Social Equity-Based Timetabling and Ticket Pricing for High-Speed Railways

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Abstract

High-speed railways are expanding rapidly over the world, providing fast, convenient, and comfortable transport. It is therefore important to ensure that people are not excluded from high-speed railway use due to relatively high ticket prices. We aim to develop a railway timetable according to passenger requirements that explicitly considers social equity by applying variable ticket prices to trains. Specifically, we want to optimize ticket prices for trains operating during off-peak hours to improve accessibility for low-income passengers to improve social justice. A mixed integer linear programming model is formulated to solve our problem. In this timetabling-based model, we not only minimize the total general travel cost from a passenger perspective but also ensure that the revenue of the train company is no less than a specified value and that social equity is maintained at a specific level. To do this, we consider the social equity constraint and the minimum revenue requirement constraint, in addition to the traditional passenger-centric (-oriented) train timetabling-related constraints. Finally, we test our model on both a hypothetical high-speed railway network and the Guangzhou–Nanning and Guangzhou–Guilin high-speed railway network in China. The results demonstrate that the proposed optimization framework can provide a convenient timetable for passengers, adequate revenue for train companies and enhance social equity to meet government targets.

Keywords: High-speed railway, train timetabling, ticket pricing, social equity

1. Introduction

High-speed railway (HSR) networks have expanded rapidly throughout the world over the past ten years, especially in China. According to the “Mid- to Long-term Railway Network Plan (Version 2016)” , the total length of China’s HSR will reach 30,000 km by 2020, and most cities with a population greater than 0.5 million will be served by a HSR. HSRs are very attractive due to their high speed, high frequency, travel-time reliability, and relatively low fare (compared with flights). They significantly improve the social economy, standard of living, and several other aspects of society, including tourism (Pagliara et al., 2015; Yin et al., 2019) and labour migration (Guirao et al., 2017, 2018). Although building a new HSR is expensive and unlikely to be economically profitable when put into operation, the social profitability is significant (Albalate and Bel, 2012). HSRs may shape the social economic landscape (Martin, 1997; Laurino et al., 2015) and reduce the regional development gaps within a country (Hu et al., 2015). Therefore, it is important to build high-speed railways, especially in large, highly populated countries, e.g., China. To date (Figure 1), China’s relatively large HSR network is more than 28,000 km long, which is greater than the total length of all HSR lines in all other countries in the world. The mainland portion of this network is shown in Figure 1.

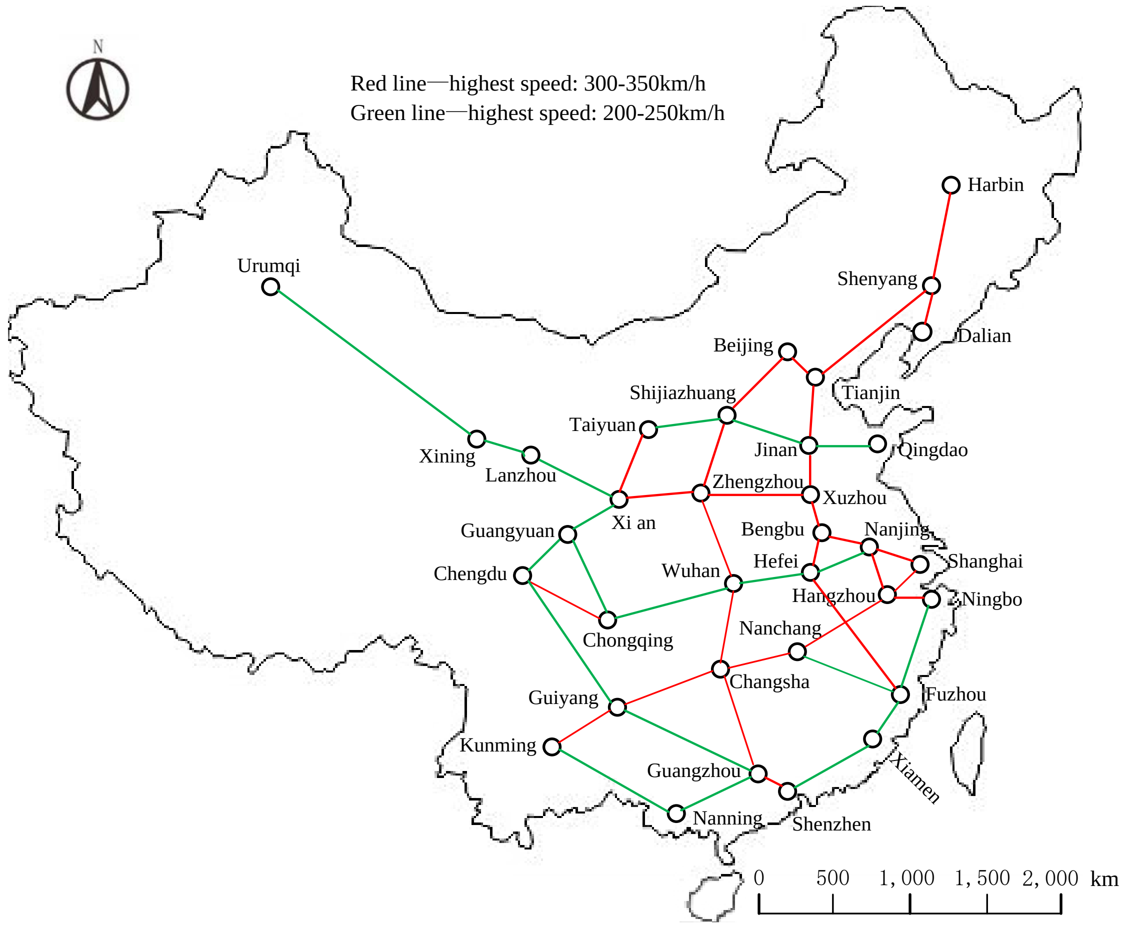


Figure 1 The HSR network of mainland China.

Although a large HSR network can improve social economic levels and peoples' standard of living, it may also bring some problems, such as social inequity. The term "transportation equity" refers to the fair or just distribution of transportation costs and benefits among current (and future) members of society (Litman, 2002). As HSRs in China (as in some other countries) are fully or partly financed by central and local government, they are a public transport mode that should be focused on social equity at both the planning (Camporeale et al., 2019; Behbahani et al., 2019) and the operating stages. However, social equity is ignored by decision makers in practice (Manaugh et al., 2015; Bills et al., 2017). From the Chinese HSR network shown in Figure 1, we can see that most HSR lines are located in the eastern and middle parts of China, whereas HSR lines in the west are rare. This spatial inequity is likely to further increase the economic gap between the rich regions (i.e., eastern and southern coastal areas) and the poor regions (i.e., western areas) of China, although the connectivity–accessibility of Chinese HSRs will become spatially balanced by 2030 (Xu et al., 2018). The labor market is highly affected by the HSR, as residents served by the HSR have better access to the labor center than those are not served, which causes equity issues for these residents (Guirao et al., 2017, 2018). Some may argue that this spatial inequity is due to passenger demand for faster and more comfortable service being higher in richer and more populous regions. However, this should not be an excuse for policymakers to exclude residents in western regions from optimal social justice.

In addition to spatial inequity in the accessibility to HSRs, we also need to consider vertical equity, that is, the distribution of transportation effects among sub-populations that differ in ability and need, such as different social and economic classes, age groups, and disabled or special needs groups (Bills et al., 2017). Based on a case study of a HSR in Spain, Pagliara et al. (2016) pointed out that many factors that cause the social exclusion of a HSR, of which economic exclusion has a significant impact in limiting users. The high cost of tickets for HSRs is the main obstacle to use for low-income peoples. The per capita expenditure of urban and rural residents in transport and communication in China from 2013 to 2017 is shown in Table 1 (China Statistical Yearbook 2018).

We can see that the average available money for transport for Chinese people is still low, especially for rural residents. If we divide the money in Table 1 into two equal parts, one for transport and the other for communication, the average expenditure of a rural resident in 2017 (754.6 yuan) is insufficient to fund a two-way trip between Beijing and Shanghai by HSR (1106 yuan for second-class seats). Therefore, railway planners and managers should pay more attention to people with low incomes to improve their access to HSRs from an equity perspective.

Table 1. Per capita expenditure of urban and rural residents in transport and communication in China.

Year	2013	2014	2015	2016	2017
Rural residents (yuan)	874.9	1012.6	1163.1	1359.9	1509.1
Urban residents (yuan)	2317.8	2637.3	2895.4	3173.9	3321.5

In this study, we aim to improve the equity of high-speed railways by creating a new HSR timetable based on the existing HSR infrastructure that is convenient for passengers, especially low-income passengers. The ticket price for a train using a segment is a dynamic variable, which can help reduce the cost of some trains for low-income passengers. A mixed-integer linear programming model is developed for social equity-based passenger-centric timetabling and ticket pricing. The three key objectives influencing train-timetabling are that: passengers want to minimize their travel cost, train companies want to improve their revenue, and the government needs to maintain social equity. These three objectives are not usually consistent, and thus trade-offs are needed to optimize the timetabling outcome. In our model, we minimize the total general passenger travel cost, including the in-vehicle time cost, waiting-time cost, transfer cost, deviation-time cost, and travel-fare cost from a passenger's perspective. We ensure that the revenue of the train companies is larger than a required value, and that the equity between different passenger classes is no less than a required level that is expected by the government. These criteria are managed by adding certain constraints to the model, similar to the ε-constraints used in many previous studies. We test our model on both a hypothetical HSR network and a real-world Chinese HSR network to determine how considering equity affects timetable construction and subsequent passenger-choice behavior.

Our research makes three contributions to the study of railway timetabling. First, we introduce the equity as a consideration in the train-timetabling problem, which is a novel inclusion in railway-timetabling study. To do so, we develop a method to quantitatively evaluate the equity problem in timetabling. Second, we develop a mixed-integer linear programming model to solve the social equity-based HSR timetabling and ticket-pricing problem. Finally, we test our model on a real-world HSR network in China to investigate the influence of social equity on the timetable and on passenger-choice behavior.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. In Section 3, we briefly introduce the demand model, the equity evaluation in train timetabling, and the revenue analysis. Section 4 describes our problem. Section 5 presents the computational results. Finally, we conclude this study and discuss some future research directions in Section 6.

2. Literature review

Because the aim of this study was to introduce the equity issue in passenger-centric train timetabling with dynamic ticket pricing, our literature review focuses on social equity in transportation, passenger-centric train timetabling, and ticket pricing. Therefore, we discuss some equity issues in transportation problems in Section 2.1 and review some research on the passenger-centric train-timetabling problem in Section 2.2. Finally, in Section 2.3 we discuss how the pricing strategy is used in railway transportation.

2.1. Social equity in transportation

Equity (also called justice and fairness) refers to the distribution of positive and negative impacts (i.e., benefits and costs) and whether that distribution can be considered fair and appropriate (Litman, 2002). Based on this definition, transportation equity refers to the fair or just distribution of transportation costs and benefits among current and future members of society. Transportation-planning decisions can have significant and diverse equity impacts (Litman, 2002). Therefore, considering the equity issue in transportation planning is necessary and important. However, transportation planning is usually done to improve efficiency by minimizing the travel time or maximizing profit; the equity issue is usually ignored because transportation equity analysis is more challenging than efficiency analysis. There are different types of equity, each with numerous impacts and ways to be measured, and various ways in which people can be grouped for equity analysis (Litman, 2002). The two main aspects of equity considered in transportation planning are horizontal equity and vertical equity (Litman, 2002; Camporeale et al., 2019). Horizontal equity refers to the distribution of the effects of an action among individuals, groups or geographic areas considered equal in capability and requirements. Based on this definition, public policies should avoid favoring one person or group over others, but this definition fails to consider existing social inequalities. Vertical equity refers to the distribution of benefits among groups with different requirements, where a distribution is regarded as fair if it provides more resources to disadvantaged individuals or groups than they would otherwise receive (Krumholz and Forester, 1990). A particular policy may be equitable in terms of horizontal equity but inequitable in terms of vertical equity (Camporeale et al., 2019).

Incorporating equity principles in the development of transportation systems has attracted increasing attention from transportation planners and decision-makers, especially in the last few decades (Bertolaccini, 2013). Cost-benefit analyses (CBA) are usually used in ex ante evaluations of transport policy options, but these are not suitable for evaluating policies aimed at countering social-exclusion (van Wee and Geurs, 2011; Levinson, 2002). The Gini coefficient (Gini, 1912) was originally used to describe the inequality of income or wealth distribution among a nation's residents, and has since been revised to evaluate other aspects of equity. Delbosc and Currie (2011) used a Gini coefficient-based method to measure how well transit supply meets transit demand in Melbourne, Australia. In similar examples, a Gini coefficient-based method was also used to evaluate travel-time reliability by Lee et al. (2017), to model horizontal and vertical equity in public transport design or distribution problems by Camporeale et al. (2019) and Tierra et al. (2017), and to analyze the equity of transit commuters' accessibility by Ben-Elia and Benenson (2019).

In most of the above studies, the Gini coefficient was applied to evaluate the equity level of an existing transport system. However, it is more desirable to integrate the equity issue in the designing or planning stage by using Gini coefficient-based methods. Ruiz et al. (2017) used this method to improve bus-service levels and social equity by optimizing bus frequency. This method was also used in hazmat facility location and routing by Romero et al. (2016) and in transit network design by Meng and Yang (2002). When grouping people for vertical equity analysis, income level is a critical variable. Therefore, adapting transit fares to make them affordable to residents with varying incomes is a good way to improve equity. Zhou et al. (2019) investigated how transit fare influences equity, and they applied their method to analyze the transport system in Brisbane, Australia. Recently, Gallo (2018) tried to improve the equity of an urban transport system by applying a taxi fare-based strategy. Briefly, an urban transit system is not evenly distributed in a city, and therefore it is inequitable if all taxi fares are charged the same way. Therefore, lowering taxi fares for services on origin–destination (OD) pairs that are not connected at all or that are connected with a low-quality transit service can help rebalance accessibility between zones of a city.

