risks in FX markets. A version of Verdelhan (2010)'s equation (4) can be recovered if we assume that SDFs across countries feature a correlation close to one, which is indeed what we estimate as we will show in the next section (Figure 4).

The left plot in Figure 3 shows that our estimated model matches the relationship in equation (12) very well. The cross-country correlation between $ECT^{US}_{-US/+J}$ and the SDF volatility in country J is -79%. Column 2 in Panel A in Table 2 shows that the relationship is highly statistically significant with a t-statistic of 4.14. The relationship is robust to controlling for inflation (t-statistic of 3.33, column 2 in Panel B) and for inflation and interest rates (t-statistic of 2.34, column 2 in Panel C). Indeed, regression (7) in our estimation approach by construction implies this strong relationship, provided that the model matches exchange rate volatilities and average carry trade

[15] Verdelhan (2010) uses the definition of the interest rate in currency J $r_{t,J} = -\ln E_t\left[M_{t+1,J}\right] = -E_t\left[m_{t+1,J}\right] - \frac{1}{2}Var_t\left[m_{t+1,J}\right]$ with SDF $M_{t+1,J}$ and log-SDF $m_{t+1,J} = \ln M_{t+1,J}$ and defines exchange rate growths as the differences in log-SDFs. The expected log carry trade return is $E\left[\ln CT_{t+dt,-I/+J}\right] = r_{t,J} - r_{t,I} + E_t\left[m_{t+1,J}\right] - E_t\left[m_{t+1,I}\right] = \frac{1}{2}Var_t\left[m_{t+1,I}\right] - \frac{1}{2}Var_t\left[m_{t+1,J}\right]$.

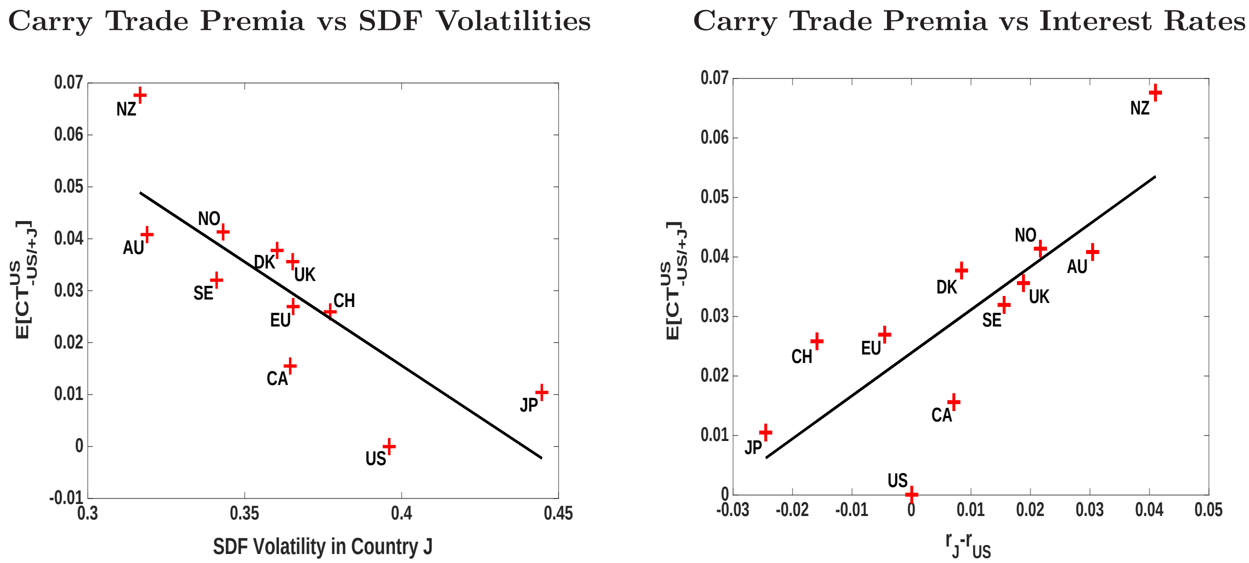

Figure 3: Left: Cross-country relationship between estimated SDF volatility in country J and average carry trade return of borrowing USD and lending currency J earned by US investor. Right: Cross-country relationship between average interest rate differential between country J and USA and average carry trade return of borrowing USD and lending in currency J earned by an investor in country J.

returns in the data. Thus, the strong empirical relationship in Figure 3 can be viewed as a check of the goodness of the fit and the suitability of our estimation approach. Since the variance of $CT^{US}_{t+dt,-US/+J}$ is basically constant across countries J, we also find the same relationship between the Sharpe ratio of the carry trade $CT^{US}_{t+dt,-US/+J}$ and the SDF volatility in country J (column 3 in Panel A, B and C in Table 2).

The plot on the right in Figure 3 and columns 4 and 5 (Panels A and B) in Table 2 show the well-known strong positive relationship between expected carry trade returns $ECT^{US}_{-US/+J}$ and Sharpe ratio and the average interest rate differential between country J and the US. The relationship between carry trade returns and interest rates is similarly strong as the relationship between the carry trade returns and the estimated SDF volatilities. Both relationships are highly statistically significant and the cross-sectional regression fit is more than 60% in all specifications. Finally, columns 2 and 3 in Panel C suggest that the SDF volatility, interest rate and inflation all add information to explain the cross-section of expected carry trade returns and Sharpe ratio (i.e., the slope coefficients on all three variables are significant).

4 Out-of-Sample Results

In the following we investigate the time-series of our estimated SDFs $\widehat{M}_{t,J}$ (Section 4.1), decompose them into permanent and transitory components and check the out-of-sample validity of our estimates using stock and long term bond prices (Section 4.2). We further study the importance of the identified risks ($\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$) and SDFs $\widehat{M}_{t,J}$ to price the cross-section of international stock returns (Section 4.3) and the relationship to financial stress indicators and macroeconomic fundamentals (Section 4.4). Since all these tests use data that were not used as inputs in the estimation of our FX risks and SDFs, these tests are out-of-sample.

4.1 Times-Series of SDFs

Figure 4 plots the time series of the natural logarithm of all 11 country-specific SDFs, $\ln(\widehat{M}_{t,J})$.[16] In our model $\ln(\widehat{M}_{t,J})$ follows a random walk with drift, where the permanent shocks are given by the changes in the n-dimensional Brownian motion dZ_t multiplied by the negative of the market price of risk vector η_J and the drift is equal to the negative of the interest rate $r_J dt$. Empirically, augmented Dickey-Fuller tests suggest that the log SDF (levels) $\ln(\widehat{M}_{t,J})$ are integrated of order 1, which is consistent with the model setup. That is, across all 11 countries the augmented Dickey-Fuller test statistics for $\ln(\widehat{M}_{t,J})$ are always larger than -2.543 (p-values are above 32%) suggesting that we cannot reject the Null hypothesis that $\ln(\widehat{M}_{t,J})$ is non-stationary. Moreover, the same test statistics for the SDF growths $\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}} \approx \ln(\widehat{M}_{t+dt,J}) - \ln(\widehat{M}_{t,J})$ are highly statistically significant and always below -86.398 (p-values are below 0.1%), suggesting that we reject the Null hypothesis that SDF growths are non-stationary.[17]

There is a strong co-movement between the SDFs across all countries. We estimate correlations of daily growths of the SDFs between any country pair I and J, $Corr\left(\frac{d\widehat{M}_{t,I}}{\widehat{M}_{t,I}}, \frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}}\right)$ in our sample and find that all estimates are above 95%. An almost perfect correlation implies that the market price of risk vectors are very similar across countries. However, in our model shocks (changes in Brownian motion dZ_t) have a permanent effect on SDFs, and as long as market price of risk vectors are not exactly identical, SDFs are not cointegrated, i.e., any linear combination of two SDFs is non-stationary. Empirically, we test for a cointegration relationship between $\ln(\widehat{M}_{t,J})$ in country J and $\ln(\widehat{M}_{t,US})$ in the USA. Therefore, we regress $\ln(\widehat{M}_{t,J})$ on a constant and $\ln(\widehat{M}_{t,US})$ and

[16] Note that we are plotting levels $\ln(\widehat{M}_{t,J})$, not the growths $\frac{d\widehat{M}_{t,J}}{\widehat{M}_{t,J}} \approx \ln(\widehat{M}_{t+dt,J}) - \ln(\widehat{M}_{t,J})$.

[17] See Table C.2 in the Appendix for details.

investigate whether the regression errors have a unit root (Engle and Granger, 1987). Augmented Dickey-Fuller tests reveal that for 6 out of 10 regressions the Null hypothesis that the errors are non-stationary cannot be rejected on the 10% level (test statistics larger than -2.704), i.e., we cannot reject the hypothesis that these SDFs are not cointegrated with the SDF in the USA. In contrast, for Australia, Eurozone, New Zealand and Switzerland we find significant Dickey-Fuller statistics (on the 1% level), suggesting that these SDFs are cointegrated with the SDF in the USA.

In summary, consistent with the theoretical diffusion model the estimated SDFs are integrated of order one (i.e. SDF growths are stationary). For the questions whether the SDFs are cointegrated, the empirical evidence is mixed. Our theoretical model assumes that SDFs are not cointegrated, but if market prices of risk vectors across countries are similar (i.e. SDF growth are highly correlated across countries), then it is difficult to distinguish a model with versus without a cointegration relationship.

The observation of highly correlated SDFs is consistent with the finding of Brandt et al. (2006), who conclude that since the exchange rate is equal to the ratio of (projected) country-specific SDFs[18] the correlation between the (projected) SDFs has to be close to one to match the smooth exchange rate process in the data. Remember that since we estimate SDFs from FX market returns, our constructed SDFs are always in the space spanned by asset returns, i.e., they are SDFs projected onto the FX market risk space.

The 5 largest quarterly increases in the estimated SDFs across the world are in the last quarter of 1998, third and fourth quarter of 2008, second quarter of 2010 and third quarter of 2011. The large increase in SDFs in the last quarter of 1998 is subsequent to the Asian financial crisis in the second half of 1997 and the Russian sovereign default and the bailout of Long-Term Capital Management in 1998. The surge in the SDFs in the second half of 2008 coincides with the collapse of Lehman Brothers and the concurrent turmoil in financial markets. The increases in 2010 and 2011 can be explained by the first two bailouts of Greece during the European sovereign debt crisis. The time-series of SDFs further shows a substantial and steady increase in the late 1990s and early 2000s, which relates to the burst of the of the Dot-com bubble in the early 2000s. Although we do not have a formal test to analyze these events and the time series pattern, we interpret it as first suggestive evidence in favor of our estimates.

[18] If markets are fully integrated and free of arbitrage Maurer and Tran (2017a,b) prove that the ratio of projected country-specific SDFs is always equal to the exchange rate in a diffusion setting (as considered in our paper). They further prove that risk entanglement in FX markets is a necessary and sufficient condition to break this strong relation and possibly allow for a low correlation between projected SDFs while still ensuring a smooth exchange rate process.

Time Series of Country-Specific log SDFs, $\ln(\widehat{M}_{t,J})$

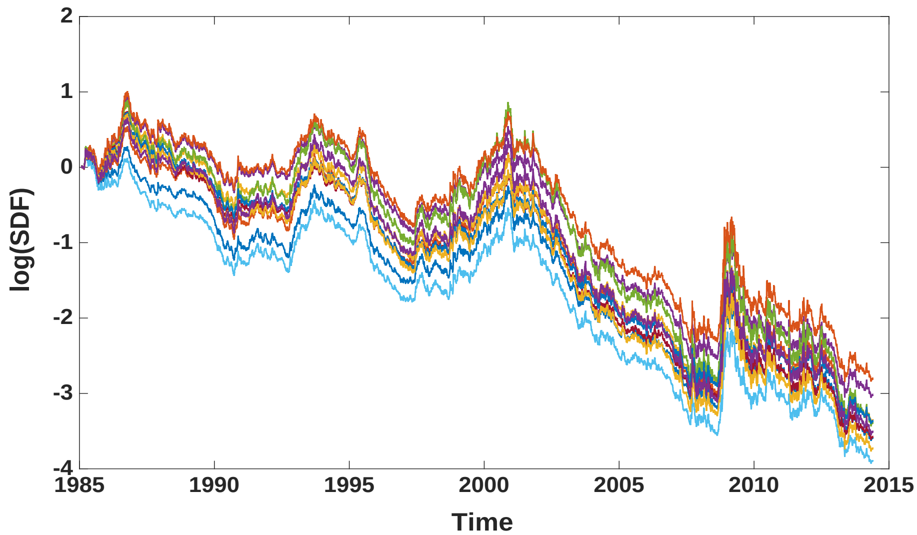

Figure 4: Time Series of the ln of country-specific SDFs, $\ln(\widehat{M}_{J,t})$ of 11 developed countries estimated according to (9).

4.2 Decomposition of SDFs into Permanent and Transitory Components

Alvarez and Jermann (2005) and Hansen and Scheinkman (2009) show how a SDF $\widehat{M}_{t,J}$ can be decomposed into a permanent (martingale) component $\widehat{M}^P_{t,J}$ and a transitory component $\widehat{M}^T_{t,J}$, $\widehat{M}_{t,J} = \widehat{M}^P_{t,J}\widehat{M}^T_{t,J}$. We decompose our estimated SDFs into permanent and transitory components following Christensen (2017) who proposes a non-parametric approach to solve the Perron-Frobenius eigenfunction problem in Hansen and Scheinkman (2009) given a time-series of state variables and the SDF. We use the two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$, which are proxies for changes in Brownian motion dZ_t in our model, as state variables in our decomposition. Details of the decomposition procedure are provided in Appendix D. An alternative approach to decompose the SDF is to use the fact that the transitory component is equal to the return of a bond with infinite maturity (for instance Sandulescu et al. (2017) choose this approach). An advantage of using the non-parametric approach of Christensen (2017) is that we can use out-of-sample tests using stock and bond return data to validate our estimated SDFs and permanent and transitory components since these estimations are based on only FX market data.

Volatility Bound Tests

In our theoretical model changes in the diffusion dZ_t are always permanent shocks to the SDF. However, in the data our estimated SDFs may still feature some transitory changes due to the time variation in the interest rate (drift of the SDF) or due to an autocorrelation in our constructed PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$. We find that the standard deviation of the permanent component $\frac{d\widehat{M}^P_{t,J}}{\widehat{M}^P_{t,J}}$ is roughly seven times larger than the standard deviation of the transitory component $\frac{d\widehat{M}^T_{t,J}}{\widehat{M}^T_{t,J}}$ across all countries J. The annualized standard deviation of the estimated permanent component $\frac{d\widehat{M}^P_{t,J}}{\widehat{M}^P_{t,J}}$ ranges between 32% (New Zealand) and 45% (Japan) across countries with an average of 37%. In contrast, the annualized standard deviation of the transitory component $\frac{d\widehat{M}^T_{t,J}}{\widehat{M}^T_{t,J}}$ ranges between 4.4% (New Zealand) and 6.2% (Japan) across countries with an average of 5.1%. We find a slightly negative correlation between permanent and transitory components. The correlation coefficient ranges between -0.23 and -0.21 with an average -0.22.

