Should Investors Join the Index Revolution? Evidence from Around the World*

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ABSTRACT

Over the past fifteen years, passive investing has seen 1.5 trillion dollars of fund inflows while active investing has seen 500 billion of outflows. These numbers are in line with the tenets of passive investing, which assert it is close to impossible to consistently outperform the market. We therefore ask in this paper whether there are truly no viable alternatives to indexing and passive investing. We devise a simple actively-managed strategy based on a new version of the minimum variance portfolio that outperforms comparable stock indices around the world with on average 20.2% higher raw returns, 46.7% higher risk-adjusted returns, and 28.4% smaller drawdowns. Furthermore, it exhibits 32.4% lower portfolio turnover than the $1/N$ strategy of DeMiguel et al. (2009) around the world. Not only does this actively-managed portfolio have higher returns at lower risk (the well-known risk-return puzzle), it also displays higher returns at higher skewness levels (i.e. lower downside risk) and thus presents a novel skewness-return puzzle. Moreover, the portfolio also has lower recession risk. Our evidence thus suggests that the principles of passive investing should be questioned and that more effort in the actively-managed fund industry should be devoted to the exploration and application of similar strategies to overcome the industry's decades-long underperformance.

JEL Classification: G11, G14, G15

Key words: Passive investing, market efficiency, minimum variance portfolio, smart beta, portfolio management, international financial markets

"Passive investing will end with active panic."—Sven Henrich[1]

In 2007, Warren Buffett made a one-million-dollar bet that a passive index fund tracking the S&P 500 index would outperform actively-managed hedge funds over a ten year period. Finance professional Ted Seides accepted the challenge, under which he could freely pick a set of hedge funds. In May 2017, eight months before the expiration of the bet, Seides conceded defeat. His chosen actively-managed funds had returned an average of 2.2% per year while the passive index fund had averaged 7% per year. A one million dollar investment in the hedge funds would have gained $220,000 while the same investment in the index fund would have earned $854,000. Although being anecdotal evidence, this story forcefully illustrates the increasingly widespread acceptance of passive indexing as a means to achieve better performance than through active investing.

We therefore ask in this paper whether it is true that there are no viable alternatives to indexing and passive investing, which urges investors to buy passive index-tracking funds due to their consistent average outperformance (so far) over actively managed funds (Malkiel, 2015). While passive investing has been advocated for decades, it has been gaining increased traction due to an ever-widening gap between fund flows of active and passive equity funds, with a cumulative difference of 2.5 trillion dollars over the last 15 years.[2]

To address our main question about alternatives to indexing, we investigate the performance of an actively-managed investment strategy around the world versus the world's major stock market indices in Australia, Germany, Hong Kong, Japan, South Africa, the United Kingdom, and the United States. Specifically, we employ a new

[1]https://twitter.com/NorthmanTrader/status/950832914723426306, accessed January 9, 2018.
[2]From BofAML Global Investment Strategy, EPFR Global.

version of the minimum variance investment strategy.

Using the same investment universe as the stock market index in each economy, we show that our form of minimum variance investing consistently has higher risk-adjusted returns than the respective total return index. (A total return index reinvests all cash distributions such as dividends.) Importantly, this outperformance is achieved with a very simple approach that is easy to implement in the real world without resorting to any non-price data such as fundamental or macroeconomic data. Our investment strategy does not short-sell and requires low portfolio turnover by design, besides having low trading costs due its focus on the most liquid stocks in each economy. Furthermore, unlike other strategies that are based on data mining and therefore might not continue to work in the future, our strategy is based on mathematics.

We view our strategy as a simple benchmark because it creates a lower boundary on the risk-adjusted returns achievable with active investing. We therefore demonstrate that it is possible to consistently outperform passive indexing around the world by using a simple actively-managed strategy. This finding stands in stark contrast to the tenets of indexing, which argues that it is very difficult or even impossible for investors to do better in the long run than putting their money in passive index funds. Our evidence thus suggests that the principles of passive indexing should be questioned.

Given the simplicity of our portfolio both in terms of its mathematical construction as well as its implementation (i.e. investing long-only in some of the most liquid stocks), there are likely other, yet to be discovered, actively-managed strategies with similar or better performance. More effort in the actively-managed fund industry should be devoted to the exploration and application of similar strategies to overcome the industry's decades-long underperformance.

To construct our version of the minimum variance portfolio, we use a three-pronged approach. First, we compute the standard minimum variance portfolio weights based on the investment universe of the economy's respective main stock market index. We do not allow for negative portfolio weights (i.e. short sales) to be consistent with passive index funds, which are long-only. Furthermore, we disallow short sales to avoid complications such as potentially high borrowing costs (especially in some international markets) as well as costs due to low liquidity on the short side (e.g. it might be difficult to find a sufficient number of stocks to borrow for shorting). To estimate the covariance matrix we use the shrinkage estimator of Ledoit and Wolf (2003) on the standard estimation window of 120 months.

Second, to reduce transaction costs and portfolio turnover, we smooth the time series of portfolio weights. The basic idea is that although we use shrinkage estimation, there is residual noise, so we use exponentially-weighted averages of portfolio weights to further wash out this noise. This procedure has two upsides, namely reducing the noise in the portfolio weights and furthermore making the weights more stable over time, thus lowering portfolio turnover and trading costs.

Third, we simplify the portfolio by eliminating exceedingly small stock positions. Specifically, we remove stocks that have a portfolio weight of less than three percent. (The results are robust to using a different percentage as well.) The basic idea is that those small portfolio positions increase the workload of the portfolio manager while not adding significantly to the portfolio's performance in terms of lower risk and higher return. Furthermore, small portfolio weights tend to fluctuate more percentage-wise due to a higher impact of estimation error. To avoid these problems, we remove stocks from the portfolio if their weight is less than three percent to make the portfolio easier to implement in the real world.

Before going on to introduce our detailed results, we want to emphasize that besides questioning the prevalence of passive indexing, our study has broader implications for market efficiency, the efficient allocation of resources in the real economy, and economic growth and development. While passive indexing has advantages such as investors paying lower management fees and not having to worry about underperforming the market (partial equilibrium), there are also substantial drawbacks for capital markets and the economy as a whole (general equilibrium). As more fund flows are allocated to passively-managed investment vehicles, less funds are under active management which tries to identify and exploit misprized stocks and market inefficiencies (Bond and García, 2019). As a result, there is less price discovery and markets become less efficient, with potentially far-reaching consequences for the allocation of capital as well as economic growth and development. Furthermore, with investors herding into the same indices, the probability of excessive market valuations with loose relations to fundamentals increases correspondingly, with potentially severe repercussions for financial market stability and ensuing fallouts for the real economy such as corporate investing, consumer spending, and potentially deeper recessions.

We also would like to briefly discuss management fees, arguing that our actively managed strategy should command slightly lower management fees than a passive counterpart. Essentially all a portfolio manager in our strategy has to do at the end of each month is push a button to run a few lines of programming code on the most recent stock return data to get the portfolio weights for the next month. For the remaining parts of the month he has nothing much to do (unless a stock gets delisted for some reason, which happens very infrequently due to our focus on large index constituents). One could thus argue that our active portfolio manager has slightly less work to do because he has fewer open positions than the index tracker, who needs to have all (or most of) the index's stocks in his portfolio. In summary, we do not believe that our actively managed strategy should command higher management fees than a passive counterpart.

Our results around the world show that minimum variance investing outperforms passive indexing by economically significant amounts while bearing lower risk. On a raw returns basis, the average return of the minimum variance portfolios around the world is 11.0%, an increase of 20.2% over the average return of 9.2% from the stock indices. Sharpe ratios (measures of risk-adjusted returns) increase by 46.7% when switching from an economy's index to its minimum variance portfolio while drawdowns decrease by 28.4%. If we calculate risk-adjusted returns as alphas (i.e. the intercepts of regressions of portfolio returns on the market and Fama-French five factors), we find average annualized alphas of 4.4% around the world, which are in four out of seven economies statistically significant. Furthermore, the betas (which measure the riskiness of the portfolio with respect to the overall market) are less than one, ranging between 0.24 and 0.81. Again this indicates that minimum variance portfolios around the world are less risky than the market while bearing higher risk-adjusted returns.

We then compare the minimum variance portfolios to the $1/N$ strategy of DeMiguel et al. (2009), which equal-weights all stocks in the investment universe and has been shown to perform very well out of sample. The strategy's name "$1/N$" comes from the assumption that N stock are in the investment universe, so the portfolio weights are $1/N$ for all stocks due to equal-weighting. Comparing minimum variance to $1/N$, we find that risk-adjusted returns as measured by Sharpe ratios are on average 29.9% higher for the minimum variance portfolios than for the $1/N$ portfolios around the world. If we measure risk-adjusted returns as annualized alphas, we obtain an increase of 163% when moving to the minimum variance strategy from $1/N$. We investigate next whether this outperformance is driven by higher portfolio turnover and find that the minimum variance turnover is 32.4% lower than that of the $1/N$ strategy. We thus find that minimum variance investing outperforms $1/N$ while having lower portfolio turnover and correspondingly smaller trading costs.

After establishing the outperformance of our minimum variance strategy, we next turn to potential explanations. First we investigate recession risk, since it is possible that the outperformance is simply compensation for bearing higher portfolio risk during recessions (i.e. when investors need the money most). However, we find the opposite, namely that the returns of minimum variance portfolios around the world decrease by a *smaller* amount than the returns of the respective stock market indices during recessions. We thus establish that the outperformance is not driven by recession risk.

Another kind of risk that could drive our results is downside risk as measured by relatively low skewness (specifically, negative skewness). The basic idea is that stocks with low skewness are at at a greater risk of extremely negative returns. Investors want to get compensated for holding this downside risk by gaining higher returns. Following Schneider et al. (2020), we use option-implied skewness because it is more precisely estimated than historical skewness based on past stock returns. We then split the investment universe into terciles based on skewness and re-estimate the minimum variance portfolios on those three subsamples. If the minimum variance strategy has higher returns due to more downside risk, then the portfolio on the low-skewness subsample should outperform the one on the high-skewness subsample. However, our findings do not support this hypothesis. In fact, we find the opposite, namely that the high-skewness subsample outperforms the one on the low-skewness subsample. This paper is therefore to our knowledge the first that documents for the minimum variance portfolio that not only the risk-return tradeoff does not hold, but that furthermore the skewness-return tradeoff is also violated.

While the results discussed so far are based on time-series variation, we next explore cross-sectional variation across 41 different economies around the world. Our goal is to obtain a deeper understanding of the cross-sectional determinants of minimum variance portfolio risk-adjusted returns (as measured by Sharpe ratios) as well as market efficiency more broadly speaking. We operate under the assumption that the minimum variance portfolios' risk-adjusted returns provide a lower boundary on the kinds of risk-adjusted returns achievable by other actively-managed strategies. (In other words, the best actively-managed strategies should have at least the same kinds of returns as the ones documented in this paper.) If a given stock market is more efficient, this boundary should be lower than for a less efficient stock market, because in a more efficient stock market it should be more difficult to achieve outperformance. Consistent with this hypothesis, we find analyst coverage, institutional ownership, and the education level of an economy's population are negatively related to to risk-adjusted returns (and positively related to higher market efficiency). In contrast, we do not find any effect for financial constraints. Finally, we also investigate the legal origins of an economy (La Porta et al., 1997, 1998) and find that common law economies have between 28 and 51 percentage points higher Sharpe ratios than civil law economies.

In Section 1 we review the related literature. Section 2 provides an overview of the different data sources used and details the construction of our version of the minimum variance portfolio. We then go on to describe our main results in Section 3. Finally, Section 4 concludes.

1 Related Literature

This paper is related to several strands of the literature. First and foremost, there is a large literature on efficient markets, surveyed by Fama (1970) and Malkiel (2003). The common advice put forth by Malkiel (page 60) is "not literally to throw darts, but instead throw a towel over the stock pages—that is, to buy a broad-based index fund [...] that charged very low expenses."

The second strand is on the high-risk, low-return puzzle dating back to Black (1972) and Black et al. (1972). Low-risk stocks, according to several measures such as idiosyncratic volatility or CAPM beta, tend to have higher stock returns than high-risk stocks. This is exactly the opposite of what is predicted by finance theory, with early work including Markowitz (1952). Haugen and Heins (1975) were among the first to explicitly challenge the view that risk premia have manifested themselves in realized rates of return (see also Haugen and Baker (1991), Ang et al. (2006), Ang et al. (2009), and Baker and Haugen (2012)). One reason for the good performance is that the estimation error for the sample mean of the returns is so large that it is better to ignore them and only focus on estimating covariances (Merton (1980)). Although (idiosyncratic) volatility is conceptually very different to raw stock returns, it still matters because volatility can predict returns (Goyal and Santa-Clara (2003) and Malkiel and Xu (2006)). Even a portfolio that only depends on estimated historical variances and ignores covariances and expected returns confirms the high-risk, low-return puzzle (Blitz and Vliet (2007)). The minimum variance portfolio has also been shown to be a useful addition to the Markowitz two-fund rule (Kan and Zhou (2007)). It has also been shown that a three-fund rule including the minimum variance portfolio performs well relative to the $1/N$ rule (DeMiguel et al. (2009) and Kan et al. (2016)).

One possible explanation for the high-risk, low-return puzzle is that institutional investors have fixed-benchmark mandates that are typically capitalization-weighted, which creates demand and subsequent overpricing for high volatility stocks (Baker et al. (2011)). This result could also be related to herding (Wermers (1999)). Similar results linking delegated portfolio management to the low volatility effect can also be found in international stock markets around the world (Blitz et al. (2013)). Funding constraints can also play a role because constrained investors bid up high-beta assets (Frazzini and Pedersen (2014)). Additionally, when investor disagreement is high and short-sale constraints severe, high-beta stocks are more sensitive to this disagreement and are priced higher (Hong and Sraer (2016)). Another explanation is that existing risk factors are not sufficient to control for the risk inherent in low-risk stocks due to the higher moments (e.g. skewness) of low-volatility returns (Schneider et al. (2020)), which have been shown to predict returns (Conrad et al. (2013)). While the high-risk, low return puzzle applies to individual stocks and not the minimum variance portfolio in general, it has been shown that both are related since the minimum variance portfolio tends to pick up exactly these low-risk stocks (Scherer (2011)). Compared with our strong preliminary results, Li et al. (2014) find weaker results for large-cap stocks using a related but different low volatility strategy.