When it comes to high-speed railways (HSRs), little attention has been paid to equity issues. HSRs are constructed first in densely populated areas with high economic development levels. Because the HSR is a relatively expensive transport model, it is mainly used for labor and tourism purposes. The labor market of cities along an HSR changes before and after HSR operation (Ureña et al., 2009, Guirao et al., 2017, 2018). Accordingly, the migration and commuting of residents between small cities and metropolis may change. For example, more people might choose to live in small cities and work in the metropolis because of low housing fares and fast HSR transit (Haas and Osland, 2014). This labour market migration affects the issue of social equity. Residents in cities who are not served by HSRs may be disadvantaged because of less access to labour centers in the metropolis than those in cities served by HSRs. This kind of spatial equity of HSRs in South Korea and China was analyzed in Kim and Sultana (2015) and Xu et al. (2018), respectively. In addition, the influence of operating a new HSR line in Europe, in terms of economic potential and daily accessibility, was investigated by Gutiérrez (2001), and the impact of HSRs on regional income inequality in China and Korea was analyzed by Jiang and Kim (2016). However, to the best of our knowledge, no research has been performed on equity issues in HSR designing and planning.

2.2. Passenger-centric train timetabling

A train timetable is the most important document for guiding daily train operations, as it specifies the detailed departure and arrival times of each train at each station. Numerical studies have investigated the train timetabling problem. We refer interested readers to Cacchiani and Toth (2012) for discussions of comprehensive nominal and robust train timetabling and to Parbo et al. (2016) for coverage of different aspects of operator- and passenger-based models for timetabling. In this section, we focus on passenger-centric train timetabling, which is closely related to our research. As mentioned by Robenek et al. (2016), studies on non-periodic timetabling have focused more on the operational aspects of the problem than on finding the most suitable departure times for passengers.

Compared to operation-oriented train timetabling, passenger-centric train timetabling lays more emphasis on passenger convenience, such as by minimizing the passenger waiting time or the total passenger travel-cost. In some studies, the waiting time for passengers was minimized based on the assumption that the paths for passengers are known in advance (Nachtigall, 1996; Peeters, 2003; Liebchen, 2008; Groβmann et al., 2012). However, in practice, passengers tend to choose their paths based on the supply. Nachtigall et al. (1998) dedicated a chapter to a periodic timetabling model with integrated passenger routing. Recently, some studies have been based on the assumption that passengers choose the shortest path when they plan their journey, and thus integrated train timetabling and passenger routing [see Schmidt and Schöbel (2015), Robenek et al. (2016), and Binder et al. (2017)]. Aside from Robenek et al. (2016) and Binder et al. (2017), all previous passenger-oriented train timetabling studies used a deterministic demand as an input. However, demand depends on supply, and a discrete-choice model is usually applied to estimate the number of passengers for various timetables, whereby the model with the highest number of passengers using trains is selected (Cordone and Redaelli, 2011). Robenek et al. (2016) and Binder et al. (2017) assumed that passengers choose the path with the highest utility, and thus considered the in-vehicle time, waiting time, number of transfers, and arrival deviation in the utility function. Sato et al. (2013) maximized passenger satisfaction for train timetabling, and their elements in the satisfaction function were similar to those used in the utility function by Robenek et al. (2016). These elements were further extended to include congestion by Kanai et al. (2011).

However, if we can ignore a passenger's convenience or the service cost, a more balanced timetable should be obtainable the perspective of train companies and the government. In Robenek et al. (2016, 2018) and Binder et al. (2017), both the passenger travel cost from a passenger's perspective and the operating cost from the train company's perspective were minimized in a passenger-centric timetabling and rescheduling approach. Chow and Pavlides (2018) also analyzed several objectives in timetabling, including train running-time, passenger waiting-time, punctuality of train service, and crowdedness of trains. To the best of our knowledge, however, social equity from the government's perspective has not been considered in train timetabling.

2.3. Revenue management and pricing

Given a potential number of passengers and a train timetable, a train company wants to maximize its revenue while optimizing the ticket price to attract custom. An appropriate ticket-pricing strategy can not only attract more passengers to travel by train but also improve revenue. It also can help improve social equity, as mentioned previously. However, revenue management is ignored by railway companies, unlike by airlines. This is due to the greater flexibility inherent to traveling by train because it involves no check-in procedures and the fact that segments of the journey cannot be considered independent as many journeys are composed of multiple adjacent segments (Armstrong and Meissner, 2010). Recently, because of competition with other transport modes, e.g., airlines and buses, railway companies have paid more attention to revenue management by reducing their operating cost and improving ticket income. Railway companies in some European countries tried to apply low-cost-airline pricing strategies to European railways to improve their revenue (Sauter-Servaes and Nash, 2007). Si et al. (2009) developed a bi-level programming model to optimize both the train service level (including the price) and passenger travel cost, by considering the competition of an intercity railway line in China with buses and cars for custom. The influence of MTR's entering the Swedish passenger railway market on train-ticket price was analyzed in Vigren (2017), and it was found that the average price had steadily declined since the entry of MTR.

To improve railway revenue, railway passenger choice behavior was analyzed by Nuzzolo et al. (2000). They used a nested-logit choice model to simulate the effects of variations in medium–long-distance railway service parameters (e.g., timetable, travel time, and prices). This passenger-choice behavior was found to be closely related to the ticket price. For example, one of the important strategies for pricing is to use different prices for peak periods and off-peak periods, which can not only improve the revenue but also flatten large passenger demand in the peak period. Chinese HSRs have tried using discount off-season train tickets on some lines since 2012, the impacts of which were analyzed in Chen and Liu (2013) and Zheng and Liu (2016). In addition to the effects of a discount, Qin et al. (2019) investigated the effect of increasing the ticket price in peak periods. A similar study of the effects of pricing policies and demand elasticity on service choice was conducted for the Dutch railway system (van Vuuren, 2002); it was found that increasing the price in peak periods was more efficient than reducing the price in off-peak periods. In addition, the manner in which ticket price and allocation affected the overall demand in an overcrowded condition was analyzed by Whelan and Johnson (2003) and Bharill and Rangaraj (2008). Similarly, a multinomial logit model and latent class model were applied by Hetrakul and Cirillo (2014) to analyze the effect of ticket-purchase timing decisions on ticket pricing and seat allocation.

All the above mentioned railway pricing studies focused on revenue management or passenger service-choice. Recently, Robenek et al. (2018) analyzed the integration of ticket pricing with railway timetabling in the planning stage to improve revenue. However, the pricing problem may cause equity problems, which was not considered by Robenek et al. (2018).

3. Models for demand, social equity, and revenue

3.1. A demand model

Passenger travel-choice can be hierarchically divided into mode choice and route choice. As discussed in Robenek et al. (2018), we assume that the mode choice is fixed. Therefore, we can predict the railway market share, and that the railway company wants to maximize its profit within the given maximum potential demand. In practice, the maximum demand is not exactly known, and it is normally predicted by the railway research institute or the railway company. We assume that the maximum demand is given. However, potential railway passengers might choose different modes of transport, such as a bus, or they may decide against traveling if the railway service is not convenient for them. We focus on HSRs with high ticket fees, which are normally higher than bus tickets for the same OD pair. Some low-income passengers are likely to choose buses because they care more about saving money (i.e., the ticket price) than about saving some travel time. Thus, it is necessary to take low-income passengers into account to encourage more potential railway passengers to travel by train. We introduce two possible strategies to achieve this: improving the quality of the train timetable to make it more convenient for passengers, and the reducing the ticket price of some trains to attract more low-income passengers.

The demand for HSRs has an unbalanced temporal distribution. We denote high-demand periods as peak periods and relatively low-demand periods as off-peak periods. Usually, early mornings and late evenings are off-peak periods for high-speed trains because it is not convenient for passengers to travel by train at those times. For example, passengers prefer to take a train departing at 10:00 a.m. than one departing at 07:00 a.m. if they can arrive at their destination no later than the desired time and the ticket price is the same. During off-peak periods, the train occupancies are quite low. For example, 13% of trains on the Beijing–Shanghai HSR line in 2014 had a load factor < 65% during off-peak periods (Zheng and Liu, 2016), that is, a large number of seats were empty during this period, which equates to a loss for the railway company. Inspired by the pricing strategy used in road transport and air transport to adapt to temporal demand, we can also use an appropriate pricing strategy to adapt railway passenger demand in a temporal dimension. Specifically, properly reducing ticket price in the off-peak period can not only increase seat occupancy but also increase the revenue of a railway company. Besides, raising the ticket price appropriately in the peak period can also increase the revenue of the railway company and adjust the highly passenger demand.

The demand can be divided according to passenger groups, and according to the OD pair and the desired arrival time. A passenger group is denoted by (i,t), where $i \in I$ is the OD pair and $t \in T_i$ is the desired arrival time. In this study, we consider heterogeneous passengers because passengers in the same group (i,t) with different characteristics may have different choice behaviors. Income is the most critical characteristic, and we thus focus on social equity for different classes of passengers. Accordingly, we further divide the passenger group (i,t) into several further groups, according to passengers' income level. We introduce parameter $\tau \in \Pi$ to denote the income level of passengers, where Π consists of all the possible income levels of passengers in group (i,t). Therefore, *a passenger group is denoted by* (i,t,τ) *in this research*. The total number of passengers in a group (i,t,τ) is $n_{it\tau}$. We assume that all passengers in a group (i,t,τ) use the same path and take the same trains during their journey, i.e., passenger-group splitting is not allowed. If a passenger group cannot travel by the same path due to a capacity limitation or a low path-utility, it will give up traveling by high-speed train and use other modes, such as buses, with a certain utility $U^o_{it\tau}$ (general travel cost $C^o_{it\tau}$). Normally, the speed of a bus is slower than that of a high-speed train, but the ticket price for a bus is lower. High-income travelers therefore prefer high-speed trains, whereas more low-income travelers choose buses.

As timetabling follows line planning, we assume that the train lines and their frequencies of use are known to the railway operator when the timetable is being compiled. A line $(l \in L)$ is from a specific origin to a specific destination consisting of a certain number of stations and segments $\left(s \in S^l\right)$ between two stations, where L is the line set and S^l is the segment set of line l. A certain number of trains operate on each line l, and we number these trains according to their sequence of departure from the first station of the line. All trains of line l comprise a train set V^l, $v \in V^l$. Thus, train v belonging to line l is denoted by a combination of indices (l,v).

For each OD pair $i \in I$, a set of paths P_i connects the origin to the destination. A path may include one line or several lines. Here, P_i is the set of all physical paths of OD pair i. The available path for a specific passenger group (i,t,τ) is time dependent, i.e., it is determined by the physical path P_i and the time (timetable). Therefore, the path set $P_{it\tau}$ of passenger group (i,t,τ) corresponds to the specific trains used by passengers in this group. Overall, $P_{it\tau}$ is determined by both the connectivity of the train lines and the schedule of trains on the corresponding lines.

A passenger chooses how to travel on an HSR from his/her origin to his/her destination based on his/her required time of arrival at the destination. Thus, the so-called passenger route-choice problem can be expressed as: if the timetable for an HSR network is given, how can one determine the trains that are taken by a passenger? A widely used method to predict passenger route-choice is discrete choice analysis (Ben-Akiva and Lerman, 1985), which operates on the basis that each passenger acts to maximize his/her own utility function. That is, a passenger chooses the available option that has the highest utility. The utility function of each option has several attributes, such as the travel time and cost and the characteristics of a passenger, e.g., income, age, or travel purpose. For each passenger group (i,t,τ) using path $p \in P_i$ to travel from his/her origin to his/her destination, we define the utility as $U^p_{it\tau}$. In this study, we consider the following attributes (below) in the utility function, which are similar to those used by Robenek et al. (2016). For a more detailed explanation of each attribute, we refer the reader to Robenek et al. (2016).

- **In-vehicle-time:** total time (minutes) that passengers spend within the various trains along their path.
- **Waiting time**: time (minutes) that passengers spend waiting for a transfer train at a transfer station during their journey.

- **Number of transfers**: the total number of times that passengers transfer from one train to another during their journey.
- **Early-arrival time**: number of minutes before their preferred arrival time that passengers arrive at their destination. If passengers arrive at their destination on time or late, this value is zero.
- **Late-arrival time:** number of minutes after their preferred arrival time that passengers arrive at their destination. If passengers arrive at their destination on time or earlier, this value is zero.
- **Travel fee**: the total money (yuan) that passengers spend to buy train tickets, where yuan is the Chinese unit of currency (1 yuan is ~0.14 USD).

As we consider both travel time and travel cost, we need to either convert time into money or vice versa to ensure consistency in the utility function. We recall our considering that heterogeneous passengers in each OD pair and the same path can have different utilities for different passengers, e.g., for high-income passengers and low-income passengers. Then, given that our goal is to encourage low-income passengers to travel by high-speed trains, their utility of a selected path is lower if we convert the travel cost into time because their value of time is lower than that of high-income passengers. Therefore, we convert the travel cost into time in the utility function. The utility function of path p for passenger group (i,t,τ) is thus defined as follows (Equation (1)):

$$U_{it\tau}^{p} = -(r_{it\tau}^{p} + \beta_1 \times w_{it\tau}^{p} + \beta_2 \times u_p + \beta_3 \times e_{it\tau}^{p} + \beta_4 \times \Delta_{it\tau}^{p} + \beta_5 \times f_{it\tau}^{p}) \quad \text{[min]} \tag{1}$$

In Equation (1), the notations $r_{it\tau}^{p}$, $w_{it\tau}^{p}$, u_p, $e_{it\tau}^{p}$, $\Delta_{it\tau}^{p}$, and $f_{it\tau}^{p}$ denote the in-vehicle time, waiting time, number of transfers, early arrival, late arrival, and travel cost of a passenger group (i,t,τ) using path p, respectively. These notations are explained in Table 8 in Appendix A. Parameters β_1~β_5 convert the waiting time, transfer cost, early-arrival time, late-arrival time, and travel cost into the in-vehicle time cost. The values of parameters β_1~β_4 are determined from previous studies, which will be explained later. Parameter β_5 corresponds to the Value-Of-Time (VOT), i.e., the willingness-to-pay for a travel-time saving. Note that we consider passengers of different income levels in a passenger OD. Thus, the VOT differs for passengers of different incomes. We assume that a passenger group (i,t,τ) chooses path p that has the highest utility $U_{it\tau}^{p}$ under the capacity limit. The utility function can also be expressed as the total general travel cost, which is shown in Equation (1a), below. From Equations (1) and (1a), we can see that high utility equates to a low general travel cost.

$$C_{it\tau}^{p} = r_{it\tau}^{p} + \beta_1 \times w_{it\tau}^{p} + \beta_2 \times u_p + \beta_3 \times e_{it\tau}^{p} + \beta_4 \times \Delta_{it\tau}^{p} + \beta_5 \times f_{it\tau}^{p} \quad \text{[min]} \tag{1a}$$

3.2. A social equity model

We aim to devise a railway timetable that incorporates consideration of the social equity issue, which, to the best of our knowledge, has not been addressed in previous studies of HSRs, which have been conducted either from the operator's perspective, to minimize the operational cost, or from the passenger's perspective, to improve passenger convenience or minimize their travel cost. Thus, the equity aspect has been neglected but remains important to address. Because HSR as public transport is financed mainly by central or local government, it should be available for all residents, especially low-income people without private vehicles. Our goal is to create a timetable and ticket-pricing strategy to keep the utility of low-, medium-, and high-income passengers at a similar level so that more low-income passengers will use high-speed trains.