Alvarez and Jermann (2005) derive bounds (from observable stock and long term bond returns) on the variation of the two components and show that the permanent component is very volatile while the transitory component is much less important. In particular, they construct the following three bounds:

$$L_t\left(\frac{M^P_{t+dt,J}}{M^P_{t,J}}\right) \geq E_t\left[\ln(R_{t+dt,J})\right] - E_t\left[\ln(R_{t+dt,\infty,J})\right] \tag{13}$$

$$\frac{L\left(\frac{M^P_{t+dt,J}}{M^P_{t,J}}\right)}{L\left(\frac{M_{t+dt,J}}{M_{t,J}}\right)} \geq \min\left\{1, \frac{E\left[\ln\left(\frac{R_{t+dt,J}}{1+r_{t,J}}\right)\right] - E\left[\ln\left(\frac{R_{t+dt,\infty,J}}{1+r_{t,J}}\right)\right]}{E\left[\ln\left(\frac{R_{t+dt,J}}{1+r_{t,J}}\right)\right] + L\left(\frac{1}{1+r_{t,J}}\right)}\right\} \tag{14}$$

$$\frac{L\left(\frac{M^T_{t+dt,J}}{M^T_{t,J}}\right)}{L\left(\frac{M_{t+dt,J}}{M_{t,J}}\right)} \leq \frac{L\left(\frac{1}{R_{t+dt,\infty,J}}\right)}{E\left[\ln\left(\frac{R_{t+dt,J}}{1+r_{t,J}}\right)\right] + L\left(\frac{1}{1+r_{t,J}}\right)}, \tag{15}$$

where $R_{t+dt,J}$ is the $[t, t+dt]$ holding period gross return of the stock market index in country J, $R_{t+dt,\infty,J}$ is the $[t, t+dt]$ holding period gross return of the (default free) long term bond with infinite maturity in country J, $r_{t,J}$ is the risk-free short rate (rate of return) at time t in country J, and $L_t(x) = \ln(E_t[x]) - E_t[\ln(x)]$ is the entropy of random variable x.[19]

We compute bounds (13), (14) and (15) using stock and bond data for all 11 countries in our analysis and check whether they hold for our estimated SDFs $\widehat{M}_{t,J}$ and permanent and transitory

[19] Entropy is a risk measure and if x is log-normally distributed then $L(x) = \frac{1}{2}Var(x)$.

components $\widehat{M}^P_{t,J}$ and $\widehat{M}^T_{t,J}$.[20] Remember that our estimates only use spot and forward exchange rate data and the time series of the US short term interest rate. Thus, the bound tests (using stock and long term bond data) are out-of-sample tests. We use monthly data from 1984-2014 (to match our FX data) of the MSCI total return indices to proxy stock market returns $R_{t+dt,J}$. We follow Lustig et al. (2017) and approximate the long term bond returns $R_{t+dt,\infty,J}$ (with infinite maturity) using the total return indices of 10-year government bonds provided by Global Financial Data. They show that this approximation is reasonable in the context of popular affine term structure models. All returns are denominated in local currency. Details about the data are provided in Tables E.4 and E.5 in the Appendix.

Table 3: SDF Volatility Bound Tests

	Bound (13)		Bound (14)		Bound (15)	
	(1)	(2)	(3)	(4)	(5)	(6)
Country	$L_t\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)$	Lower Bound	$\frac{L\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)}{L\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)}$	Lower Bound	$\frac{L\left(\frac{\widehat{M}^T_{t+dt,J}}{\widehat{M}^T_{t,J}}\right)}{L\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)}$	Upper Bound
Australia	0.0535	0.0058	1.0469	0.0815	0.0204	0.0446
Canada	0.0698	0.0128	1.0460	0.2020	0.0207	0.0368
Denmark	0.0681	0.0232	1.0425	0.2468	0.0210	0.0243
Euro	0.0701	0.0086	1.0427	0.1107	0.0210	0.0166
Japan	0.1041	0.0492	1.0458	0.4654	0.0211	0.0276
New Zealand	0.0527	-0.0069	1.0471	-0.1686	0.0205	0.0664
Norway	0.0617	-0.0131	1.0421	-0.4859	0.0211	0.0737
Sweden	0.0610	0.0230	1.0423	0.2681	0.0211	0.0193
Switzerland	0.0747	0.0955	1.0429	0.6401	0.0211	0.0038
UK	0.0700	0.0169	1.0434	0.2739	0.0210	0.0450
USA	0.0825	0.0393	1.0461	0.4558	0.0209	0.0379

Notes: The table reports the entropies of the estimated SDFs and their permanent and transitory components as well as the lower and upper bounds of Alvarez and Jermann (2005) estimated from stock and bond return data for all 11 countries in our analysis. Columns 1 and 2 report values for the bound in (13), 3 and 4 the values for the bound in (14), and 5 and 6 the values for the bound in (15). All reported quantities are annualized.

Table 3 reports the results. The odd columns provide estimates of the entropies on the left hand side of the conditions (13), (14) and (15), while the even columns report the lower and upper

[20]Note that we compute the unconditional version of (13), which is less tight than the theoretical conditional bound that has to hold at every point in time.

bounds estimated from stock and bond returns. The first lower bound (13) on the entropy of the estimated permanent component $L_t\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)$ holds in all countries except for Switzerland, which appears to be due to the exceptionally large average excess return of the Swiss stock market index between 1984 and 2014 and may be attributed to noise in the estimation of the expected return. The second lower bound (14) on the entropy of the estimated permanent component relative to the entropy of the SDF $\frac{L\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)}{L\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)}$ holds in all 11 countries. The entropy of the permanent component is always larger than the entropy of the SDF, which is consistent with the fact that the permanent and transitory components are negatively correlated. Finally, the upper bound (15) on the entropy of the transitory component relative to the entropy of the SDF $\frac{L\left(\frac{\widehat{M}^T_{t+dt,J}}{\widehat{M}^T_{t,J}}\right)}{L\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)}$ holds for 8 of our 11 countries but is violated in case of Europe, Sweden and Switzerland.

Long Term Bond Yields

In the SDF decomposition we obtain an estimate of the eigenvalue ρ in the Perron-Frobenius eigenfunction problem. $-\ln(\rho)$ may be interpreted as the yield on a long term bond with infinite maturity (Christensen, 2017). We use these implied yields from our decomposition of the SDFs and compare them to the average yields (in local currency) of the 10-year government bonds across all countries. The 10-year bond yield data is again from Global Financial Data. Since our SDFs are estimated from FX market data and do not use any information about long term bonds, our comparison is an out-of-sample validation of our SDF estimation.

Figure 5 shows a striking positive cross-country relationship with a correlation of 91% between the average (annualized) 10-year bond yields and the (annualized) yields extracted from our estimated SDFs. The slope in a regression of the data on the implied yields is 0.74, which is statistically different from 0 with a t-statistic of 9.35.[21] The R^2 of the regression is 91% suggesting that the SDFs estimated from FX market data are able to explain a large fraction of the cross-country variation in long term bond yields. This is strong out-of-sample evidence in favor of our estimated SDFs.

[21] The constant term in the regression is 0.029 and significantly different from 0.

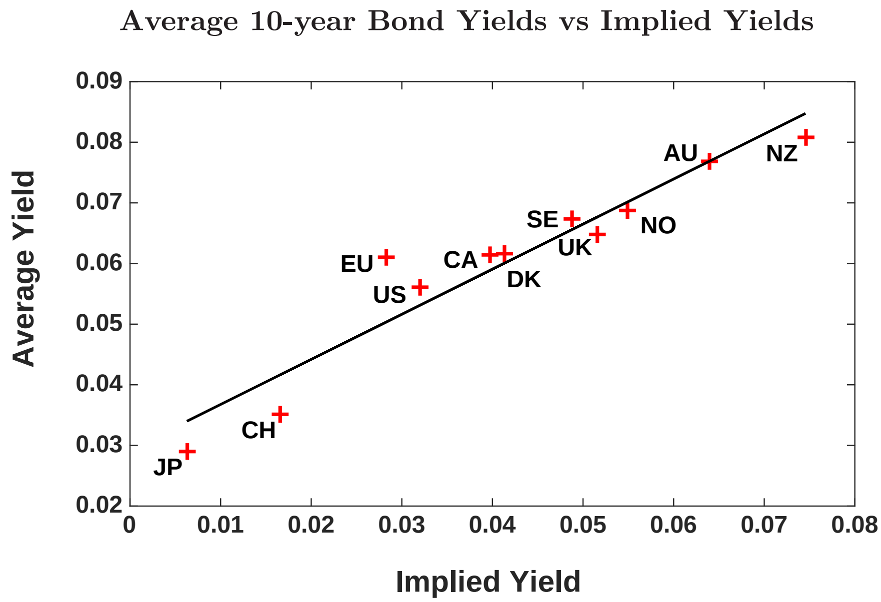

Figure 5: Plot of average 10-year government bond yields (in local currency) against implied yields $-\ln(\rho)$ (in local currency) obtained from the SDF decomposition of Christensen (2017) (red points). The black line in the Figure is the regression fit when regressing (20).

Long Term Bond Excess Returns and Permanent Components in SDFs

Lustig et al. (2017) derive an identity (Proposition 1 in their paper) between long term bond excess returns across different countries (denominated in USD) and entropies of the permanent components of these countries' SDFs,

$$E_t\left[rx^{US}_{t+dt,\infty,J}\right] = E_t\left[rx_{t+dt,\infty,US}\right] + L_t\left(\frac{M^P_{t+dt,US}}{M^P_{t,US}}\right) - L_t\left(\frac{M^P_{t+dt,J}}{M^P_{t,J}}\right), \tag{16}$$

with $rx^{US}_{t+dt,\infty,J} = \ln\left(\frac{R_{t+dt,\infty,J}}{1+r_{t,J}}\right) + \ln\left(CT^{US}_{t+dt,-US/+J}\right)$ and $rx_{t+dt,\infty,US} = \ln\left(\frac{R_{t+dt,\infty,US}}{1+r_{t,US}}\right)$. The left hand side (LHS) is the expected log excess return of the long term bond (with infinite maturity) in country J denominated in USD.[22] The right hand side (RHS) is the the expected log excess return of the long term bond in the USA (denominated in USD) plus the difference in the entropies of the permanent components of the SDFs in the USA versus country J.

Lustig et al. (2017) do not have estimates of the entropies of the permanent SDF components and thus cannot directly test their theoretical relationship (16). Instead they use it as a bound on

[22] Note that adding the carry trade premium to the expected log excess return of the long term bond denominated in local currency changes the denomination to USD.

Long Term Bond Excess Returns and Permanent Components in SDFs

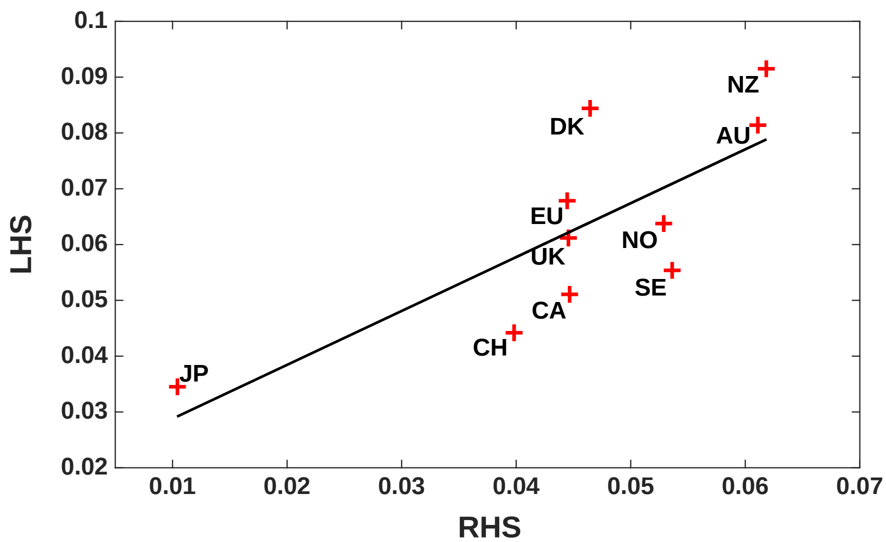

Figure 6: Plot of $LHS = E\left[rx^{US}_{t+dt,10,J}\right]$ against $RHS = E_t\left[rx_{t+dt,10,US}\right] + L_t\left(\frac{\widehat{M}^P_{t+dt,US}}{\widehat{M}^P_{t,US}}\right) - L_t\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)$ (red points) and fitted regression line $LHS = a + b * RHS + \epsilon_J$ (black line), where $E\left[rx^{US}_{t+dt,10,J}\right]$ is the average excess return of the 10-year bond in country J denominated in USD, $E\left[rx_{t+dt,10,USD}\right]$ is the 10-year bond in the USA, $L_t\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)$ is the entropy of the estimated permanent component of the SDF in country J.

how much entropies of the permanent SDF components may differ across countries and investigate which models in the literature satisfy this bound.

In contrast, we have estimates of the entropies of the permanent components across countries and can test the relationship directly. We use again the 10-year bond return data from Global Financial Data and the permanent components from the decomposition of our estimated SDFs. We directly test relationship (16) using the cross-sectional regression,

$$E\left[rx^{US}_{t+dt,10,J}\right] = a + b\left[E_t\left[rx_{t+dt,10,US}\right] + L_t\left(\frac{\widehat{M}^P_{t+dt,US}}{\widehat{M}^P_{t,US}}\right) - L_t\left(\frac{\widehat{M}^P_{t+dt,J}}{\widehat{M}^P_{t,J}}\right)\right] + \epsilon_J, \tag{17}$$

where the excess returns of the 10-year government bonds $rx^{US}_{t+dt,10,J}$ and $rx_{t+dt,10,US}$ are again approximations of the excess returns of the long term bonds with infinite maturity (as discussed earlier). If (16) holds, then we should find constant $a = 0$ and slope $b = 1$.[23] Figure 6 visualizes regression (17) and shows a striking positive relationship between the RHS and LHS of equation (16). We estimate the constant term a equal to 0.019 and not statistically significantly different from 0 (t-statistic of 1.39). The slope coefficient b is equal to 0.97, statistically significantly different from 0 (t-statistic of 3.38) but not different from 1 (t-statistic of 0.12). Thus, we conclude that the theoretical equation (16) of Lustig et al. (2017) holds for the permanent components extracted from our estimated SDFs. This is strong out-of-sample evidence in favor of our estimation.

4.3 International Stock Returns

In our first set of tests we take market prices of PC risks $\widehat{\gamma}^J_1$ and $\widehat{\gamma}^J_2$ estimated from the FX data as given and estimate implied equity premia from covariations between stock returns and PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$. In the second set of tests we use Fama and MacBeth (1973) regressions to estimate market prices of PC risk from international stock returns.

Pricing Stocks using Market Prices of PC Risks Estimated from FX Data

We use again the monthly MSCI stock market return indices denominated in local currency (as in section 4.2). We assume that country J's stock market excess return denominated in its local

[23]Note that if the hypothesis that $E\left[rx^{US}_{t+dt,\infty,J}\right] = E\left[rx_{t+dt,\infty,US}\right]$ was true and differences in average excess returns of 10-year bonds are just noise, then we should not find any significant relationship in our regression. We deem it unlikely that the noise in average excess returns is correlated with the differences in entropies of the permanent components because our estimated SDFs and the constructed permanent components do not use any long term bond data.

currency J is described by a diffusion process,

$$R_{t+dt,J} = (\mu_J - r_J)\, dt + \sigma_J^T dZ_t, \tag{18}$$

where $R_{t+dt,J}$ is the realized stock market excess return of country J in local currency, $\mu_J - r_J$ is the equity premium, σ_J is the exposure to Brownian motion risk sources dZ_t. We estimate stock market J's exposures $\sigma_{1,J}$ and $\sigma_{2,J}$ to FX market risks $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ using the time-series regression,

$$R_{t+dt,J} = \alpha_J + \sum_{k=1}^{2} \sigma_{k,J} \overline{\Pi}_{t,k} + \epsilon_{t,J}, \tag{19}$$

where α_J equals the average stock excess return and $\epsilon_{t,J}$ captures all the risk not spanned by $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$.

The implied annualized expected excess return on J's stock market (i.e., implied equity premium) measured in its home currency J is,

$$ER_J \quad = \quad \mu_J - r_J = -\frac{1}{dt}\text{Cov}_t\left(\frac{d\widehat{M}_{J,t}}{\widehat{M}_{t,J}}, R_{t+dt,J}\right) = \sum_{k=1}^{2} \widehat{\gamma}_k^J \sigma_{k,J}. \tag{20}$$

Thus, we estimate the implied ER_J using the market prices $\widehat{\gamma}_1^J$ and $\widehat{\gamma}_2^J$ estimated in FX markets (8) and the stock market loadings $\sigma_{1,J}$ and $\sigma_{1,J}$ on $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ obtained from the time-series regression (19). Hence, (20) presents an expression for the equity premium *implied* by our estimated SDFs $\widehat{M}_{t,J}$ in FX markets. Column 3 in Panel A in Table 4 reports these estimates. Column 4 reports the percentage of the variance of $R_{t+dt,J}$ explained by the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ in the time-series regression (19), and columns 1 and 2 report averages and volatilities of stock market excess returns in the data.