In general, it has been shown that idiosyncratic volatility has increased over time (Campbell et al. (2001) and Xu and Malkiel (2003)), which is related to institutional ownership and which makes it more difficult to achieve a given level of diversification. However, no final consensus has been reached about whether the high-risk, low-return puzzle even exists, with contradicting evidence found by Ghysels et al. (2005) and Fu (2009), for example. Furthermore, idiosyncratic risk seems to only be prized in a subset of stocks, e.g. firms that respond with a delay to new information (Hou and Moskowitz

(2005)).

The third strand of the literature is about the efficient construction and implementation of stock portfolios that are optimal in some sense. The framework of Markowitz (1952) is elegant in theory, but the problem is that model inputs (i.e. the true covariance matrix and means of the returns) are unknown and have to be estimated from the data, which leads to imprecise input assumptions (Michaud (1989) and Best and Grauer (1991)). One solution, proposed by Jagannathan and Ma (2003), is to impose additional constraints on the portfolio weights which amounts to using a shrinkage estimator. In fact, to obtain reliable estimates one would need around 6,000 months for a portfolio consisting of 50 assets (DeMiguel et al. (2009)).

Jorion (1986) proposes a Bayes-Stein estimator, building on the inadmissibility result of the sample mean from Stein (1956). Related, Avramov (2002) shows that a Bayesian approach can predict stock returns. For a survey on Bayesian portfolio analysis see Avramov and Zhou (2010). MacKinlay and Pastor (2000) use the mispricing from a missing risk factor that is embedded in the covariance matrix to improve portfolio selection. Asset allocation frameworks that use uncertainty sets / confidence intervals instead of point estimates to obtain best worst-case results are proposed by Tütüncü and Koenig (2004) and Garlappi et al. (2007) and are related to robust mean-variance analysis (Maccheroni et al. (2013)). To reduce the estimation error, it can even be shown that imposing no-short-sale constraints (which should hurt performance according to theory) empirically end up improving performance due to a reduction in estimation error (Jagannathan and Ma (2003)). Tu and Zhou (2011) propose that similar to model averaging, a combination of sophisticated strategies overcomes some of the estimation problems of the traditional Markowitz framework.

Another angle of attack proposed in the literature is to improve the inputs to mean-variance optimization, specifically the estimation of the covariance matrix (see for example Ledoit and Wolf (2003), Schäfer and Strimmer (2005), and Gerber et al. (2015)). Alternatively it is possible to directly estimate the inverse of the covariance matrix through sparse hedging restrictions (Goto and Xu (2015)).

Fourth, there is a large literature on asset prices in international markets. Rouwenhorst (1999) shows that local factors driving emerging markets are qualitatively similar to those from developed markets. Similar results are found in Fama and French (2012), especially for size and value/growth. However, some economies have higher synchronous stock price movement due to weaker investor property rights (Morck et al. (2000)) or lack of transparency (Jin and Myers (2006)).

2 Data

2.1 Data Sources

We use Compustat Global Security Daily for international stock return data and convert it to monthly frequency. By "monthly frequency" we mean that the time series have one observation per month. The monthly stock returns include cash distributions such as dividends and are adjusted for stock splits. The date range in Compustat Global is from January 1986 until June 2017, although some economies cover a shorter date range. Furthermore, we lose 120 months at the beginning of each economy's sample for the construction of the minimum variance portfolio, see Section 2.2. For each economy, we require that each company is incorporated and headquartered in that economy as well as traded in the local currency. For U.S. monthly stock returns we use data from the Center for Research in Security Prices (CRSP). We restrict the date range to January 1977 until December 2016 due to the unavailability of total return stock index data before that time period.

Our choice of economies is based on the sizes of their GDP and financial markets as well as selecting pertinent developed economies from each continent around the world. Although potentially more economies could be added, we believe our selection provides a representative overview of some of the most important financial markets in the world.

Stock index return data is from Bloomberg. Unless noted otherwise, we use the total return version of each stock market index throughout the paper. All cash distributions such as dividends are therefore reinvested. This ensures a fair comparison with the investment strategies which also reinvest all cash distributions. For each economy we use the most well-known stock market index. For the U.S., instead of a single index, we include two indices in our analysis, the Dow Jones Industrial Average (DJIA) and the S&P 500. The reason is that the U.S. has the world's largest financial markets, so we would like to assess the performance of the minimum variance portfolio against both a narrow (DJIA with 30 stocks) and a broad index (S&P 500 with 500 stocks).

Stock index constituents are from Compustat Global Index Constituents for international data and for the U.S. we use Compustat North America Index Constituents. At the beginning of the sample period there is not always data available on index constituents. In this case we use the N largest stocks based on market capitalization, where N corresponds to the number of stocks in the index. For example, for the FTSE 100 Index, $N = 100$.

The Fama-French factors are from Kenneth French's online data library. For each of the non-U.S. economies we use the closest available version of the international research returns, e.g. for Germany we use the Fama-French European 5 factors. Since we take the perspective of a local investor, we convert all return data into local currencies. Foreign exchange data as well as recession data is from the Federal Reserve Bank of

St. Louis.

Stock options data is from OptionMetrics, analyst coverage is from Thomson Reuters I/B/E/S, institutional ownership data is from Thomson Reuters Institutional 13F Holdings, education levels are from the World Bank Education Statistics (EdStats), and legal origins data is from Andrei Shleifer's website[3] based on La Porta et al. (2008).

After merging all data, Table 1 shows the dates included for each economy. The earliest date for all economies is November 1987 and the last date included is June 2017. The coverage varies by economy due to the various time periods included in the databases mentioned in this section.

2.2 Construction of the Minimum Variance Portfolio

For each economy, at the end of month t, we calculate the portfolio weights valid for investing in month $t+1$ according to the following procedure. We begin by extracting all stocks that have, at time t, a 120-month trading history available and furthermore are stock index constituents. Using a 120-month estimation window is standard in the literature (Ledoit and Wolf, 2003), although our results are robust to different window lengths as well. We remove stocks that have missing returns in more than 5% of the observations in this window. If a company has several securities outstanding, we keep the one with the highest liquidity according to the past six-month average value traded (trading volume times price). Based on this 120-month estimation window, we then calculate the covariance matrix estimate $\hat{\Sigma}_{t+1}$ of the stock returns according to Ledoit and Wolf (2003). We use subscript $t+1$ to indicate that this covariance matrix is going to be used for investing in month $t+1$ (even though it is estimated using stock

[3]https://scholar.harvard.edu/shleifer/publications/economic-consequences-legal-origins, accessed January 28, 2017.

return data up until the end of month t). Given $\hat{\Sigma}_{t+1}$, we use quadratic programming to compute the portfolio weights $w_{t+1} = (w_{t+1,1}, \ldots, w_{t+1,N})^T$ valid for investing in month $t+1$ according to the usual portfolio optimization problem, which minimizes the variance of the portfolio subject to the usual constraints:

$$w_{t+1}^T \hat{\Sigma}_{t+1} w_{t+1} \longrightarrow \min! \qquad \text{subject to} \quad w_{t+1} \geq 0 \text{ and } \sum_{i=1}^{N} w_{t+1,i} = 1. \qquad (1)$$

We require that the weights w_{t+1} are nonnegative to disallow short sales. The reason is that (a) we want to keep the portfolio as simple as possible, (b) short-selling is often much more costly than having a long position due to liquidity and trading costs (which can be substantial in some international markets and furthermore finding a stock to borrow is sometimes not easy), and (c) we want this investment strategy to be broadly applicable in the sense that mutual funds (who cannot short-sell) or even retail investors can implement this strategy in addition to more unconstrained institutional investors such as hedge funds, who can short-sell. The constraint that the portfolio weights sum to one means that we stay 100% invested in the stock market at all times.

This procedure gives a sequence of portfolio weights telling us exactly the fraction of net worth we could invest into each stock at each point in time. However, there are several potential problems if we simply use the w_{t+1}'s coming out of the portfolio optimization. First, it is well-known that the covariance estimate $\hat{\Sigma}$ is a noisy estimate of the true covariance matrix Σ of stock returns. Second, we would like to reduce trading costs and avoid having to trade in and out of stocks too frequently due to large changes in portfolio weights. To solve both problems, we follow Jin (2015) and smooth the portfolio weights by taking an exponentially weighted moving average of current and past portfolio weights. Specifically, we use a smoothed portfolio weight vector $\tilde{w}_{t+1}$

for our *actual* portfolio allocation, which is computed as

$$\tilde{w}_{t+1} = \rho w_{t+1} + (1 - \rho)\tilde{w}_t. \qquad (2)$$

We follow Jin (2015) and choose the smoothing parameter $\rho = \frac{1}{15}$. Our results are robust to other values of ρ as well. Finally, to avoid very small portfolio weights, which can complicate portfolio management due to small position sizes and can drive up trading costs, we eliminate stocks from the portfolio that have a weight of less than three percent and distribute their weights to the remaining stocks in the portfolio. It should be noted that the three-percent exclusion rule applies to the minimum variance portfolio weights, not to the investment universe. In other words, even if a stock has less then three percent weight in the index, it is nonetheless included in the investment universe and could potentially end up in the minimum variance portfolio.

3 Results

3.1 Comparison with Stock Market Index

Figure 1 shows cumulative returns around the world. For each economy, the figure illustrates the performance of the absolute return version of the respective stock market index as well as the minimum variance portfolio in that economy. In all markets except for Japan, the minimum variance portfolio outperforms the index on a raw return basis. Furthermore, a cursory inspection of the figure reveals that despite its outperformance, the minimum variance portfolio seems to have lower volatility than the stock market index. There is no risk-return tradeoff in the sense that the higher returns of the minimum variance portfolio are riskier than the lower returns of the index.

To make this qualitative assessment more specific, we next investigate performance metrics in Table 2. Corroborating our earlier results from Figure 1, we find that in all economies except for Japan the returns of the minimum variance portfolio are higher than the returns of the respective stock index. On a risk-adjusted basis using the Sharpe ratio, we find a stronger result in the sense that in all markets (including Japan) the Sharpe ratio of the portfolio is higher than the index. On average, the Sharpe ratio increases by 46.7% when switching from the index to the portfolio. Finally, the maximum drawdown, which is an alternative measure of the riskiness of a strategy, is consistently smaller for the portfolio compared to the index, with the exception of Australia. Specifically, the maximum drawdown is on average 28.4% lower for the portfolio than the index.

In Table 3 we compute risk-adjusted excess returns for minimum variance portfolios in each economy. Specifically, for each economy, we regress the minimum variance excess returns on the excess returns of the economy's market and the Fama-French factors (Fama and French, 2015). The intercept of this regression represents risk-adjusted excess returns and is shown in the rows labeled "alpha." For easier interpretation, we show annualized alphas in Table 3.

We find that the alphas of the minimum variance portfolios are positive in all economies around the world. We find insignificantly positive alphas in Australia (2.5%), Japan (2.2%), and the United States (1.4% for the Dow Jones Industrial Average investment universe and 1.1% for the investment universe consisting of stocks in the S&P 500). In contrast, significantly positive alphas are obtained in Germany (6.9%), Hong Kong (7.5%), South Africa (10.1%), and the United Kingdom (3.7%). In summary, the average annualized risk-adjusted excess returns (alphas) are with 4.4% economically meaningful and in four out of our seven economies statistically significant.

Consistent with being a low-risk investment strategy, the minimum variance portfolio's exposure to the market is significantly lower than one, as shown in the rows labeled *Market*$-r_f$. In the full model specification that includes the Fama-French factors, the highest coefficient for the market is 0.81 in Australia and the lowest is 0.24 in Hong Kong. This result demonstrates that the outperformance of the minimum variance portfolio is not the result of leveraging up on the market and exploiting the general tendency of markets to rise over the long run.

The small-minus-big (SMB) coefficient, when statistically significant, shows up negative. This result establishes that the minimum variance outperformance is not acquired by loading up on small stocks and exploiting the size effect. This makes sense inasmuch our investment universe consists of the constituent stocks of the respective economy's main stock market index, which typically contains large stocks. Even so, the negative SMB coefficient suggests that the minimum variance strategy's outperformance is not primarily due to picking the smallest stocks within this investment universe.

The coefficient on high-minus-low (HML) is mainly insignificant. When significant, it has a negative value. It thus confirms that the outperformance of the minimum variance portfolio is not based on the value effect, where stocks with high book-to-market ratios on average outperform stocks with low such ratios.

The robust-minus-weak (RMW) coefficient is significantly negative in Japan, significantly positive in the U.S., and insignificant in the remaining economies. RMW contains the returns of a portfolio exploiting the fact that stocks with robust operating profitability on average outperform stocks with weak operating profitability. A positive RMW coefficient means that the minimum variance portfolio's outperformance can in part be attributed to containing stocks with robust operating profitability. Our results thus show that in Japan the minimum variance portfolio does not capitalize on this effect, while in the U.S. it does. The result for Japan may partially be explained by the fact that the RMW effect in Japan has reversed in the 2000s decade. However, this decade constitutes only about half of our sample period for Japan (see Figure 1), and in the other half the RMW effect was not reversed. This suggests that in Japan the minimum variance portfolio's outperformance cannot be attributed to picking stocks with robust operating profitability. In total, when looking at all economies, we find that the outperformance of the minimum variance portfolio cannot consistently be explained by the operating profitability effect.

The coefficient on conservative-minus-aggressive (CMA) is significantly positive in Hong Kong, the UK, and the U.S., and insignificant in the remaining economies. CMA is a portfolio exploiting the fact that on average firms with conservative corporate investment outperform firms with aggressive investment. We thus find that in three out of our seven economies the minimum variance portfolio's outperformance can in part be explained by loading up on companies that invest less than their peers.