No direct answer on how to model the social equity for HSR timetabling can be found from previous research, but relevant ideas can be borrowed from other research domains. Many previous studies have modeled social equity using a Gini coefficient-based method, and the Gini coefficient of concentration and the Lorenz curve have historically been used to measure equality in the distribution of good. They can be used not only to measure income distribution but also other kinds of equity, by altering what is shown on the vertical axis (Levinson, 2002). For example, a revised version was used to assess equity in hazmat facility location and routing by Romero et al. (2016), where their goal was to ensure an equal per capita burden of exposure to hazmat across different income groups. Similarly, Meng and Yang (2002) considered the equity issue in road-network design by limiting the travel-cost improvement of any passenger OD in any design scenario to a given level. Inspired by Romero et al. (2016) and Meng and Yang (2002), we adopt the following method to formulate the equity constraint in HSR timetabling and pricing.

For each passenger OD pair with the same desired arrival time (i,t), we ensure that the general travel cost $(C_{it\tau})$ of any passenger group (i,t,τ) cannot exceed the average general travel cost of all the passenger groups belonging to the same OD pair (i,t). As mentioned previously, we divide the passengers belonging to the same (i,t) into several passenger groups based on their income levels, and the general travel cost of a passenger group (i,t,τ) consisting of high-income passengers is lower than that of a passenger group $\left(i,t,\tau'\ \right)$ consisting of low-income passengers. Therefore, we propose to restrict the general travel cost of low-income passengers to a low level to improve their ability to access high-speed trains. The equity is modeled by Inequality (2) and Equation (3).

$$\max_{\tau\in\Pi} \left\{\frac{C_{it\tau}}{\bar{C}_{it}}\right\} \le \varepsilon \qquad \forall i\in I,\ t\in T_i \tag{2}$$

$$\bar{C}_{it} = \frac{\sum_{\tau\in\Pi} C_{it\tau}}{|\Pi|} \qquad \forall i\in I,\ t\in T_i \tag{3}$$

In Inequality (2), ε is a parameter indicating the equity level for passenger groups within each OD pair with the same desired arrival time (i,t). The exact value of ε can be properly set by the government and railway manager according to common practice. This inequality means that the general travel cost $C_{it\tau}$ of any passenger group (i,t,τ) cannot exceed the average general travel cost $\bar{C}_{it}$ of all the passenger groups with the same origin, destination, and desired arrival time, i.e., all passenger groups belonging to the same (i,t). Therefore, it ensures that the general travel cost of the passenger group consisting of low-income people is at an acceptable level. As mentioned previously, the general travel cost of low-income travelers is higher than that of high-income travelers within the same (i,t) because their VOT is smaller. Equation (3) can be used to calculate the average general travel cost of all the passenger groups within the same (i,t). $C_{it\tau}$ is not restricted to a specific path p as is $C_{it\tau}^{p}$ (defined previously) because we do not know which path p the passenger group actually chooses. Similarly, different passenger groups with the same (i,t) may use different paths. Thus, $\bar{C}_{it}$ does not correspond to a specific path p.

3.3. The revenue analysis

High-speed trains are operated by many train companies, and each company must be able to afford the operating costs, such as the cost of multiple train units (rolling stock) and the cost of train drivers. Although the cost of constructing HSRs is borne mostly by central or local governments, the daily operating cost is borne by the train company. A train company wants to maximize its revenue by operating trains, where this revenue is the money that is obtained by selling tickets, minus the railway operating cost. The operating cost is assumed to be fixed with a given line plan, and thus the train company aims to maximize the ticket revenue as follows (Equation (4)):

$$\max\ \sum_{i\in I}\sum_{t\in T_i}\sum_{\tau\in\Pi}\sum_{p\in P_i} f_{it\tau}^{p}\times n_{it\tau} \tag{4}$$

where $f_{it\tau}^{p}$ is the ticket price for passenger group (i,t,τ) using path p and $n_{it\tau}$ is the volume of passenger group (i,t,τ). We have modeled the revenue of railway companies globally in formula (4), which is suitable for situations in which all of the involved railway companies are public. This is the case for the Chinese HSR. To maintain the revenue at a required value, we model it as a constraint, as follows (Equation (4a)):

$$\sum_{i\in I}\sum_{t\in T_i}\sum_{\tau\in\Pi}\sum_{p\in P_i} f_{it\tau}^{p}\times n_{it\tau} \ge R \tag{4a}$$

where R is the minimum required revenue for a train company, which should be greater than the total operating cost. For example, if R is 15% greater than the total operating cost, this means that the profit is 15%.

4. Problem description

We focus on the passenger-centric train-timetabling and ticket-pricing problem while also explicitly considering social equity. The train-timetabling problem is not new and has been extensively investigated (Cacchiani and Toth, 2012; Parbo et al., 2016). The core of the problem is to determine the proper departure and arrival times of each train at each station to prevent conflict, so that either the total travel time or the travel cost is minimized. If we take dynamic passenger demand into account at the timetabling stage, we can obtain a passenger-centric timetable in which the passenger travel cost is minimized, or the passenger's convenience is maximized. Normally, the ticket price for paths of the same OD is constant. Therefore, the travel cost is not considered at the timetabling stage. However, motivated by competition with air transport, fluctuating ticket prices may help either flatten demand, increase revenue, or attract more passengers. Thus, in this study, the ticket price is not kept constant but rather is set as a decision variable, meaning that we also have to determine the appropriate ticket price for a train traveling on a given segment. By reducing the ticket price for some trains, it is possible that the fare of some train paths will be reduced, as will the general travel-cost of these paths. In this context, the ability of low-income passenger groups to access HSR increases. That is, more low-income passengers will be able to afford to travel by high-speed trains by choosing lower-price trains. The social equity of HSRs is thus improved, to some extent. Therefore, by considering the variable ticket price and the social equity level in passenger-centric timetabling, the problem to be resolved becomes social equity-based passenger-centric train timetabling and ticket pricing.

A train corresponds to a line $l \in L$, where the running segments and the stations either for dwelling or passing are given by the line plan. Many trains may operate on the same line. Therefore, trains belonging to the same line are ordered according to their departure time from the origin of the line, and the specific train of a line is denoted by the combined indices of train line and train order (l, v), where $v \in V^l$ is the order of the train within line l. Similar to Robenek et al. (2016), we assume the running time of a train in a segment and the dwell time of a train at a station are constant and known. However, the stopping patterns of trains within the same line differ in Chinese railways. To this end, the running time for different trains from the same origin to the same destination can be different, i.e., it is train specific. This is different from Robenek et al. (2016). Therefore, the travel time $\bar{r}_i^{plv}$ of OD pair i on path p using train (l, v) consisting of the running times between stations and the dwell times at stations are deterministic (i.e., a given value), and we only need to determine the departure time of each train from its origin $d_{l,v}$ in the train-timetabling problem. The departure and arrival times of a train (l, v) at other stations can be calculated by $d_{l,v}$ and $\bar{r}_i^{plv}$, respectively. A line is unidirectional in this study, i.e., trains in the opposite direction serve a different line. Note that the conflicts among trains are not considered in this research because the timetabling problem is well-solved in many previous studies, and thus conflicts are not our main focus. Instead, we aim to devise a passenger-centric timetable for train companies, and we leave the train-conflict solution to railway infrastructure managers, as examined by Robenek et al. (2016, 2018).

A train path consists of one or several lines and is served by one or several trains. As the line plan is given, the set of path P_i for passenger OD $i \in I$ is known. The possible path set $P_{it\tau}$ for passenger group (i, t, τ) is related to the actual timetable. If path $p \in P_{it\tau}$ includes several trains, the connection between two consecutive trains must be reasonable, i.e., the arrival time of the previous train at the transfer station must be at least m minutes before the departure time of the successor train from the same station, where m is the minimum transfer time. We can remove all of the unreasonable paths from the path set to reduce the problem size. Each passenger group (i, t, τ) is assumed to use the path with the highest utility, if possible, and the utility function is defined in Section 3.1 by Equation (1). Because each train has a limited capacity, the total number of passengers on a train in each segment cannot exceed the capacity. If the spare capacity of a train is insufficient to accommodate all the passengers in a passenger group, no-one in the passenger group can board the train because the passenger group is not allowed to be split. If a passenger group cannot choose any path due to limited train capacity or if the utilities of all possible paths for the group are too low, the group gives up traveling by train and chooses other options.

Our objective is to minimize the total general passenger travel cost (maximize the total utility). To improve the equity of HSRs, we use a variable ticket fare. The total ticket fare f_p of path p is calculated by adding up the fare of each train (l, v) on path p using each segment s, f_s^{lv}. One can also use other structures to calculate the ticket fare. We are interested in attracting low-income passengers to high-speed trains by adapting the ticket price when the revenue of the train companies is at a required level. The equity of HSR access is considered by ensuring that the utility of low-income passengers is not smaller than that of high- and medium-income passengers by a given, known amount. Therefore, more high-speed trains are available for low-income passengers. We formulate our problem by a mixed-integer linear programming model partly based on the model in Robenek et al. (2016). The notations used and the detailed model are given in Appendix A.

5. Case study

In this section, we describe the experimental results from using our approach on a hypothetical railway network and on part of a real-world Chinese HSR network: the Guangzhou–Nanning and Guangzhou–Guilin HSR network. The model was coded in Optimization Programing Language, and IBM ILOG CPLEX 12.8 was utilized as the solver, with CPLEX parameters set to their default values. All our experiments were run on an Intel Core i7-7700 processor CPU5 @3.60GHz (i.e., 3.60GHz, 16.0GB RAM desktop).

5.1. A small case study

We first tested our model on a small hypothetical railway network consisting of two HSR lines, one having stations 1, 2 and 3 and the other having stations 2 and 4, meaning that the lines share station 2. There were three segments between two of these four stations, as seen in Figure 2. In the figure, stations are denoted by circles with the station number inside the circle, and a segment is denoted by a line connecting two stations with a segment number beside the line. We assumed that four trains run on line 1 (from station 1 to station 3) and that three trains run on line 2 (from station 2 to station 4) in our planning horizon. Thus, there were seven trains in total, and each train stopped at each station that it visited. We also assumed that the running time of each train in each segment was 30 minutes (min) and that the dwell time for each train at each visited station was 5 min. The passenger OD information is shown in Table 2. There were three OD pairs—one from station 1 to station 3 on line 1, another from station 1 to station 2 on line 1, which then crosses to station 4 on line 2, and the third from station 2 to station 4 on line 2. These OD pairs are shown in the "Route" column in Table 2. Each OD pair was further divided into several time-dependent OD pairs based on their desired arrival time at the destination, shown in the "Desired arrival time" column of Table 2. For each time-dependent OD pair, passengers were divided into high-, medium-, and low-income groups; the volume of each type of passenger is given by the "Passenger volume" column of Table 2. Therefore, we had 30 passenger groups in total, classified according to their origin, destination, desired arrival time, and income class.

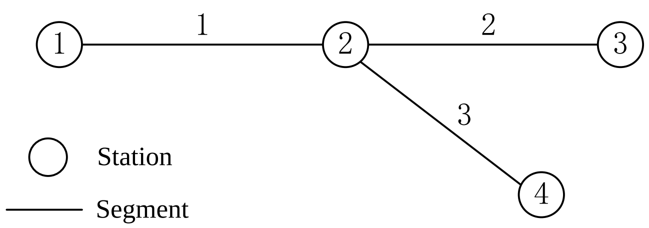


Figure 2 A small railway network.

Table 2 Passenger information

OD pairs	Time-dependent OD pairs	Route	Desired arrival time (min)	Passenger volume		
				High	Medium	Low
1	1	1—2—3	70	10	10	20
	2	1—2—3	80	30	20	10
	3	1—2—3	90	30	25	15

	4	1—2—3	100	20	30	20
2	5	1—2—4	70	5	5	5
	6	1—2—4	85	10	5	5
	7	1—2—4	95	10	10	5
3	8	2—4	90	10	20	15
	9	2—4	105	25	20	10
	10	2—4	120	30	20	10

The minimum departure headway (h) was 5 min, and the minimum transfer time (m) was also 5 min. The values of $\beta_1 - \beta_4$ in the utility function, obtained according to the previous study of Robenek et al. (2016), were 2.5, 10, 0.5, and 1, respectively. However, β_5 is related to the VOT for the various passengers, as mentioned previously. Therefore, each class of passenger had its own VOT. In this case, we assumed that the values of β_5 for the three types of passengers were 0.5, 1.0, and 1.5, respectively. The value of ε reflects the equity level of passengers, which we set as 1.2. However, we also changed the value of ε to see the results obtained with different equity levels. The capacity of a train was assumed to be 80 passengers ($\mu_v^l \times q = 80$). If a passenger group could not travel by any train, the general cost was assumed to be 300 min for passengers in OD pairs 1 and 2 and 200 min for passengers in OD pair 3. Finally, the value of M was 500. The ticket price for a train running on segment s was between 32 (f_s^{min}) and 46 (f_s^{max}) yuan.