Stock markets across all countries negatively covary with the SDFs and the FX market implied equity premia ER_J are positive. The implied premia have on average a magnitude of 30% of the average realized excess returns. This is a substantial amount considering that the FX market risks $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ capture only slightly more than 10% of the total stock market return variation in the time series. Moreover, the correlation between the cross-country variation in implied premia ER_J and the average of realized excess returns $R_{t+dt,J}$ is 67%. Figure 7 illustrates the strong positive cross-country relationship. Regressing average realized excess returns $R_{t+dt,J}$ on the implied premia ER_J yields a statistically significant regression coefficient of 3.35 with a t-statistic of 2.89. We

Table 4: Country-Specific Stock Markets and SDFs

	Panel A: Stock Market Returns in Country J and M_J			
	(1)	(2)	(3)	(4)
Country	Average Stock Excess Return	Volatility of Excess Return	Implied Equity Premium ER_J	Percentage of Variance Explained
Australia	0.062	0.177	0.014	8.5
Canada	0.062	0.164	0.019	12.3
Denmark	0.090	0.201	0.027	17.1
Euro	0.073	0.219	0.027	11.5
Japan	0.103	0.245	0.030	7.3
New Zealand	0.037	0.200	0.017	14.1
Norway	0.026	0.185	0.015	5.8
Sweden	0.078	0.251	0.031	14.6
Switzerland	0.144	0.247	0.026	9.5
UK	0.059	0.185	0.018	9.6
USA	0.085	0.165	0.022	12.6

	Panel B: Stock Market Returns in Country J and first two PCs			
	(1)	(2)	(3)	(4)
Country	Premium Earned from Risk of $\overline{\Pi}_{t,1}$	Premium Earned from Risk of $\overline{\Pi}_{t,2}$	% of Variance Explained by $\overline{\Pi}_{t,1}$	% of Variance Explained by $\overline{\Pi}_{t,2}$
Australia	0.004	0.010	4.9	3.5
Canada	0.006	0.013	6.9	5.2
Denmark	0.014	0.013	12.0	4.8
Euro	0.011	0.015	5.9	5.3
Japan	0.008	0.021	2.4	4.8
New Zealand	0.006	0.011	10.3	3.6
Norway	0.006	0.009	2.3	3.3
Sweden	0.014	0.018	8.1	6.3
Switzerland	0.014	0.012	6.9	2.5
United Kingdom	0.009	0.009	7.2	2.2
USA	0.006	0.015	6.4	6.1

Notes: Country-specific OLS time-series regression $R_{t,J} = \alpha_J + \sum_{K=1}^{2} \beta_{J,K} \overline{\Pi}_{t,K} + \epsilon_{t,J}$ to examine the effects of exchange rate risks captured by the first two PCs $\overline{\Pi}_{t,1}$, $\overline{\Pi}_{t,2}$ on country J's stock market excess returns $R_{t,J}$. Panel A reports the average and volatility of country J's stock market excess returns (denominated in local currency; column 1 & 2), the implied equity premium in (20) (column 3), and the regression R^2 or percentage of stock market return variance explained by the two PCs combined (column 4). Panel B reports the impact of each PC separately, i.e., the implied equity premia due to exposure to the first and second PC (columns 1 & 2), and the percentage of stock market return variance explained by the first and second PC (column 3 & 4). Excess stock returns are computed from monthly country-specific MSCI Total Return Index series. All reported returns and volatilities are annualized.

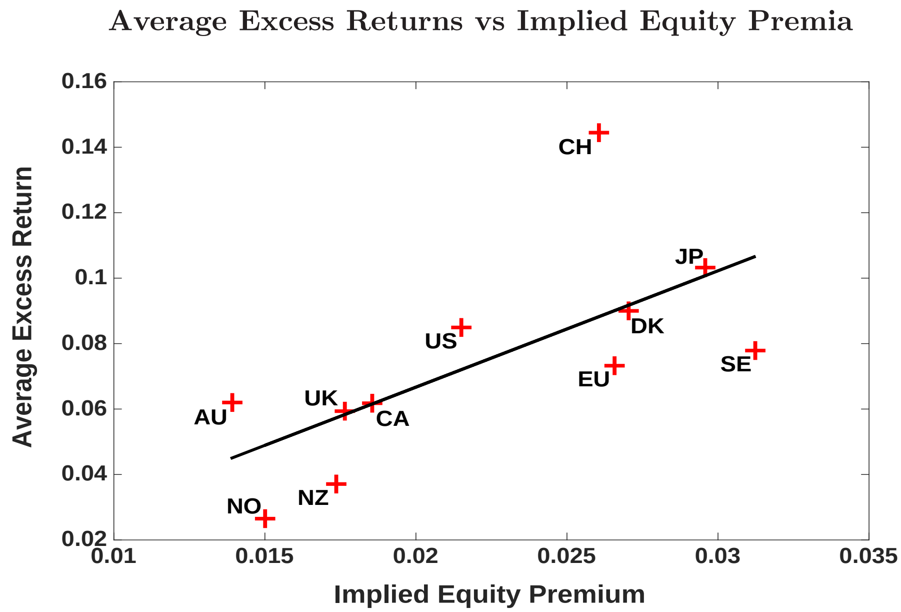

Figure 7: Plot of average stock market excess returns (in local currency) against implied risk premia (in local currency) according to (20) (red points). The black line shows the regression fit when regressing average excess returns on the implied premia.

conclude that while FX market risks $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ only capture a small part of the time-series variation in stock returns, they are able to explain a substantial amount of priced stock market risks, i.e., a substantial part of international equity premia. These estimates lend support to the validity of our construction of country-specific SDFs from FX market returns and demonstrate that FX market risks are important for pricing stocks.

To investigate the importance of the individual PCs Panel B in Table 4 decomposes the SDFs and columns 1 and 2 report the implied premia stock market J earns due to exposure to $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$, i.e., $\widehat{\gamma}_1^J \sigma_{1,J}$ and $\widehat{\gamma}_2^J \sigma_{2,J}$. Columns 3 and 4 report the percentage of stock market return variance captured by $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$. All stock markets load negatively on $\overline{\Pi}_{t,1}$ and positively on $\overline{\Pi}_{t,2}$. Remember that the market price $\widehat{\gamma}_1^J$ ($\widehat{\gamma}_2^J$) of $\overline{\Pi}_{t,1}$ ($\overline{\Pi}_{t,2}$) is negative (positive) and thus an increase (decrease) in $\overline{\Pi}_{t,1}$ ($\overline{\Pi}_{t,2}$) is bad news to the marginal investor. Hence, stocks are risky and earn a positive premium for an exposure to the two PCs. Though, we find that $\overline{\Pi}_{t,1}$ generally explains more of the time-series variation in stock returns than $\overline{\Pi}_{t,2}$, the premia paid due to risk exposure is slightly larger for $\overline{\Pi}_{t,2}$. This is because the market price of risk $\widehat{\gamma}_2^J$ is estimated to be substantially larger than $\widehat{\gamma}_1^J$ in the FX market data (Table 1).

Next, we repeat the above analysis (i.e., estimation of (19) and (20)) for the US only and

Table 5: US Stock Market and SDF

Historical data (1984-2014):						
Average US Stock Market Excess Return			0.085			
Volatility of US Stock Market Excess Return			0.165			
	(1)	(2)	(3)	(4)	(5)	(6)
Horizon	Implied Equity Premium	Premium Earned from PC 1	Premium Earned from PC 2	Percentage of Variance Explained	% of Variance Explained by PC 1	% of Variance Explained by PC 2
1-day Return	0.016	0.005	0.012	4.9	3.5	3.4
5-day Return	0.023	0.007	0.016	10.0	7.5	6.6
10-day Return	0.023	0.007	0.016	10.0	8.0	6.1
20-day Return	0.023	0.008	0.015	11.1	9.6	5.7
60-day Return	0.024	0.009	0.015	15.0	13.5	6.6
125-day Return	0.024	0.010	0.015	15.9	14.2	7.1

Notes: US stock return OLS time-series regression $R_{US,t} = \alpha_{US} + \sum_{K=1}^{2} \beta_{US,K} \Pi_{t,K} + \epsilon_{US,t}$ to examine the effects of exchange rate risks captured by the first two PCs $\Pi_{t,1}$, $\Pi_{t,2}$ on US stock market excess returns $R_{US,t}$ over diverse holding periods (1, 5, 10, 20, 60, 125 days). Columns 1-3 report the equity premia implied by both PCs together and each PC separately. Columns 4-6 report the percentage of stock market return variance explained by both PCs together and each PC separately. We use daily returns of a value-weighted US stock market portfolio including all stocks in CRSP. Reported results are for overlapping windows. All reported returns and volatilities are annualized.

investigate how our results are affected by changes in the data frequency from monthly to 1, 5, 10, 20, 60 and 125 trading day holding periods.[24] We use the daily value-weighted-index from CRSP. Table 5 reports for the diverse holding periods the implied US equity premium (column 1), the premia earned due to exposure to $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ (columns 2 and 3), the percentage of US stock market return variance explained by the US SDF $\widehat{M}_{t,US}$ (column 4), and the percentage of stock return variance captured by $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ (columns 5 and 6). The percentage of stock return variance explained by our estimated SDF is only 4.9% at the daily frequency, roughly 10% at 5-, 10- and 20-trading day frequencies, and increases to slightly more than 15% at 60- and 125-trading day frequencies. $\overline{\Pi}_{t,1}$ explains slightly more of the time-series variation than $\overline{\Pi}_{t,2}$, and the difference increases at longer horizons. Except for the daily frequency, the implied premia are quite stable across the diverse data frequencies. The implied (annualized) premium by the overall SDF is 1.6% at the daily frequency and 2.3%-2.4% for frequencies between 5 and 125 trading days, which is about 30% of the average realized stock market excess return in the US. The premium earned due to risk exposure to $\overline{\Pi}_{t,1}$ is slightly less than 1% and about 1.5% for $\overline{\Pi}_{t,2}$. Thus, $\overline{\Pi}_{t,1}$ is slightly more important to explain the time-series of returns but $\overline{\Pi}_{t,2}$ is more important to price the US stock market. Overall, the data frequency does not seem to matter much as long as we use a lower frequency than daily data. As shown above, $\overline{\Pi}_{t,1}$ is somewhat similar to the Carry factor while $\overline{\Pi}_{t,2}$ captures a more distinct risk. Thus, similar to the discussion in section 3.2, it is interesting that $\overline{\Pi}_{t,2}$, which is less studied in the literature, appears more important for pricing.

Pricing Stocks using Market Prices of PC Risks Estimated from Stock Data

The test assets for the following Fama and MacBeth (1973) regressions are the 220 international stock portfolios provided by Kenneth French[25] from 1984-2014 (to match our FX data). The data covers the following 22 countries: Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Malaysia, Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, UK and USA. For each country we have 10 portfolios: one value weighted stock market index, four high and four low valuation ratio portfolios (using the ratios book-market, earnings-price, cash earnings-price, dividend yield), and one zero-dividend portfolio. We take the perspective of a US investor in our estimations and thus, denominate all test assets in USD.

[24] This roughly corresponds to daily, weekly, bi-weekly, monthly, quarterly and semi-annual returns.

[25] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

Table 6: Time-Series Regressions of International Stock Markets on US SDF

	Unconditional		60-Month Rolling Windows		
	(1)	(2)	(3)	(4)	(5)
	β_J	R^2	$Mean(\beta_{J,t})$	$Std(\beta_{J,t})$	$Mean(R^2)$
Austria	-0.120^{***}	23%	-0.090	0.097	21%
Australia	-0.136^{***}	31%	-0.104	0.083	26%
Belgium	-0.087^{***}	17%	-0.068	0.076	18%
Canada	-0.083^{***}	18%	-0.062	0.066	14%
Denmark	-0.084^{***}	18%	-0.062	0.075	16%
Finland	-0.104^{***}	11%	-0.077	0.080	14%
France	-0.091^{***}	17%	-0.069	0.072	18%
Germany	-0.101^{***}	19%	-0.080	0.065	18%
Hong Kong	-0.082^{***}	9%	-0.059	0.070	13%
Ireland	-0.115^{***}	24%	-0.085	0.089	17%
Italy	-0.098^{***}	13%	-0.075	0.075	18%
Japan	-0.001	0%	0.022	0.065	11%
Malaysia	0.015	0%	0.016	0.006	0%
Netherlands	-0.096^{***}	21%	-0.070	0.082	17%
New Zealand	-0.103^{***}	19%	-0.085	0.071	25%
Norway	-0.128^{***}	22%	-0.100	0.084	18%
Singapore	-0.090^{***}	11%	-0.066	0.078	14%
Spain	-0.099^{***}	15%	-0.069	0.069	14%
Sweden	-0.115^{***}	19%	-0.092	0.081	18%
Switzerland	-0.070^{***}	15%	-0.051	0.051	17%
UK	-0.082^{***}	19%	-0.058	0.060	18%
USA	-0.054^{***}	11%	-0.037	0.049	12%
Mean	-0.088	16%	-0.065	0.070	16%

Notes: Monthly OLS time-series regressions of each country J's stock market excess return $R_{t,J}$ (denominated in USD) on the US SDF $\widehat{M}_{US}$ estimated according to (9) (and re-scaled to set its variance equal to 1), $R_{t,J} = \alpha_J + \beta_J \frac{1}{\|\widehat{\gamma}_J\|^2} \frac{d\widehat{M}_{t,US}}{\widehat{M}_{t,US}} + \varepsilon_{t,J}$. α_J is a constant, $\varepsilon_{t,J}$ is an error, β_J measures the exposure of stock market J to the US SDF. Columns 2 and 3 report slope coefficient β_J and regression R^2 for unconditional regressions (i.e., one regression per country for entire time-series). Columns 3, 4 and 5 report the averages and standard deviations of the slope coefficients $\beta_{J,t}$ and the average regression R^2 of regressions of 60-month rolling windows for each country J. We use Monthly data from 1984 to 2014. Significance of the slope coefficients in column 1 at the 1%, 5% and 10% level are indicated by ***, ** and *. Robust standard errors are estimated following Newey and West (1987).

We estimate market prices of risk for our estimated US SDF $\widehat{M}_{t,US}$ and other popular factors.[26] Following Fama and French (2015) we use their five global factors, which include a world stock market ($WMkt$), size (SMB), book-market (HML), operating profitability (RMW) and investment (CMA) factor. Moreover, we also include global momentum (MOM). Data for SMB, HML, RMW and CMA, MOM is only available starting in July 1990. Thus, all estimations cover the time period 1984-2014, except when we work with the global Fama-French and momentum factors our sample period shortens to 1990-2014. Following Brusa et al. (2015) we further control for the Dollar (DOL) and Carry (CAR) factors. We normalize all factors so that they have an annual variance of 1. This normalization is non-material but useful to compare the magnitude of estimated market prices across factors in the second stage regression.

In the first stage we estimate for each test asset j the month t conditional factor loadings $\beta_{t,i,j}$ using time-series regression over the past 60 months,

$$R_{\tau,j} = \alpha_{t,j} + \sum_i \beta_{t,i,j} F_{\tau,i} + \varepsilon_{\tau,j}, \tag{21}$$

where $R_{\tau,j}$ denotes the realized excess return of asset j, $F_{\tau,i}$ the return of factor i, $\tau \in \{t-61, \ldots, t-1\}$, $\alpha_{t,j}$ is the time-series abnormal return and $\varepsilon_{\tau,j}$ an error. Using rolling windows allows us to take into account time variations in factor loadings. In the second stage we then estimate the month t conditional market prices $\gamma_{t,i}$ of factors i using the cross-sectional regression,

$$R_{t,j} = \sum_i \beta_{t,i,j} \gamma_{t,i} + \alpha^*_{t,j}, \tag{22}$$

where $\alpha^*_{t,j}$ is the cross-section abnormal return. Finally, we take the time-series average of $\gamma_{t,i}$ as an estimate of the market price of risk of factor i.