3.2 Comparison with $1/N$ Portfolio and Results on Portfolio Turnover

In this section we compare the minimum variance portfolio's performance to that of the $1/N$ portfolio of DeMiguel et al. (2009). We then go on to investigate how the performances are linked to portfolio turnover and trading costs.

3.2.1 Comparing Minimum Variance and $1/N$ Portfolios

The $1/N$ portfolio of DeMiguel et al. (2009) has received a lot of attention because it has been shown to perform very well out-of-sample. The basic idea is to create a portfolio that simply equal-weights each stock in the investment universe, thus giving each stock the weight $1/N$, where N is the number of stocks in the investment universe. In our case, for better comparison with the minimum variance portfolio, the investment universe consists of each economy's stock index constituents (i.e. the same stocks available to the minimum variance portfolio). Furthermore, to level the playing field, the $1/N$ portfolio reinvests all cash distributions such as dividends, just like the minimum variance portfolio does.

We take a first look at the results in Figure 2, where we plot the cumulative returns of both portfolios in each economy around the world. In Germany, Hong Kong, South Africa, and the UK, the minimum variance portfolio outperforms the $1/N$ portfolio, while in Australia, Japan, and the U.S., the $1/N$ portfolio outperforms. Meanwhile, a visual inspection also suggests that the minimum variance returns are consistently less risky than the $1/N$ returns. To better understand risk-adjusted returns and further investigate the risk borne by investors in both strategies, we next consider performance metrics.

In Table 4 we present annualized returns, annualized volatilities, Sharpe ratios, and maximum drawdown figures for both the minimum variance portfolio as well as the $1/N$ portfolio. Annualized returns echo the findings from Figure 2, where the minimum variance portfolio has higher returns than $1/N$ in Germany, Hong Kong, South Africa, and the UK, while it has a lower return in the remaining economies. The volatility on the other hand is consistently lower for the minimum variance portfolio. Likewise, with the exception of Australia, the minimum variance strategy persistently has a lower drawdown than $1/N$. When we therefore adjust the returns for risk, we obtain Sharpe ratios for the minimum variance portfolio that are with the exception of Japan consistently higher than those of $1/N$. Taking the average increase over all economies, we find that the Sharpe ratio of the minimum variance portfolio increases by 29.9% compared to the $1/N$ portfolio, which is an economically meaningful gain in risk-adjusted returns.

In Table 5 we calculate an alternative version of risk-adjusted returns by running regressions of the portfolios' excess returns on the market and the Fama-French factors (Fama and French, 2015). The intercept, or "alpha," represents the risk-adjusted excess return. For easier interpretation we have annualized the alphas in the tables.

When comparing the risk-adjusted excess returns of minimum variance and $1/N$ in Table 5, we find consistently higher returns for the minimum variance portfolio. The minimum variance alphas are significantly positive in Germany, Hong Kong, South Africa, and the UK when controlling for the Fama-French factors and furthermore in the U.S. when controlling for the market alone. Averaging across all economies and controlling for the Fama-French factors, the minimum variance alphas are 4.4% per year while the $1/N$ alphas amount to 1.7%. For an investor moving to the minimum variance strategy from the $1/N$ strategy, this corresponds to an increase of 163% in the risk-adjusted excess returns. In fact, in two economies the differences in alphas are large enough to cause a split in statistical significance in the sense that $1/N$ becomes insignificant while minimum variance stays significant. In summary, we find that the minimum variance portfolio performs significantly better than the $1/N$ portfolio on a risk-adjusted basis around the world.

3.2.2 Portfolio Turnover and Trading Costs

To probe whether the risk-adjusted outperformance of the minimum variance portfolio comes at the cost of higher turnover, we report annualized portfolio turnover statistics in Table 6. Portfolio turnover for month t is calculated as the sum of the absolute values of all trades at the end of month t across all open positions, i.e.

$$PT_t = \sum_{i=1}^{N} \left| \hat{w}_{t+1,i} - \hat{w}_{t+,i} \right|, \tag{3}$$

where $\hat{w}_{t+,i}$ is the portfolio weight of stock i at the end of month t (after taking into account changes in portfolio weights due to different stock returns throughout month t) and $\hat{w}_{t+1,i}$ is the portfolio weight at the beginning of month $t+1$. Specifically, the weights $\hat{w}_{t+,i}$ at the end of month t are based on the weights $\hat{w}_{t,i}$ at the beginning of the month, adjusted for the effects of stock returns throughout the month so that $\hat{w}_{t+,i} = \hat{w}_{t,i}(1+r_{t,i})/\sum_{j=1}^{N}\hat{w}_{t,j}(1+r_{t,j})$, where $r_{t,i}$ is the return of stock i in month t. At the beginning of the following month $t+1$ we have for the $1/N$ strategy $\hat{w}_{t+1,i} = 1/N$ and for the minimum variance strategy we have $\hat{w}_{t+1,i} = \tilde{w}_{t+1,i}$, where $\tilde{w}_{t+1,i}$ is defined in equation (2) in Section 2.2. The numbers reported in Table 6 are computed as $\frac{12}{T}\sum_{t+1}^{T} PT_t$, which is the average of equation (3) over all months multiplied by twelve, i.e. the average annualized portfolio turnover.

To interpret these turnover numbers we consider a highly stylized example. Assume for simplicity that there are only two stocks in the investment universe ($N = 2$) and that all stock returns are zero (so that $\hat{w}_{t,i} = \hat{w}_{t+,i}$). Furthermore, assume that each month we invest all our net worth into one stock only, and that each following month we alternate between the two stocks. This means we go for stock $i = 1$ from, say, $w_{t+,1} = 100\%$ to $w_{t+1,1} = 0\%$ while for stock $i = 2$ we do the opposite, i.e. $w_{t+,2} = 0\%$ to $w_{t+1,2} = 100\%$. In this case we can see that $PT_t = 200\%$, which corresponds to exchanging all stocks in the portfolio. If we add these monthly trades up for a whole year, we obtain an annualized portfolio turnover of 2,400%. This is of course an extremely stylized example, but the main point goes through under more realistic assumptions (e.g. more than two stocks) as well.

This stylized example provides an upper bound and reference point for a portfolio that trades as much as possible by exchanging all stocks each month. It thus helps to put the reported numbers in Table 6 in perspective. For example, a reported number of 100% means that each year about 4.2% (= 100/2400) of all stocks in the portfolio are exchanged. So it is important to keep in mind that the highest turnover for the minimum variance portfolio (85.1% in South Africa) means that approximately 3.5% (= 85.1/2400) of the stocks in the portfolio are exchanged each year. This result shows that trading costs are not a major issue, especially given that the investment universe consists of the major stock index in each economy, i.e. the largest and most liquid stocks that have the lowest trading costs to begin with.

Table 6 shows that around the world the minimum variance portfolio has a consistently lower turnover than the $1/N$ portfolio. On average, the minimum variance turnover is 58.1% while $1/N$ has 85.9%. The minimum variance's turnover is thus 32.4% smaller than that of the $1/N$ portfolio. This is an economically significant reduction in trading activity necessary to implement this strategy, with an approximately proportional amount of savings in transaction costs.

In general, we find the portfolio turnover of the minimum variance portfolio to be relatively low, with a maximum turnover of 85.1% in South Africa and a minimum turnover of 30.0% in Hong Kong. We attribute this low turnover to three factors. First, we we use the standard estimation window of 120 months, as detailed in Section 2.2. If we move this window forward by one month (to get the portfolio weights for the next month), we drop one observation at the end and add one observation at the beginning, which means that both windows share 119 observations, so they are to 99% (= 119/120) identical. The statistical properties of both windows therefore do not change very much, which translates into more stable portfolio weights and lower portfolio turnover. Second, the covariance matrix estimator of Ledoit and Wolf (2003) is a shrinkage estimator, so it tends to be more stable over time. Since the covariance matrix is the only input that varies for the optimization problem (1), having a stable covariance matrix means that the optimization output (i.e. the portfolio weights) also tend to be more stable, leading again to a lower portfolio turnover. Third, the smoothed portfolio weights from equation (2) further contribute to fewer changes in these weights and thus again to a lower turnover.

3.3 Performance Decomposition

To find out where the value added of the new investing strategy is coming from, we compare it to the pure minimum variance strategy and then add our additional components step by step. In this subsection we focus on the Dow Jones Industrial Average only.

First, we begin with the pure minimum variance strategy, which has an annualized return of 10.1%, a volatility of 12.1%, a Sharpe ratio of 83.1%, and a maximum drawdown of 33.0%. Second, we re-run the strategy but now exclude small positions with portfolio weights below 3%. While this step makes it easier to manage the portfolio (especially if there is a large investment universe) and is therefore relevant from a practical implementation perspective, the performance degrades slightly in terms of the Sharpe ratio, while on the other hand the maximum drawdown improves slightly. The return is 9.7%, the volatility is 12.2%, the Sharpe ratio is 80.1%, and the maximum drawdown is 32.5%. Third, we add shrinkage to the covariance matrix (Ledoit and Wolf, 2003). The performance improves with an annualized return of 10.7%, a volatility of 12.4%, a Sharpe ratio of 86.0%. On the other hand, the maximum drawdown deteriorates to 35.2%. Fourth, we average the portfolio weights over time to reduce trading costs and portfolio turnover. The overall performance improves across the board with an annualized return of 11.2%, a volatility of 12.5%, a Sharpe ratio of 89.6%, and a maximum drawdown of 33.3% (the same numbers as reported in Table 2).

To summarize, shrinkage and averaging the portfolio weights over time are the biggest contributors to an improved portfolio performance relative to the pure minimum variance strategy. On the other hand, the exclusion of stocks with small portfolio weights is detrimental to the Sharpe ratio, although it makes it easier to manage the portfolio and it reduces the maximum drawdown.

3.4 Recession Risk

The next question we ask is whether the minimum variance portfolio has lower returns than the market during recessions. If true, the good performance of the minimum variance portfolio can be explained as compensation required by investors for holding a portfolio that goes down the very moment they do *not* want it to go down, i.e. during recessions. If, on the other hand, the minimum variance portfolio is less sensitive to recessions than the market, we can conclude that its outperformance is not driven by risk premia for recession risk.

We therefore regress Table 7 the returns of the minimum variance portfolio and the returns of the market on recession dummy variables. For each economy, we use its own recession dummy (if available) as well as dummies for the two largest economies, the U.S. and China. Furthermore, to capture other cross-economy spillover effects, we add the "recession anywhere" dummy REC that is one if there is a recession in any of the economies in our sample. Due to potential multicollinearity problems we run a separate regression for each recession dummy.

For all economies with the exception of Australia, we find that the absolute values of the coefficients on all recession dummies are consistently smaller for the minimum variance portfolio than for the stock market index. For example, for the U.S. in panel (h), we find that a recession in the U.S. (captured by *USREC*) lowers the monthly minimum variance return by 1.10% while it lowers the monthly return of the S&P 500 by 1.94%.

Furthermore, there is a stark difference in statistical significance for some economies such as Hong Kong, Japan, South Africa, and the U.S. (S&P 500 investment universe). For example, in panel (h), the U.S. recession dummy *USREC* is significant at the 1% level for the S&P 500, but only significant at the 10% level for the minimum variance portfolio. In other economies such as Hong Kong the recession dummy *CHNRECM* (for China) becomes even insignificant for the minimum variance portfolio, while staying highly significant for the Hang Seng Index.

We therefore confirm that the outperformance of the minimum variance portfolio is not driven by higher risk premia due to recession risk. On the contrary, we find that around the world, minimum variance portfolios are less sensitive to recessions than the corresponding stock market index and actually have higher (i.e. less negative) returns in recessions than the index.

To add further texture, we find around the world that the most important recession indicators are those of the U.S. and China for both the minimum variance portfolios as well as the respective stock indices. In many economies such as Australia, Germany, and the UK, recession dummies of the U.S. or China are more important than those of the own economy. For example, the recession dummy for Australia (*AUSRECDM*) is insignificant, while the recession dummies for both the U.S. (*USREC*) and China (*CHNRECM*) are highly statistically as well as economically significant, with a recession in the U.S. or China lowering returns between 1.46% and 2.87% per month. So although stock indices suffer more strongly than minimum variance portfolios, we can confirm that both can be influenced by recessions occurring in the U.S. or China, sometimes more so than by recessions in their home economy.

3.5 Downside Risk and Skewness

In this section we explore whether the outperformance of the minimum variance portfolio is driven by compensation for downside risk in the form of relatively low skewness (specifically, negative skewness). The basic idea is that investors holding stocks that have a long left tail (i.e. low skewness) are at a greater risk of extremely negative outcomes and would like to get compensated with higher returns for holding this risk (Kraus and Litzenberger, 1976; Harvey and Siddique, 2000). If the minimum variance portfolio loads up on this type of risk by holding more stocks with low skewness it could explain the portfolio's outperformance. On the other hand, if we find that minimum variance portfolio returns decrease when including more stocks with low skewness, we can conclude that its outperformance is not driven by higher downside risk (i.e. low skewness).

To investigate this hypothesis, we need a reliable measure of skewness. Schneider et al. (2020) show that an option-implied skewness measure is often more precise than a skewness measure based on historical stock returns. The basic idea is in principle very similar to the well-known concept of implied volatility, where volatility is backed out of prices of traded options. Option-implied skewness is in principle similar except that it uses higher moments.

