




Confinement transition in the QED₃-Gross-Neveu-XY universality classLukas Janssen ¹, Wei Wang ^{2,3}, Michael M. Scherer,⁴ Zi Yang Meng,^{2,5,6} and Xiao Yan Xu ⁷¹*Institut für Theoretische Physik and Würzburg-Dresden Cluster of Excellence ct.qmat, Technische Universität Dresden, 01062 Dresden, Germany*²*Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*³*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China*⁴*Institute for Theoretical Physics, University of Cologne, 50937 Cologne, Germany*⁵*Department of Physics and HKU-UCAS Joint Institute of Theoretical and Computational Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China*⁶*Songshan Lake Materials Laboratory, Dongguan, Guangdong 523808, China*⁷*Department of Physics, University of California at San Diego, La Jolla, California 92093, USA*

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The coupling between fermionic matter and gauge fields plays a fundamental role in our understanding of nature, while at the same time posing a challenging problem for theoretical modeling. In this situation, controlled information can be gained by combining different complementary approaches. Here, we study a confinement transition in a system of N_f flavors of interacting Dirac fermions charged under a $U(1)$ gauge field in $2 + 1$ dimensions. Using quantum Monte Carlo simulations, we investigate a lattice model that exhibits a continuous transition at zero temperature between a gapless deconfined phase, described by three-dimensional quantum electrodynamics, and a gapped confined phase, in which the system develops valence-bond-solid order. We argue that the quantum critical point is in the universality class of the QED₃-Gross-Neveu-XY model. We study this field theory within a $1/N_f$ expansion in fixed dimension as well as a renormalization group analysis in $4 - \epsilon$ space-time dimensions. The consistency between numerical and analytical results is revealed from large to intermediate flavor number.

DOI: [10.1103/PhysRevB.101.235118](https://doi.org/10.1103/PhysRevB.101.235118)**I. INTRODUCTION**

The coupling between fermionic matter and gauge fields is of fundamental importance in both high-energy and condensed-matter physics. In the latter context, gauge fields can emerge as a consequence of fractionalization in quantum materials. Prominent examples include spin liquids [1,2] and deconfined quantum critical points [3–8] in frustrated magnets. As such states are characterized by topological order, it is often difficult to identify in an experiment or a simulation the relevant low-energy excitations and their characteristic properties.

In many cases, however, it is possible to tune the system by nonthermal external parameters, such as pressure or magnetic field, through a zero-temperature transition between an exotic phase with topological order and a conventionally ordered phase. If this quantum phase transition is continuous, the presence of fractionalized low-energy excitations on the topologically ordered side of the transition leaves characteristic fingerprints on the pertinent quantum critical behavior. These are in principle measurable and can be used to identify the topological order. To take advantage of this fact, it is decisive to gain a comprehensive understanding of the quantum universality classes that involve fluctuating gauge fields.

Recent work on a system of fermions coupled to a discrete \mathbb{Z}_2 gauge field in $2 + 1$ dimensions has demonstrated the existence of such a quantum transition between a

deconfined phase with gapless fermionic excitations at weak coupling and a symmetry-broken confined phase with gapped fermionic excitations at strong coupling [9–14]. Systems of gapless fermions coupled to gauge fields with continuous gauge groups are of significant current interest as well. An important example is $(2 + 1)$ -dimensional quantum electrodynamics (QED₃) with $U(1)$ gauge group, both in its compact and noncompact version [15–32]. This theory is a natural candidate for the low-energy description of gapless $U(1)$ spin liquids, which may be realizable in certain planar magnets [33–39].

Using quantum Monte Carlo (QMC) simulations, the phase diagram of a lattice model of fermions coupled to a compact $U(1)$ gauge field has recently been mapped out by some of the present authors [40,41]. The model has a free parameter that can be used to drive a transition from a $U(1)$ deconfined (UID) phase with nodal dispersion of the fermionic excitations to confined phases in which spin and/or lattice symmetries are spontaneously broken (see Fig. 1). The UID phase represents a lattice realization of QED₃, while the confined phases describe different conventionally ordered phases.