5.1.1. The influence of ticket revenue

As train operation has to consider both the revenue from the train company's perspective and the general travel cost from the passengers' perspective, we first investigated the possible trade-off between these two requirements. Our goal was to maximize the total utility (minimize the total general travel cost) of all the passengers while keeping the revenue of the train company at a reasonable level. Therefore, we changed the minimum revenue that a company requires to see how this would influence the general passenger travel cost. The values of minimum revenue were 15,000, 20,000, 25,000, 30,000, and 32,000 yuan. We set the largest revenue to 32,000 yuan instead of 35,000 yuan because no feasible solution existed under our given ticket price interval for each segment when 35,000 yuan was used. In this test, the equity constraints were not considered. The results are given in Table 3.

Table 3. Various ticket revenues for different ticket prices.

Revenue (yuan)	Objective (min)	Ticket price (yuan)	Time (s)	Gap (%)
15,000	48457	32;32;32;32;32;32;32;32;32;32;32	82	0
20,000	48457	32;32;32;32;32;32;32;32;32;32;32	117	0
25,000	48875	32;32;35;46;32;32;32;32;32;32;32	128	0
30,000	52812	32;32;46;46;46;46;32;32;32;46;46	300	0.11
32,000	54901	32;32;46;46;46;46;43.9;32;46;46;46	300	0.75

In Table 3, the first column is the minimum ticket revenue and the second column shows the objective values (i.e., the total general passenger travel cost). The third column gives the optimal ticket fare (f_s^{lv}) for each train in each segment. We assumed that there were four trains running on line 1 from station 1 to station 3 using segments 1 and 2. Thus, the first eight numbers show the ticket price of these four trains on segments 1 and 2. In addition, we had three trains running from station 2 to station 4 using segment 3. Therefore, the last three numbers in the third column are the ticket prices for these three trains using segment 3. The fourth column is the computation time, which we limited to 300 s (5 min), and the last column is the optimality gap between the best upper bound and the best lower bound within 5 min.

We can see that when the revenue is no more than 20,000 yuan, the objective value remained at 48457, and the ticket price for each segment equaled the minimum allowed value (32 yuan). This occurred because the passenger's travel cost is low when the required revenue is low, i.e., all passengers can use the path at the lowest general travel cost. However, with increasing required revenue, the total general travel cost also increased, and the ticket price for some trains in some segments increased to ensure that the higher revenue could be achieved. The relationship between revenue and total passenger general travel cost is shown in Figure 3. When we increased the minimum required revenue further, to 35,000 yuan, no feasible solution existed because we limited the maximum ticket price for trains in each segment. Figure 3 shows that when the required revenue is less than 25,000 yuan, the total passenger travel cost increases slowly with increasing revenue. However, when the revenue is higher than 25,000 yuan, the total travel cost increases dramatically with increasing revenue. The is because, in the former case, most passengers can use a short path with a low ticket price, whereas when the revenue is more than 25,000 yuan (i.e., 30,000 or 32,000 yuan), the ticket price for most trains must rise and the total passenger travel costs increase accordingly. It is clear that a trade-off exists between the revenue required by the train company and the total general travel cost able to be borne by passengers. The government and the railway company must therefore find a balance between these two key variables. We also found that the computation time increased with increased minimum required revenue. However, all the test cases were solved within 300 s (5 min) with a gap smaller than 1%.

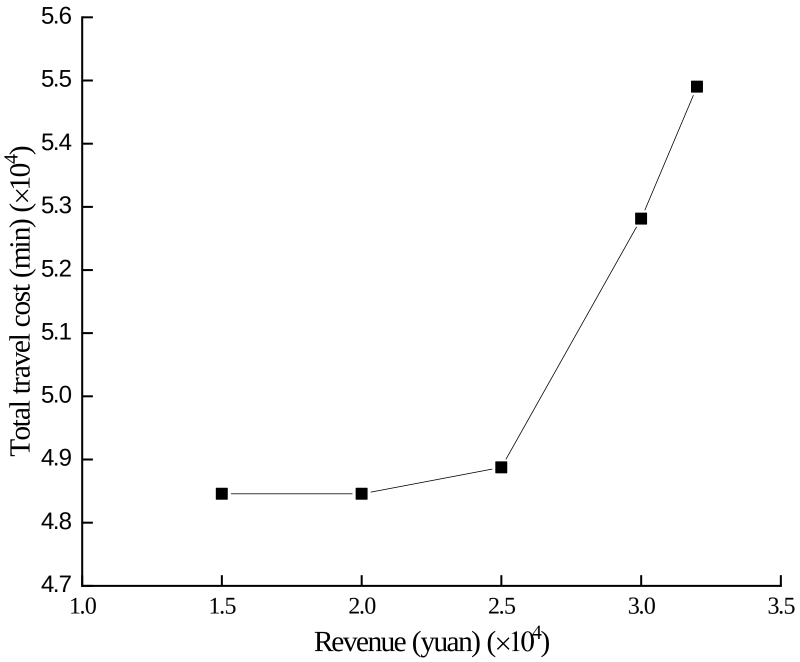


Figure 3. The relationship between total passenger travel cost and revenue.

5.1.2. Social equity analysis

To consider social equity when making an HSR timetable, we introduced an equity constraint (14) in our model. In this subsection, we describe how the equity constraint affected the solutions. We tested our model on a small case with various equity levels (ε). We set the value of ε to a number from 1.0 to 1.5 in intervals of 0.1. The values of all other parameters remained the same as those explained previously. We wanted to determine the relationship between total general passenger travel cost and social equity levels under fixed minimum required train company revenues, i.e., 25,000 and 30,000 yuan. The results are given in Table 4. Except for the first and second columns, the meaning of other columns is similar to those in Table 3.

Table 4. Solutions with different social equity levels.

Revenue (yuan)	Equity level (ε)	Objective (min)	Ticket price (yuan)	Time (s)	Gap (%)
	1.0	62142	32;32;32;43.3;32;32;32;32;32;32;32	4	0
	1.1	55073	32;32;32;46;32;32;32;32;32;32;32	5	0

25,000	1.2	50228	32;32;46;32.6;32;32;32;32;32;32;32	51	0
	1.3	48900	32;32;35.3;46;32;32;32;32;32;32;32	167	0
	1.4	48875	32;32;35.3;46;32;32;32;32;32;32;32	241	0
	1.5	48875	32;32;35.3;46;32;32;32;32;32;32;32	126	0
30,000	1.0	63239	46;46;32;32;46;46;33.9;32;32;32;46	66	0
	1.1	56269	46;46;32;32;46;46;35.9;32;32;32;46	140	0
	1.2	53212	32;32;46;46;46;46;32;32.2;46;32;46	300	1.63
	1.3	52831	32;32;46;46;46;46;32;32;32.1;46;46	300	1.05
	1.4	52812	32;32;46;46;46;46;32;32;32;46;46	300	1.22
	1.5	52984	32;32;46;46;32;32;46;46;46;32;46	300	1.13

It can be seen that the equity level affected the total general passenger travel cost. When the government wants to improve the equity of an HSR, the travel cost for high-income passengers may increase because special attention is paid to low-income passengers. When ε equals 1, it means that the travel cost for low-income passengers is not higher than that of medium- or high-income passengers in the same OD pair. In such a scenario, we would say that the equity is perfect. However, with increasing ε, the general travel cost of low-income passengers becomes higher than that of medium- or high-income passengers. Thus, we say the equity of an HSR worsens under these conditions. Overall, when the value of ε increases, the equity level of an HSR decreases. From Table 4 and Figure 4, we can see that the total general passenger travel cost decreases significantly when the value of ε increases from 1.0 to 1.3, and it remains stable when the value of ε increases from 1.3 to 1.5 with a minimum required revenue of either 25,000 or 30,000.

Although the total general passenger travel cost increases with increasing equity level, such an increase might be beneficial to low-income passengers. We show the relationship between the general passenger travel costs of low-, medium-, and high-income passengers and the social equity level in Figures 5 and 6 with minimum revenues of 25,000 and 30,000, respectively. We take ODs 5, 6, 7, and 8 as an example in Figure 5, and ODs 2, 5, 6, and 10 as an example in Figure 6. We can see that the general travel costs for the low-, medium-, and high-income passenger group of the same OD were the same when ε is 1.0. With increasing ε, the difference among the general travel costs of the low-, medium-, and high-income passenger group of the same OD increased. Specifically, the general travel cost for the low-income passenger group increases and those for the medium-income and high-income passenger groups decreased. Therefore, a higher equity level can help reduce the general travel cost of low-income passengers and encourage more low-income passengers to travel by high-speed train. There is a trade-off between the total general passenger travel cost and social equity. If a higher equity level is required, the total general passenger travel cost tends to be higher, but the difference of general travel cost for low-, medium-, and high-income passengers decreases. The government and train company can choose a reasonable value of ε to achieve a balance.

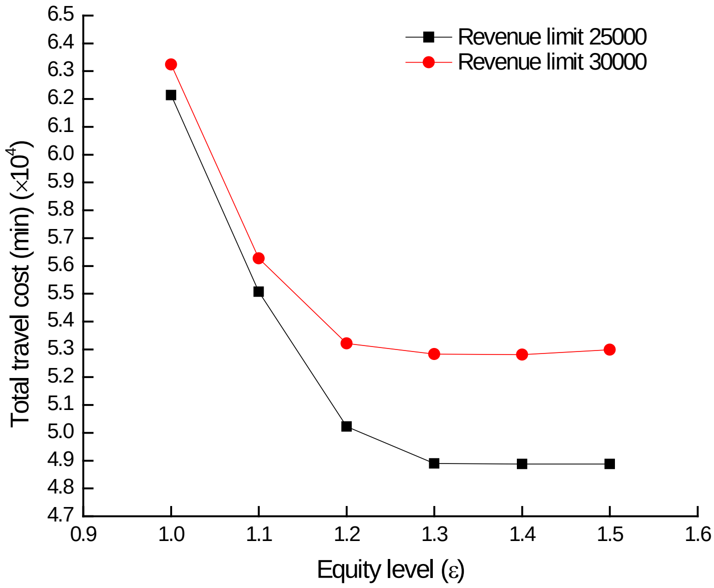


Figure 4. The total general passenger travel cost at various equity levels with a given minimum operation cost.

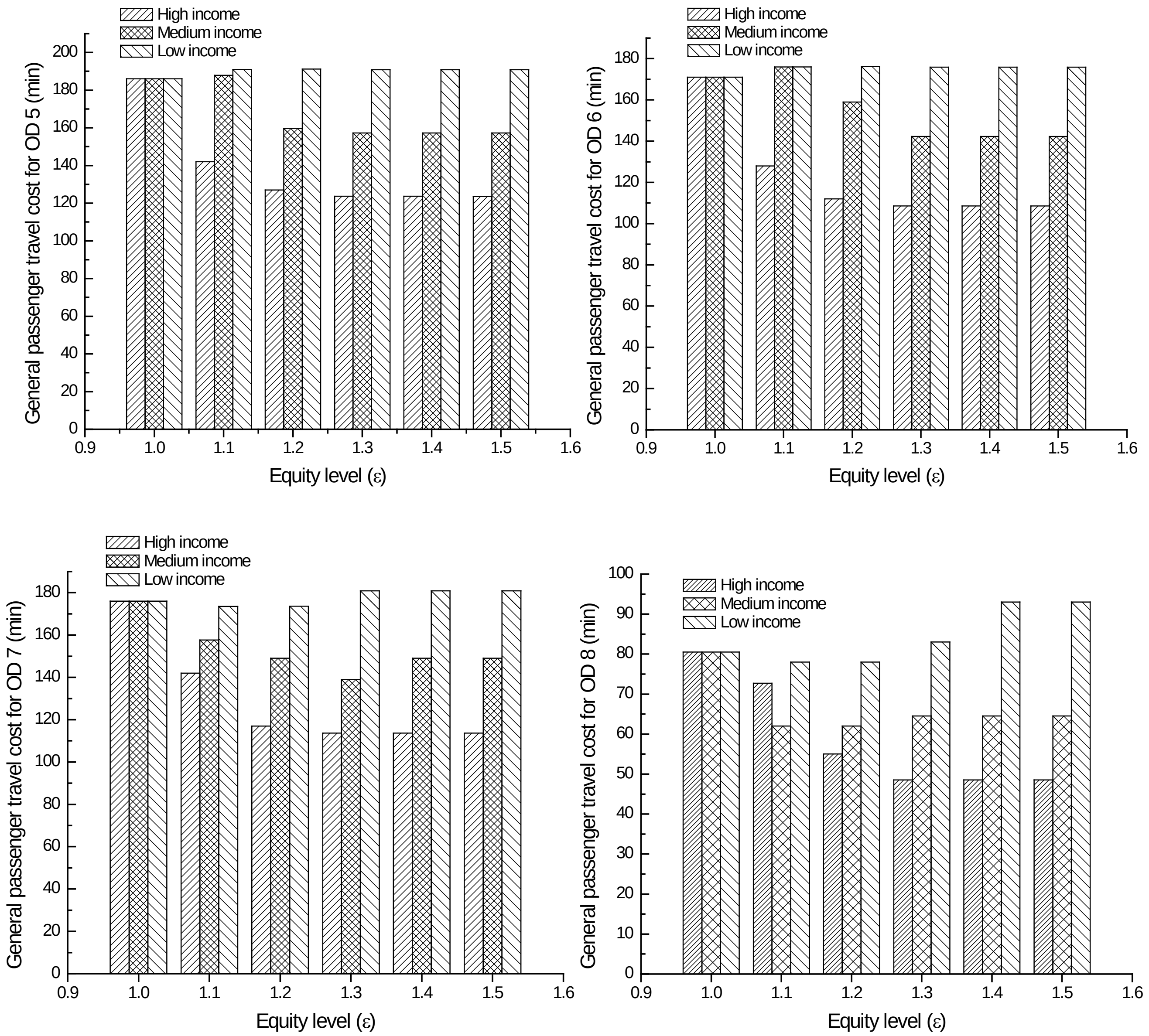


Figure 5. The general travel cost for passengers at different equity levels with a minimum operating cost of 25,000 yuan.