Following Schneider et al. (2020), to make our measure closer to central skewness (net of variance effects), we calculate option implied skewness as

$$\frac{SKEW_{t,T}}{VAR_{t,T}^{\frac{3}{2}}},$$

where $VAR_{t,T}$ and $SKEW_{t,T}$ are defined as portfolios of out-of-the-money put and call options measuring option-implied variance and skewness:

$$VAR_{t,T} = \frac{2}{p_{t,T}} \left(\int_0^{F_{t,T}} \frac{\sqrt{\frac{K}{F_{t,T}}} P_{t,T}(K)}{K^2} dK + \int_{F_{t,T}}^{\infty} \frac{\sqrt{\frac{K}{F_{t,T}}} C_{t,T}(K)}{K^2} dK \right), \tag{4}$$

$$SKEW_{t,T} = \frac{1}{p_{t,T}} \left(\int_{F_{t,T}}^{\infty} \log\left(\frac{K}{F_{t,T}}\right) \frac{\sqrt{\frac{K}{F_{t,T}}} C_{t,T}(K)}{K^2} dK - \int_0^{F_{t,T}} \log\left(\frac{F_{t,T}}{K}\right) \frac{\sqrt{\frac{K}{F_{t,T}}} P_{t,T}(K)}{K^2} dK \right). \tag{5}$$

Here $p_{t,T}$ is the price at time t of a zero coupon bond with maturity at time T, $F_{t,T}$ is the forward price of the stock (contracted at time t for delivery at time T), $P_{t,T}(K)$ and $C_{t,T}(K)$ are prices of a European put and European call option with strike K on the stock. We calculate the measures in equations (4) and (5) using the volatility surface (for the European option prices), the zero coupon yield curve (for the zero coupon bond $p_{t,T}$), and standardized options (for the forward price $F_{t,T}$), all from OptionMetrics. (We only have OptionMetrics data for the U.S., so in this part we do not focus on international evidence due to data limitations.) Given this data we then use numerical integration to calculate time series of $VAR_{t,T}$ and $SKEW_{t,T}$ for all stocks in our investment universe, where we use a remaining time to expiration of 30 days (i.e. $T-t=30$). So our measure of implied skewness reflects the option market's aggregate opinion on skewness over the next 30 days.

We then split the stocks in the investment universe on each date into terciles based on skewness. On the following date we form three minimum variance portfolios, one for each skewness subsample. If skewness is driving our results, we should see higher minimum variance portfolio returns in the low-skewness subsample (i.e. the one with more downside risk) than in the high-skewness subsample (i.e. the one with less downside risk). In other words, if our results are due to downside risk, the minimum variance returns should be a decreasing function of skewness.

In Figure 3 we visualize the cumulative returns of the three portfolios. For comparison we also plot the Dow Jones Industrial Average. The plots show that instead of decreasing, the minimum variance returns actually increase with skewness. In the low-skewness subsample, the minimum variance returns are less than the stock index, in the mid-skewness sample they are about the same, and in the high-skewness sample they are significantly larger than those of the stock index. We thus find that based on visual inspection the outperformance of the minimum variance portfolio is not driven by risk premia due to downside risk.

In Table 8 we calculate performance metrics for all three skewness subsets. As indicated graphically before in Figure 3, the annualized returns are an increasing function of skewness, going up to 11.7% from 7.8% when moving from low to high skewness, an increase of 50.0%. Volatility only increases slightly so that the Sharpe ratio (a measure of risk-adjusted returns) is also an increasing function of skewness. Specifically, the Sharpe ratio increases to 75.8% from 55.0% when going from low to high skewness, an increase of 37.8%. We thus establish that the minimum variance portfolio's performance is an increasing function of skewness not only in terms of raw returns but also for risk-adjusted returns.

We explore risk-adjusted returns in Table 9 by calculating the alphas of the three skewness subsets. Again we find that the alphas are increasing in skewness, although they are not always statistically significant. Even so, if the outperformance of the minimum variance portfolio is due to downside risk premia, we would expect the low skew alpha to be significantly positive. However, we find the exact opposite, with the high-skew alpha being the only one that is significant and positive.

In summary, we do find no evidence that the outperformance of the minimum variance portfolio is due to premia for downside risk (i.e. low skewness). Furthermore, instead of finding no effect of skewness, the puzzle deepens because the outperformance is concentrated in stocks with high skewness, i.e. low downside risk. We thus find that the minimum variance portfolio has higher risk-adjusted returns while at the same time having lower downside risk (as measured by skewness). We do not further investigate this puzzle here but note that it might be related to fixed-benchmark mandates (Baker et al., 2011), funding constraints (Frazzini and Pedersen, 2014), and speculative overpricing due to short-sale constraints (Hong and Sraer, 2016).

3.6 Cross-Sectional Evidence Across Economies

To complement the tests presented thus far (which exploit intra-economy time series variation) we bolster our research to examine cross-sectional variation in this section, across different economies in our sample. Specifically, the question we explore is what drives risk-adjusted returns (as measured in this section by Sharpe ratios) of minimum variance portfolios across economies. In a broader sense we also investigate the efficiency of capital markets around the world under the assumption that in more efficient markets the risk-adjusted returns of the minimum variance portfolios are lower.

To obtain a large cross-sectional sample, we include all economies from Compustat Global having at least twenty years of data, which results in 75 economies. In order to exclude exceedingly small stock markets, we further screen all economies to include at least 50 stocks in the last observation period, which further reduces the number of economies to 51. To have a fair comparison between economies, and because we do not have index constituent data available for all economies, we use a simple and uniform rule to specify the investment universe. Specifically, for each economy, the investment universe includes at each point in time the largest 30 stocks based on market capitalization. Some economies trade fewer stocks, in which case we remove the economy if at any point in time less than five stocks are traded. This screen brings down the total number of economies to 41. The remaining steps of the construction of the minimum variance portfolio are the same as described in Section 2.2. Once we have computed the minimum variance portfolio returns for each economy, we then calculate the Sharpe ratios (which are a measure of risk-adjusted returns) of these portfolios for each economy and use these Sharpe ratios as the dependent variables in the cross-economy cross-sectional regressions in this section.

While questions of causality are eminently important, we take a more modest approach in the results contained in this section. We view our results as a first step in obtaining a deeper understanding of the drivers of minimum variance risk-adjusted returns across different economies, while acknowledging that there are potential endogeneity issues left to be addressed in future work.

In a more expansive sense we also probe the efficiency of markets around the world and their determinants. Given that the minimum variance investment strategy has been around for more than half a century (Markowitz, 1952), it is reasonable to assume it can be viewed as a benchmark providing a lower boundary on the risk-adjusted returns achievable by other actively-managed strategies. Specifically, if the stock market of a given economy is more efficient, this boundary (proxied by the minimum variance risk-adjusted return / Sharpe ratio) should be lower than that of a less efficient stock market. In other words, under this assumption, market efficiency and minimum variance Sharpe ratios should be inversely related.

A visual comparison of Sharpe ratios' magnitudes across various economies can be found in Figure 4. It shows that risk-adjusted average returns (as measured by Sharpe ratios) vary widely from economy to economy. The top-ranked economies are Sri Lanka, Pakistan, Thailand, New Zealand, and Mexico with an average Sharpe ratio of 1.40, while at the bottom we have Greece, Jordan, Australia, Poland, and Japan with an average Sharpe ratio of 0.18.

The number of observations used to compute the Sharpe ratios in each economy varies due to data availability in Compustat Global and ranges from 132 months to 258 months, with a mean number of 217 months (unreported results, excluding the initial 120-month estimation window). To control for this effect, we add a variable containing an economy's sample size (in months) to all regressions in this section.

In Table 10 we investigate whether and how analyst coverage is related to risk-adjusted returns (Sharpe ratios) of the minimum variance portfolio across economies. More analysts should be associated with a more efficient market and thus with lower risk-adjusted returns. Indeed we find a significantly negative coefficient on analyst coverage, implying that in economies covered by a larger amount of analysts the risk-adjusted returns of the minimum variance portfolio are lower than in economies with fewer analysts. A one-standard-deviation increase in analyst coverage yields a ten percentage point decrease in the economy's Sharpe ratio, which is an economically significant increase.

We next investigate the effect of institutional ownership in Table 11. An economy with more institutional shareholders should have more efficient capital markets because these investors are more sophisticated than retail investors. Consistent with this argument, we find a significantly negative coefficient on institutional ownership. In terms of economic significance, a one-standard-deviation increase in institutional ownership produces a twelve percentage point decrease in the economy's Sharpe ratio.

In Table 12 we explore the effects of education levels such as completion rates of primary school. Higher education levels in an economy should be associated with higher financial literacy and thus more efficient markets. Consistent with this hypothesis, we find a significantly negative coefficient of education on an economy's Sharpe ratio. A one-standard-deviation increase in primary completion rates yields a 16 percentage point decrease in an economy's Sharpe ratio, highlighting an economically meaningful effect of education.

We next delve into the question of whether differences in financial constraints have bearing on risk-adjusted returns and market efficiency. Economies with a higher degree of financial constraints make it difficult for firms to invest and grow due to their inability to obtain sufficient outside financing. Stock markets in such economies could be more inefficient due to higher information asymmetries regarding the investment and growth prospects of its firms. In Table 13 we therefore regress minimum variance Sharpe ratios on aggregate measures of financial constraints. We compute the *KZ index* based on Kaplan and Zingales (1997) and Lamont et al. (2001) and the *WW index* based on Whited and Wu (2006) for each firm in our sample. For each economy, we then aggregate each index by taking the median value (for a given economy, we first aggregate across all firms by date, and then aggregate across dates), thus arriving at two measures of financial constraints for each economy. The results in Table 13 however are inconclusive, with the KZ index having an insignificantly positive sign and the WW index being insignificantly negative. We thus do not find evidence that financial constraints affect the performance of minimum variance portfolios and market efficiency around the world.

Finally we also investigate legal origins in Table 14. The basic idea is that legal protection of outside investors limits the extent of expropriation corporate insiders can inflict on them. Legal protection thus boosts financial development, e.g. the size of the stock market (La Porta et al., 1997, 1998). Legal protection varies based on the legal origin of an economy, which broadly falls into common law (originating in English law) and civil law (originating in Roman law) with its subdivisions of French, German, socialist, and Scandinavian civil law.

If financial development is influenced by legal origins, we hypothesize that market efficiency can depend on legal origins as well. We therefore regress minimum variance Sharpe ratios on dummy variables indicating each economy's legal origin in Table 14. We find that common law economies (originating from English law) have significantly higher Sharpe ratios than civil law economies. Depending on the regression specification, common law economies have between 28 and 51 percentage points higher risk-adjusted returns than common law economies. (These numbers are based on the reference category of the dummy variables being French or German as shown in the second and third columns in Table 14.) A similar result is obtained for economies with Scandinavian origin, where the increase in Sharpe ratios is between 26 and 49 percentage points. For French legal origins the results are mixed because depending on the regression specification we obtain both significantly positive and negative coefficients, depending on which reference category is chosen when selecting the dummy variables. If the reference category is chosen to be (English) common law (the regression in the first column), then being an economy with French origins means that risk-adjusted returns are 28 percentage points lower compared to English law. For German law economies, the effect is universally and significantly negative, with German law economies having risk-adjusted returns that are between 23 and 51 percentage points lower compared to economies with other legal origins. In summary we find that English and Scandinavian origins command higher Sharpe ratios while French and German origins have lower Sharpe ratios, with the difference both statistically as well as economically significant.

3.7 Robustness

To gain confidence that the calibration was not data mined, we vary the parameters of our strategy and present the results in Table 15. The table demonstrates that the default values of the parameters never correspond to the highest Sharpe ratio. In other words, we have not chosen the parameters to "data mine" the best possible results.

Another robustness check is to use a different estimator for the covariance matrix. A simple yet efficient way to compute the covariance is through an exponentially weighted moving average (EWMA) covariance model.[4] Using the EWMA covariance estimator, we obtain a Sharpe ratio of 88.2% with our minimum variance strategy applied to the investment universe of the Dow Jones Industrial Average. This is a slight decrease compared to the shrinkage estimator (89.6%), yet still lies well above the Sharpe ratio of the index (77.4% from Table 2).

Finally, as Sharpe ratios can be sensitive to the return interval used, we would like to investigate how they vary given longer return intervals. To this end, we estimate the Sharpe ratios based on two-month and three-month return intervals for the investment universe based on the Dow Jones Industrial Average. We find that the Sharpe ratios increase slightly if longer return intervals are used. Specifically, the Sharpe ratio for the one-month return interval (used for all tests in this paper) is 89.6%, for two months it is 89.8%, and for three months it is 98.4%. As most investors have horizons that exceed one month, it is thus expected that performance would improve for those investors.

[4] https://vlab.stern.nyu.edu/docs/correlation/EWMA-COV, accessed January 2, 2018.

4 Conclusion

In this paper we present a novel version of the minimum variance portfolio and show that it outperforms various stock markets around the world as well as the $1/N$ strategy of DeMiguel et al. (2009) both on a raw as well as on a risk-adjusted basis. We therefore question the strong trend over recent years towards passive investing, which posits that investors should pour their money into passive index funds due to their outperformance over actively-managed funds (Malkiel, 2015). There are also broader implications for market efficiency as well as economic growth and development since indexing can lead to less price discovery and less efficient markets as well as lower financial market stability due to herding effects.

Our version of the minimum variance portfolio starts with the classical minimum variance weights obtained using the shrunk covariance matrix from Ledoit and Wolf (2003). We then smooth out the portfolio weights over time using an exponentially-weighted average, thus further reducing estimation noise and lowering portfolio turnover. Finally, we remove exceedingly small portfolio weights, mainly to make the strategy easier to implement, but also to further reduce estimation noise in the small portfolio weights.

We find that the raw returns of our strategy are on average 20.2% higher than those of the respective stock market indices around the world. Risk adjusted returns are 46.7% larger and drawdowns 28.4% smaller. Annualized alphas are 4.4%, with betas ranging between 0.24 and 0.81, thus again confirming that our strategy has higher returns and lower risk than stock indices around the world. Comparing our strategy to the $1/N$ strategy of DeMiguel et al. (2009), we again find higher returns and lower risk on top of 32.4% lower portfolio turnover, demonstrating that this outperformance is not acquired by incurring higher trading costs.

Investigating recession risk, we find that our minimum variance portfolio has higher returns than the market during recessions. When exploring downside risk measured with option-implied skewness, we find that the higher returns are not due to investors getting compensated for holding stocks with lower skewness (i.e. stocks with high downside risk). Instead, we are to the best of our knowledge the first to find there is not only a risk-return puzzle, but also a skewness-return puzzle in the sense that our portfolio has higher returns on high-skewness stocks (i.e. low downside risk). Using cross-sectional evidence across 41 economies, we then discover that risk-adjusted returns are lower for economies with higher analyst coverage, higher institutional holdings, and higher education levels. These discoveries are consistent with more efficient markets in those economies. Finally, we find evidence that legal origins of the economies around the world matter, with common law economies having between 28 and 51 percentage points higher Sharpe ratios than civil law economies.