Table 6 reports factor loadings of the 22 stock market portfolios estimated in the first stage regression (21) for a model with only the estimated US SDF $\widehat{M}_{t,US}$ as a pricing factor. To save space we only report factor loadings for the 22 stock markets and omit the other 198 portfolios (Tables for the other 198 test assets are available on request). Notice that in the regressions in Table 6 all stock market returns are denominated in USD and the pricing factor is the US SDF, which is different to the analysis in Table 4 where we investigate the relationship between stock market returns denominated in local currencies and local SDFs. Thus, factor loadings and regression fits

[26] This test of estimating the market prices of risk of the US SDF $\widehat{M}_{t,US}$ can be understood similarly to tests of the market portfolio when testing the CAPM.

differ between Tables 4 and 6. Column 1 in Table 6 reports estimates of $\beta_{i,j}$ in an unconditional regression, i.e., one time-series regression for each test asset instead of rolling windows, and column 2 the corresponding regression fit. Column 3 and 4 report the average and standard deviation of conditional factor loadings $\beta_{t,i,j}$ from estimations in rolling windows as described in (21), and column 5 reports the average regression fit in the rolling window estimations.

Every country's stock market has a strong negative exposure to the US SDF, except for the Japanese and Malaysian stock markets, which appear orthogonal to the SDF. Remember that an increase in the SDF indicates bad times, i.e., the market price of risk in the SDF is by definition negative and the SDF is counter-cyclical. A negative exposure means that these stock markets drop in bad times. Thus, they are risky and will be compensated with a positive premium.

Notice that the Japanese stock market denominated in JPY loads negatively on the Japanese SDF (Table 4) and earns a positive premium. But the loading of the JPY/USD exchange rate (i.e., the currency exposure of the Japanese stock market when denominated in USD) is opposite (i.e., $CT^{US}_{t+dt,-US/+JP}$ earns a negative premium) and offsets the exposure of the local market to the priced risk. In contrast, the currency and the stock market exposures to priced risk are the same for other countries and thus, the regression fit increases in Table 6 compared to the analysis in Table 4. Regression fits in the rolling window estimations (column 5) are similar to the ones in the unconditional regressions. While column 3 in Table 6 shows that the average factor loadings in the rolling window regressions are similar to the loadings in the unconditional regressions, column 4 displays substantial variations in the conditional loadings. This finding is consistent with the estimations in Brusa et al. (2015), albeit their pricing factors differ from ours.

Table 7 reports the market prices γ_i estimated in the second stage cross-sectional regressions (i.e., averages of conditional market prices $\gamma_{t,i}$ in regression (22)). The regression includes all 220 test assets. The first row reports estimated market prices of the US SDF $\widehat{M}_{t,US}$ across several model specifications: model with $\widehat{M}_{t,US}$ as single factor (column 1), $\widehat{M}_{t,US}$, DOL and CAR (column 2), $\widehat{M}_{t,US}$ and $WMkt$ (column 3), $\widehat{M}_{t,US}$, DOL, CAR, $WMkt$ (column 4), $\widehat{M}_{t,US}$, 5 global Fama-French factors and MOM (column 5), and all nine factors combined (column 6). As aforementioned all factors are normalized to have an annual volatility of 1. Hence, the estimated market price γ_i is theoretically equal to the Sharpe ratio of an asset which perfectly negatively correlates with the pricing factor (i.e., a factor mimicking asset). This normalization makes the interpretation of the magnitudes of the estimated market prices and comparisons across pricing factors more convenient.

Table 7: Cross-Sectional Regressions of International Stock Markets on US SDF

	(1)	(2)	(3)	(4)	(5)	(6)
γ_M	-0.80^{**}	-0.64^{**}	-0.71^{***}	-0.64^{***}	-0.40^{**}	-0.42^{**}
	(2.24)	(2.12)	(3.50)	(3.22)	(2.00)	(2.04)
γ_{DOL}		0.59^{*}		0.44^{**}		0.31
		(1.81)		(2.02)		(1.39)
γ_{CAR}		0.79^{***}		0.76^{***}		0.78^{***}
		(2.66)		(3.27)		(3.37)
γ_{WMkt}			0.37^{*}	0.37^{*}	0.37	0.42^{*}
			(1.81)	(1.76)	(1.57)	(1.76)
γ_{SMB}					0.13	0.06
					(0.53)	(0.25)
γ_{HML}					0.35	0.33
					(1.59)	(1.49)
γ_{RMW}					0.55^{**}	0.57^{**}
					(2.39)	(2.50)
γ_{CMA}					-0.07	-0.11
					(0.27)	(0.47)
γ_{MOM}					-0.07	-0.10
					(0.33)	(0.50)
N Assets	220	220	220	220	220	220
# significant α^*:						
1% level	10	5	8	8	8	5
5% level	27	24	23	25	11	13
10% level	42	36	40	41	15	26
$MAPE$	5.14	4.48	4.65	4.25	3.25	3.16
$RMSE$	7.32	6.10	6.91	6.21	5.00	4.86

Notes: Fama and MacBeth (1973) cross-sectional regressions $R_{t,j} = \sum_i \beta_{t,i,j}\gamma_{t,i} + \alpha^*_{t,j}$ at each time t. Conditional factor loadings $\beta_{t,i,j}$ are estimated in time-series regressions $R_{\tau,j} = \alpha_{t,j} + \sum_i \beta_{t,i,j} F_{\tau,i} + \varepsilon_{\tau,j}$ for $\tau \in \{t-61, \ldots, t-1\}$ using 60-month rolling windows. We test the following factors $F_{t,i}$: US SDF ($\widehat{M}_{t,US}$), Dollar (DOL), carry (CAR), world stock market portfolio ($WMkt$), 4 global Fama and French (1992) factors (SMB, HML, RMW, CMA) and global momentum (MOM). We normalize all factors such that the annual volatility is 1. $\alpha^*_{t,j}$ is the abnormal return of asset j in the cross-sectional regression. The reported market prices γ_i are annualized time-series averages of $\gamma_{t,i}$. Significance of the market prices at the 1%, 5% and 10% level are indicated by ***, ** and *. N Assets indicates the number of test assets j. For each of 22 countries we have 10 portfolios: one country-specific stock market portfolio, two Book/Market, two Earnings/Price, two Cashflow/Price and 3 Dividend Yield sorted portfolios. Monthly returns (from 1984 to 2014) are provided by Kenneth French on his website. # significant α^* reports the number of test assets with significant average abnormal returns at the 1%, 5%, 10% level according to the pricing model under consideration. $MAPE$ is the annualized mean absolute pricing error (α^*) in percentage. $RMSE$ is the annualized root mean square pricing error (α^*) in percentage. FOr columns 1-4 we have data from 1984 to 2014. The global Fama-French factors in columns 5 and 6 are only available since July 1990.

The market price γ_M of the US SDF is statistically significant and economically large across all six model specifications. It is negative as expected, i.e., an asset which positively (negatively) correlates with the SDF is considered a hedge (risk) and is compensated with a negative (positive) premium. The magnitude decreases after controlling for various other factors. In particular, in the single factor model (column 1) the market price of the SDF is -0.80 and it adjusts to (still a large value of) -0.42 after controlling for all other factors. Interestingly, the adjustment is not very large after controlling for DOL and CAR, i.e., it is still -0.64. This further illustrates and enforces the discussion in Section 3.2 that important dimensions of the SDF we estimate from FX data are not in the space spanned by DOL and CAR. Besides the SDF, CAR and RMW remain important factors with statistically and economically significant market prices. The market price of $WMkt$ is only significant on the 10% level. All other factors do not earn a significant risk premium in the cross-section of international stock returns. The fact that the SDF estimated from FX data does not crowed out all other factors means that there are some important risks which our PCs do not pick up. On the upside, the risks of our FX market PCs appear important outside of FX markets, besides several prominent factors described in the literature.

Overall, we conclude that the US SDF $\widehat{M}_{t,US}$ is an important pricing factor in the cross-section of international stock returns but it does not explain all the priced risks. In particular, CAR and RMW (and $WMkt$) appear to carry important pricing information (for international stock returns) in addition to the priced risks captured by the SDF estimated from FX data.

Next, we decompose the SDF and investigate the pricing implications of the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ of exchange rate growths separately. As above we use the two stage Fama and MacBeth (1973) regressions (21) and (22) but remove the US SDF $\widehat{M}_{t,US}$ and instead use the two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ as new pricing factors. Notice that the SDF is a linear combination of the two PCs, and thus, if the relative market prices $\gamma_{\overline{\Pi}_{t,1}}$ and $\gamma_{\overline{\Pi}_{t,2}}$ estimated in the cross-section of stock returns is the same as $\widehat{\gamma}_1^{US}$ and $\widehat{\gamma}_2^{US}$ in FX markets, then the regressions using the US SDF or the two PCs are identical. Empirically, the analysis involving the two PCs allows for more flexibility than the regression using the US SDF.

Table 8 reports the factor loadings of each of our 22 stock markets (denominated in USD) on the two PCs.[27] In columns 1 and 2 are factor loadings in unconditional regressions, i.e., one regression for each country's stock market (analogous to column 1 in Table 6). Columns 3-6 report time-series

[27]To save space we only report the results for the 22 market portfolios. Tables for all other 198 portfolios are available upon request.

Table 8: Time-Series Regressions of International Stock Markets on first two PCs

	Unconditional		60-Month Rolling Windows			
	(1)	(2)	(3)	(4)	(5)	(6)
	$\beta_{1,J}$	$\beta_{2,J}$	$Mean(\beta_{1,J,t})$	$Std(\beta_{1,J,t})$	$Mean(\beta_{2,J,t})$	$Std(\beta_{2,J,t})$
Austria	−0.018	0.101***	−0.011	0.018	0.089	0.090
Australia	−0.102***	0.089***	−0.097	0.032	0.079	0.069
Belgium	0.007	0.080***	0.006	0.028	0.072	0.071
Canada	−0.062***	0.054***	−0.068	0.029	0.041	0.051
Denmark	−0.008	0.071***	−0.005	0.032	0.060	0.073
Finland	−0.061***	0.076***	−0.054	0.050	0.070	0.080
France	−0.007	0.079***	−0.016	0.047	0.068	0.072
Germany	−0.008	0.087***	−0.017	0.059	0.080	0.062
Hong Kong	−0.068***	0.051***	−0.074	0.032	0.039	0.066
Ireland	−0.033**	0.092***	−0.028	0.026	0.078	0.081
Italy	−0.016	0.082***	−0.025	0.041	0.074	0.075
Japan	0.008	0.003	0.011	0.072	−0.033	0.063
Malaysia	−0.087*	−0.043**	−0.127	0.028	−0.056	0.007
Netherlands	−0.015	0.081***	−0.014	0.040	0.067	0.079
New Zealand	−0.081***	0.069***	−0.079	0.032	0.067	0.056
Norway	−0.040***	0.101***	−0.041	0.025	0.093	0.079
Singapore	−0.072***	0.057***	−0.074	0.056	0.041	0.070
Spain	−0.012	0.084***	−0.013	0.045	0.072	0.079
Sweden	−0.044***	0.088***	−0.052	0.047	0.081	0.076
Switzerland	0.006	0.064***	0.008	0.035	0.053	0.051
UK	−0.018*	0.068***	−0.019	0.036	0.050	0.055
USA	−0.042***	0.035***	−0.044	0.032	0.023	0.045
Mean	−0.035	0.067	−0.038	0.038	0.055	0.066

Notes: Monthly OLS time-series regressions of each country J's stock market excess return $R_{t,J}$ (denominated in USD) on the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$, $R_{t,J} = \alpha_J + \beta_{1,J}\overline{\Pi}_{t,1} + \beta_{2,J}\overline{\Pi}_{t,2} + \varepsilon_{t,J}$. α_J is a constant, $\varepsilon_{t,J}$ is the error, $\beta_{1,J}$ and $\beta_{2,J}$ measure the exposures of stock market J to $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$. Columns 1 and 2 report slope coefficient $\beta_{1,J}$ and $\beta_{2,J}$ for unconditional regressions (i.e., one regression per country for entire time-series). Columns 3-6 report the averages and standard deviations of the slope coefficients $\beta_{t,1,J}$ and $\beta_{t,2,J}$ of regressions of 60-month rolling windows for each country J. We use Monthly data from 1984 to 2014. Significance of the slope coefficients in column 1 at the 1%, 5% and 10% level are indicated by ***, ** and *. Robust standard errors are estimated following Newey and West (1987).

averages and standard deviations of conditional factor loading in regressions of 60-month rolling windows (analogous to columns 3 and 4 in Table 6). We observe that all stock markets (with the exception of the Japanese and the Malaysian markets) have a significant positive exposure to the second PC. Our analysis in section 3.2 suggests that the market price of the second PC is positive and thus, it is negatively related to the SDF, i.e., the second PC is pro-cyclical. In turn, this means that an asset that is positively exposed to the second PC is risky and is compensated by a positive premium, which is what we generally expect about stock markets. Exposures to the first PC are mostly negative but they are significant for only half of the investigated stock markets. The first PC's market price is negative when estimated in FX markets, which implies a positive relationship with the SDF and the first PC is counter-cyclical. In turn, an asset which negatively correlates with the first PC is risky and earns a positive premium, which is again what we generally expect for stock markets. We further observe that average conditional loadings are similar to that of the unconditional estimates and there is a large time-series variation in conditional factor loadings (columns 2-6).

Table 9 is analogous to Table 7 and reports the estimated market prices γ_i in the cross-section of international stock returns. As expected the market price of $\overline{\Pi}_{t,1}$ is negative and the one of $\overline{\Pi}_{t,2}$ is positive across all model specifications. This is in line with the estimates of market prices from FX market data in Table 1, i.e., $\overline{\Pi}_{t,1}$ ($\overline{\Pi}_{t,2}$) positively (negatively) affects the SDF and an asset that loads negatively (positively) on $\overline{\Pi}_{t,1}$ ($\overline{\Pi}_{t,2}$) is risky and is compensated with a positive premium. Though the sign is consistent for both PCs, the estimated price of risk is only statistically significant for $\overline{\Pi}_{t,2}$. The estimated price of risk $\gamma_{\overline{\Pi}_{t,1}}$ is between -0.25 and -0.08 across the diverse model specifications. The magnitude of the price of risk $\gamma_{\overline{\Pi}_{t,2}}$ decreases from 0.74 in a model with only the two PCs as pricing factors to (still a large value of) 0.41 after controlling for all other pricing factors. These values are comparable to (and not statistically significantly different from) the estimated market prices from FX data in Table 1, i.e., -0.143 for $\overline{\Pi}_{t,1}$ and 0.369 for $\overline{\Pi}_{t,2}$. Thus, the two PCs are priced similarly in stock and FX markets. Consistent with the analysis using the US SDF $\widehat{M}_{t,US}$, we find again that CAR and RMW (and $WMkt$) are important factors in addition to the two PCs from exchange rate growths.