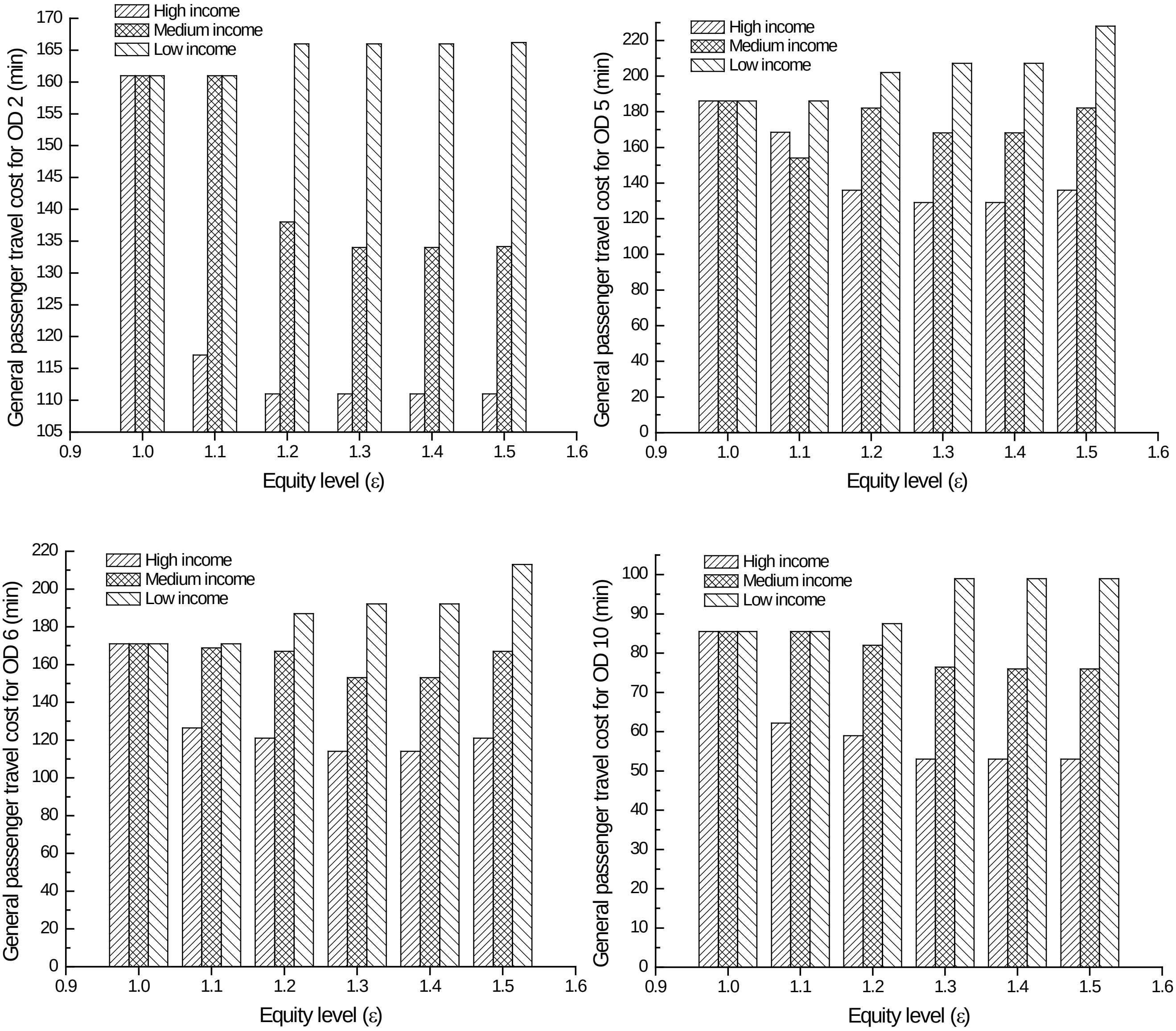


Figure 6. The general travel cost for passengers at different equity levels with a minimum operation cost of 30,000 yuan.

5.1.3. Social equity and pricing

We also investigated ticket pricing for passengers of different income levels with various social equity levels, again on the small case with the same parameter values as described in the previous two subsections. The minimum required revenue was set to 30,000 yuan. To obtain good solutions for all cases, we did not limit the computation time to 5 min but allowed it to run longer until the relative gap was less than 1%. The results are given in Table 5. The first three columns are the same as those in Table 4. The fourth column shows the total ticket revenue for high-, medium-, and low-income passengers. The last column shows the average ticket price per person per running minute for high-, medium-, and low-income passengers. The detailed ticket prices for high-, medium-, and low-income passengers are shown in Figure 7.

Table 5. Ticket prices with different social equity levels.

Revenue	Equity level	Objective	Ticket revenue (yuan)	Ticket price [yuan/(person × min)]

(yuan)	(ε)	(min)	High	Medium	Low	High	Medium	Low
	1.0	63239	13187	10535	6277	1.49	1.30	1.07
	1.1	56269	12998	10684	6317	1.47	1.32	1.08
30,000	1.2	53212	13435	10325	6240	1.52	1.27	1.07
	1.3	52831	12848	10597	6553	1.45	1.31	1.12
	1.4	52812	13223	10044	6732	1.49	1.24	1.15
	1.5	52984	13105	10140	6755	1.48	1.25	1.15

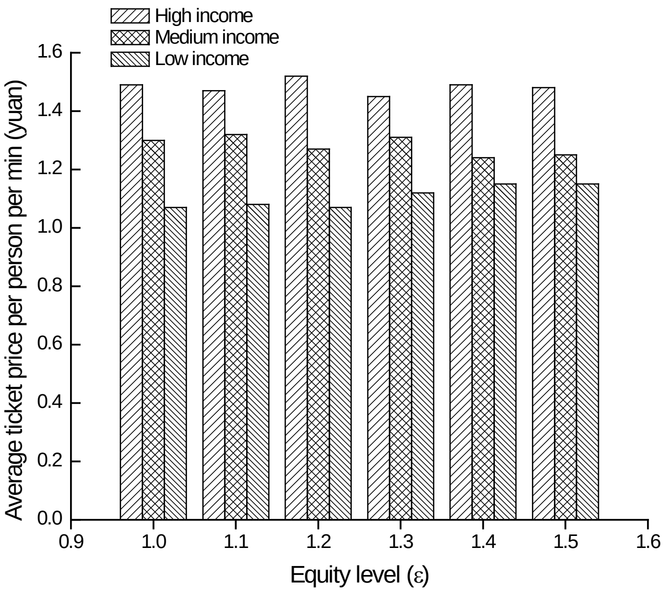


Figure 7. The average ticket price for high-, medium-, and low-income passengers at various equity levels.

From Table 5 and Figure 7, we can see that low-income passengers prefer to choose trains with a lower ticket price, whereas high-income passengers prefer to choose trains with a higher ticket price. This is because high-income passengers do not care about the high ticket price and choose the most convenient trains for them, which is consistent with what we expected. The average ticket prices of medium- and low-income passengers are 86% and 75% of those of high-income passengers. With increasing ε, the ticket price for low-income passengers increases, but the influence is not very obvious in the small case used herein.

5.2. A large case study

5.2.1. A description of the railway network and passenger demand

We tested our model on a part of a Chinese HSR network: the Guangzhou–Nanning and Guangzhou-Guilin HSR network (Figure 8). This railway network consists of two double-track HSR lines: the Guangzhou–Nanning HSR line and the Guangzhou–Guilin HSR line. The former is 563 km long and has 14 stations from Guangzhou South Station to Nanning East Station, whereas the latter is 438 km long consisting of 11 stations from Guangzhou South Station to Guilin North Station. Therefore, there are 21 stations in total and 20 segments between any two consecutive stations on this HSR network with a total length of 1001 km. Note that we do not consider the Guilin West Station on the Guangzhou –Guilin HSR line. The trains to Guilin West Station are assumed to end at Guilin North. Each station is shown by a circle in the figure, and terminal stations are denoted by double circles. The length of each segment is given beside it in the figure. The network consists of two bidirectional lines. Therefore, we have four unidirectional train lines, numbered from 1 to 4 in Figure 8. For more information about this network, we refer to the Chinese railway official website www.12306.cn.

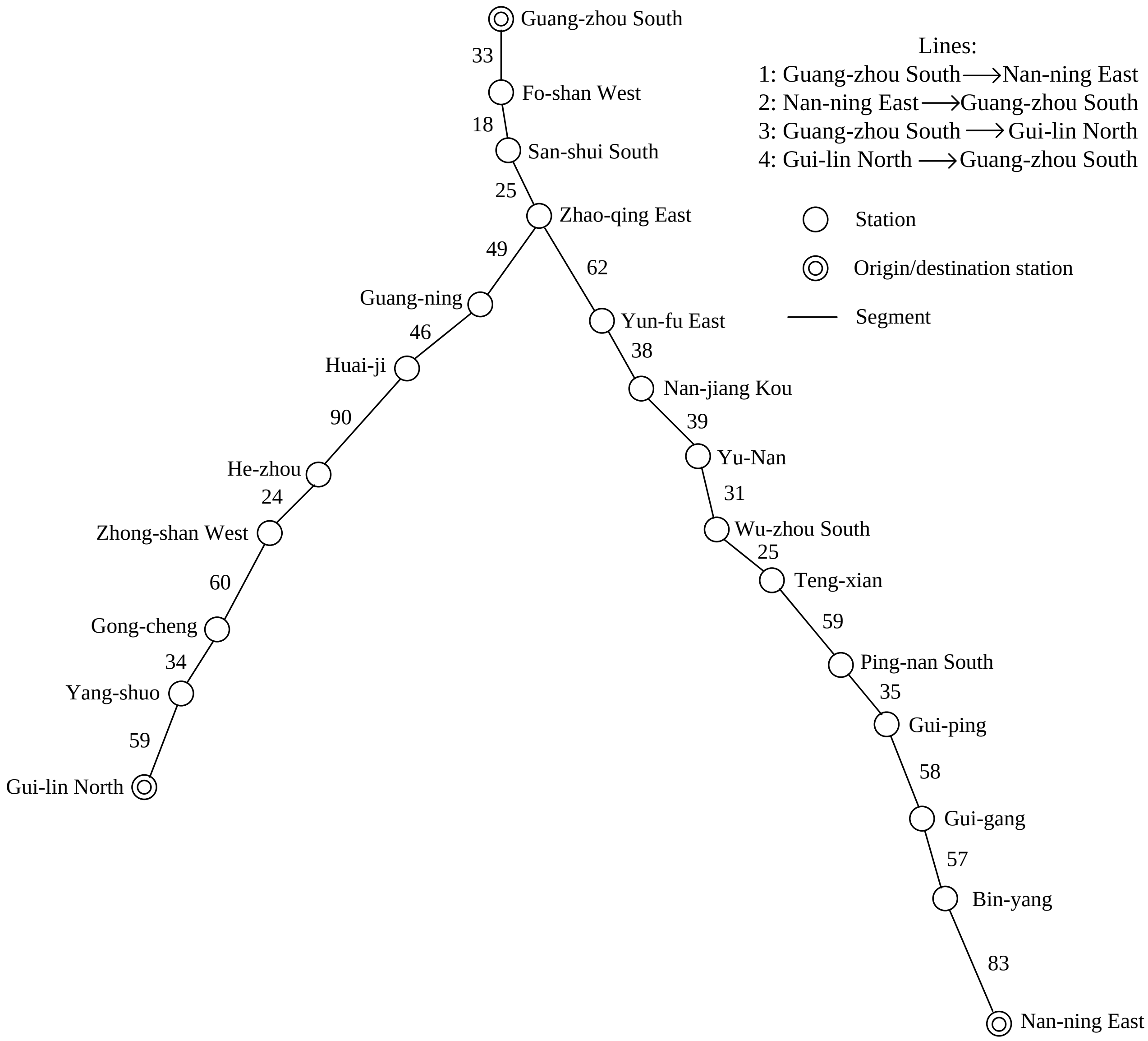


Figure 8. A part of south China HSR network.

According to the timetable of March 2020, 56 trains run from Guangzhou South to Nanning East (or Nanning), 55 trains run from Nanning East (or Nanning) to Guangzhou South, 61 trains run from Guangzhou South to Guilin North (or Guilin West), and 61 trains run from Guilin North (or Guilin West) to Guangzhou South per day. Therefore, there are 233 trains operating on this railway network during a single day. In this study, we considered a planning period of three hours from 06:00 a.m. to 09:00 a.m., which includes the morning peak period. The information for trains in our considered period is shown in Table 6. As the timetable is usually made separately for each direction, we only considered trains running in one direction of these two lines, from Guangzhou South to Nanning East and from Guangzhou South to Guilin North. Our model can handle trains in both directions simultaneously, but it would require a longer computation time. The maximum design speed of Nanning-Guangzhou HSR line is 250 km/h and that of Guilin-Guangzhou HSR line is 300 km/h. However, the daily operating speeds of both lines are 250 km/h. From the timetable given in Table 6, we can see that it is a non-periodic timetable, where lines are busy during some periods but not in others. Based on the actual timetable, we calculated the running time of each segment according to the length of the segment and the running speed of trains; the results are given in Table 7. The required dwell time in each stop station was assumed to be 2 min, and if a train stopped at an intermediate station, the additional acceleration and deceleration times were both taken as 2 min. As the timetable for Chinese HSRs is non-cyclic, the stopping pattern of trains in the same line can be different. The train-stopping pattern is determined at the line planning stage, and we used the same stopping pattern for each train as that in the actual timetable. Therefore, the values of parameters b_i^{plv} and $\bar{r}_i^{plv}$ were calculated by the running time, the dwell time (also the additional acceleration and deceleration times), and the stopping pattern of trains.

Table 6. The train lines of the studied HSR network.

Train lines	From	To	Trains	Departure time			
1	Guangzhou South	Nanning East	9	06:52	07:36	07:42	07:56
				08:06	08:22	08:27	08:32
				08:47			
2	Guangzhou South	Guilin North (West)	10	06:57	07:02	07:07	07:13
				07:23	07:53	08:05	08:17
				08:42	08:47		

Table 7. The running time in each segment between two stations.