In summary, we find compelling evidence around the world that questions the prevalence of passive investing in stock market indices. We discover that a simple actively-managed investing strategy can outperform stock market indices and equally-weighted $1/N$ portfolios on a risk-adjusted basis around the world. This finding is in stark contrast to the tenets of indexing, which argue that it is very difficult or even impossible for investors to do better in the long run than investing in passively-managed index-tracking funds. Given the simplicity of our portfolio and the fact that its foundations have been around for more than half a century (Markowitz, 1952), we believe more research is called for into the exploration and application of related strategies having the capacity to overcome the decades-long underperformance of active investing. For example, market timing could be used to boost risk-adjusted returns further (Maurer et al., 2019). Furthermore, more effort in the actively-managed fund industry should be devoted to the exploration and application of similar strategies to overcome the industry's decades-long underperformance.

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Table 1: Years Included

This table shows the years covered for each economy included in this study. The coverage differs among various economies due to limitations in our data sources. Further details about the data sources can be found in Section 2.

Economy	First Date Included	Last Date Included
Australia	May 2000	Jun 2017
Germany	Jan 1996	Jun 2017
Hong Kong	Nov 2004	Jun 2017
Japan	Feb 2002	Jun 2017
South Africa	Jul 2002	Jun 2017
United Kingdom	Jan 1996	Jun 2017
United States (DJIA Investment Universe)	Nov 1987	Dec 2016
United States (S&P 500 Investment Universe)	Feb 1988	Dec 2016

Table 2: Performance Metrics

This table shows portfolio performance metrics around the world. For each economy, we report the return, volatility, Sharpe ratio, and maximum drawdown for both the minimum variance portfolio as well as the respective stock market index. For all indices we use total return versions that reinvest all cash distributions such as dividends.

(a) Australia

	Min. Var.	ASX 200
Annualized Return	9.7%	8.1%
Annualized Volatility	13.4%	12.8%
Sharpe Ratio	72.7%	63.3%
Maximum Drawdown	60.3%	47.2%

(b) Germany

	Min. Var.	DAX
Annualized Return	13.7%	8.1%
Annualized Volatility	18.5%	21.7%
Sharpe Ratio	73.8%	37.4%
Maximum Drawdown	50.1%	68.3%

(c) Hong Kong

	Min. Var.	HSI
Annualized Return	11.2%	9.2%
Annualized Volatility	11.8%	20.9%
Sharpe Ratio	94.5%	44.1%
Maximum Drawdown	19.4%	57.5%

(d) Japan

	Min. Var.	Nikkei 225
Annualized Return	4.9%	6.2%
Annualized Volatility	13.2%	19.1%
Sharpe Ratio	36.9%	32.7%
Maximum Drawdown	48.8%	57.2%

Table 2: Performance Metrics (cont.)

(e) South Africa

	Min. Var.	JSE FTSE Top 40
Annualized Return	17.4%	13.6%
Annualized Volatility	14.8%	16.9%
Sharpe Ratio	118.2%	80.5%
Maximum Drawdown	26.6%	43.4%

(f) United Kingdom

	Min. Var.	FTSE 100
Annualized Return	9.6%	6.8%
Annualized Volatility	12.5%	13.7%
Sharpe Ratio	76.3%	49.8%
Maximum Drawdown	31.1%	44.4%

(g) United States (DJIA Investment Universe)

	Min. Var.	DJIA
Annualized Return	11.2%	11.0%
Annualized Volatility	12.5%	14.2%
Sharpe Ratio	89.6%	77.4%
Maximum Drawdown	33.3%	47.2%

(h) United States (S&P 500 Investment Universe)

	Min. Var.	S&P 500
Annualized Return	10.3%	10.2%
Annualized Volatility	12.1%	14.3%
Sharpe Ratio	84.7%	71.6%
Maximum Drawdown	25.9%	50.9%

Table 3: Risk-Adjusted Excess Returns

This table shows regressions of each economy's minimum variance portfolio on the respective stock market index and the Fama-French factors. The intercept ("alpha") corresponds to the risk-adjusted excess returns of the minimum variance portfolio. For easier interpretation, we present the alphas as annualized excess returns. For all indices we use total return versions that reinvest all cash distributions such as dividends. Numbers in parentheses are t-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

(a) Australia

	Min. Var.	Min. Var.
alpha (annualized)	2.67	2.52
	(1.13)	(1.01)
Market $-$ r_f	0.85***	0.81***
	(27.67)	(12.78)
SMB		−0.15**
		(−2.11)
HML		0.04
		(0.44)
RMW		0.07
		(0.96)
CMA		−0.05
		(−0.43)
R^2	0.79	0.80
N	205	205

(b) Germany

	Min. Var.	Min. Var.
alpha (annualized)	5.96**	6.89***
	(2.57)	(2.96)
Market $-$ r_f	0.75***	0.71***
	(26.37)	(19.14)
SMB		−0.23***
		(−2.77)
HML		−0.03
		(−0.29)
RMW		−0.07
		(−0.86)
CMA		0.16
		(1.26)
R^2	0.73	0.75
N	256	256

Table 3: Risk-Adjusted Excess Returns (cont.)

(c) Hong Kong

	Min. Var.	Min. Var.
alpha (annualized)	8.38***	7.54**
	(2.61)	(2.30)
Market $-$ r_f	0.18***	0.24***
	(4.18)	(4.38)
SMB		-0.24**
		(-2.20)
HML		-0.26*
		(-1.74)
RMW		-0.17
		(-1.08)
CMA		0.34**
		(2.04)
R^2	0.10	0.17
N	151	151

(d) Japan

	Min. Var.	Min. Var.
alpha (annualized)	2.37	2.18
	(0.71)	(0.72)
Market $-$ r_f	0.37***	0.38***
	(6.41)	(7.20)
SMB		0.06
		(0.57)
HML		-0.23**
		(-2.15)
RMW		-0.45***
		(-4.11)
CMA		0.12
		(0.83)
R^2	0.18	0.37
N	184	184

Table 3: Risk-Adjusted Excess Returns (cont.)

(e) South Africa

	Min. Var.	Min. Var.
alpha (annualized)	3.89	10.11***
	(1.25)	(3.73)
Market $-$ r_f	0.90***	0.62***
	(25.02)	(12.57)
SMB		-0.25**
		(-2.31)
HML		-0.17
		(-1.32)
RMW		-0.13
		(-1.07)
CMA		-0.04
		(-0.21)
R^2	0.78	0.85
N	179	179

(f) United Kingdom

	Min. Var.	Min. Var.
alpha (annualized)	3.39**	3.73**
	(2.01)	(2.16)
Market $-$ r_f	0.74***	0.76***
	(24.37)	(17.74)
SMB		-0.09
		(-1.47)
HML		-0.15**
		(-2.01)
RMW		-0.03
		(-0.40)
CMA		0.22**
		(2.43)
R^2	0.70	0.71
N	257	257

Table 3: Risk-Adjusted Excess Returns (cont.)

(g) United States (DJIA Investment Universe)

	Min. Var.	Min. Var.
alpha (annualized)	2.36*	1.41
	(1.74)	(1.04)
Market $-$ r_f	0.72***	0.76***
	(26.33)	(26.99)
SMB		-0.12***
		(-2.98)
HML		-0.11**
		(-2.22)
RMW		0.13**
		(2.58)
CMA		0.21***
		(2.97)
R^2	0.67	0.70
N	350	350

(h) United States (S&P 500 Investment Universe)

	Min. Var.	Min. Var.
alpha (annualized)	4.59**	1.17
	(2.24)	(0.60)
Market $-$ r_f	0.37***	0.51***
	(9.06)	(11.95)
SMB		-0.12**
		(-2.15)
HML		0.01
		(0.09)
RMW		0.23***
		(3.05)
CMA		0.51***
		(4.72)
R^2	0.19	0.34
N	347	347

Table 4: Performance Metrics of Minimum Variance and $1/N$ Portfolios

This table shows portfolio performance metrics around the world. For each economy, we report the return, volatility, Sharpe ratio, and maximum drawdown for both the minimum variance portfolio and the $1/N$ portfolio. The investment universe is the same for both portfolios, i.e. the constituents of the main stock index of each economy. Both portfolios' returns include reinvestment of all cash distributions such as dividends.

(a) Australia (ASX 200 Investment Universe)

	Min. Var.	1/N
Annualized Return	9.7%	9.9%
Annualized Volatility	13.4%	15.2%
Sharpe Ratio	72.7%	65.0%
Maximum Drawdown	60.3%	52.8%

(b) Germany (DAX Investment Universe)

	Min. Var.	1/N
Annualized Return	13.7%	11.1%
Annualized Volatility	18.5%	22.1%
Sharpe Ratio	73.8%	50.4%
Maximum Drawdown	50.1%	60.8%

(c) Hong Kong (HSI Investment Universe)

	Min. Var.	1/N
Annualized Return	11.2%	10.4%
Annualized Volatility	11.8%	21.8%
Sharpe Ratio	94.5%	47.4%
Maximum Drawdown	19.4%	52.9%

(d) Japan (Nikkei 225 Investment Universe)

	Min. Var.	1/N
Annualized Return	4.9%	9.2%
Annualized Volatility	13.2%	20.3%
Sharpe Ratio	36.9%	45.2%
Maximum Drawdown	48.8%	57.5%

Table 4: Performance Metrics of Minimum Variance and $1/N$ Portfolios (cont.)

(e) South Africa (JSE FTSE Top 40 Investment Universe)

	Min. Var.	1/N
Annualized Return	17.4%	13.7%
Annualized Volatility	14.8%	16.9%
Sharpe Ratio	118.2%	81.0%
Maximum Drawdown	26.6%	28.6%

(f) United Kingdom (FTSE 100 Investment Universe)

	Min. Var.	1/N
Annualized Return	9.6%	8.9%
Annualized Volatility	12.5%	14.5%
Sharpe Ratio	76.3%	61.1%
Maximum Drawdown	31.1%	47.8%

(g) United States (DJIA Investment Universe)

	Min. Var.	1/N
Annualized Return	11.2%	11.5%
Annualized Volatility	12.5%	15.4%
Sharpe Ratio	89.6%	74.9%
Maximum Drawdown	33.3%	57.0%

(h) United States (S&P 500 Investment Universe)

	Min. Var.	1/N
Annualized Return	10.3%	12.1%
Annualized Volatility	12.1%	15.6%
Sharpe Ratio	84.7%	77.5%
Maximum Drawdown	25.9%	54.2%

Table 5: Risk-Adjusted Excess Returns of Minimum Variance and $1/N$ Portfolios

This table shows regressions of each economy's minimum variance portfolio and $1/N$ portfolio on the respective stock market index and the Fama-French factors. The investment universe for both portfolios is that of the respective stock market index. The intercept ("alpha") corresponds to the risk-adjusted excess returns of the minimum variance portfolio. For easier interpretation, we present the alphas as annualized excess returns. For all portfolios we reinvest all cash distributions such as dividends. Numbers in parentheses are t-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

(a) Australia (ASX 200 Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	2.67	2.52	1.67	2.07
	(1.13)	(1.01)	(0.93)	(1.11)
Market $-$ r_f	0.85***	0.81***	1.03***	1.02***
	(27.67)	(12.78)	(43.80)	(21.56)
SMB		−0.15**		0.12**
		(−2.11)		(2.34)
HML		0.04		0.12*
		(0.44)		(1.82)
RMW		0.07		0.08
		(0.96)		(1.45)
CMA		−0.05		−0.28***
		(−0.43)		(−3.19)
R^2	0.79	0.80	0.90	0.91
N	205	205	205	205

(b) Germany (DAX Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	5.96**	6.89***	2.73**	2.86**
	(2.57)	(2.96)	(2.27)	(2.49)
Market $-$ r_f	0.75***	0.71***	1.00***	1.01***
	(26.37)	(19.14)	(67.87)	(54.91)
SMB		−0.23***		0.05
		(−2.77)		(1.24)
HML		−0.03		0.04
		(−0.29)		(0.88)
RMW		−0.07		−0.22***
		(−0.86)		(−5.17)
CMA		0.16		0.18***
		(1.26)		(2.73)
R^2	0.73	0.75	0.95	0.96
N	256	256	256	256

Table 5: Risk-Adjusted Excess Returns of Minimum Variance and $1/N$ Portfolios (cont.)

(c) Hong Kong (HSI Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	8.38***	7.54**	1.47	2.42
	(2.61)	(2.30)	(0.69)	(1.18)
Market $-$ r_f	0.18***	0.24***	0.98***	0.90***
	(4.18)	(4.38)	(33.60)	(26.41)
SMB		−0.24**		−0.15**
		(−2.20)		(−2.28)
HML		−0.26*		0.16*
		(−1.74)		(1.70)
RMW		−0.17		−0.37***
		(−1.08)		(−3.63)
CMA		0.34**		−0.21**
		(2.04)		(−2.04)
R^2	0.10	0.17	0.88	0.91
N	151	151	151	151

(d) Japan (Nikkei 225 Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	2.37	2.18	2.61**	1.22
	(0.71)	(0.72)	(2.01)	(1.05)
Market $-$ r_f	0.37***	0.38***	1.04***	1.06***
	(6.41)	(7.20)	(45.80)	(51.72)
SMB		0.06		0.19***
		(0.57)		(4.96)
HML		−0.23**		0.21***
		(−2.15)		(5.10)
RMW		−0.45***		−0.23***
		(−4.11)		(−5.43)
CMA		0.12		−0.15***
		(0.83)		(−2.67)
R^2	0.18	0.37	0.92	0.94
N	184	184	184	184

Table 5: Risk-Adjusted Excess Returns of Minimum Variance and $1/N$ Portfolios (cont.)