Table 9: Cross-Sectional Regressions of International Stock Markets on first two PCs

	(1)	(2)	(3)	(4)	(5)	(6)
$\gamma_{\Pi_{t,1}}$	-0.12	-0.06	-0.25	-0.17	-0.08	-0.24
	(0.38)	(0.25)	(1.02)	(0.70)	(0.31)	(0.89)
$\gamma_{\Pi_{t,2}}$	0.74^{**}	0.65^{**}	0.65^{***}	0.59^{***}	0.38^{*}	0.41^{*}
	(2.39)	(2.41)	(3.18)	(2.92)	(1.85)	(1.97)
γ_{DOL}		0.62^{**}		0.41^{*}		0.26
		(2.10)		(1.95)		(1.17)
γ_{CAR}		0.72^{**}		0.73^{***}		0.80^{***}
		(2.56)		(3.17)		(3.55)
γ_{WMkt}			0.36^{*}	0.38^{*}	0.40^{*}	0.43^{*}
			(1.74)	(1.82)	(1.68)	(1.81)
γ_{SMB}					0.18	0.12
					(0.77)	(0.53)
γ_{HML}					0.33	0.36
					(1.50)	(1.63)
γ_{RMW}					0.56^{**}	0.55^{**}
					(2.44)	(2.41)
γ_{CMA}					-0.14	-0.17
					(0.57)	(0.69)
γ_{MOM}					-0.04	-0.09
					(0.18)	(0.47)
N Assets	220	220	220	220	220	220
# significant α^*:						
1% level	9	10	7	8	7	4
5% level	21	27	22	20	12	14
10% level	42	43	42	35	19	29
$MAPE$	5.17	4.49	4.55	4.08	3.22	3.14
$RMSE$	7.64	6.20	6.77	6.02	4.93	4.78

Notes: Fama and MacBeth (1973) cross-sectional regressions $R_{j,t} = \sum_i \beta_{i,j,t}\gamma_{i,t} + \alpha^*_{j,t}$ at each time t. Conditional factor loadings $\beta_{i,j,t}$ are estimated in time-series regressions $R_{j,\tau} = \alpha_{j,t} + \sum_i \beta_{i,j,t} F_{i,\tau} + \varepsilon_{j,\tau}$ for $\tau \in \{t-61, t-1\}$ using 60-month rolling windows. We test the following factors F_i: first two PCs (Π_1, Π_2), Dollar (DOL), carry (CAR), world stock market portfolio $(WMkt)$, 4 global Fama and French (1992) factors (SMB, HML, RMW, CMA) and global momentum (MOM). We normalize all factors such that the annual volatility is 1. $\alpha^*_{t,j}$ is the abnormal return of asset j in the cross-sectional regression. The reported market prices γ_i are annualized time-series averages of $\gamma_{t,i}$. Significance of the market prices at the 1%, 5% and 10% level are indicated by ***, ** and *. N Assets indicates the number of test assets R_j. For each of 22 countries we have 10 portfolios: one country-specific stock market portfolio, two Book/Market, two Earnings/Price, two Cashflow/Price and 3 Dividend Yield sorted portfolios. Monthly returns (from 1984 to 2014) are provided by Kenneth French on his website. # significant α^* reports the number of test assets with significant average abnormal returns at the 1%, 5%, 10% level according to the pricing model under consideration. $MAPE$ is the annualized mean absolute pricing error (α^*) in percentage. $RMSE$ is the annualized root mean square pricing error (α^*) in percentage. For columns 1-4 we have data from 1984 to 2014. The global Fama-French factors in columns 5 and 6 are only available since July 1990.

4.4 Financial Stress Indicators and Macroeconomic Fundamentals

We now analyze the correlation of various financial stress indicators with our estimated SDF growths $\frac{d\widehat{M}_{t,US}}{\widehat{M}_{t,US}}$ in the USA and the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$. All our financial stress data is specific to the USA and thus we restrict our analysis to the US SDF.

Our first set of financial stress variables are the Chicago Federal Reserve Bank Financial Condition Index and its four sub-indices: Risk, Credit, Leverage, and Non-Financial Leverage.[28] We use monthly changes in these indices in our analysis.

Our second set of stress variables proxy for volatility. Following Menkhoff et al. (2012a), we construct a monthly FX market volatility measure as the average of absolute daily exchange rate changes within a month and across currencies. We denote monthly changes in the FX market volatility by ΔFX Volatility. We download monthly data for the S&P 500 Volatility Index (VIX) from the CBOE.[29] ΔVIX indicates monthly changes in the VIX. He et al. (2016) provide data on the capital ratio of primary dealers and use this variable as a proxy for risk in an intermediary asset pricing model. ΔIntermediary Capital Ratio denotes monthly changes of their measure. Finally, we use six volatility measures provided by Giglio et al. (2016), who aggregate risk measures of the top 20 financial institutions. ΔVolatility (Top 20 Fin) is the monthly change in the average return volatility of the top 20 financial institutions. ΔTurbulence (Top 20 Fin) is the monthly change in the average of the returns' recent covariance relative to a longer-term covariance (Kritzman and Li, 2010). ΔSize Concentration (Top 100 Fin) is the monthly change in the Herfindal index of the size distribution among the top 100 financial institutions.

Our third set of variables is measuring tail risk. We use three measures provided by Giglio et al. (2016). ΔCatFin (Top 20 Fin) is the monthly change in the cross-sectional value-at-risk measure of Allen et al. (2012). Note that while standard value-at-risk measures typically use a time series of returns (of a firm or an index) to estimate a potential loss, CatFin uses the cross-section of returns at a point in time, and thus, estimates systemic risk instead of individual firm risk. ΔBook and ΔMarket Leverage (Top 20 Fin) are monthly changes in average book and market leverage.

Fourth, we look at two illiquidity risk measures. ΔFX Illiquidity is the monthly change in the FX market illiquidity measure of Karnaukh et al. (2015), which is constructed from high frequency

[28] We have also tested Financial Condition Indices from the St. Louis Fed and Kansas City Fed and the results are almost the same. We do not report these estimates for brevity.

[29] VIX data is only available starting in January 1990.

exchange rates against the USD.[30] Moreover, we use ΔAmihud which is the monthly change in the average stock illiquidity of the top 20 financial institutions using the measure of Amihud (2002).

Fifth, we look at credit risk measures. ΔDefaut Spread is the monthly change in the difference between BAA and AAA corporate bond yields. ΔTED Spread is the monthly change in the difference between the 3-month LIBOR and T-Bill interest rates. ΔTerm Spread is the monthly change in the difference between the 10-year and 3-month US Treasury yields.

Finally, we look at contagion risks within the financial industry and use five measures provided by Giglio et al. (2016). ΔAbsorption (Top 20 Fin) is the monthly change in the fraction of return variance of the top 20 financial institution explained by the first 3 PCs (of the 20 return time-series) (Kritzman et al., 2010). ΔCoVaR (Top 20 Fin) is the monthly change in the average CoVaR measure by Adrian and Brunnermeier (2016). CoVaR measures systemic risk as the value-at-risk of the financial system conditional on an institution being in distress. ΔDynamic Causality Idx (Top 20 Fin) is the monthly change in the fraction of significant Granger-causality relationships among the returns of the top 20 financial institutions (Billio et al., 2012). ΔInternational Spillover is the monthly change in the index of Diebold and Yilmaz (2009), which measures comovement in macroeconomic quantities across countries. ΔMES (Top 20 Fin) is the monthly change in the average of the top 20 financial institutions' expected returns conditional on the financial system being in its lower tail (Acharya et al., 2017).

With the exception of ΔIntermediary Capital Ratio and ΔTerm Spread, all these financial stress measures are counter-cyclical, i.e., an increase (or a positive change) indicates bad times. ΔIntermediary Capital Ratio is pro-cyclical, i.e., a positive realization is good news because an increase in the capital ratio of intermediaries relaxes constraints in an intermediary asset pricing model (He et al., 2016). Moreover, an increase in the slope of the yield curve (i.e., a positive value for ΔTerm Spread) predicts increases in future GDP growth and it is pro-cyclical (Ang et al., 2006).

Remember that the SDF is counter-cyclical, i.e., an increase (or a positive realization in $\frac{d\widehat{M}_{t,US}}{\widehat{M}_{t,US}}$) indicates bad times. Moreover, the first (second) PC carries a negative (positive) market price of risk and thus is positively (negatively) related to the SDF and counter-cyclical (pro-cyclical) (see Table 1). Thus, a positive realization in the first PC indicates bad times, while a positive realization in the second PC indicates good times.

Table 10 shows that the sign of the correlation coefficients is consistent with our interpretation of

[30] The FX illiquidity data is only available starting in January 1991.

Table 10: Financial Stress Indicators

	(1) $\widehat{M}_{US}$	(2) $\overline{\Pi}_1$	(3) $\overline{\Pi}_2$
Federal Reserve Bank Indicators:			
ΔChicago Fed Fin Con Idx	0.35***	0.29***	-0.26***
ΔChicago Fed Fin Con Idx (Risk)	0.33***	0.24***	-0.26***
ΔChicago Fed Fin Con Idx (Credit)	0.34***	0.31***	-0.24***
ΔChicago Fed Fin Con Idx (Leverage)	0.27***	0.17***	-0.22***
ΔChicago Fed Fin Con Idx (Non-Fin Leverage)	-0.00	0.05	0.02
Volatility:			
ΔFX Volatility	0.28***	0.19***	-0.22***
ΔVIX	0.38***	0.31***	-0.30***
ΔVolatility (Top 20 Fin)	0.21***	0.26***	-0.12**
ΔTurbulence (Top 20 Fin)	0.06	0.08	-0.03
ΔIntermediary Capital Ratio	-0.25***	-0.30***	0.15***
ΔSize Concentration (Top 100 Fin)	0.17***	0.03	-0.17***
Tail Risk:			
ΔCatFin (Top 20 Fin)	0.15***	0.21***	-0.08
ΔBook Leverage (Top 20 Fin)	0.03	-0.11*	-0.07
ΔMarket Leverage (Top 20 Fin)	0.16***	0.16***	-0.10*
Illiquidity:			
ΔFX Illiquidity	0.32***	0.12*	-0.29***
ΔAmihud (Top 20 Fin)	0.11*	0.05	-0.09*
Credit:			
ΔDefault Spread	0.21***	0.10*	-0.18***
ΔTED Spread	0.11**	0.11**	-0.08
ΔTerm Spread	-0.12**	-0.03	0.11**
Contagion:			
ΔAbsorption (Top 20 Fin)	0.06	0.12**	-0.02
ΔCoVaR (Top 20 Fin)	0.22***	0.24***	-0.14**
ΔDynamic Causality Idx (Top 20 Fin)	0.10*	0.15***	-0.05
ΔInternational Spillover	0.04	-0.01	-0.05
ΔMES (Top 20 Fin)	0.18***	0.15***	-0.13**

Notes: Monthly correlations between changes in financial stress indicators and the SDF growth in the USA and the first two PCs. Significance of the correlation coefficients at the 1%, 5% and 10% level are indicated by ***, ** and *. Details of all financial stress indicators are in the main text.

the variables. The US SDF positively correlates with all stress indicators except for ΔIntermediary Capital Ratio and ΔTerm Spread, for which the correlation coefficient is negative. 15 out of 24 correlation coefficients are significant on the 1% level and 4 coefficients are significant on the 10% level (but not on the 1% level). The SDF is strongly positively correlated to changes in the Chicago Fed Financial Conditions Index and its risk, credit and leverage sub-indices (correlation coefficients ranging between 27% and 35%). It is however orthogonal to the sub-index capturing non-financial leverage. We find similarly strong correlations between these indices and the first and the second PC. Note that the correlation for the first PC is positive and for the second it is negative, which is consistent with the interpretation that an increase in the first (second) PC is bad (good) news. Since the Chicago Fed Index is a combination of 105 financial activity variables, we further investigate some of its components.

The estimated SDF is positively related to changes in the FX market volatility, the VIX, the average volatility of the top 20 financial institutions and the size concentration in the financial industry. The SDF is negatively correlated to changes in the intermediary capital ratio. We find no significant relationship between our SDF and changes in turbulence (which captures the the current covariance between returns compared to the long run) and book leverage. We conclude that our SDF captures important volatility dimensions. While both PCs are related to the volatility variables, the first PC is more exposed to changes in the intermediary capital ratio and the second PC is stronger related to changes in the size concentration in the financial industry.

The SDF is also positively related to changes in the tail risk variables CatFin index and market leverage of the top 20 financial firms. There is no significant relationship between the SDF and the book leverage. Thus, our SDF captures important tail risks in the financial industry. Interestingly, only the first PC is significantly related to these tail risk variables.

Our SDF is further related to changes in FX market illiquidity and average illiquidity of the top 20 financial firms. The correlations are positive as expected. Thus, the SDF is strongly related to measures of illiquidity. We find that the first PC is only weakly related to changes in FX market illiquidity and not significantly related to changes in illiquidity of financial firms. In contrast, these correlations are stronger and significant for the second PC.

The SDF is positively correlated to default and TED spreads and negatively correlated to the term spread, which is consistent with our expectation. Interestingly, the first PC significantly correlates with changes in the TED spread, but the correlation to changes in the default spread is weak and the correlation to changes in the term spread is insignificant. In contrast, the second PC has a stronger and significant correlation with both changes in the default and term spread while its correlation with the TED spread is insignificant.

Finally, we find that our SDF is significantly correlated with changes in the CoVaR and the MES indices on the 1% level and the Dynamic Causality Index on the 10% level. It appears unrelated to changes in the Absorption and International Spillover measures. Thus, there is some evidence that the SDF is related to contagion measures. The first PC is stronger correlated to most contagion measures than the second PC. Both PCs are seem unrelated to changes in the International Spillover measure.

In summary, our SDF estimated from FX market data correlates with a broad set of financial stress indicators, capturing volatility, tail risk, illiquidity, credit and contagion risk in financial markets. While several stress indicators correlate similarly with the first and the second PC, there are some differences. The first PC is associated with the TED spread and quantities that measure volatility, tail and contagion risks. The second PC is associated the default and term spreads and quantities that measure volatility and illiquidity.

Next, we explore the relationship between our country-specific SDFs and PCs and macroeconomic fundamentals. We consider the following 10 quantities: GDP growth (ΔGDP), change in output gap ($\Delta OutputGap$), consumption growth ($\Delta Consumption$), capital formation growth ($\Delta CapitalFormation$), industrial production growth ($\Delta IndProduction$), manufacturing growth ($\Delta Manufacturing$), construction growth ($\Delta Construction$), change in the unemployment rate ($\Delta Unemployment$), change in the overnight rate ($\Delta OvernightRate$), and change in the 10-year government bond rate ($\Delta Long - TermRate$). All variables are per capita (except unemployment and interest rates) and adjusted for inflation (except unemployment). Output gap is estimated as the difference between GDP and its smooth trend using a Hodrick and Prescott (1997) filter with a smoothing factor of 1600 as suggested for quarterly data. The data for all 11 countries, for which we have estimated country-specific SDFs, is provided by the OECD and is available on a quarterly frequency for our entire time horizon, 1984-2014.

Remember that the SDF is counter-cyclical, i.e., an increase in country J's SDF is a bad shock for country J. Following a bad shock we expect GDP, output gap, consumption, capital formation, industrial production manufacturing and construction to drop in country J, i.e., a negative correlation to the local SDF. Similarly, following a bad shock growth prospects are lower and we expect short and long term interest rates to drop, implying a negative correlation as well. The exception is unemployment, which we expect to increase in response to a bad shock, implying a positive correlation to the SDF.

We use lead-lag within panel regressions to investigate the effect of a change in a country-specific SDF $\widehat{M}_{t,J}$ from quarter $t-1$ to t on future changes in macroeconomic quantities in the corresponding country from quarter t to $t+h$,

$$Y_{t,t+h,J} = c_J + \theta \frac{\widehat{M}_{t,J} - \widehat{M}_{t-1,J}}{\widehat{M}_{t-1,J}} + \sum_{k=1}^{4} \delta_k Y_{t-k,t-k+1,J} + \varepsilon_{t,t+h,J}, \tag{23}$$

where $Y_{t,t+h,J}$ is the change or growth of a macroeconomic quantity in country J over h quarters from t to $t+h$, c_J is a country-specific constant, $\frac{\widehat{M}_{t,J}-\widehat{M}_{t-1,J}}{\widehat{M}_{t-1,J}}$ is the growth rate of the SDF in country J over the quarter $t-1$ to t estimated according to (9), $Y_{t-k,t-k+1,J}$ are past realizations of the macroeconomic quantity to control for potential auto-correlation in Y_J, $\varepsilon_{t,J}$ is the regression error. Some of the macroeconomic quantities are persistent and we find that four quarterly lags are sufficient to remove all auto-correlation (in most cases less than 4 lags are sufficient). Since we work with overlapping observations we estimate standard errors following the approach of Hodrick (1992). We further cluster errors within time to account for correlation across countries. Column 1 in Table 11 reports the slope coefficient estimate θ, column 2 the corresponding $t-$statistics, and column 3 the goodness of the regression fit. Table 11 has four panels reporting results for regressions with $h = \{1, 2, 3, 4\}$.

We observe that the sign of the regression coefficient θ is in all regressions as expected, i.e., implying a negative correlation between the SDF and all quantities except for unemployment. However, there is a lot of noise and only the coefficient on the change in the long-term interest rate is statistically significant when $h = 1$. For longer horizons of 2, 3 or 4 quarters ($h = \{2, 3, 4\}$), several of the regression coefficients become statistically significant at the 5% or 10% level. Overall, we take this as evidence that our estimated SDFs from FX data reflect future changes in macroeconomic fundamentals.