Meanwhile, several analytical works, such as renormalization group (RG) and large- N_f calculations, have made quantitative predictions for the universal behavior of quantum critical fermion systems coupled to gauge fields [42–52]. However, while there has been tremendous progress in the case of the ungauged $(2 + 1)$ -dimensional Gross-Neveu-like

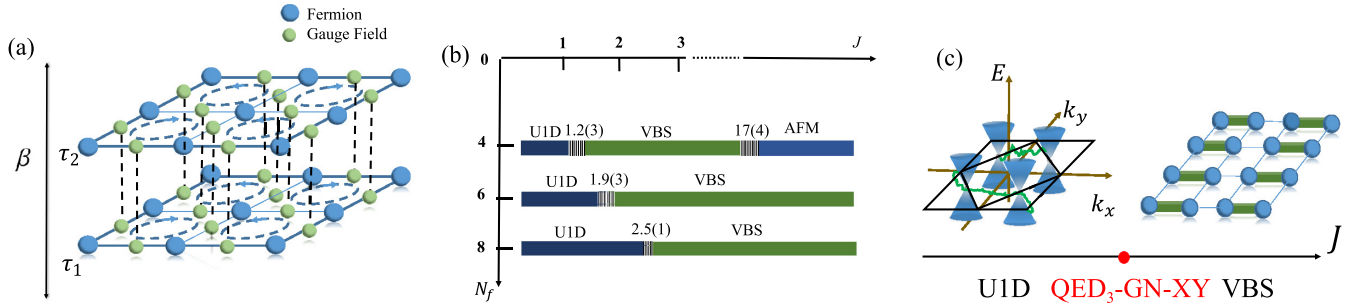


FIG. 1. (a) Illustration of the lattice model in Eqs. (1) and (2). Blue balls represent fermions and green balls represent the U(1) gauge field, coupled to the nearest-neighbor fermion hopping. The flux term is represented by blue dashed circles in each plaquette. The gauge fields fluctuate from $\phi_{ij}(\tau_1)$ at imaginary time slice τ_1 to $\phi_{ij}(\tau_2)$ at time slice τ_2 . (b) Zero-temperature phase diagram as a function of the coupling J , parametrizing the strength of the gauge field fluctuations, for different fermion flavor numbers N_f . In this work, we study the transition between the UID phase and the confined VBS phase. (c) Illustration of the confinement transition from UID to VBS. In the UID phase, the fermions form Dirac cones and interact via a U(1) gauge field, representing a lattice realization of QED₃. In the VBS phase, the fermions form gapped spin singlets. We argue that this transition is in the QED₃-Gross-Neveu-XY universality class.

transitions [53–67], the situation with fluctuating gauge fields remains significantly less clear. In particular, a quantitative comparison of universal properties between analytical predictions from the continuum quantum field theory and measurements in corresponding lattice simulations has so far not been possible. Such a comparison is the goal of this work.

The remainder of the paper is organized as follows: In Sec. II, we review the lattice model studied in Ref. [40] and present additional evidence for a continuous transition between deconfined and confined phases. The continuum field theory that we propose to describe the universal behavior of the confinement transition is discussed in Sec. III. Section IV contains the comparison of the field-theory predictions with the numerical results. Conclusions are drawn in Sec. V.

II. LATTICE MODEL AND QMC RESULTS

We simulate a square-lattice model described by the Euclidean action $S = \int_0^\beta d\tau (L_F + L_\phi)$ with

$$L_F = \sum_{\langle ij \rangle, \alpha} \psi_{i\alpha}^\dagger [(\partial_\tau - \mu)\delta_{ij} - t e^{i\phi_{ij}}] \psi_{j\alpha} + \text{H.c.}, \quad (1)$$

$$L_\phi = \frac{4}{JN_f \Delta \tau^2} \sum_{\langle ij \rangle} \{1 - \cos[\phi_{ij}(\tau + 1) - \phi_{ij}(\tau)]\} + \frac{1}{2} K N_f \sum_{\square} \cos(\text{curl } \phi), \quad (2)$$

where $\psi_{i\alpha}^\dagger$ and $\psi_{i\alpha}$ denote fermion creation and annihilation operators at lattice site i and flavor index $\alpha = 1, \dots, N_f$. The compact U(1) gauge field lives on nearest-neighbor bonds $\langle ij \rangle$ and is denoted by ϕ_{ij} . The nearest-neighbor fermion hopping amplitude t is modulated with phase ϕ_{ij} , thereby inserting magnetic flux into each plaquette. The expression $\text{curl } \phi$ sums the gauge fields in each elemental plaquette \square and the coupling $K > 0$ stabilizes a π flux in each plaquette [see Fig. 1(a)].