(e) South Africa (JSE FTSE Top 40 Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	3.89	10.11***	−0.07	3.17
	(1.25)	(3.73)	(−0.03)	(1.24)
Market − r_f	0.90***	0.62***	0.99***	0.85***
	(25.02)	(12.57)	(32.79)	(18.18)
SMB		−0.25**		0.06
		(−2.31)		(0.59)
HML		−0.17		−0.23*
		(−1.32)		(−1.91)
RMW		−0.13		−0.16
		(−1.07)		(−1.42)
CMA		−0.04		0.00
		(−0.21)		(0.02)
R^2	0.78	0.85	0.86	0.88
N	179	179	179	179

(f) United Kingdom (FTSE 100 Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	3.39**	3.73**	1.93	2.52**
	(2.01)	(2.16)	(1.45)	(1.98)
Market − r_f	0.74***	0.76***	0.99***	0.97***
	(24.37)	(17.74)	(41.49)	(30.90)
SMB		−0.09		0.23***
		(−1.47)		(4.87)
HML		−0.15**		0.10*
		(−2.01)		(1.92)
RMW		−0.03		−0.24***
		(−0.40)		(−4.71)
CMA		0.22**		−0.11
		(2.43)		(−1.56)
R^2	0.70	0.71	0.87	0.89
N	257	257	257	257

Table 5: Risk-Adjusted Excess Returns of Minimum Variance and $1/N$ Portfolios (cont.)

(g) United States (DJIA Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	2.36*	1.41	0.12	0.10
	(1.74)	(1.04)	(0.23)	(0.21)
Market $- r_f$	0.72***	0.76***	1.07***	1.06***
	(26.33)	(26.99)	(102.55)	(103.96)
SMB		−0.12***		0.02
		(−2.98)		(1.39)
HML		−0.11**		0.14***
		(−2.22)		(7.64)
RMW		0.13**		−0.05***
		(2.58)		(−2.91)
CMA		0.21***		−0.05**
		(2.97)		(−1.97)
R^2	0.67	0.70	0.97	0.97
N	350	350	350	350

(h) United States (S&P 500 Investment Universe)

	Min. Var.	Min. Var.	$1/N$	$1/N$
alpha (annualized)	4.59**	1.17	1.68*	−0.90
	(2.24)	(0.60)	(1.67)	(−1.19)
Market $- r_f$	0.37***	0.51***	1.03***	1.08***
	(9.06)	(11.95)	(50.94)	(65.64)
SMB		−0.12**		0.30***
		(−2.15)		(13.55)
HML		0.01		0.20***
		(0.09)		(7.04)
RMW		0.23***		0.18***
		(3.05)		(6.48)
CMA		0.51***		0.07
		(4.72)		(1.62)
R^2	0.19	0.34	0.88	0.94
N	347	347	347	347

Table 6: Annualized Portfolio Turnover

This table shows annualized portfolio turnovers for the minimum variance portfolio and the $1/N$ portfolio. For a description of the construction of the portfolio turnover measure see Section 3.2.

Economy	Investment Universe	Min. Var.	$1/N$
Australia	ASX 200	60.9%	117.0%
Germany	DAX	55.3%	71.8%
Hong Kong	HSI	30.0%	71.9%
Japan	Nikkei 225	63.7%	77.2%
South Africa	JSE FTSE Top 40	85.1%	118%
United Kingdom	FTSE 100	61.1%	91.4%
United States	DJIA	46.2%	60.2%
United States	S&P 500	62.3%	79.6%
Average Portfolio Turnover		58.1%	85.9%

Table 7: Recession Risk

This table shows time series regressions of minimum variance portfolio returns and the returns of the economy's stock market index (shown in alternating columns) on recession dummy variables. In addition to each economy's own recession dummy (if available, shown in the row below "*Intercept*"), we add recession indicators for the world's two largest economies, the U.S. (*USREC*) and China (*CHNRECM*). *REC* is a dummy that is one if there is a recession in any of the following economies: Australia, China, Euro Area, Japan, the UK, the U.S., or South Africa. Due to potential multicollinearity, we only include one recession dummy in each regression specification. Coefficients are multiplied by 100 for readability. Numbers in parentheses are t-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

(a) Australia (ASX 200 Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	1.01***	0.84**	1.22***	0.98***	1.47***	1.40***	1.40**	1.71***
	(2.89)	(2.49)	(4.35)	(3.59)	(4.22)	(4.22)	(2.35)	(3.02)
AUSRECDM	−0.39	−0.28						
	(−0.71)	(−0.54)						
USREC			−2.87***	−2.02***				
			(−3.64)	(−2.64)				
CHNRECM					−1.46***	−1.62***		
					(−2.72)	(−3.16)		
REC							−0.68	−1.24*
							(−1.02)	(−1.96)
R^2	0.00	0.00	0.06	0.03	0.04	0.05	0.01	0.02
Num. obs.	206	206	206	206	206	206	206	206

(b) Germany (DAX Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	1.89***	1.56***	1.55***	1.19***	1.68***	1.53***	1.85**	2.33***
	(4.03)	(2.84)	(4.49)	(2.93)	(3.58)	(2.79)	(2.40)	(2.60)
DEUREC	−1.36**	−1.40*						
	(−2.04)	(−1.80)						
USREC			−3.32***	−3.40***				
			(−3.05)	(−2.66)				
CHNRECM					−0.93	−1.37*		
					(−1.40)	(−1.76)		
REC							−0.78	−1.83*
							(−0.92)	(−1.83)
R^2	0.02	0.01	0.04	0.03	0.01	0.01	0.00	0.01
Num. obs.	256	256	257	257	257	257	257	257

Table 7: Recession Risk (cont.)

(c) Hong Kong (HSI Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	1.05***	1.23**	1.10***	2.24***	0.64	2.17**
	(3.57)	(2.38)	(2.96)	(3.51)	(1.22)	(2.34)
USREC	−0.90	−2.64*				
	(−1.04)	(−1.75)				
CHNRECM			−0.35	−2.98***		
			(−0.62)	(−3.11)		
REC					0.42	−1.73
					(0.67)	(−1.59)
R^2	0.01	0.02	0.00	0.06	0.00	0.02
Num. obs.	152	152	152	152	152	152

(d) Japan (Nikkei 225 Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	0.73**	1.03**	0.60**	0.92**	0.59*	1.35***	0.78	1.13
	(2.01)	(1.97)	(2.03)	(2.18)	(1.68)	(2.69)	(1.31)	(1.32)
JPNRECP	−0.68	−0.96						
	(−1.18)	(−1.16)						
USREC			−1.31	−2.71**				
			(−1.39)	(−2.00)				
CHNRECM					−0.33	−1.90**		
					(−0.57)	(−2.28)		
REC							−0.39	−0.61
							(−0.59)	(−0.62)
R^2	0.01	0.01	0.01	0.02	0.00	0.03	0.00	0.00
Num. obs.	184	184	185	185	185	185	185	185

Table 7: Recession Risk (cont.)

(e) South Africa (JSE FTSE Top 40 Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	1.97***	1.92***	1.64***	1.43***	1.89***	1.88***	2.36***	2.89***
	(5.04)	(4.33)	(4.94)	(3.76)	(4.75)	(4.16)	(3.60)	(3.90)
ZAFREC	−1.49**	−2.05***						
	(−2.24)	(−2.71)						
USREC			−2.03*	−2.42**				
			(−1.93)	(−2.01)				
CHNRECM					−1.22*	−1.87**		
					(−1.87)	(−2.52)		
REC							−1.20	−2.22***
							(−1.60)	(−2.62)
R^2	0.03	0.04	0.02	0.02	0.02	0.03	0.01	0.04
Num. obs.	179	179	180	180	180	180	180	180

(f) United Kingdom (FTSE 100 Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	1.01***	0.88***	1.03***	0.86***	0.99***	1.02***	0.88*	1.09*
	(3.35)	(2.67)	(4.39)	(3.36)	(3.10)	(2.94)	(1.67)	(1.90)
GBRRECDM	−0.41	−0.56						
	(−0.91)	(−1.13)						
USREC			−1.98***	−2.28***				
			(−2.68)	(−2.82)				
CHNRECM					−0.32	−0.78		
					(−0.70)	(−1.59)		
REC							−0.06	−0.56
							(−0.10)	(−0.88)
R^2	0.00	0.01	0.03	0.03	0.00	0.01	0.00	0.00
Num. obs.	258	258	258	258	258	258	258	258

Table 7: Recession Risk (cont.)

(g) United States (DJIA Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	1.11***	1.15***	0.90***	1.05***	0.75	0.81
	(5.51)	(5.06)	(3.38)	(3.47)	(1.60)	(1.53)
USREC	−1.62**	−2.03***				
	(−2.50)	(−2.77)				
CHNRECM			0.11	−0.19		
			(0.29)	(−0.44)		
REC					0.25	0.18
					(0.49)	(0.31)
R^2	0.02	0.02	0.00	0.00	0.00	0.00
Num. obs.	350	350	350	350	350	350

(h) United States (S&P 500 Investment Universe)

	Min. Var.	Index	Min. Var.	Index	Min. Var.	Index
(Intercept)	0.99***	1.09***	0.86***	1.08***	0.34	0.88
	(5.01)	(4.72)	(3.29)	(3.52)	(0.73)	(1.62)
USREC	−1.10*	−1.94***				
	(−1.75)	(−2.63)				
CHNRECM			0.05	−0.37		
			(0.13)	(−0.84)		
REC					0.65	0.02
					(1.28)	(0.03)
R^2	0.01	0.02	0.00	0.00	0.00	0.00
Num. obs.	347	347	347	347	347	347

Table 8: Skewness and Performance Metrics

This table shows performance metrics of the U.S. minimum variance portfolio on different subsets of the investment universe based on skewness. Specifically, the universe consists of the DJIA index constituents split up into terciles (shown in the three columns) according to their option-implied skewness (details in Section 3.5). Low skewness corresponds to high downside risk while high skewness corresponds to low downside risk.

	Low Skew	Mid Skew	High Skew
Annualized Return	7.8%	8.6%	11.7%
Annualized Volatility	14.2%	14.0%	15.5%
Sharpe Ratio	55.0%	61.7%	75.8%
Maximum Drawdown	41.2%	32.8%	39.0%

Table 9: Skewness and Risk-Adjusted Returns

This table shows regressions of the U.S. minimum variance portfolio on the market and the Fama-French five factors. The intercept ("alpha") corresponds to risk-adjusted excess returns. For easier interpretation, we present the alphas as annualized excess returns. The investment universe holds the constituents of the Dow Jones Industrial Average, split up into terciles according to option-implied skewness (details in Section 3.5). The different columns relate to different skewness subsets of the investment universe. Low skewness corresponds to high downside risk while high skewness corresponds to low downside risk. Numbers in parentheses are *t*-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Low Skew	Mid Skew	High Skew	Low Skew	Mid Skew	High Skew
alpha (annualized)	0.66	1.02	3.61**	−0.21	−0.25	2.44
	(0.38)	(0.78)	(2.23)	(−0.12)	(−0.20)	(1.54)
Market − r_f	0.80***	0.86***	0.92***	0.85***	0.91***	0.96***
	(24.13)	(34.22)	(29.56)	(24.26)	(35.54)	(29.49)
SMB				−0.14***	−0.08**	−0.10**
				(−2.83)	(−2.11)	(−2.09)
HML				−0.08	−0.10**	−0.01
				(−1.32)	(−2.13)	(−0.24)
RMW				0.16***	0.18***	0.12**
				(2.62)	(3.97)	(2.07)
CMA				0.13	0.17***	0.20**
				(1.57)	(2.83)	(2.57)
R^2	0.71	0.83	0.78	0.74	0.86	0.81
Num. obs.	244	244	244	244	244	244

Table 10: Analyst Coverage

This table shows cross-sectional regressions (across economies) of the minimum variance Sharpe ratio (a measure of risk-adjusted returns) on the number of analysts. *Analysts* is the number of analysts covering an economy's stock market, scaled to have a mean of zero and a standard deviation of one for easier interpretation. "# *months*" is a control variable consisting of the number of observations (months) we have available in Compustat Global for a given economy. Standard errors are adjusted according to White (1980) and numbers in parentheses are z-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Sharpe Ratio	Sharpe Ratio	Sharpe Ratio
(Intercept)	0.75^{***}	0.65^{*}	0.68^{*}
	(13.62)	(1.67)	(1.76)
Analysts	-0.10^{**}		-0.10^{**}
	(−2.43)		(−2.25)
# months		0.00	0.00
		(0.27)	(0.19)
R^2	0.07	0.00	0.07
Num. obs.	41	41	41

Table 11: Institutional Ownership

This table shows cross-sectional regressions (across economies) of the minimum variance Sharpe ratio (a measure of risk-adjusted returns) on institutional ownership holdings. *Shares* is the sum of shares held by institutional investors in an economy divided by the number of shares outstanding, scaled to have a mean of zero and a standard deviation of one for easier interpretation. "*# months*" is a control variable consisting of the number of observations (months) we have available in Compustat Global for a given economy. Standard errors are adjusted according to White (1980) and numbers in parentheses are z-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Sharpe Ratio	Sharpe Ratio	Sharpe Ratio
(Intercept)	0.72^{***}	0.48	0.52^{*}
	(13.68)	(1.28)	(1.66)
Shares	-0.12^{***}		-0.12^{***}
	(−2.59)		(−2.65)
# months		0.00	0.00
		(0.66)	(0.66)
R^2	0.20	0.02	0.21
Num. obs.	19	19	19

Table 12: Education Levels

This table shows cross-sectional regressions (across economies) of the minimum variance Sharpe ratio (a measure of risk-adjusted returns) on education levels. *Education* is the "primary completion rate, both sexes (%)" from World Bank Education Statistics (EdStats), scaled to have a mean of zero and a standard deviation of one for easier interpretation. "# *months*" is a control variable consisting of the number of observations (months) we have available in Compustat Global for a given economy. Standard errors are adjusted according to White (1980) and numbers in parentheses are z-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Sharpe Ratio	Sharpe Ratio	Sharpe Ratio
(Intercept)	0.76^{***}	0.61	0.61^{*}
	(13.58)	(1.54)	(1.72)
Education	-0.16^{***}		-0.16^{***}
	(−3.24)		(−3.11)
# months		0.00	0.00
		(0.40)	(0.47)
R^2	0.18	0.01	0.19
Num. obs.	37	37	37