Finally, we investigate the effects of the two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ separately on macroeconomic quantities. We use similar within panel regressions as in (23) but replace the SDF by the two PCs and control for the exchange rate between J and the US,

$$Y_{t,t+h,J} = c_J + \sum_{K=1}^{2} \theta_K \overline{\Pi}_{t-1,t,K} + \vartheta \frac{EX_{t,J/US} - EX_{t-1,J/US}}{EX_{t-1,J/US}} + \sum_{k=1}^{4} \delta_k Y_{t-k,t-k+1,J} + \varepsilon_{t,t+h,J}. \quad (24)$$

While the SDF in the regressions (23) was country-specific, the PCs in (24) are not. Thus, controlling for exchange rates addresses this issue. Though this is conceptually important, empirically the results are qualitatively the same (and quantitatively very similar) whether we control for exchange rates or not. The estimations of market prices of risk of the two PC from either FX or stock returns (sections 3.2 and 4.3) suggest that an increase (decrease) in $\overline{\Pi}_{t,1}$ ($\overline{\Pi}_{t,2}$) is a bad shock. Thus, we expected negative (positive) regression coefficients θ_1 (θ_2) for all macroeconomic variables except for unemployment, for which we expect the opposite.

Table 12 shows that our intuition is confirmed in the data and the sign on all regression coefficient is as expected. None of the regression coefficients on $\overline{\Pi}_{t,2}$ is statistically significant except for the coefficients on the change in the long-term interest rate at short horizons $h = \{1, 2\}$, which are significant on the 5% level. In contrast, we find that most of the coefficients on $\overline{\Pi}_{t,1}$ are highly statistically significant (on the 1% level). Moreover, the relationship appears much stronger at longer horizons, i.e., coefficients are more significant for $h = \{3, 4\}$. This is an interesting finding. First, it appears that some of the results in regressions (23) (Table 11) are relatively modest because $\overline{\Pi}_{t,2}$ is not strongly associated with most macroeconomic fundamentals (except for the long term interest rate) and the SDF puts a larger weight on $\overline{\Pi}_{t,2}$ than $\overline{\Pi}_{t,1}$. Second, while $\overline{\Pi}_{t,2}$ is more important for pricing FX and stock market returns (i.e., estimated market prices are larger in magnitude for $\overline{\Pi}_{t,2}$), $\overline{\Pi}_{t,1}$ is much stronger associated with a broad set of macroeconomic quantities. Third, the results for $\overline{\Pi}_{t,1}$ suggest that it captures news about economic growth, especially at a horizons of 3 to 4 quarters. In contrast, $\overline{\Pi}_{t,2}$ seems to capture short term (1 to 2 quarters) changes in bond markets (long term interest rate) but the association with quantities that capture economic growth are insignificant.

Overall, we conclude that the country-specific SDFs $\widehat{M}_{t,J}$ estimated from FX market data according to (9) are related to fundamentals, which is important out-of-sample evidence in favor of our estimation approach. The first PC $\overline{\Pi}_{t,1}$ of exchange rate growths is strongly associated with a broad set of fundamentals and appears to capture economic growth at a horizon of 2 to 4 quarters. The second PC $\overline{\Pi}_{t,2}$ is related to short term changes in the long term interest rate.

Table 11: Macroeconomic Panel Regressions: SDFs

1 Quarter ahead ($h = 1$)			
	(1) Coefficient	(2) ($t-$stat)	(3) R^2 (in %)
Δ GDP	-0.00808	(-1.25)	4.43
Δ Output Gap	-0.00766	(-1.47)	3.95
Δ Consumption	-0.00562	(-1.53)	11.30
Δ Capital Formation	-0.01939	(-1.19)	3.16
Δ Ind Production	-0.02155	(-1.18)	8.36
Δ Manufacturing	-0.02431	(-1.09)	7.16
Δ Construction	-0.01608	(-1.25)	3.43
Δ Unemployment	0.03413	(1.06)	11.02
Δ Overnight Rate	-0.57474	(-1.58)	1.25
Δ Long-Term Rate	-0.45161**	(-2.53)	9.55

2 Quarters ahead ($h = 2$)			
	(1) Coefficient	(2) ($t-$stat)	(3) R^2 (in %)
Δ GDP	-0.01432	(-1.47)	5.82
Δ Output Gap	-0.01432*	(-1.84)	7.32
Δ Consumption	-0.00807*	(-1.69)	11.91
Δ Capital Formation	-0.03752	(-1.41)	5.87
Δ Ind Production	-0.04019	(-1.41)	7.68
Δ Manufacturing	-0.04777	(-1.38)	8.37
Δ Construction	-0.02442	(-1.14)	4.88
Δ Unemployment	0.07862	(1.47)	13.41
Δ Overnight Rate	-0.92907*	(-1.91)	2.08
Δ Long-Term Rate	-0.65821***	(-2.77)	10.85

3 Quarters ahead ($h = 3$)			
	(1) Coefficient	(2) ($t-$stat)	(3) R^2 (in %)
Δ GDP	-0.01574*	(-1.70)	5.06
Δ Output Gap	-0.01454**	(-2.00)	9.02
Δ Consumption	-0.00968**	(-2.02)	13.70
Δ Capital Formation	-0.04759*	(-1.77)	6.85
Δ Ind Production	-0.03925	(-1.47)	5.85
Δ Manufacturing	-0.04785	(-1.49)	6.55
Δ Construction	-0.03017	(-1.40)	5.91
Δ Unemployment	0.09377*	(1.66)	11.60
Δ Overnight Rate	-0.93429*	(-1.84)	2.14
Δ Long-Term Rate	-0.52517*	(-1.70)	9.62

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	4 Quarters ahead ($h = 4$)		
	(1) Coefficient	(2) ($t-$stat)	(3) R^2 (in %)
Δ GDP	-0.01644	(-1.63)	4.24
Δ Output Gap	-0.01549*	(-1.93)	10.08
Δ Consumption	-0.00917*	(-1.76)	13.91
Δ Capital Formation	-0.04974*	(-1.77)	6.62
Δ Ind Production	-0.03751	(-1.29)	4.59
Δ Manufacturing	-0.04319	(-1.24)	5.02
Δ Construction	-0.03262	(-1.42)	6.09
Δ Unemployment	0.10713*	(1.84)	10.35
Δ Overnight Rate	-0.96566*	(-1.80)	2.72
Δ Long-Term Rate	-0.36099	(-1.03)	7.31

Notes: Quarterly within panel regressions $Y_{t,t+h,J} = c_J + \theta \frac{\widehat{M}_{t,J} - \widehat{M}_{t-1,J}}{\widehat{M}_{t-1,J}} + \sum_{k=1}^{4} \delta_k Y_{t-k,t-k+1,J} + \varepsilon_{t,t+h,J}$, where $Y_{t,t+h,J}$ is the change or growth of a macroeconomic quantity in country J over h quarters from t to $t+h$, c_J is a country-specific constant, $\frac{\widehat{M}_{t,J} - \widehat{M}_{t-1,J}}{\widehat{M}_{t-1,J}}$ is the growth rate of the SDF in country J over quarter $t-1$ to t estimated according to (9), $Y_{t-k,t-k+1,J}$ are past realizations of the macroeconomic quantity which captures the persistence in Y_J, $\varepsilon_{t,t+h,J}$ is the regression error. Column 1 reports the slope coefficient estimate θ, 2 the $t-$statistics of θ, and 3 the regression R^2 in percentage points. Significance of the slope coefficients at the 1%, 5% and 10% level are indicated by ***, ** and *. Errors are clustered within time and adjusted for overlapping observations according to Hodrick (1992).

Table 12: Macroeconomic Panel Regressions: PCs

			1 Quarter ahead ($h = 1$)		
	(1) $\overline{\Pi}_{t-1,t,1}$	(2) ($t-$stat)	(3) $\overline{\Pi}_{t-1,t,2}$	(4) ($t-$stat)	(5) R^2 (in %)
Δ GDP	-0.00187	(-1.49)	0.00210	(1.01)	4.79
Δ Output Gap	-0.00183*	(-1.71)	0.00189	(1.09)	4.85
Δ Consumption	-0.00131	(-1.41)	0.00141	(1.21)	11.94
Δ Capital Formation	-0.00527	(-1.31)	0.00390	(0.76)	3.64
Δ Ind Production	-0.00541*	(-1.75)	0.00511	(0.83)	8.63
Δ Manufacturing	-0.00648*	(-1.66)	0.00560	(0.77)	7.64
Δ Construction	-0.00467	(-1.46)	0.00374	(0.93)	3.59
Δ Unemployment	0.00876	(1.16)	-0.00769	(-0.76)	11.74
Δ Overnight Rate	-0.02542	(-0.21)	0.18245	(1.52)	2.79
Δ Long-Term Rate	-0.11646*	(-1.67)	0.12785**	(2.02)	10.72
			2 Quarters ahead ($h = 2$)		

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Table 12 – continued from previous page

	(1) $\overline{\Pi}_{t-1,t,1}$	(2) ($t-$stat)	(3) $\overline{\Pi}_{t-1,t,2}$	(4) ($t-$stat)	(5) R^2 (in %)
Δ GDP	-0.00390**	(-2.26)	0.00343	(1.10)	6.25
Δ Output Gap	-0.00380***	(-3.03)	0.00341	(1.32)	8.24
Δ Consumption	-0.00259**	(-2.32)	0.00160	(1.07)	12.97
Δ Capital Formation	-0.01425***	(-2.67)	0.00622	(0.76)	7.49
Δ Ind Production	-0.01039**	(-2.44)	0.00988	(1.04)	7.77
Δ Manufacturing	-0.01264**	(-2.34)	0.01173	(1.05)	8.79
Δ Construction	-0.00829*	(-1.88)	0.00389	(0.56)	6.17
Δ Unemployment	0.01959*	(1.86)	-0.01781	(-1.09)	14.48
Δ Overnight Rate	-0.19167	(-1.53)	0.23971	(1.62)	2.63
Δ Long-Term Rate	-0.14375*	(-1.85)	0.20071**	(2.29)	11.23

3 Quarters ahead ($h = 3$)

	(1) $\overline{\Pi}_{t-1,t,1}$	(2) ($t-$stat)	(3) $\overline{\Pi}_{t-1,t,2}$	(4) ($t-$stat)	(5) R^2 (in %)
Δ GDP	-0.00618***	(-3.25)	0.00295	(1.03)	6.18
Δ Output Gap	-0.00501***	(-3.62)	0.00294	(1.24)	10.14
Δ Consumption	-0.00462***	(-3.35)	0.00125	(0.85)	15.10
Δ Capital Formation	-0.02097***	(-3.59)	0.00663	(0.82)	9.21
Δ Ind Production	-0.01409***	(-3.08)	0.00775	(0.91)	6.70
Δ Manufacturing	-0.01713***	(-3.04)	0.00978	(0.98)	7.68
Δ Construction	-0.01208**	(-2.48)	0.00413	(0.59)	7.48
Δ Unemployment	0.03398***	(3.10)	-0.01676	(-0.99)	12.83
Δ Overnight Rate	-0.23713	(-1.52)	0.21221	(1.43)	2.77
Δ Long-Term Rate	-0.13554	(-1.45)	0.16768	(1.37)	9.89

4 Quarters ahead ($h = 4$)

	(1) $\overline{\Pi}_{t-1,t,1}$	(2) ($t-$stat)	(3) $\overline{\Pi}_{t-1,t,2}$	(4) ($t-$stat)	(5) R^2 (in %)
Δ GDP	-0.00771***	(-3.12)	0.00272	(0.86)	5.77
Δ Output Gap	-0.00640***	(-3.63)	0.00282	(1.07)	11.60
Δ Consumption	-0.00478***	(-2.83)	0.00115	(0.70)	14.84
Δ Capital Formation	-0.02216***	(-2.96)	0.00684	(0.83)	8.34
Δ Ind Production	-0.01691***	(-2.85)	0.00609	(0.66)	5.90
Δ Manufacturing	-0.01996***	(-2.67)	0.00695	(0.65)	6.62
Δ Construction	-0.01451**	(-2.31)	0.00417	(0.57)	7.51
Δ Unemployment	0.05070***	(3.47)	-0.01487	(-0.87)	12.14
Δ Overnight Rate	-0.39396*	(-1.92)	0.14097	(0.91)	3.73
Δ Long-Term Rate	-0.18872**	(-2.01)	0.08231	(0.63)	7.95

Continued on next page

Notes: Quarterly within panel regressions $Y_{t,t+h,J} = c_J + \sum_{K=1}^{2} \theta_K \overline{\Pi}_{t-1,t,K} + \vartheta \frac{EX_{t,J/US} - EX_{t-1,J/US}}{EX_{t-1,J/US}} + \sum_{k=1}^{4} \delta_k Y_{t-k,t-k+1,J} + \varepsilon_{t,t+h,J}$, where $Y_{t,t+h,J}$ is the change or growth of a macroeconomic quantity in country J over h quarters from t to $t+h$, c_J is a country-specific constant, $\overline{\Pi}_{t-1,t,K}$ is the change in PC K over quarter $t-1$ to t, $\frac{EX_{t,J/US} - EX_{t-1,J/US}}{EX_{t-1,J/US}}$ is the exchange rate growth over quarter $t-1$ to t, $Y_{t-k,t-k+1,J}$ are past realizations of the macroeconomic quantity which captures the persistence in Y_J, $\varepsilon_{t,t+h,J}$ is the regression error. Columns 1 and 3 report the slope coefficient estimates θ_1 and θ_2, 2 and 4 the $t-$statistics of θ_1 and θ_2, and 5 the regression R^2 in percentage points. Significance of the slope coefficients at the 1%, 5% and 10% level are indicated by ***, ** and *. Errors are clustered within time and adjusted for overlapping observations according to Hodrick (1992).

5 Conclusion

We use PCA on 55 bilateral exchange rates of 11 developed currencies to identify two major risk sources in FX markets. Including all bilateral exchange rates is important because it focuses the PCA on global risks. In contrast, if only exchange rates quoted against some base currency (e.g., the USD) are used, then the PCA is biased towards risks specific to the base currency, even though such risks may not necessarily be important from a global or other countries' perspectives. We find that our identified risk sources (i.e., first two PCs of all bilateral exchange rate growths) have some overlap with the Carry and Dollar factors but the relation to the Dollar is weaker. We use a cross-sectional regression of FX returns to estimate market prices of our risk sources and construct FX market implied country-specific SDFs. We show that currencies with lower interest rates have more volatile SDFs, and the carry trade of borrowing currencies with more volatile SDFs and lending currencies with less volatile SDFs is profitable. Furthermore, we decompose our SDFs into permanent and transitory components and show that the theoretical bounds of Alvarez and Jermann (2005) are generally satisfied. We further document that model implied long term bond yields line up well with yields observed in the data. In addition, the theoretical relationship derived by Lustig et al. (2017) between long term bond excess returns and entropies of permanent SDF components across countries holds in our estimated model. Moreover, we show that our FX market implied SDFs are able to price international stock returns and are related to important financial stress indicators and macroeconomic fundamentals. Finally, we find that the second PC is more important to price risks in both FX and stock markets than the first PC but the first PC is stronger associated with a broad set of macroeconomic fundamentals than the second PC. Moreover, the first PC is associated with the TED spread and quantities that capture current volatility, tail risk and contagion risk as well as future economic growth. In contrast, the second PC is associated with the default and term spreads and variables measuring volatility and illiquidity. The second PC is mostly unrelated to future economic growth but has a significant association with short term changes in the long term interest rate.

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Pricing Risks across Currency Denominations
Online Appendices

Thomas A. Maurer **Thuy-Duong Tô** **Ngoc-Khanh Tran**

A Stationarity

In our model we assume (A4) sufficient stationarity in the exchange rate processes. In particular, the assumption is that the composition of the PCs and the market prices of risks are constant through time. We validate this assumption using bootstrapping. We resample (with replacements) our monthly exchange rate growth data and construct the first two PCs and estimate the market price of risks. We repeat this 10,000 times and construct the distributions of the loadings of the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ on our 11 currencies and the FX market implied market prices $\hat{\gamma}_1^J$ and $\hat{\gamma}_2^J$ associated with the two PCs.