ID	Segment	Run time (min)	ID	Segment	Run time (min)
1	Guangzhou South–Foshan West	8	11	Guiping– Guigang	14
2	Foshan West –Sanshui South	5	12	Guigang–Binyang	14
3	Sanshui South–Zhaoqing East	6	13	Binyang–Nanning East	20
4	Zhaoqing East–Yunfu East	15	14	Zhaoqing East–Guangning	12
5	Yunfu East–Nanjiang Kou	10	15	Guangning–Huaiji	12
6	Nanjiang Kou–Yunan	10	16	Huaiji–Hezhou	22
7	Yunan–Wuzhou South	8	17	Hezhou–Zhongshan West	6
8	Wuzhou South–Tengxian	6	18	Zhongshan West–Gongcheng	15
9	Tengxian–Pingnan South	15	19	Gongcheng–Yangshuo	9
10	Pingnan South–Guiping	9	20	Yangshuo–Guilin North	15

As mentioned previously, the timetable for a Chinese HSR is non-periodic, and the frequency of trains is determined by passenger demand. Passenger demand is normally obtained by a passenger survey or forecast, whereas detailed passenger OD information is difficult to determine. In this study, we generated the passenger OD demand based on the actual train timetable (e.g., train frequency and train-stopping pattern) and the scale of each city (e.g., the population of the city) where a station was located. The details of how we generated the passenger OD are given in Appendix B. We generated 531 passenger groups in total, which included high-, medium-, and low-income passenger groups.

Because the stopping pattern of each train is different, if a passenger group wants to use a train, this train must stop at the passengers' boarding and alighting stations. Thus, we did not allow passengers to choose a train without stops at their boarding or alighting station. The ticket price was determined based on the length of a segment. According to actual practice, the ticket fare of each segment was about 1.25 times the running time (see the running time in Table 7) for our considered HSR network. Therefore, in this study, we allowed the ticket price for a segment to vary between 0.65 times the segment running time (f_s^{min}) and 1.85 times the segment running time (f_s^{max}), which are approximately 50% down and up the practical value. We assumed that the values of $\beta_1 - \beta_4$ were 2.5, 10, 0.5, and 1, respectively, and that β_5 for the three types of passengers was 0.5, 1.0, and 1.5, which are the same values as those used in the small case study. In this situation, high-income passengers' salary is three times as that of low-income passengers, which is reasonable for residents living along these two lines. The capacity of each train was 630 ($\mu_v^l \times q = 630$). In practice, some long trains with 16 carriages consisted of two short trains also operate on this network, but we do not know the detail information. Therefore, we assume all the trains have the same capacity 630 here, which can be adapted if the detailed information is available. The value of $C^o_{it\tau}$ for a passenger group (i, t, τ) that cannot use any train was set to 750 min and the value of M was 1,000.

5.2.2. Test results.

Based on the case and parameter settings, we tested our model on this large-scale railway network. We tested with different revenue requirements, where the minimum required revenue was set to either 900,000 or 950,000 yuan. The computation time-limit was 6 h. We allowed the value of ε to vary from 1.0 to 1.5 with an interval of 0.1 to see the influence of equity levels, and we obtained near-optimal solutions within the time limit for all cases.

First, we found that the total general passenger travel cost decreased with increasing ε, which means that if we want to improve the equity for passengers of different income levels, the total general passenger cost must increase. This can be regarded as the cost required to achieve a better equity level. The relationship between total general passenger travel cost and the equity level under revenues of 900,000 and 950,000 yuan are shown by subfigures (a) and (b), respectively, in Figure 9. In subfigures (a), a small increase is existed when $\varepsilon = 1.4$ because of a larger optimality gap within the given time limit. Second, we investigated the impact of equity levels on the travel cost for passengers of various incomes. We found that when ε is 1.0, the average general travel cost per person for passengers of different incomes was approximately the same. However, with increasing ε, the difference in the average general travel cost for high-, medium-, and low-income passengers increased (Figure 10). Specifically, the general travel cost for high-income and medium-income passengers decreased when we did not emphasize on the equity issue, and both groups used paths with high utility.

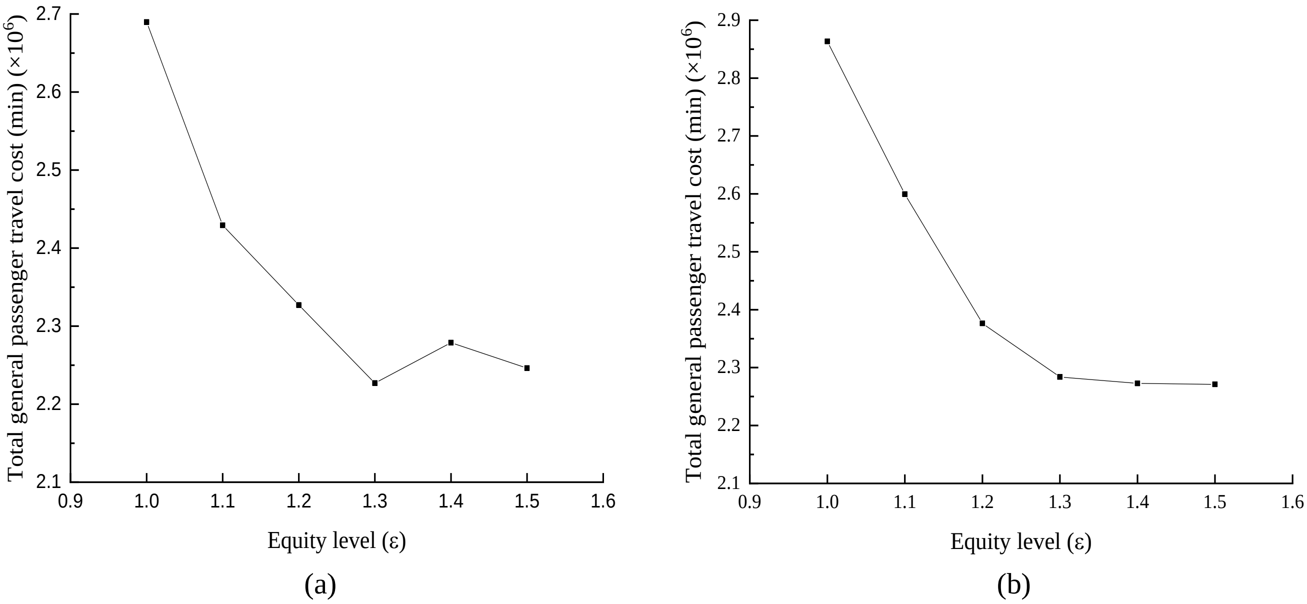
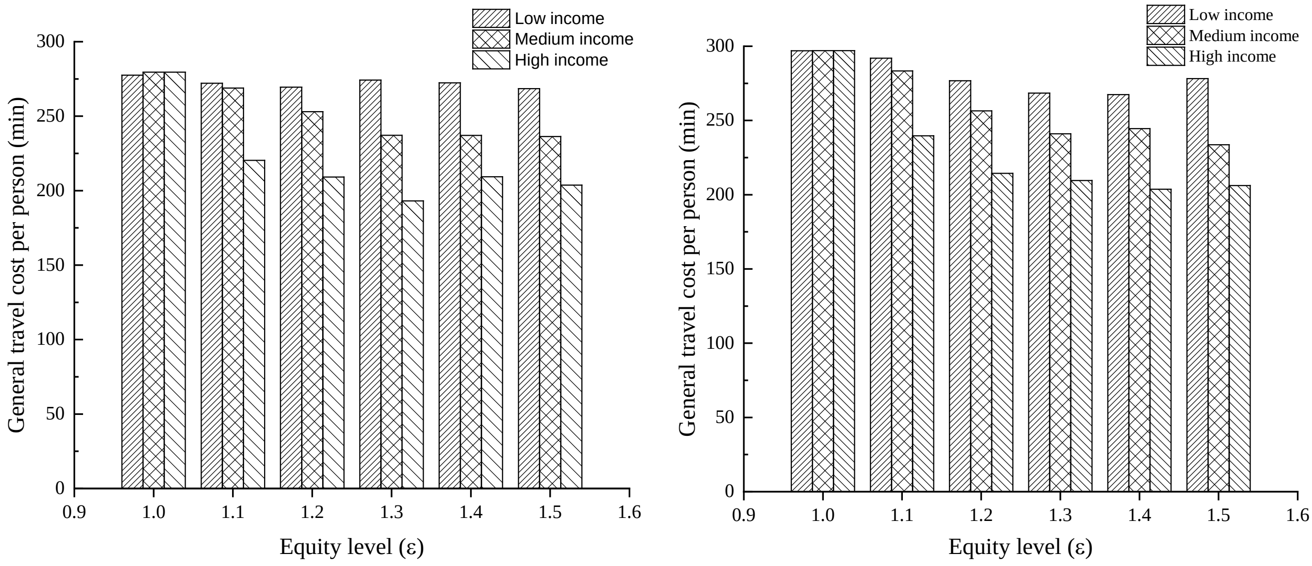


Figure 9. The total general passenger travel cost with various equity levels for revenues of (a) 900,000 and (b) 950,000 yuan.



(a) (b)

Figure 10. The general travel cost per person for different income passengers with various equity levels for revenues of (a) 900,000 and (b) 950,000 yuan.

We also investigated the ticket prices for passengers of different income levels. To do so, we specified the same passenger volumes of different income-levels. Then, we decreased the passenger OD volumes of high- and medium-income passengers that travel between large stations from 30 to 20, increased the passenger OD volume of low-income passengers that travel between large stations and medium stations from 10 to 15, and decreased the passenger OD volume of low-income passengers that travel between medium stations from 20 to 10 as shown in Table 10. Therefore, the volumes of passengers for high-income, medium-income and low-income passengers in each OD pair remained the same. In this case, if the ticket prices for passengers of different income levels are the same, the total ticket revenue should be identical. The minimum required revenue is set to 900,000.

All of the other parameters maintained the same values as before, and the computational time was limited to 6 h. We obtained the total ticket revenues for passengers of different income levels. For comparison purpose, we calculated the average ticket price per person per kilometer for different income-class passengers. The results were shown in Figure 11. We could see that the average ticket price for low-income passengers was much lower than that of high-income and medium-income passengers. The average ticket price for high-income passengers was the highest. Specifically, the average ticket prices for low- and medium-income passengers are 79% and 87% of those of high-income passengers in this case. This was because low-income passengers were more concerned about the ticket price than high-income passengers. There is no obvious regulation for the average ticket price affected by the equity level. The average ticket price fluctuated with the increase of the value of ε for all the three types of passengers.

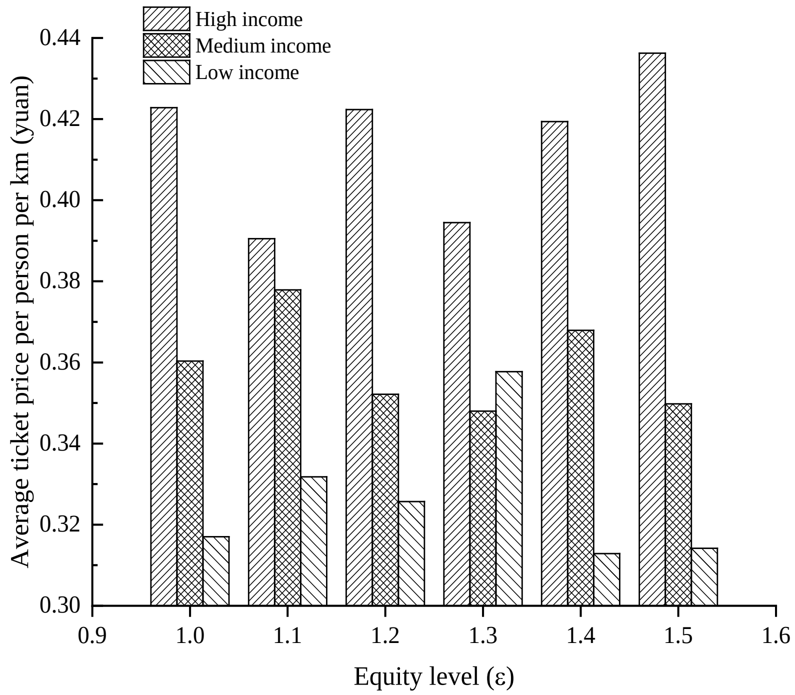


Figure 11 The average ticket price per person per kilometer for different income passengers

6. Conclusions

In this study we considered the equity issue in HSR planning. Specifically, we aimed to generate a passenger-centric train timetable while explicitly considering the equity of passengers of different income levels. Three aspects were considered: minimizing the general travel cost for passengers from the passengers' perspective, maximizing the revenue from the train companies' perspective, and improving social equity from the government's perspective. By including these last two aspects in the constraints, we obtained a mixed-integer linear programming model having a single objective. The influences of revenue for train operation companies and social equity required by the government on passenger travel cost were investigated. It was found that trade-offs among these three aspects could be achieved by selecting appropriate revenue and equity parameters. A dynamic ticket-pricing strategy was applied to alter the social equity by reducing the general travel cost for low-income passengers.

Our model was tested on both a hypothetical railway network and the Guangzhou–Nanning and Guangzhou–Guilin HSR network in China. The results for the hypothetical railway network showed that minimizing the general passenger travel-cost and maximizing the revenue of train operation companies are in conflict, and if the revenue is less than 25,000 yuan, the total general travel-cost for passengers is low. In addition, the social equity level significantly affects the general passenger travel cost. Specifically, if we want to achieve a better social equity level, the total general passenger travel cost must be increased, but the differences between the general travel costs for high-, medium-, and low-income passengers must decrease. A higher equity level means a relatively lower travel cost for low-income passengers. Also, low-income passengers tend to choose trains with lower prices, whereas high-income passengers do not. Our test with a real-world HSR network demonstrated that our model can solve an actual problem within a reasonable length of computation time. The results also showed that our equity-based timetabling model can help reduce the differences among passengers of various income classes.

Our equity-based timetabling model is complicated. Although we considered the detailed passenger route choice and the equity issue in the model, the conflicts among trains were not considered. Therefore, it will be necessary to take train conflict-prevention into account in future studies to ensure that our timetable is also useful for railway infrastructure managers. In addition, more efficient algorithms are necessary for larger-scale problems.

Finally, it is interesting to consider other external costs from a societal perspective beyond equity, such as environmental costs, by extending our model in future research.

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Appendix A. Notations and model formulation.

Table 8. Notations used in the model.