In the $J \rightarrow 0$ limit, the gauge field has no imaginary time dynamics. The ground state in this limit is characterized by gapless Dirac excitations, as both L_F and the flux term in L_ϕ with $K > 0$ favor a π flux in each plaquette. The Brillouin

zone contains two Dirac cones per flavor and the number of (irreducible) Dirac fermions is $2N_f$.

Turning on a small finite J allows the U(1) gauge field to fluctuate. This model has been studied by sign-problem-free QMC simulations [40]. For $N_f = 2, 4, 6, 8$, the low-temperature phase for small J has been found to be characterized by a gapless spectrum and emergent scale invariance. This phase represents a lattice realization of QED₃ with deconfined excitations and as such has been dubbed UID. In particular, for small J , the simulations suggested no evidence for confining monopole proliferation [41].

Upon increasing J beyond a certain threshold, however, the fermions acquire a mass, monopoles of the compact gauge field start to proliferate, and the fermions exhibit a confining potential [see Fig. 1(b)]. For $N_f = 4, 6, 8$, the confined phase is described by a valence bond solid (VBS), characterized by an ordered array of spin singlets, thereby breaking lattice translation and rotation symmetries spontaneously.

To improve the understanding of the UID-VBS transition, we have carried out additional QMC simulations near the transition point and on larger lattices. We also calculated larger N_f cases with $N_f = 10$ and 12. Technical details of the simulations, including the update scheme, the determination of the phase diagram, and the calculation of dynamical properties near the transition, are given in Refs. [40,41]. Figure 2 shows the numerical data of VBS correlation ratios [68]

$$r_{\text{VBS}} = 1 - \frac{\chi_D(\vec{M} + \delta\vec{q})}{\chi_D(\vec{M})}, \quad (3)$$

where $\vec{M} = (\pi, 0)$, $\delta\vec{q} = (\frac{2\pi}{L}, 0)$, for different flavor numbers N_f in the vicinity of the UID-VBS phase transition. In the above equation, χ_D denotes the VBS structure factor, given as

$$\chi_D(\vec{k}) = \frac{1}{L^4} \sum_{ij} (\langle D_i D_j \rangle - \langle D_i \rangle \langle D_j \rangle) e^{i\vec{k} \cdot (\vec{r}_i - \vec{r}_j)}, \quad (4)$$

where the dimer operator $D_i = \sum_{\alpha\beta} S_\beta^\alpha(i) S_\alpha^\beta(i + \hat{x})$ is defined along the \hat{x} direction, and the $S_\beta^\alpha(i)$ is the spin operator defined as $S_\beta^\alpha(i) = \psi_{i\alpha}^\dagger \psi_{i\beta} - \frac{1}{N_f} \delta_{\alpha\beta} \sum_\gamma \psi_{i\gamma}^\dagger \psi_{i\gamma}$. In the