We first analyze the stationarity of the decomposition of the first two PCs. Figure 8 reports the averages and the intervals spanned by the 5 and 95 percentiles of the loadings of the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ on our 10 exchange rates J/USD and (denoted by US) 1 minus the sum of the loadings on the 10 exchange rates. The 90% confidence interval is small indicating that the composition of the first two PCs does not vary much across the bootstrap samples.

Next, we analyze the distribution of the market prices of risks of the first two PCs. Figure 9 reports the averages and the intervals spanned by the 5 and 95 percentiles of $\hat{\gamma}_1^J$ and $\hat{\gamma}_2^J$. Although the confidence intervals are wider than in the case of the PC decompositions, the variation of estimated market prices across bootstrap samples is relatively small. The standard deviations across bootstrap samples are more than one order of magnitude smaller than the averages of $\hat{\gamma}_1^J$ and $\hat{\gamma}_2^J$. This indicates that the estimated market prices are relatively stable across bootstrap samples. Moreover, Figure 10 further characterizes the distribution of $\hat{\gamma}_1^{US}$ and $\hat{\gamma}_2^{US}$ from a US perspective in more detail. Figure 10 confirms that the estimates across bootstrap samples are very much concentrated around the mean.

To sum up, the decomposition of our two PCs and the estimated market prices do not vary much across the bootstrap samples. We interpret this in favor of our assumption (A4) which requires

Decomposition of the First Two Principal Components

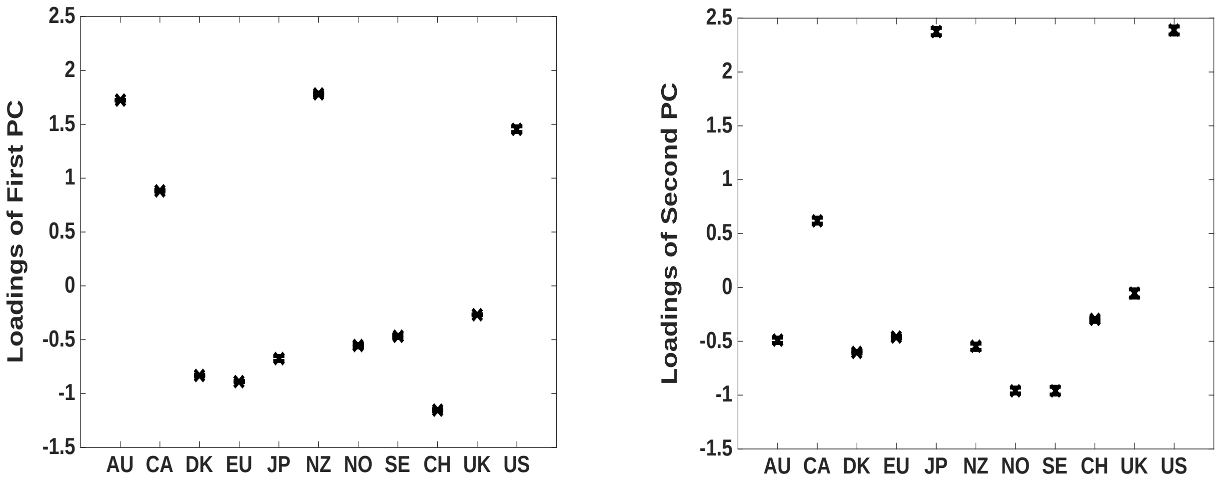

Figure 8: Averages (indicated by x) and intervals spanned by the 5 and 95 percentiles of the loadings of the first PC $\overline{\Pi}_{t,1}$ (left) and second PC $\overline{\Pi}_{t,2}$ (right) on the 10 exchange rates J/USD and (denoted by US) 1 minus the sum of the loadings on the 10 exchange rates. Averages and percentiles are constructed from 10,000 bootstrap samples.

sufficient stationarity to estimate our proposed model.

Market Prices of the First Two Principal Components

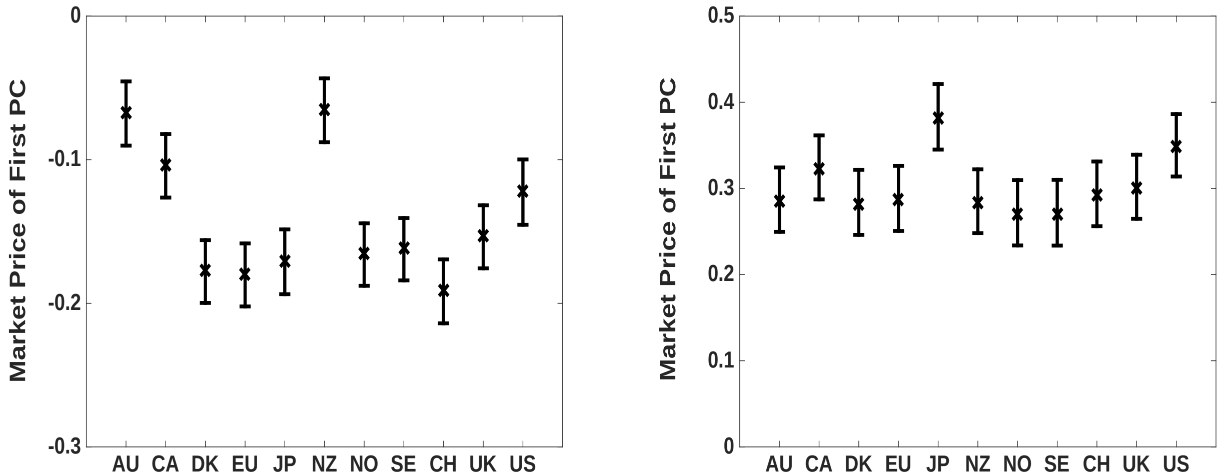

Figure 9: Averages (indicated by x) and intervals spanned by the 5 and 95 percentiles of the market prices $\hat{\gamma}_1^J$ (left) and $\hat{\gamma}_2^J$ (right) for 11 developed countries J. Averages and percentiles are constructed from 10,000 bootstrap samples.

Market Prices of the First Two Principal Components in the USA

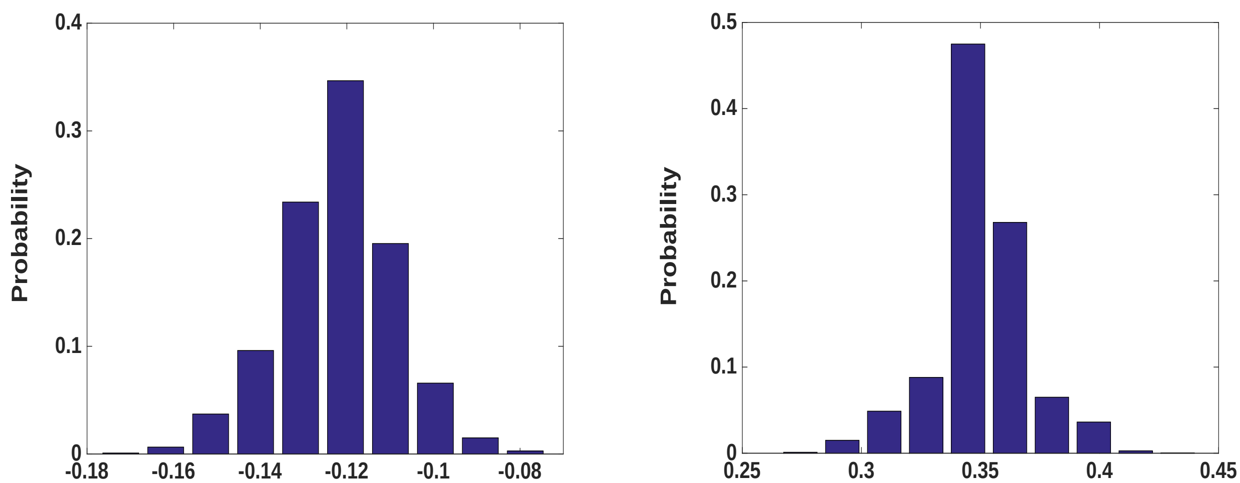

Figure 10: Distribution of US market prices $\hat{\gamma}_1^{US}$ (left) and $\hat{\gamma}_2^{US}$ (right) constructed from 10,000 bootstrap samples.

B Eigenvalue and Growth Ratios

Table B.1 provides values of the Eigenvalue Ratio $ER(k)$ and the Growth Ratio $GR(k)$ for $k \in \{1, \ldots, k_{max}\}$ (with $k_{max} = 6$) in our PCA on 55 bilateral exchange rate growths. $ER(k)$ and $GR(k)$ both reach the maximum at $k = 2$.

Table B.1: Eigenvalue and Growth Ratios

k	$ER(k) = \frac{\lambda_k}{\lambda_{k+1}}$	$GR(k) = \frac{\ln(1+\lambda_k/V(k))}{\ln(1+\lambda_{k+1}/V(k+1))}$
1	1.587	1.050
2	1.713	1.151
3	1.430	1.040
4	1.189	0.896
5	1.205	0.871
6	1.588	1.040

Notes: The Table provides the value of the Eigenvalue Ratio $ER(k) = \frac{\lambda_k}{\lambda_{k+1}}$ and the Growth Ratio $GR(k) = \frac{\ln(1+\lambda_k/V(k))}{\ln(1+\lambda_{k+1}/V(k+1))}$ for the first $k_{max} = 6$ PCs of the $P = 55$ bilateral exchange rate growths of 11 developed currencies, where $V(j) = \sum_{i=j+1}^{P} \lambda_i$ and λ_j is the eigenvalue associated with the jth PCs.

C Time-Series of SDFs

Table C.2 provides augmented Dickey-Fuller test statistics for the time series $\ln\left(\widehat{M}_{t,J}\right)$, $\ln\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)$ and the errors $e_{t,J}$ from the Engle and Granger (1987) regression $\ln\left(\widehat{M}_{t,J}\right) = a + b\ln\left(\widehat{M}_{t,US}\right) + e_{t,J}$.[31] All tests include a constant and linear time trend.

[31] The table reports critical values for a Dickey-Fuller test including a constant and a linear trend. The critical values are very similar to the values from surface response functions provided by MacKinnon (1996) and our conclusions are unaffected, whether we use the standard Dicky-Fuller or the critical values of MacKinnon (1996). The critical values provided by MacKinnon (1996) are -3.9638, -3.4126 and -3.1279 for significance on the 1%, 5% and 10% levels.

Table C.2: Dickey-Fuller Tests

Country	$\ln\left(\widehat{M}_{t,J}\right)$	(p-value)	$\ln\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)$	(p-value)	$e_{t,J}$	(p-value)
Australia	-2.024	(0.581)	-86.406***	(0.000)	-4.200***	(0.005)
Canada	-2.171	(0.508)	-86.476***	(0.000)	-2.618	(0.287)
Denmark	-2.543	(0.324)	-86.551***	(0.000)	-2.297	(0.446)
Euro	-2.474	(0.358)	-86.552***	(0.000)	-4.499***	(0.002)
Japan	-2.104	(0.542)	-86.537***	(0.000)	-2.704	(0.244)
New Zealand	-2.052	(0.567)	-86.398***	(0.000)	-5.859***	(0.001)
Norway	-2.427	(0.381)	-86.553***	(0.000)	-1.914	(0.636)
Sweden	-2.521	(0.335)	-86.554***	(0.000)	-1.900	(0.642)
Switzerland	-2.443	(0.373)	-86.547***	(0.000)	-4.840***	(0.001)
UK	-2.364	(0.412)	-86.553***	(0.000)	-1.687	(0.748)
USA	-2.142	(0.523)	-86.494***	(0.000)		

Notes: provides augmented Dickey-Fuller test statistics for the time series $\ln\left(\widehat{M}_{t,J}\right)$, $\ln\left(\frac{\widehat{M}_{t+dt,J}}{\widehat{M}_{t,J}}\right)$ and the errors $e_{t,J}$ from the Engle and Granger (1987) regression $\ln\left(\widehat{M}_{t,J}\right) = a + b\ln\left(\widehat{M}_{t,US}\right) + e_{t,J}$. All tests include a constant and linear time trend. Significance of the slope coefficients at the 1%, 5% and 10% level are indicated by ***, ** and *.

D Non-Parametric Decomposition of the SDF

In this seciton we describe the non-parametric approach of Christensen (2017) to decompose a SDF into a permanent and transitory component. We keep the our description brief and refer to Christensen (2017) and his references for details.

Alvarez and Jermann (2005) introduce a decomposition of SDF M_t into a permanent component M_t^P and a transitory component M_t^T such that $M_t = M_t^P M_t^T$ or in terms of growths,

$$\frac{M_{t+\tau}}{M_t} = \frac{M_{t+\tau}^P}{M_t^P}\frac{M_{t+\tau}^T}{M_t^T}.$$

Let the n-dimensional state variable X be a time-homogeneous, strictly stationary, and ergodic Markov process. $\mathbb{M}$ is a one-period pricing operator such that the price at time t of an asset with payoff $\psi(X_{t+1})$ at time $t+1$ is

$$\mathbb{M}\psi(x) = E\left[\frac{M_{t+1}(X_{t+1})}{M_t(X_t)}\psi(X_{t+1})\middle| X_t = x\right].$$

Hansen and Scheinkman (2009) provide conditions such that solving the Perron-Frobenius eigenfunction problem $\mathbb{M}\phi(x) = \rho\phi(x)$, (where the eigenvalue ρ is a positive scalar and the eigenfunction ϕ is positive) yields the decomposition

$$\frac{M^P_{t+\tau}}{M^P_t} = \rho^{-\tau}\frac{M_{t+\tau}}{M_t}\frac{\phi(X_{t+\tau})}{\phi(X_t)}, \qquad \frac{M^T_{t+\tau}}{M^T_t} = \rho^{\tau}\frac{\phi(X_t)}{\phi(X_{t+\tau})}.$$

The permanent component $M^P_t = E_t\left[M^P_{t+\tau}\right]$ is a martingale and $-\ln(\rho)$ may be interpreted as the yield on a long term bond (with infinite maturity).

Christensen (2017) proposes a sieve approach to reduce the infinite-dimensional eigenfunction problem to a low-dimensional eigenvector problem. In particular, he defines the basis functions $b_{k1}, \ldots, b_{kk}$ to approximate the state space spanned by X. The projection of the eigenfunction problem onto the linear subspace spanned by $b_{k1}, \ldots, b_{kk}$ is

$$\begin{aligned} G_k^{-1}M_k c_k &= \rho_k c_k \\ G_k &= \mathbb{E}[b^k(X_t)b^k(X_t')] \\ M_k &= \mathbb{E}[b^k(X_t)\frac{M_{t+1}(X_{t+1})}{M_t(X_t)}b^k(X_{t+1})'], \end{aligned}$$

where ρ_k is the largest real eigenvalue and c_k the associated eigenvector of matrix $G_k^{-1}M_k$. ρ_k and $\phi_k(X) = b^k(X)'c_k$ with vector $b^k(x) = (b_{k1}(x), b_{k2}(x), \ldots, b_{kk}(x))'$ are the approximate solution of the eigenvalue ρ and eigenfunction ϕ solving the Perron-Frobenius problem.

In our application we use the first two PCs $\overline{\Pi}_{t,1}$ and $\overline{\Pi}_{t,2}$ as the 2-dimensional state variable X. We choose Hermite polynomials of degree five as basis functions for each PC, then construct a tensor product basis from the univariate bases and discard tensor product polynomials whose total degree is order six or higher. The resulting sparse basis $b_{k1}, \ldots, b_{kk}$ has dimension $k = 15$.