Notations	Description	Type
L	Set of train lines given by line plan, $l \in L$	Set
L^p	Set of train lines used by path p, $L^p \subset L$	Set
V^l	Set of available trains for line l	Set
(l, v)	The vth train serving line l	Index
I	Set of passenger OD pair, $i \in I$	Set
T_i	Set of desired arrival time of OD pair i, $t \in T_i$	Set
Π	Set of passenger income levels for passengers in the same OD pair with the same desired arrival time, $\tau \in \Pi$	Set
(i, t, τ)	A passenger group in OD pair i with a desired arrival time t having an income level τ	Index
P_i	Set of possible paths of OD pair i, $p \in P_i$	Set
$P_{it\tau}$	Set of available paths of passenger group (i, t, τ)	Set
S	Set of segments of HSR network, $s \in S$	Set
S^l	Set of segments used by line l, $S^l \subset S$	Set
S^{lp}	Set of segments used by line l on path p, $S^{lp} \subset S$	Set
$n_{it\tau}$	Number of passengers in group (i, t, τ)	Parameter
$\bar{a}_{it\tau}$	Desired arrival time of passenger group (i, t, τ)	Parameter
b_i^{plv}	Time to travel from the origin station of line l using train (l, v) on path p to a boarding station of passenger traveling between OD pair i	Parameter
$\bar{r}_i^{plv}$	Time on board of train (l, v) for OD pair i along path p	Parameter
μ_v^l	The number of carriage units used by train (l, v)	Parameter
ε	The amount by which the general travel cost of low-income passengers cannot exceed the average general travel cost of all passenger groups of the same (i, t)	Parameter
q	The capacity of each carriage	Parameter
h	The minimum train departure headway	Parameter
m	The minimum transfer time	Parameter
f_s^{min}	The minimum fare of segment s	Parameter
f_s^{max}	The maximum fare of segment s	Parameter
$U_{it\tau}^o$ $(C_{it\tau}^o)$	The utility (general travel cost) of passenger group (i, t, τ) if it cannot use any path p	Parameter

M	A large positive number	Parameter
$d_{l,v}$	The time train v of line l departs from the first station	Decision
$x_{it\tau}^{p}$	Whether passenger group (i,t,τ) chooses path p; 1 = yes, 0 = otherwise	Decision
$y_{it\tau}^{plv}$	Whether passenger group (i,t,τ) take train v on line l when choosing path p; 1 = yes, 0 = otherwise	Decision
$z_{it\tau}^{plvs}$	Whether passenger group (i,t,τ) take train v on line l pass segment s when choosing path p; 1 = yes, 0 = otherwise	Decision
o_{s}^{lv}	Occupation of train v on line l in segment s	Decision
$w_{it\tau}^{p}$	Waiting time of passenger group (i,t,τ) using path p	Decision
$w_{it\tau}^{plvl'v'}$	Waiting time of passenger group (i,t,τ) using path p to transfer from train (l,v) on line l to train (l',v') on line l'	Decision
$r_{it\tau}^{p}$	The in-vehicle time of passenger group (i,t,τ) using path p	
$e_{it\tau}^{p}$	The earliness of arrival of passenger group (i,t,τ) using path p	Decision
$\Delta_{it\tau}^{p}$	The lateness of arrival of passenger group (i,t,τ) using path p	Decision
f_{s}^{lv}	The ticket cost of train v of line l traveling on segment s	Decision
$\bar{f}_{it\tau}^{plvs}$	The ticket cost for passenger group (i,t,τ) using train (l,v) on path p passing segment s	Decision
$f_{it\tau}^{p}$	The ticket cost of passenger group (i,t,τ) using path p	Decision
$U_{it\tau}^{p}$ $(C_{it\tau}^{p})$	The utility (general travel cost) of passenger group (i,t,τ) using path p	Decision
$U_{it\tau}$ $(C_{it\tau})$	The utility (general travel cost) of passenger group (i,t,τ)	Decision
$\bar{U}_{it}$ $(\bar{C}_{it})$	The average utility (general travel cost) of all passenger groups in OD pair i with desired arrival time t	Decision

Our goal is to create a good timetable for both passengers and train operators while explicitly considering social equity. As railway timetabling follows railway line planning, we assume the railway line plan is given. Therefore, the train lines, the train frequency within each line, and the stopping pattern are known when we create the train timetable. When creating the timetable, we need to consider different aspects. First, the general travel cost for passengers should be minimized from the passengers' perspective. Second, the train operation companies aim to maximize their revenue. Third, social equity must be considered from the government's point of view. These three aspects are usually in conflict and cannot be fulfilled simultaneously. In this study, we minimize the total general travel cost for passengers in the objective function of our model and include the revenue and the social equity in the constraints. Therefore, our model can be formulated as follows:

$$\min \sum_{i\in I}\sum_{t\in T_i}\sum_{\tau\in\Pi}\sum_{p\in P_i} C_{it\tau}\times n_{it\tau} \tag{5}$$

$$C_{it\tau}\geq C_{it\tau}^{p}-M\left(1-x_{it\tau}^{p}\right) \quad \forall i\in I,\ t\in T_i,\ \tau\in\Pi,\ p\in P_i \tag{6}$$

$$C_{it\tau}\geq C_{it\tau}^{o}\left(1-\sum_{p\in P_i} x_{it\tau}^{p}\right) \quad \forall i\in I,\ t\in T_i,\ \tau\in\Pi \tag{7}$$

$$\sum_{p\in P_i} x_{it\tau}^{p}\leq 1 \quad \forall i\in I,\ t\in T_i,\tau\in\Pi \tag{8}$$

$$\sum_{v\in V^l} y_{it\tau}^{plv}=x_{it\tau}^{p} \quad \forall i\in I,\ t\in T_i,\tau\in\Pi,\ p\in P_i, l\in L^p \tag{9}$$

$$o_{s}^{lv}=\sum_{i\in I}\sum_{t\in T_i}\sum_{\tau\in\Pi}\sum_{p\in P_i} z_{it\tau}^{plvs}\times n_{it\tau} \quad \forall l\in L,\ v\in V^l, s\in S^l \tag{10}$$

$$\mu_{v}^{l}\times q\geq o_{s}^{lv} \quad \forall l\in L,\ v\in V^l, s\in S^l \tag{11}$$

$$f_{it\tau}^{p}=\sum_{l\in L^p}\sum_{v\in V^l}\sum_{s\in S^{lp}} \bar{f}_{it\tau}^{plvs} \quad \forall i\in I,\ t\in T_i,\tau\in\Pi,\ \forall p\in P_i \tag{12}$$

$$d_{l,v}\leq d_{l,v+1}-h \quad \forall l\in L,\ v\in V^l: v<|V^l| \tag{13}$$

$$\frac{C_{it\tau}}{\sum_{\tau'\in\Pi} C_{it\tau'}/|\Pi|}\leq\varepsilon \quad \forall i\in I,\ t\in T_i,\tau\in\Pi \tag{14}$$

$$z_{it\tau}^{plvs}=y_{it\tau}^{plv} \quad \forall i\in I,\ t\in T_i,\tau\in\Pi, p\in P_i, l\in L^p, v\in V^l, s\in S^{lp} \tag{15}$$

$$f_s^{lv} - M \times \left(1 - y_{it\tau}^{plv}\right) \le \bar{f}_{it\tau}^{plvs} \le f_s^{lv} \quad \forall i \in I, t \in T_i, \tau \in \Pi, \forall p \in P_i,\ l \in L^p, v \in V^l, s \in S^{lp} \tag{16}$$

$$\bar{f}_{it\tau}^{plvs} \le M \times y_{it\tau}^{plv} \quad \forall i \in I, t \in T_i, \tau \in \Pi, \forall p \in P_i,\ l \in L^p, v \in V^l, s \in S^{lp} \tag{17}$$

$$\sum_{i\in I} \sum_{t\in T_i} \sum_{\tau\in\Pi} \sum_{p\in P_i} f_{it\tau}^{p} \times n_{it\tau} \ge R \tag{18}$$

$$f_s^{min} \le f_s^{lv} \le f_s^{max} \quad \forall l \in L, v \in V^l, s \in S^l \tag{19}$$

$$x_{it\tau}^{p} \in \{0,1\} \quad \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i \tag{20}$$

$$y_{it\tau}^{plv} \in \{0,1\} \quad \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i, l \in L^p,\ v \in V^l \tag{21}$$

$$z_{it\tau}^{plvs} \in \{0,1\} \quad \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i, l \in L^p, v \in V^l, s \in S^{lp} \tag{22}$$

$$d_{l,v} \ge 0 \quad \forall l \in L,\ v \in V^l \tag{23}$$

$$o_s^{lv} \ge 0 \quad \forall l \in L,\ v \in V^l, s \in S^l \tag{24}$$

$$f_s^{lv} \ge 0 \quad \forall l \in L,\ v \in V^l, s \in S^l \tag{25}$$

$$\bar{f}_{it\tau}^{plvs} \ge 0 \quad \forall i \in I, t \in T_i, \tau \in \Pi, \forall p \in P_i,\ l \in L^p, v \in V^l, s \in S^{lp} \tag{26}$$

$$f_{it\tau}^{p},\ C_{it\tau}^{p} \ge 0 \quad \forall i \in I, t \in T_i, \tau \in \Pi, p \in P_i \tag{27}$$

$$C_{it\tau} \ge 0 \quad \forall i \in I, t \in T_i, \tau \in \Pi \tag{28}$$

Objective function (5) minimizes the total general passenger travel cost, which is denoted as the cost $C_{it\tau}$ of each passenger group (i,t,τ) multiplied by the number of passengers $n_{it\tau}$ of passenger group (i,t,τ). Constraint (6) ensures that each passenger group (i,t,τ) chooses the path with the lowest general travel cost. Constraint (7) indicates that if a passenger group (i,t,τ) cannot use any path to travel by railway due to limited train capacity, its general cost is given as that of an alternative ($C_{it\tau}^{o}$), e.g., the cost of traveling by bus. Constraint (8) denotes that each passenger group can choose at most one path to travel from its origin to its destination. Constraint (9) means that if a passenger group uses a path p, it must use a certain train (l,v) on line l of path p. Constraint (10) is used to calculates the occupation of each segment s, and constraint (11) ensures that the total number of passengers on train (l,v) in segment s cannot exceed the capacity of the train. Constraint (12) is used to calculate the ticket fare of a passenger group (i,t,τ) using path p by adding the fare for each segment of path p. In constraint (13), $|V^l|$ denotes the last train on line l. This constraint removes the symmetrical solution and ensures that the departure headway between two trains of the same line in the origin cannot be smaller than the minimum headway h. Constraint (14) is the social equity constraint, which ensures that the general travel cost of a passenger group (i,t,τ) consisting of low-income passengers cannot exceed the average general travel cost of all the passenger groups with the same origin, destination, and expected arrival time by ε times. Constraint (15) builds the relationship between variables $z_{it\tau}^{plvs}$ and $y_{it\tau}^{plv}$. Constraints (16) and (17) ensure that $\bar{f}_{it\tau}^{plvs}$ receives the correct value. Specifically, if passenger group (i,t,τ) uses train (l,v) of path p ($y_{it\tau}^{plv} = 1$), its fare should be f_s^{lv}; otherwise, it is 0. Constraint (18) ensures that the revenue will be larger than a given value (R). Constraint (19) specifies the proper value of variable f_s^{lv}, which must be within a given interval. Constraints (20)–(28) show the domain of variables.

Additional constraints to model some of the attributes used in the utility function are as follows:

$$r_{it\tau}^{p} = \sum_{l\in L^p} \sum_{v\in V^l} \bar{r}_i^{plv} \times y_{it\tau}^{plv} \quad \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i \tag{29}$$

$$w_{it\tau}^{p} = \sum_{l,l' \in L^p: l' > 1} \sum_{v\in V^l, v' \in V^{l'}} w_{it\tau}^{plvl'v'} \quad \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i \tag{30}$$

$$w_{it\tau}^{plvl'v'} \ge \left(d_{l'v'} + b_i^{pl'v'}\right) - \left(d_{lv} + b_i^{plv} + \bar{r}_i^{plv} + m\right) - M\left(1 - y_{it\tau}^{plv}\right) - M\left(1 - y_{it\tau}^{pl'v'}\right)$$
$$\forall i \in I,\ t \in T_i,\ \tau \in \Pi,\ p \in P_i,\ l, l' \in L^p: l' > 1, l = l' - 1, v \in V^l, v' \in V^{l'} \tag{31}$$

$$w_{it\tau}^{plvl'v'} \le \left(d_{l'v'} + b_i^{pl'v'}\right) - \left(d_{lv} + b_i^{plv} + \bar{r}_i^{plv} + m\right) + M\left(1 - y_{it\tau}^{plv}\right) + M\left(1 - y_{it\tau}^{pl'v'}\right)$$
$$\forall i \in I,\ t \in T_i,\ \tau \in \Pi,\ p \in P_i,\ l, l' \in L^p: l' > 1, l = l' - 1, v \in V^l, v' \in V^{l'} \tag{32}$$

$$\Delta_{it\tau}^{p} \geq \left(d_{|L^p|,v} + b_i^{p|L^p|v} + \bar{r}_i^{p|L^p|v}\right) - \bar{a}_{it\tau} - M\left(1 - y_{it\tau}^{p|L^p|v}\right) \quad \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i, v \in V^{|L^p|} \tag{33}$$

$$e_{it\tau}^{p} \geq \left(\bar{a}_{it\tau} - \left(d_{|L^p|,v} + b_i^{p|L^p|v} + \bar{r}_i^{p|L^p|v}\right)\right) - M\left(1 - y_{it\tau}^{p|L^p|v}\right) \ \forall i \in I,\ t \in T_i, \tau \in \Pi, p \in P_i,\ v \in V^{|L^p|} \tag{34}$$

$$C_{it\tau}^{p} = \left(r_{it\tau}^{p} + \beta_1 \times w_{it\tau}^{p} + \beta_2 \times (|L^p|-1) \times x_{it\tau}^{p} + \beta_3 \times e_{it\tau}^{p} + \beta_4 \times \Delta_{it\tau}^{p} + \beta_5 \times f_{it\tau}^{p}\right)$$
$$\forall i \in I, t \in T_i, \tau \in \Pi, p \in P_i \tag{35}$$

$$r_{it\tau}^{p}, w_{it\tau}^{p}, \Delta_{it\tau}^{p}, e_{it\tau}^{p} \geq 0 \quad \forall i \in I,\ \ t \in T_i, \tau \in \Pi, p \in P_i \tag{36}$$

$$w_{it\tau}^{plvl'\ v'} \geq 0 \quad \forall i \in I,\ \ t \in T_i, \tau \in \Pi, p \in P_i, l, l' \in L^p: l' > 1, l = l' - 1, v \in V^l, v' \in V^{l'} \tag{37}$$

Equations (29) and (30) are used to calculate the in-vehicle time and waiting time of passenger group (i,t,τ) on path p, respectively. Inequities (31) and (32) are used to calculate the transfer waiting time between two trains (l,v) and (l',v') of passenger group (i,t,τ) along path p. Inequities (33) and (34) specify the arrival delay and earliness of passenger group (i,t,τ) on path p. Here, $|L^p|$ denotes the last line of path p. Equation (35) calculates the general travel cost of passenger group (i,t,τ) using path p. Constraints (36) and (37) show the domain of variables.