Table 13: Financial Constraints

This table shows cross-sectional regressions (across economies) of the minimum variance Sharpe ratio (a measure of risk-adjusted returns) on measures of financial constraints. *KZ Index* and *WW Index* are indices of financial constraints based on Kaplan and Zingales (1997) and Whited and Wu (2006), for easier interpretation scaled to have a mean of zero and a standard deviation of one. "*# months*" is a control variable consisting of the number of observations (months) we have available in Compustat Global for a given economy. Standard errors are adjusted according to White (1980) and numbers in parentheses are z-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Sharpe Ratio	Sharpe Ratio	Sharpe Ratio
(Intercept)	0.65	0.63	0.63
	(1.64)	(1.64)	(1.62)
KZ Index	0.06		0.05
	(0.95)		(0.85)
WW Index		−0.06	−0.06
		(−1.13)	(−1.14)
# months	0.00	0.00	0.00
	(0.25)	(0.33)	(0.31)
R^2	0.03	0.03	0.05
Num. obs.	41	41	41

Table 14: Legal Origins

This table shows cross-sectional regressions (across economies) of the minimum variance Sharpe ratio (a measure of risk-adjusted returns) on legal origins. *English*, *French*, *German* and *Scandinavian* are dummy variables indicating an economy's legal origin. "*# months*" is a control variable consisting of the number of observations (months) we have available in Compustat Global for a given economy. Standard errors are adjusted according to White (1980) and numbers in parentheses are z-statistics. Asterisks ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Sharpe Ratio	Sharpe Ratio	Sharpe Ratio	Sharpe Ratio
(Intercept)	0.98^{***}	0.70^{**}	0.47	0.96^{***}
	(2.65)	(2.07)	(1.44)	(2.60)
English		0.28^{**}	0.51^{***}	0.02
		(2.03)	(4.18)	(0.18)
French	-0.28^{**}		0.23^{**}	-0.26^{**}
	(−2.03)		(2.37)	(−2.53)
German	-0.51^{***}	-0.23^{**}		-0.49^{***}
	(−4.18)	(−2.37)		(−5.17)
Scandinavian	−0.02	0.26^{**}	0.49^{***}	
	(−0.18)	(2.53)	(5.17)	
# months	−0.00	−0.00	−0.00	−0.00
	(−0.07)	(−0.07)	(−0.07)	(−0.07)
R^2	0.28	0.28	0.28	0.28
Num. obs.	41	41	41	41

Table 15: Robustness

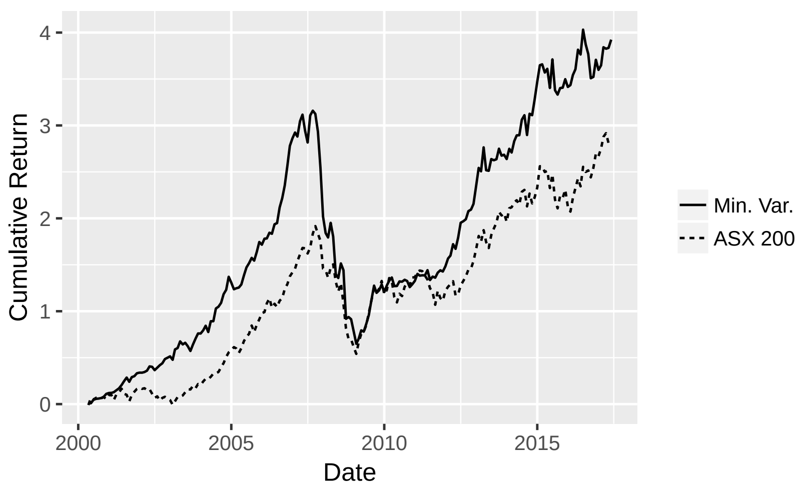
This table shows robustness tests by comparative statics. Specifically, we vary the parameters of our strategy and present the resulting Sharpe ratios for the Dow Jones Industrial Average. For reference, the default values used throughout this study are $\bar{w} = 3\%$ (stocks with a portfolio weight below this value are removed from the portfolio), $l = 120$ (the length of the estimation window in months of the covariance matrix), and $\rho = \frac{1}{15}$ (the parameter to reduce the portfolio turnover). The default values correspond to the middle column in the table below. Each row shows the parameter being varied and the corresponding Sharpe ratio below.

Sharpe Ratios				
$\bar{w} = 1\%$	$\bar{w} = 2\%$	$\bar{w} = 3\%$	$\bar{w} = 4\%$	$\bar{w} = 5\%$
92.2%	91.6%	89.6%	90.6%	88.7%
$l = 100$	$l = 110$	$l = 120$	$l = 130$	$l = 140$
88.4%	88.8%	89.6%	90.2%	95.1%
$\rho = \frac{1}{17}$	$\rho = \frac{1}{16}$	$\rho = \frac{1}{15}$	$\rho = \frac{1}{14}$	$\rho = \frac{1}{13}$
90.0%	90.1%	89.6%	89.3%	88.9%

Figure 1: Cumulative Returns

This figure shows time series of cumulative returns around the world. For each economy, we plot the minimum variance portfolio as well as the respective stock market index. The names of the stock market indices for each economy can be found in the legend on the right-hand side of the figures. For all indices we use total return versions that reinvest all cash distributions such as dividends.

(a) Australia



(b) Germany

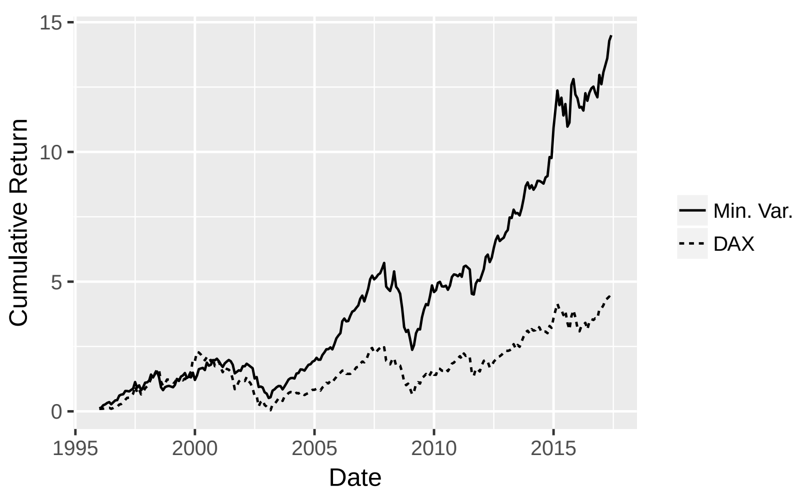
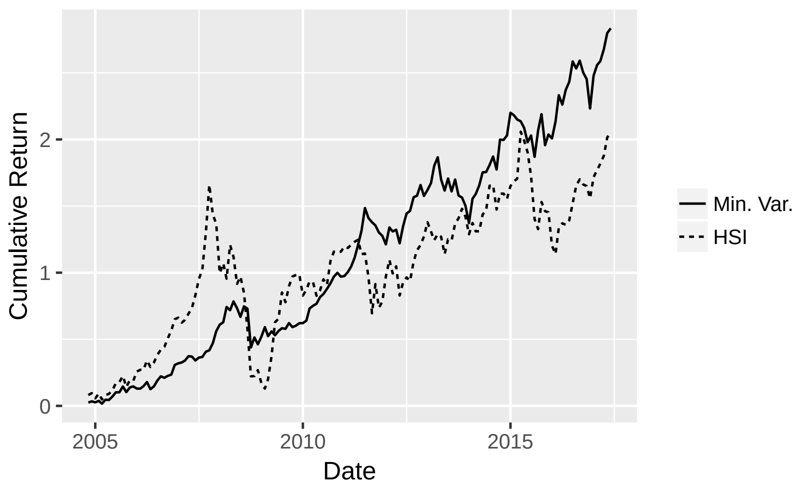


Figure 1: Cumulative Returns (cont.)

(c) Hong Kong



(d) Japan

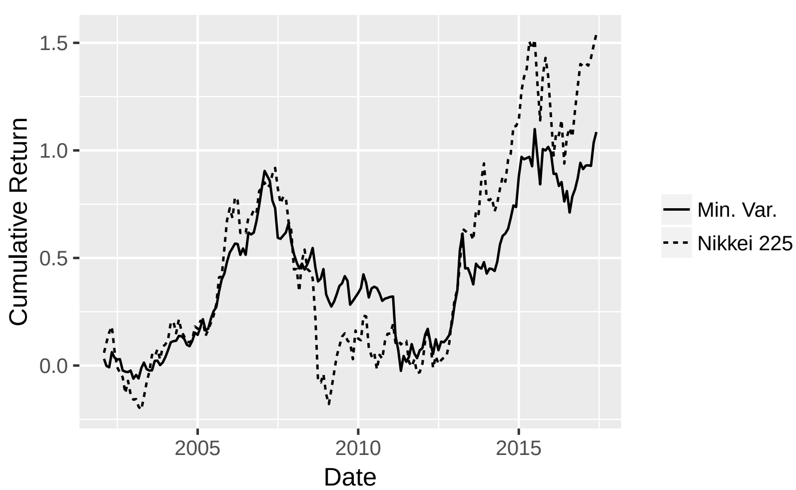
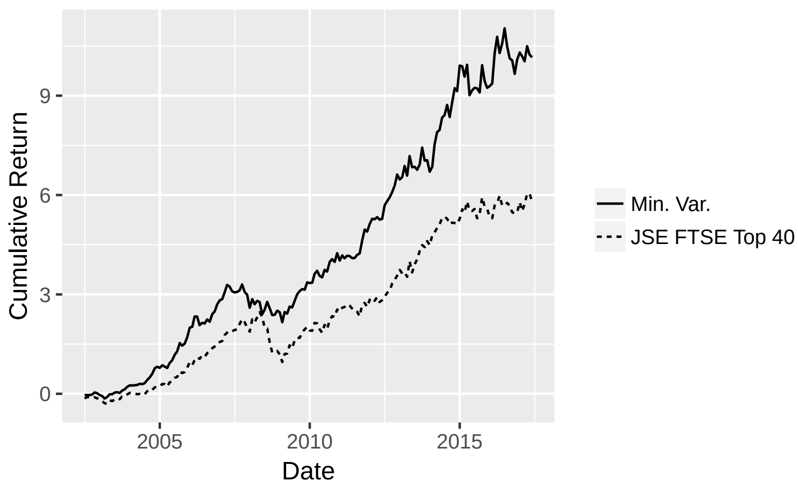


Figure 1: Cumulative Returns (cont.)

(e) South Africa



(f) United Kingdom

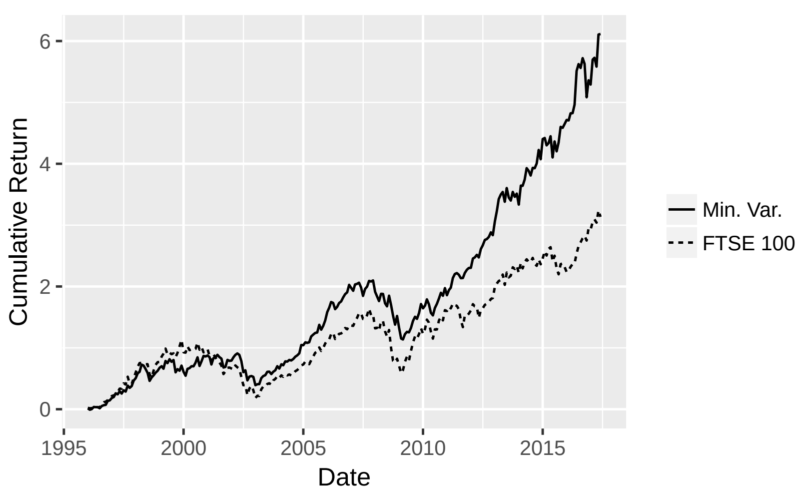
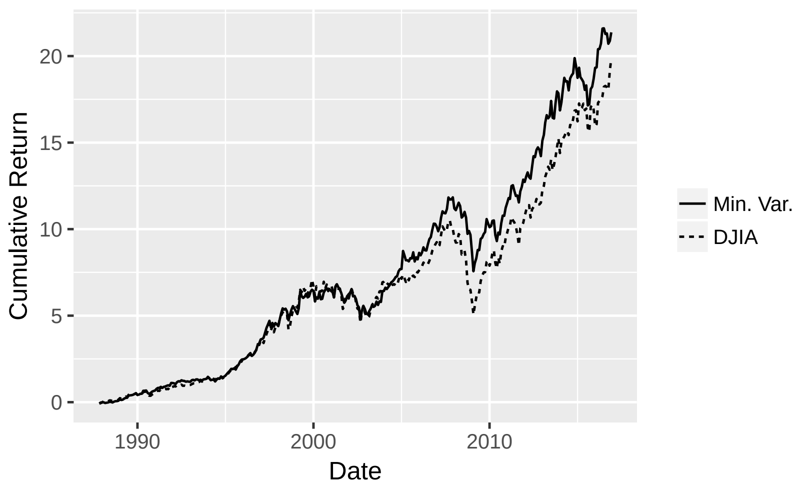


Figure 1: Cumulative Returns (cont.)