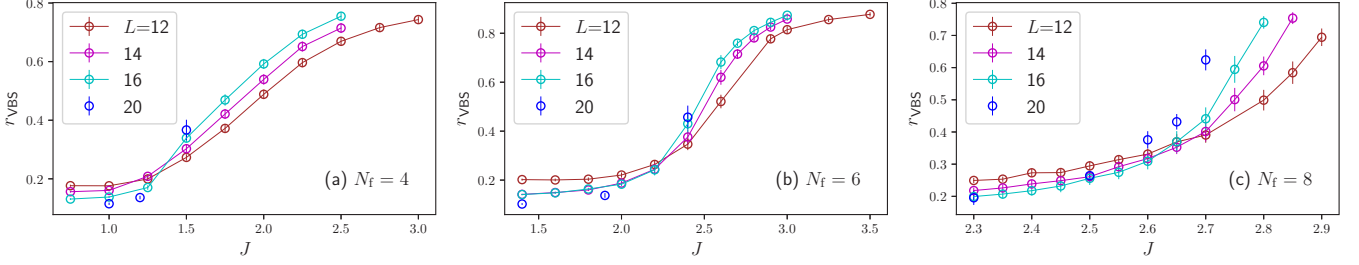


FIG. 2. Correlation ratio of VBS order parameter across the UID-VBS phase transition with linear system size up to $L = 20$, for (a) $N_f = 4$, (b) $N_f = 6$, and (c) $N_f = 8$. Correlation ratios for $N_f = 10$ ($J_c \approx 2.9$) and $N_f = 12$ ($J_c \approx 3.2$) are not shown here.

thermodynamic limit, r_{VBS} is zero (1) in the UID disordered (VBS ordered) phase. The curves on different lattices suggest a single crossing point in the thermodynamic limit, implying the existence of a continuous quantum phase transition.

More evidence for a continuous phase transition comes from the measurement of the spin and dimer correlation functions $C_S(r)$ and $C_D(r)$. Technical details concerning these measurements are deferred to the Appendix. Figure 3 shows the real-space decay of the spin and dimer correlators at the crossing points of the correlation ratios for different flavor numbers N_f . Instead of the usual exponential behavior, the correlators show a power-law decay with characteristic

exponents $2\Delta_{\text{AFM}}$ and $2\Delta_{\text{VBS}}$.¹ This constitutes further strong evidence for quantum criticality beyond the findings previously presented in Ref. [40].

The VBS order parameter has a discrete \mathbb{Z}_4 rotation symmetry, associated with the four possible VBS ordering patterns on the square lattice. At criticality, such a \mathbb{Z}_4 symmetry is usually expected to get enhanced to a continuous $U(1) \simeq O(2)$ rotation symmetry [69,70]. This suggests that the low-energy description of the quantum critical point has to include three different types of excitations: $2N_f$ flavors of (irreducible) gapless Dirac fermions ψ and $\bar{\psi}$, coupled to a $U(1)$ gauge field a_μ , and a real two-component scalar field $\vec{\phi} = (\phi^x, \phi^y)$ serving as an order parameter for spontaneous $O(2)$ symmetry breaking. The minimal theory that is consistent with these requirements is the QED₃-Gross-Neveu-XY (QED₃-GN-XY) model proposed in Ref. [40]. The purpose of the rest of this paper is to confirm that the numerical data is indeed consistent with this proposal.

III. CONTINUUM FIELD THEORY

In this section, we discuss the critical behavior of the QED₃-GN-XY model within two complementary analytical approaches: a $1/N_f$ expansion in fixed dimension and a RG analysis in $D = 4 - \epsilon$ space-time dimensions. The Lagrangian of the QED₃-GN-XY model in $D = 3$ reads

$$\mathcal{L} = \bar{\psi}_i \gamma_\mu (\partial_\mu - i a_\mu) \psi_i + \phi^a \bar{\psi}_i (\mu^a \otimes \mathbb{1}_2 \otimes \mathbb{1}_{N_f/2})_{ij} \psi_j + \frac{1}{2g^2} \phi^a (r - \partial_\mu^2) \phi^a + \lambda (\phi^a \phi^a)^2 + \frac{1}{2e^2} (\epsilon_{\mu\nu\rho} \partial_\nu a_\rho)^2, \quad (5)$$

where ψ and $\bar{\psi} = \psi^\dagger \gamma_0$ are two-component spinors, $(\phi^a) = (\phi^x, \phi^y)$, $a = 1, 2$, is a real XY order-parameter field, and the 2×2 Dirac matrices satisfy the Euclidean Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \mathbb{1}_2$, $\mu, \nu = 0, 1, 2$. In the Yukawa vertex $\mu^a \otimes \mathbb{1}_2 \otimes \mathbb{1}_{N_f/2}$, the 2×2 Pauli matrices $(\mu^a) = (\mu^x, \mu^y)$ connect the two Dirac nodes, $\mathbb{1}_2$ acts on spin indices, and $\mathbb{1}_{N_f/2}$ acts on the additional flavor indices (see Ref. [36] for details). Thus, the indices i and j run from 1 to $2N_f$, and we assume N_f even. a_μ denotes the $U(1)$ gauge field and $\epsilon_{\mu\nu\rho}$ is the totally antisymmetric tensor. In addition to the gauge symmetry, the model features a global axial $U(1)$ symmetry under which the two-component scalar field (ϕ^a) transforms as an $SO(2)$