Appendix B. Passenger demand generation

To model passenger demand, we defined passenger groups according their origin, destination, the desired arrival time, and income level. Generally, passenger travel demand is related to the number of inhabitants in a city. Therefore, we classified all the stations of the considered railway network into three levels based on passenger demand: large stations, medium stations, and small stations. From Figure 8, we can see there are 21 stations in total, which were classified into the three types shown in Table 9.

Table 9. Different classes of stations on the considered HSR network.

ID	Station type	Stations
1	Large stations	Guang-zhou South, Zhao-qing East, Wu-zhou South, Nan-ning East, Gui-lin North
2	Medium stations	Fo-shan West, Gui-ping, Gui-gang, He-zhou, Yang-shuo
3	Small stations	All other stations

Most trains stop only at large stations and medium stations; only a few trains stop at small stations. In addition, the period from 07:00 a.m. to 09:00 a.m. is the morning peak period and that from 06:00 a.m. to 07:00 a.m. is the off-peak hour. Thus, we generated three passenger OD demands for each hour between any two large stations during the peak period, which were assumed to be evenly distributed. As passengers of the same OD can be further divided into three groups based on their income levels, we obtained nine passenger OD groups between any two large stations within one hour of the peak period. For the off-peak period, passenger demand is lower, and we generated two demands between any two large stations for an hour. Therefore, we obtained six passenger groups between any two large stations in the period from 07:00 a.m. to 09:00 a.m. In total, we obtained 192 passenger groups among large stations for both the peak period and the off-peak period. Similarly, we generated two passenger ODs for each hour of the peak period and one OD for each hour of the off-peak period between any large station and medium station. Therefore, we obtained 285 passenger groups between a large station and a medium station. The demand between two medium stations is small. As a result, we generated one passenger OD for each hour between any two medium stations, which resulted in six passenger ODs and 54 passenger groups in total. We did not consider passenger demand between any two small stations because this demand is quite small.

For the passenger volume of each passenger group, we assumed that each high-income or medium-income passenger group between two large stations had 30 passengers, whereas the low-income passenger group had 20 passengers. In addition, the passenger volume of any high-income or medium-income passenger group between a large station and a medium station was 15 passengers and that of a low-income passenger group was also 10 passengers. Finally, the passenger volume of any high-income or medium-income passenger group between any two medium stations was 10 passengers and that of a low-income passenger group was 20 passengers. Because high-income inhabitants prefer to live in large cities in China, the proportion of low-income passengers in large cities is smaller than that in relatively small cities. Detailed passenger information is given in Table 10.

Table 10. Passenger OD information.

OD types	Income levels	No. of passenger groups	Volume
From large station to large station	High income	64	30
	Medium income	64	30
	Low income	64	20
From large station to medium station or from medium station to large station	High income	95	15
	Medium income	95	15
	Low income	95	10
From medium station to medium station	High income	18	10
	Medium income	18	10
	Low income	18	20
In total		531	9,640

References

Albalate, D., Bel, G., 2012. The Economics and Politics of High-Speed Rail. Lessons from Experiences Abroad. Rowman and Littlefield Publishers (Lexington Books), Lanham, MA.

Armstrong, A., Meissner, J., 2010. Railway revenue management: overview and models. Working Paper. The Department of Management Science, Lancaster University, Lancaster, U.K.

Behbahani, H., Nazari, S., Kang, M. J., Litman, T., 2019. A conceptual framework to formulate transportation network design problem considering social equity criteria. Transportation Research Part A: Policy and Practice, 125, 171-183.

Ben-Akiva, M., Lerman, S., 1985. Discrete choice analysis. The MIT Press, Cambridge, Massachusetts.

Ben-Elia, E., Benenson, I., 2019. A spatially-explicit method for analyzing the equity of transit commuters' accessibility. Transportation Research Part A: Policy and Practice, 120, 31-42.

Bertolaccini, K. L., 2013. Assessing the equity of transit supply distribution in metropolitan areas using Lorenz curves and Gini coefficients. Master’s Thesis. Paper 483.

Bharill, R., Rangaraj, N., 2008. Revenue management in railway operations: a study of the Rajdhani Express, Indian Railways. Transportation Research Part A: Policy and Practice, 42(9), 1195-1207.

Bills, T. S., Walker, J. L., 2017. Looking beyond the mean for equity analysis: examining distributional impacts of transportation improvements. Transport Policy, 54, 61-69.

Binder, S., Maknoon, Y., Bierlaire, M., 2017. The multi-objective railway timetable rescheduling problem. Transportation Research Part C: Emerging Technologies, 78, 78-94.

Cacchiani, V., Toth, P., 2012. Nominal and robust train timetabling problems. European Journal of Operational Research, 219(3), 727-737.

Camporeale, R., Caggiani, L., Ottomanelli, M., 2019. Modeling horizontal and vertical equity in the public transport design problem: a case study. Transportation Research Part A: Policy and Practice, 125, 184-206.

Chen, Y., Liu, J., 2013. Analysis on the Beijing-Shanghai high-speed railway ticket discount strategy. Applied Mechanics and Materials, 411-414, 2429-2433.

Chow, A., Pavlides, A., 2018. Cost functions and multi-objective timetabling of mixed train services. Transportation Research Part A: Policy and Practice, 113, 335-356.

Cordone, R., Redaelli, F., 2011. Optimizing the demand captured by a railway system with a regular timetable. Transportation Research Part B: Methodological, 45(2), 430-446.

Delbosc, A., Currie, G., 2011. Using Lorenz curves to assess public transport equity. Journal of Transport

Geography, 19, 1252-1259.
Gallo, M., 2018. Improving equity of urban transit systems with the adoption of origin-destination based taxi fares. Socio-Economic Planning Sciences, 64, 38-55.
Gini, C. W., 1912. Variability and mutability, contribution to the study of statistical distributions and relations. Studi Economico-Giuridici della R. Università di Cagliari, Cagliari, Sardinia.
Großmann, P., Hölldobler, S., Manthey, N., Nachtigall, K., Opitz, J., Steinke, P., 2012. Solving periodic event scheduling problems with SAT. In: Jiang, H., Ding, W., Ali, M., Wu, X. (Eds.), Advanced Research in Applied Artificial Intelligence. In: Lecture Notes in Computer Science, 7345. Springer Berlin Heidelberg, Germany, 166–175.
Guirao, B., Casado-Sanz, N., Campa, J.L., 2018. Labour opportunities provided by Spanish high-speed rail (HSR) commuting services in a period of financial crisis: an approach based on regional wage disparities and housing rental prices, Regional Studies, DOI: 10.1080/00343404.2018.1530751.
Guirao, B., Lara-Galera, A., Campa, J.L., 2017. High Speed Rail commuting impacts on labour migration: The case of the concentration of metropolis in the Madrid functional area. Land Use Policy, 66, 131-140.
Gutiérrez, J., 2001. Location, economic potential and daily accessibility: an analysis of the accessibility impact of the high-speed line Madrid–Barcelona–French border. Journal of Transport Geography, 9(4), 229-242.
Haas, A., Osland, L., 2014. Commuting, migration, housing and labour markets: Complex interactions. Urban Studies, 51(3), 463-476.
Hetrakul, P., Cirillo, C., 2014. A latent class choice-based model system for railway optimal pricing and seat allocation. Transportation Research Part E: Logistics and Transportation Review, 61, 68-83.
Hu, H., Wang, J. E., Jin, F. J., Ding, N., 2015. Evolution of regional transport dominance in China 1910–2012. Journal of Geographical Sciences, 25, 723–738.
Jiang, M., Kim, E., 2016. Impact of high-speed railroad on regional income inequalities in China and Korea. International Journal of Urban Sciences, 20(3), 393-406.
Kanai, S., Shiina, K., Harada, S., Tomii, N., 2011. An optimal delay management algorithm from passengers' viewpoints considering the whole railway network. Journal of Rail Transport Planning & Management, 1(1), 25-37.
Kim, H., Sultana, S., 2015. The impacts of high-speed rail extensions on accessibility and spatial equity changes in South Korea from 2004 to 2018. Journal of Transport Geography, 45, 48-61.
Krumholz, N., Forester, J., 1990. Making equity planning work: leadership in the public sector. Temple University Press, Philadelphia.
Laurino, A., Ramella, F., Beria, P., 2015. The economic regulation of railway networks: a worldwide survey. Transportation Research Part A: Policy and Practice, 77, 202-212.
Lee, S. B., Lee, S. M., Lee, K.Y., 2019. A Gini coefficient-based evaluation on the reliability of travel time forecasting. Journal of King Saud University - Engineering Sciences, 31(4), 314-319.
Levinson, D., 2002. Identifying winners and losers in transportation. Transportation Research Record, 1812(1), 179-185.
Liebchen, C., 2008. The first optimized railway timetable in practice. Transportation Science, 42(4), 420-435.
Litman, T., 2002. Evaluating transportation equity. World Transport Policy & Practice, 8(2), 50-65.
Manaugh, K., Badami, M. G., El-Geneidy, A. M., 2015. Integrating social equity into urban transportation planning: a critical evaluation of equity objectives and measures in transportation plans in North America. Transport Policy, 37, 167-176.
Martin F., 1997. Justifying a high-speed rail project: social value vs. regional growth. The Annals of Regional Science, 31(2), 155-174.
Meng, Q., Yang, H., 2002. Benefit distribution and equity in road network design. Transportation Research Part B: Methodological, 36(1), 19-35.
Nachtigall, K., 1996. Periodic network optimization with different arc frequencies. Discrete Applied Mathematics,

69, 1-17.
Nuzzolo, A., Crisalli, U., Gangemi, F., 2000. A behavioural choice model for the evaluation of railway supply and pricing policies. Transportation Research Part A: Policy and Practice, 34(5), 395-404.
Pagliara, F., Menicocci, F., Vassallo, J.M, Gómez, J., 2016. Social exclusion and high speed rail: the case study of Spain. XII Congreso de Ingeniería del Transporte.
Pagliara, F., Pietra, A. L., Gomez, J., Vassallo, J. M., 2015. High-speed rail and the tourism market: Evidence from the Madrid case study. Transport Policy, 37, 187-194.
Parbo, J., Nielsen, O. A., Prato, C. G., 2016. Passenger perspectives in railway timetabling: A literature review, Transport Reviews, 36(4), 500-526.
Peeters, L., 2003. Cyclic railway timetable optimization, ERIM Ph.D. Series Research in Management, Erasmus Research Inst. of Management (ERIM).
Qin, J., Qu, W., Wu, X., Zeng, Y., 2019. Differential pricing strategies of high-speed railway based on prospect theory: An empirical study from China. Sustainability, 11(14), 3804.
Robenek, T., Azadeh, S. S., Maknoon, Y., Lapparent, M., Bierlaire, M., 2018. Train timetable design under elastic passenger demand. Transportation Research Part B: Methodological, 111, 19-38.
Robenek, T., Maknoon, Y., Azadeh, S. S., Chen, J., Bierlaire, M., 2016. Passenger-centric train timetabling problem. Transportation Research Part B: Methodological, 89, 107-126.
Romero, N., Nozick, L. K., Xu, N., 2016. Hazmat facility location and routing analysis with explicit consideration of equity using the Gini coefficient. Transportation Research Part E: Logistics and Transportation Review, 89, 165-181.
Ruiz, M., Segui-Pons, J. M., Mateu-LLadó, J., 2017. Improving bus service levels and social equity through bus frequency modelling. Journal of Transport Geography, 58, 220-233.
Sato, K., Tamura, K., Tomii, N., 2013. A MIP-based timetable rescheduling formulation and algorithm minimizing further inconvenience to passengers. Journal of Rail Transport Planning & Management, 3(3), 38-53.
Sauter-Servaes, T., Nash, A., 2007. Applying low-cost airline pricing strategies to European railroads. Transportation Research Record, 1995(1), 1-8.
Schmidt, M., Schöbel, A., 2015. Timetabling with passenger routing. OR Spectrum, 37(1), 75-97.
Si, B., Zhong, M., Gao, Z., 2009. Bilevel programming for evaluating revenue strategy of railway passenger transport under multimodal market competition. Transportation Research Record, 2117(1), 1-6.
Ureña, J.M., Meneraut, P., Garmendía, M., 2009. The high-speed rail challenge for big intermediate cities: a national, regional and local perspective. Cities 26, 266–279.
van Vuuren, D., 2002. Optimal pricing in railway passenger transport: theory and practice in The Netherlands. Transport Policy, 9(2), 95-106.
van Wee, B., Geurs, K., 2011. Discussing equity and social exclusion in accessibility evaluations. European Journal of Transport and Infrastructure Research, 11(4), 350-367.
Vigren, A., 2017. Competition in Swedish passenger railway: entry in an open access market and its effect on prices. Economics of Transportation, 11–12, 49-59.
Whelan, G., Johnson, D., 2004. Modelling the impact of alternative fare structures on train overcrowding. International Journal of Transport Management, 2(1), 51-58.
Xu, W. A., Zhou, J., Yang, L., Li, L., 2018. The implications of high-speed rail for Chinese cities: connectivity and accessibility. Transportation Research Part A: Policy and Practice, 116, 308-326.
Yin, P., Pagliara, F., Wilson, A., 2019. How does high-speed rail affect tourism? A case study of the capital region of China. Sustainability, 11(2), 1-16.
Zheng, J., Liu, J., 2016. The research on ticket fare optimization for China's high-speed train. Mathematical Problems in Engineering, Article ID 5073053, 8, 2016.
Zhou, J., Zhang, M., Zhu, P., 2019. The equity and spatial implications of transit fare. Transportation Research Part A: Policy and Practice, 121, 309-324.