Following Christensen (2017), the sample estimators of matrices G_k and M_k are

$$\widehat{G} = \frac{1}{T}\sum_{t=0}^{T-1} b^k(X_t)b^k(X_t)',$$
$$\widehat{M} = \frac{1}{T}\sum_{t=0}^{T-1} b^k(X_t)\frac{\widehat{M}_{t+1}}{\widehat{M}_t}b^k(X_{t+1})'.$$

E Data Sources

Table E.3: Spot and Forward Exchange Rate Datastream Tickers

Country	Abbr.	Source	Spot Rate	Forward Rate
Australia	AU	Barclays	BBAUDSP(ER)	BBAUD1F(ER)
Canada	CA	Barclays	BBCADSP(ER)	BBCAD1F(ER)
Denmark	DK	Barclays	BBDKKSP(ER)	BBDKK1F(ER)
Eurozone	EU	WMR	EUDOLLR(ER)	EUDOL1F(ER)
Germany	DE	WMR	DMARKE$(ER)	USDEM1F(ER)
Japan	JP	Barclays	BBJPYSP(ER)	BBJPY1F(ER)
New Zealand	NZ	Barclays	BBNZDSP(ER)	BBNZD1F(ER)
Norway	NO	Barclays	BBNOKSP(ER)	BBNOK1F(ER)
Sweden	SE	Barclays	BBSEKSP(ER)	BBSEK1F(ER)
Switzerland	CH	Barclays	BBCHFSP(ER)	BBCHF1F(ER)
United Kingdom	UK	WMR	UKDOLLR(ER)	UKUSD1F(ER)

Table E.4: MSCI Total Stock Market Return Index Datastream Tickers

Country	Total Return	Country	Total Return
Australia	MSAUSTL(MSRI)	New Zealand	MSNZEAL(MSRI)
Canada	MSCNDAL(MSRI)	Norway	MSNWAYL(MSRI)
Denmark	MSDNMKL(MSRI)	Sweden	MSSWDNL(MSRI)
Eurozone	MSEMUIE(MSRI)	Switzerland	MSSWITL(MSRI)
Germany	MSGERML(MSRI)	United Kingdom	MSUTDKL(MSRI)
Japan	MSJPANL(MSRI)	United States	MSUSAML(MSRI)

Table E.5: Global Financial Data Total Return Index Tickers

Country	Total Return	Yield	Country	Total Return	Yield
Australia	TRAUSGVM	IGAUS10D	New Zealand	TRNZLGVD	IGNZL10D
Canada	TRCANGVM	IGCAN10D	Norway	TRNORGVM	IGNOR10D
Denmark	TRDNKGVM	IGDNK10D	Sweden	_RXTBD	IGSWE10D
Eurozone	TREURGVM	IGEUR10D	Switzerland	_SDGTD	IGCHE10D
Germany	TRDEUGVM	IGDEU10D	United Kingdom	TRGBRGVM	IGGBR10D
Japan	IGJPN10D	TRJPNGVM	United States	TRUSG10M	IGUSA10D

F Technical Details and Proofs

F.1 Diffusion Risk Model of FX Markets

Exchange rates: Consider a random and traded payoff Y_{t+dt} to be realized at $t+dt$ ($Y_{t+dt} \in \mathcal{F}_{t+dt}$) in units of currency I. Let the exchange rate $EX_{t,J/I}$ be the number of units of currency J which buys one unit of currency I at time t. Then time-t value Y_t of payoff Y_{t+dt} can either be computed directly in currency I (using I's SDF M_I), or in currency J (using J's SDF M_J) and exchanged back to currency I. That is,

$$E_t\left[\frac{M_{t+dt,I}}{M_{t,I}}Y_{t+dt}\right] = Y_t = \frac{1}{EX_{t,J/I}}E_t\left[\frac{M_{t+dt,J}}{M_{t,J}}\left(EX_{t+dt,J/I}Y_{t+dt}\right)\right].$$

Assuming complete financial markets so that SDFs M are unique, and the above pricing equations hold for a complete set of traded (Arrow-Debreu) assets Y. As a result, the exchange rate unambiguously is the ratio of the two SDFs, $EX_{t,J/I} = \frac{M_{t,I}}{M_{t,J}}$, $\forall t$, which is (2).

Realized carry trade excess returns: Consider the following net-zero strategy denominated in currency I: (i) at time t, borrow $\frac{M_{t,I}}{M_{t,B}}$ units of currency B (worth one unit of currency I) paying interest rate r_B, and simultaneously lend $\frac{M_{t,I}}{M_{t,L}}$ units of currency L (also worth one unit of currency I) earning interest rate r_L, (ii) at time $t+dt$, close all positions and convert the proceeds to denomination currency I. The realized excess return of the strategy is, after using differential representation (1) and applying Ito's lemma,

$$CT^I_{t+dt,-B/+L} = \frac{M_{t,I}}{M_{t,L}}(1+r_L dt)\frac{M_{t+dt,L}}{M_{t+dt,I}} - \frac{M_{t,I}}{M_{t,B}}(1+r_B dt)\frac{M_{t+dt,B}}{M_{t+dt,I}}$$

$$= \frac{M_{t,I}}{M_{t+dt,I}} \times \left[\frac{M_{t+dt,L}}{M_{t,L}}(1 + r_L dt) - \frac{M_{t+dt,B}}{M_{t,B}}(1 + r_B dt) \right]$$

$$= \frac{1}{1 - r_I dt - \eta_I^T dZ_t} \times \left[(1 - r_L dt - \eta_L^T dZ_t)(1 + r_L dt) - (1 - r_B dt - \eta_B^T dZ_t)(1 + r_B dt) \right]$$

$$= \left[1 + (r_I + ||\eta_I||^2) dt + \eta_I^T dZ_t \right] \times \left[\eta_B^T dZ_t - \eta_L^T dZ_t \right] = \eta_I^T \left(\eta_B - \eta_L \right) dt + \left(\eta_B^T - \eta_L^T \right) dZ_t,$$

which yields (3).

F.2 Details on PCA and the FX-Based SDF Estimates

We begin with the mean-zero innovations $X_{t,J/I} \equiv \left(\eta_J^T - \eta_I^T \right) dZ_t$ of exchange rate growths, (2),

$$X_{t,J/I} = \sum_i^n dZ_{t,i}(\eta_{J,i} - \eta_{I,i}) = \sum_i^n dZ_{t,i}\eta_{J/I,i}; \quad t \in [0, s]; \quad \forall J/I \in \mathcal{P}, \tag{25}$$

$$\text{where:} \qquad \eta_{J/I} \equiv (\eta_J - \eta_I) \in \mathbf{R}^n; \quad \forall J/I \in \mathcal{P}.$$

Above, $dZ_{t,i}$ is a normally distributed random variable with mean zero and variance dt, index $i \in \{1, \ldots, n\}$ denotes n such independent risks in the setting, and $\mathcal{P}$ denotes the set of $P \equiv dim\mathcal{P}$ bilateral exchange rates (i.e., currency pair $\{J/I\}$) in the data (see (26) below). For each $J/I \in \mathcal{P}$, let $s \times 1$ column vector $X_{J/I}$ denote the *demeaned* exchange rate growth time series (25), where s is the number of observations in each time series. We arrange these P time series into P columns of $s \times P$ matrix $X = [X_1; \ldots; X_P]$ (26). Hence, $\frac{1}{dt}$ is the number of observations for each exchange rate time series per year, and $s \times dt$ is the length of each time series in years. Similarly, for country each currency pair $J/I \in \mathcal{P}$, let $n \times 1$ column vector $\Delta\eta_{J/I}$ denote the differential prices of prices of n risks across the two respective currencies (3), $\Delta\eta_{k,J/I} \equiv \eta_{k,J} - \eta_{k,I}$, $k \in \{1, \ldots, n\}$. Finally, for each $t \in \{1, \ldots, s\}$, let $1 \times n$ row vector $dZ_t \equiv [dZ_{t,1}, \ldots, dZ_{t,n}]$ denote the n contemporaneous innovations. Therefore, we have explicitly,

$$X \equiv \begin{bmatrix} X_{1,1} & & X_{1,J/I} & & X_{1,P} \\ \vdots & & \vdots & & \vdots \\ X_{t,1} & \dots & X_{t,J/I} & \dots & X_{t,P} \\ \vdots & & \vdots & & \vdots \\ X_{s,1} & & X_{s,J/I} & & X_{s,P} \end{bmatrix} =$$

$$\begin{bmatrix} dZ_{1,1} & & dZ_{1,k} & & dZ_{1,n} \\ \vdots & & \vdots & & \vdots \\ dZ_{t,1} & \dots & dZ_{t,k} & \dots & dZ_{t,n} \\ \vdots & & \vdots & & \vdots \\ dZ_{s,1} & & dZ_{s,k} & & dZ_{s,n} \end{bmatrix} \times \begin{bmatrix} \Delta\eta_{1,1} & & \Delta\eta_{1,J/I} & & \Delta\eta_{1,P} \\ \vdots & & \vdots & & \vdots \\ \Delta\eta_{k,1} & \dots & \Delta\eta_{k,J/I} & \dots & \Delta\eta_{k,P} \\ \vdots & & \vdots & & \vdots \\ \Delta\eta_{n,1} & & \Delta\eta_{n,J/I} & & \Delta\eta_{n,P} \end{bmatrix} \equiv dZ \times \Delta\eta. \quad (26)$$

Since the sum of each column of X is zero (the law of large numbers), the symmetric matrix $X^T X$ is proportional to the empirical (i.e., sample) covariance matrix of the exchange rate fluctuations.

In the PCA, we solve for the eigenvalues and eigenstates of this $P \times P$ empirical covariance matrix $X^T X$ to identify and sort out the most important risks in FX markets. Because $X^T X$ is symmetric, it can be diagonalized by an $P \times P$ orthogonal matrix W (that is $W^T W = W W^T = \mathbf{1}_{P\times P}$): $W^T \left[X^T X\right] W = \mathrm{Diag}[\lambda_1; \dots; \lambda_P]$. Note that because $dZ^T dZ = s \times dt \times \mathbf{1}_{n\times n}$, then by virtue of (26), the same orthogonal matrix W also diagonalizes $\Delta\eta^T \Delta\eta$ (that is, $W^T \Delta\eta^T \Delta\eta W = s \times dt \times \mathrm{Diag}[\lambda_1; \dots; \lambda_P]$). We can also define $n \times P$ score matrix $\Pi \equiv XW$, which then satisfies, $\Pi^T \Pi = \mathrm{Diag}[\lambda_1; \dots; \lambda_P]$, or columns of matrix Π are pairwise orthogonal.

In PCA literature, elements and columns of orthogonal matrix W are respectively referred to as *loadings* and loading vectors, and columns of Π are P principal components when (without loss of generality) eigenvalues are arrange in descending order of magnitudes $\lambda_1 \geq \dots \geq \lambda_P$ (which we also do here). For the notational convenience in our asset pricing tests, however, we work with rescaled $P \times P$ loading matrix $\overline{W} \equiv W \mathrm{Diag}\left[\sqrt{\lambda_1}; \dots; \sqrt{\lambda_P}\right]$ (so that $\overline{W}^T \overline{W} = \mathrm{Diag}[\lambda_1; \dots; \lambda_P]$), rescaled $n \times P$ score matrix $\overline{\Pi} \equiv \Pi\, \mathrm{Diag}\left[\frac{1}{\sqrt{\lambda_1}}; \dots; \frac{1}{\sqrt{\lambda_P}}\right]$ (so that $\overline{\Pi}^T \overline{\Pi} = \mathbf{1}_{P\times P}$), and rescaled differential price of risk matrix $\Delta\overline{\eta} \equiv \Delta\eta W \mathrm{Diag}\left[\frac{1}{\sqrt{\lambda_1}}; \dots; \frac{1}{\sqrt{\lambda_P}}\right]$ (so that $\Delta\overline{\eta}^T \Delta\overline{\eta} = \mathbf{1}_{P\times P}$), which explain the definitions in (4). In the paper, we refer to K-th column of matrix $\overline{\Pi}$ as the K-th (rescaled) principal component.

Employing matrix (stacking) notation, the linear system determining principal factor prices γ's

(7) can be written as,

$$\begin{bmatrix} ECT_1^I \\ \vdots \\ ECT_{-B/+L}^I \\ \vdots \\ ECT_P^I \end{bmatrix} = \begin{bmatrix} \overline{W}_{1,1} & \dots & \overline{W}_{1,C/D} & \dots & \overline{W}_{1,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{W}_{B/L,1} & \dots & \overline{W}_{B/L,C/D} & \dots & \overline{W}_{B/L,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{W}_{P,1} & \dots & \overline{W}_{P,C/D} & \dots & \overline{W}_{P,P} \end{bmatrix} \begin{bmatrix} \gamma_1^I \\ \vdots \\ \gamma_{C/D}^I \\ \vdots \\ \gamma_P^I \end{bmatrix} \tag{27}$$

From this follow the OLS estimates (8).

Similar to (27), we can also stack the (time series) regression equations in the Fama-MacBeth first stage into,

$$\begin{bmatrix} CT_{1,1}^I & \dots & CT_{1,-B/+L}^I & \dots & CT_{1,P}^I \\ \vdots & \dots & \vdots & \dots & \vdots \\ CT_{t,1}^I & \dots & CT_{t,-B/+L}^I & \dots & CT_{t,P}^I \\ \vdots & \dots & \vdots & \dots & \vdots \\ CT_{s,1}^I & \dots & CT_{s,-B/+L}^I & \dots & CT_{s,P}^I \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \overline{\Pi}_{1,1} & \dots & \overline{\Pi}_{1,K} & \dots & \overline{\Pi}_{1,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{\Pi}_{t,1} & \dots & \overline{\Pi}_{t,K} & \dots & \overline{\Pi}_{t,P} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \overline{\Pi}_{s,1} & \dots & \overline{\Pi}_{s,K} & \dots & \overline{\Pi}_{s,P} \end{bmatrix}}_{\equiv \overline{\Pi}} \times \underbrace{\begin{bmatrix} b_{1,1}^I & \dots & b_{1,B/L}^I & \dots & b_{1,P}^I \\ \vdots & \dots & \vdots & \dots & \vdots \\ b_{K,1}^I & \dots & b_{K,B/L}^I & \dots & b_{K,P}^I \\ \vdots & \dots & \vdots & \dots & \vdots \\ b_{P,1}^I & \dots & b_{P,B/L}^I & \dots & b_{P,P}^I \end{bmatrix}}_{\equiv b^I} \tag{28}$$

Clearly, B/L-th column of matrix b^I denotes the P loadings of the carry trade strategy $CT_{t,-B/+L}^I$ (borrowing B, lending L, denominated in I) on P respective principal factors (i.e., P columns of the score matrix $\overline{\Pi}$). Therefore, if γ_K^I, $K \in \mathcal{P}$ denote the factor prices of the respective principal factors (nominated in currency I), and $\widehat{b}^I$'s denote the estimates of factor loadings b^I's, the expected carry trade return reads,

$$ECT_{-B/+L}^I = \sum_{K \in \mathcal{P}} \widehat{b}_{B/L,K}^I \gamma_K^I.$$

We can stack these strategies for all currency pairs in the set $\mathcal{P}$ and obtain in the matrix form,

$$\begin{bmatrix} ECT_1^I \\ \vdots \\ ECT^I_{-B/+L} \\ \vdots \\ ECT_P^I \end{bmatrix} = \underbrace{\begin{bmatrix} \widehat{b}^I_{1,1} & \dots & \widehat{b}^I_{1,K} & \dots & \widehat{b}^I_{P,1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \widehat{b}^I_{B/L,1} & \dots & \widehat{b}^I_{B/L,K} & \dots & \widehat{b}^I_{B/L,K} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \widehat{b}^I_{1,P} & \dots & \widehat{b}^I_{K,P} & \dots & \widehat{b}^I_{P,P} \end{bmatrix}}_{\widehat{b}^{I,T}} \times \underbrace{\begin{bmatrix} \gamma_1^I \\ \vdots \\ \gamma_K^I \\ \vdots \\ \gamma_P^I \end{bmatrix}}_{\gamma^I} = \left(\widehat{b}^I\right)^T \gamma^I. \tag{29}$$

This is the basis for the cross-sectional regressions in the Fama-MacBeth second stage on expected carry trade returns ECT on factor loading estimates $\widehat{b}^I$'s. From this follow the OLS principal factor price estimates, $\widehat{\gamma}^I = (\widehat{b}^I \, \widehat{b}^{I,T})^{-1} \, \widehat{b}^I \, ECT^I$, which is identical to our PCA-based estimates $(\overline{W}^T \, \overline{W})^{-1} \, \overline{W}^T \, ECT^I$ (8) (because of the identity (10) $\widehat{b}^I = \overline{W}^T$).