(g) United States (DJIA Investment Universe)



(h) United States (S&P 500 Investment Universe)

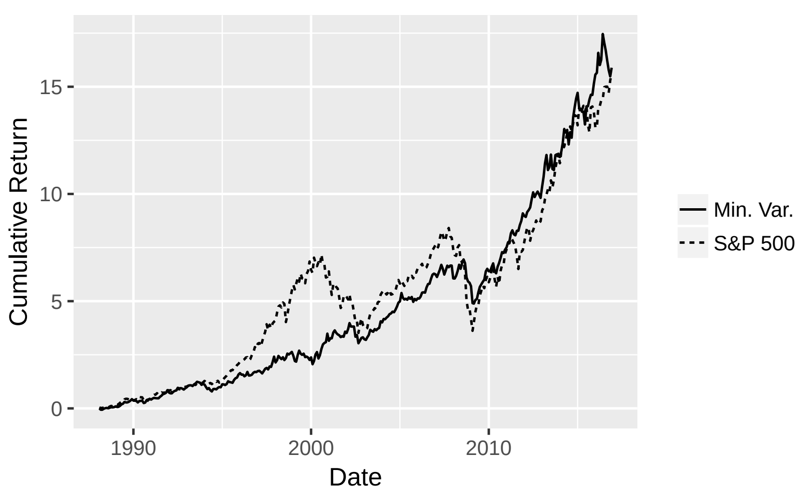
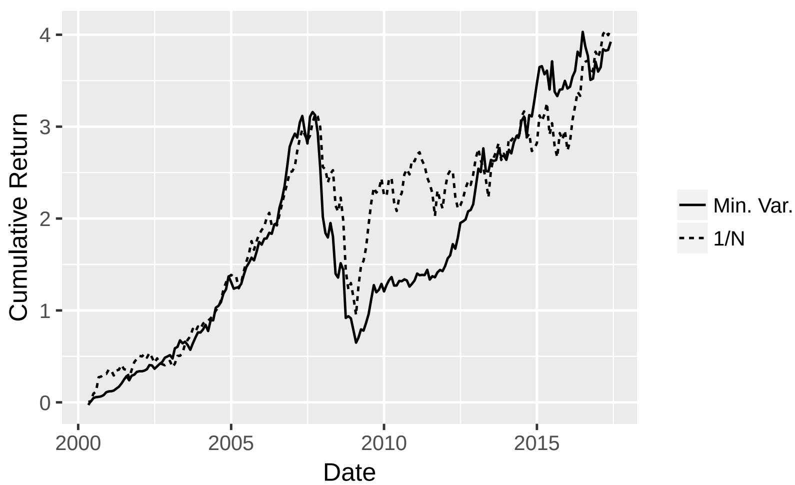


Figure 2: Cumulative Returns of Minimum Variance and $1/N$ Portfolios

This figure shows time series of cumulative returns around the world. For each economy, we plot the minimum variance portfolio and the $1/N$ Portfolio. The investment universe is the same for both portfolios, i.e. the constituents of the main stock index of each economy. Both portfolios' returns include reinvestment of all cash distributions such as dividends.

(a) Australia (ASX 200 Investment Universe)



(b) Germany (DAX Investment Universe)

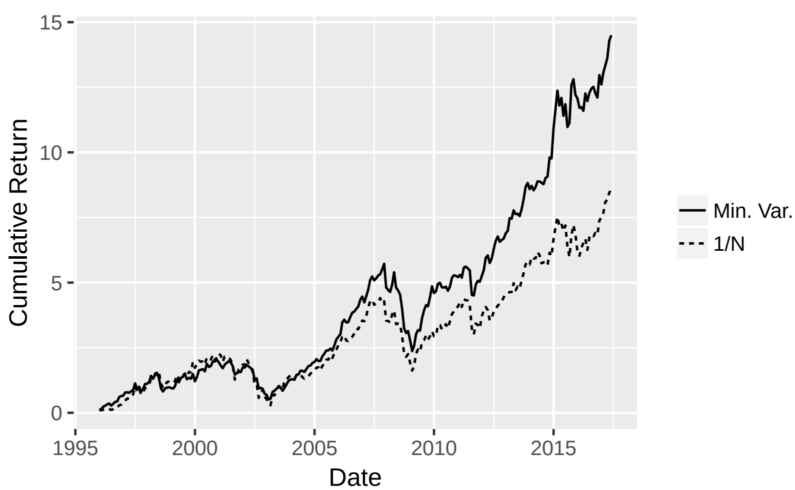


Figure 2: Cumulative Returns of Minimum Variance and $1/N$ Portfolios (cont.)

(c) Hong Kong (HSI Investment Universe)

(d) Japan (Nikkei 225 Investment Universe)

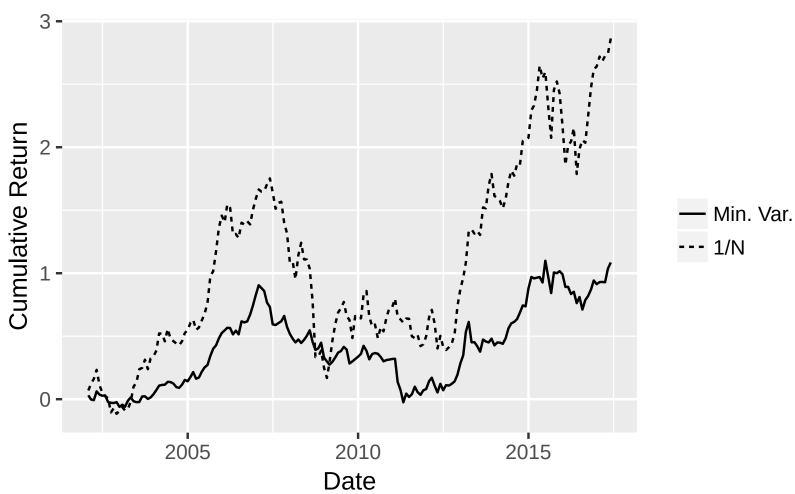
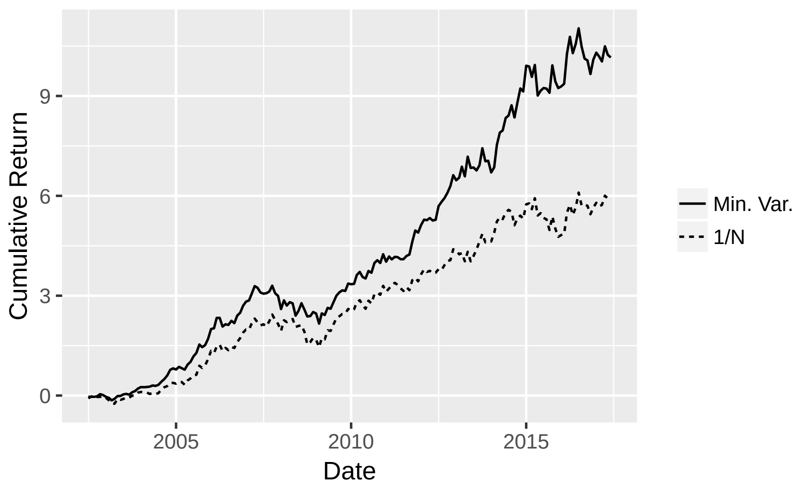


Figure 2: Cumulative Returns of Minimum Variance and 1/N Portfolios (cont.)

(e) South Africa (JSE FTSE Top 40 Investment Universe)



(f) United Kingdom (FTSE 100 Investment Universe)

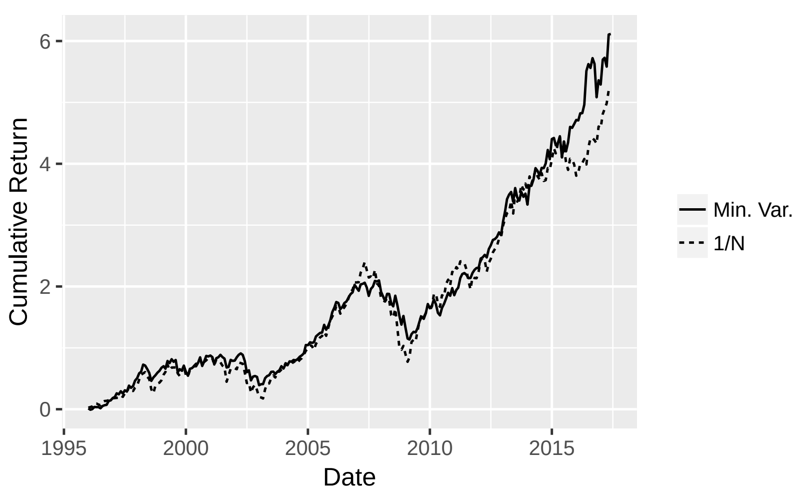
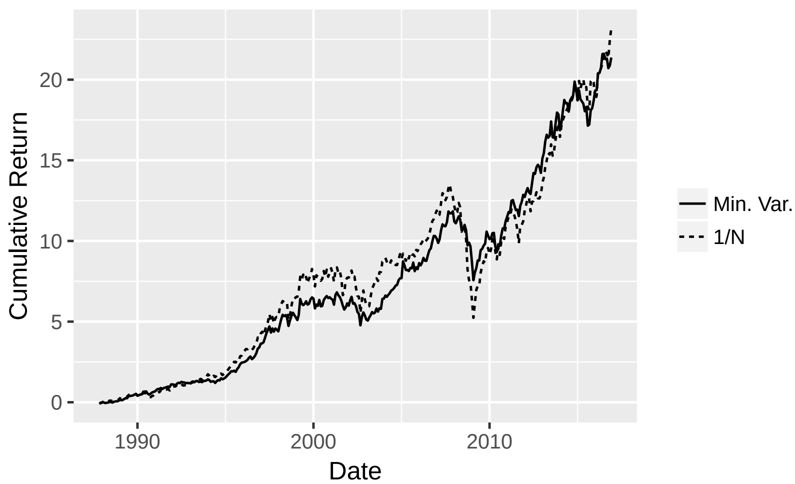


Figure 2: Cumulative Returns of Minimum Variance and 1/N Portfolios (cont.)

(g) United States (DJIA Investment Universe)



(h) United States (S&P 500 Investment Universe)

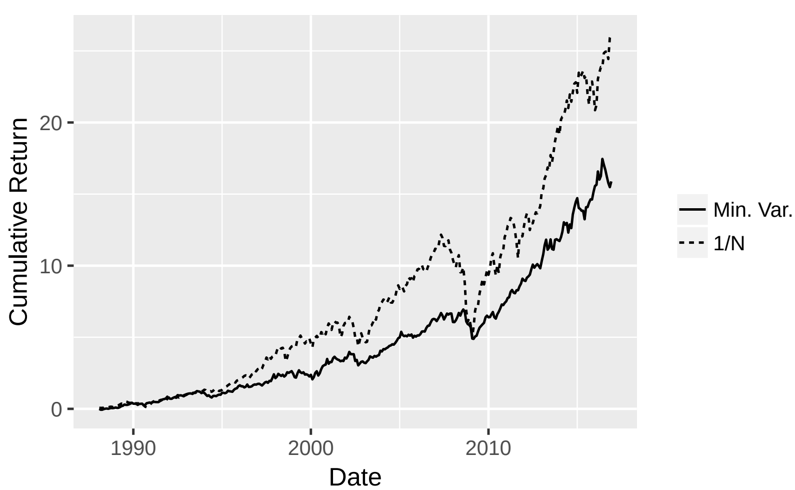
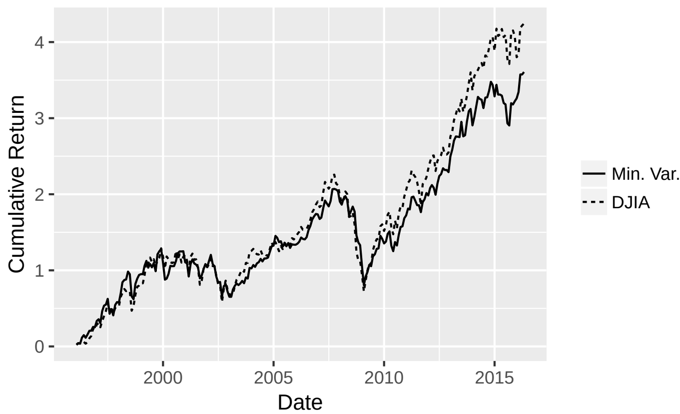


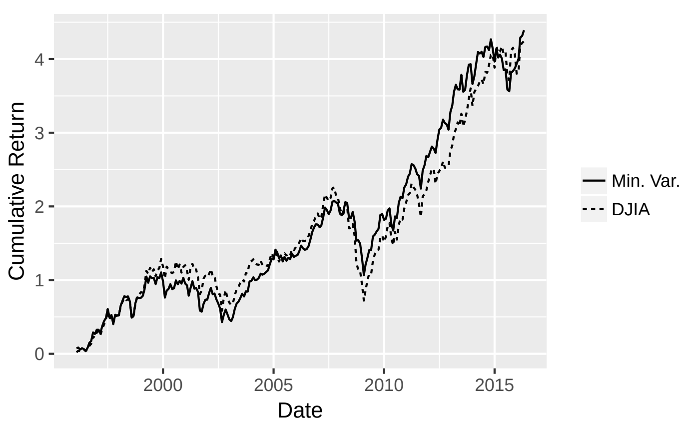
Figure 3: Downside Risk and Skewness

This figure shows U.S. minimum variance portfolio cumulative returns. The investment universe consists of the constituents of the Dow Jones Industrial Average (DJIA), split up according to option-implied skewness into terciles (details in Section 3.5). For comparison we also plot the DJIA in each panel (based on all index constituents, not just the subsets, to keep the index as a constant reference point).

(a) Low Skewness (Large Downside Risk)



(b) Mid Skewness (Medium Downside Risk)



(c) High Skewness (Low Downside Risk)

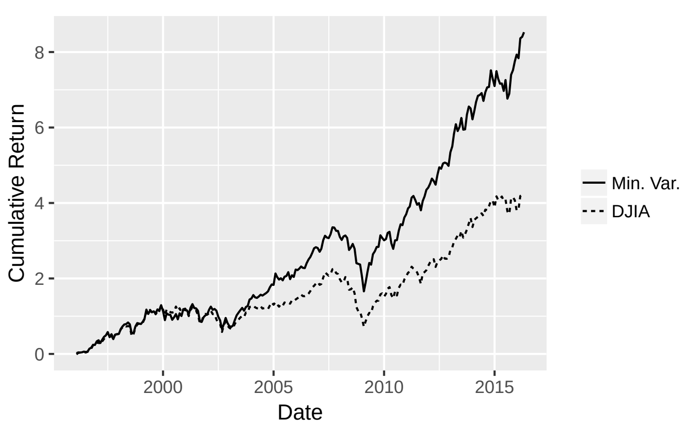


Figure 4: Sharpe Ratios of Minimum Variance Portfolios Around the World