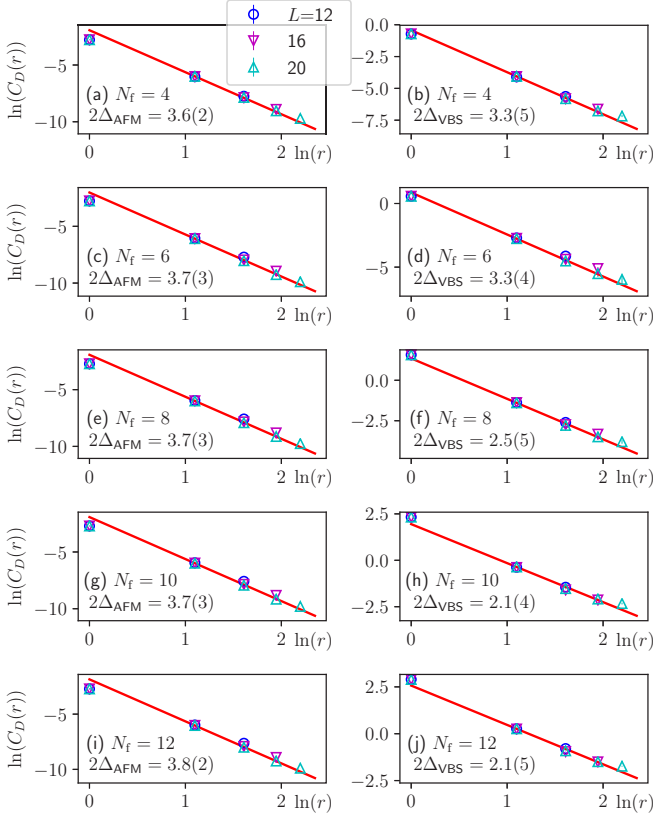


FIG. 3. Real-space decay of spin (left column) and dimer (right column) correlation function for $N_f = 4, 6, 8, 10, 12$, at the UID-VBS transition point. The scaling dimensions are measured by fitting the slopes in the log-log plot.

¹ Δ_{AFM} and Δ_{VBS} will be defined as scaling dimensions of antiferromagnetic (AFM) and VBS order parameters in Sec. III.

vector. The parameters e , g , and λ denote the charge, Yukawa, and quartic scalar couplings. The parameter r can be used to tune through the UID-VBS transition.

For $r > 0$, the scalar field ϕ^a can be integrated out and the theory describes the UID phase with enhanced $SU(2N_f)$ symmetry. For $r < 0$, the fermions are fully gapped, since $\gamma_0 \mu^a$ anticommutes with the Hamiltonian $\mathcal{H}(p) = \gamma_0 \gamma_i p_i$, and the gauge field confines due to the proliferation of monopoles [15,16,35,39]. This represents the VBS ordered phase. The critical point is given by $r = 0$ at tree level.

Besides the universal exponents that describe the critical behavior in the vicinity of the quantum phase transition, we aim at computing the correlators of the staggered magnetization

$$O_{\text{AFM}}^\alpha(\vec{r}) \sim (-1)^{r_x+r_y} S^\alpha(\vec{r}) \sim \bar{\psi}_i(\mu^z \otimes \sigma^\alpha \otimes \mathbb{1}_{N_f/2})_{ij} \psi_j, \quad (6)$$

where $\alpha = x, y, z$ denotes the spin components and $\vec{r} = r_x \hat{x} + r_y \hat{y}$ the lattice site, and the staggered dimer operator

$$O_{\text{VBS}}^a(\vec{r}) \sim [(-1)^{r_y} \vec{S}(\vec{r}) \cdot \vec{S}(\vec{r} + \hat{y}), (-1)^{r_x} \vec{S}(\vec{r}) \cdot \vec{S}(\vec{r} + \hat{x})]^a \sim \bar{\psi}_i(\mu^a \otimes \mathbb{1}_2 \otimes \mathbb{1}_{N_f/2})_{ij} \psi_j \sim \phi^a \quad (7)$$

with $a = x, y$ [36]. In the above equations, two operators O and O' are considered equivalent, $O \sim O'$, if they transform in the same way under all symmetries. At criticality, the correlation functions are given by power laws

$$\langle O_{\text{AFM}}^\alpha(\vec{r}) O_{\text{AFM}}^\alpha(0) \rangle \propto \frac{1}{r^{2\Delta_{\text{AFM}}}}, \quad (8)$$

$$\langle O_{\text{VBS}}^a(\vec{r}) O_{\text{VBS}}^a(0) \rangle \propto \frac{1}{r^{2\Delta_{\text{VBS}}}}, \quad (9)$$

where Δ_{AFM} and Δ_{VBS} are the scaling dimensions of the operators O_{AFM}^α and O_{VBS}^a . The correlation function of a microscopic lattice operator will in general be a linear combination of all operators in the conformal field theory that are equivalent to the respective lattice operator. In the long-wavelength limit, however, the operator with the lowest scaling dimension will dominate.

A. Large- N_f expansion

In the limit of large fermion flavor number N_f , the universal critical exponents and the scaling dimensions of the fermion bilinears can be computed analytically. We start by discussing the scaling dimensions Δ_{AFM} and Δ_{VBS} . The diagrams that contribute at order $\mathcal{O}(1/N_f)$ are shown in Figs. 4 and 5. We note that at this order of the $1/N_f$ expansion, mixed diagrams that involve both scalar- and gauge-field propagators are finite and therefore do not contribute to the scaling dimensions. This can be understood as a consequence of the Ward identity associated with the $U(1)$ gauge symmetry, in analogy to the QED_3 -Gross-Neveu-Ising case [45,46].

The gauge-field contributions to the scaling dimensions (Fig. 4) are well known [29,34,36,46],

$$\Delta_{\text{AFM}}|_{\text{QED}_3} = \Delta_{\text{VBS}}|_{\text{QED}_3} = 2 - \frac{32}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2). \quad (10)$$

We note that Aslamazov-Larkin diagrams vanish due to $\text{tr}(\mu^z) = \text{tr}(\mu^a) = 0$ [46]. Both operators have exactly the same scaling dimensions as a consequence of the enhanced

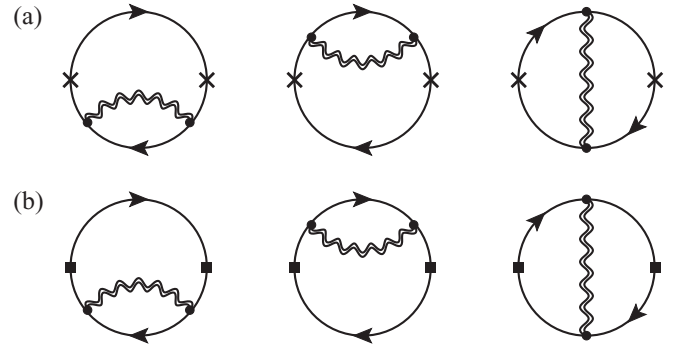


FIG. 4. Diagrams that contribute to the scaling dimensions Δ_{AFM} (a) and Δ_{VBS} (b) at $\mathcal{O}(1/N_f)$ from gauge fluctuations. Solid (wiggly) lines correspond to fermion-field (gauge-field) propagators at $\mathcal{O}(1/N_f^0)$. The cross (\times) denotes $\mu^z \otimes \sigma^\alpha \otimes \mathbb{1}_{N_f/2}$ insertions and the square (\blacksquare) denotes $\mu^a \otimes \mathbb{1}_2 \otimes \mathbb{1}_{N_f/2}$ insertions. Aslamazov-Larkin diagrams (not shown) vanish due to $\text{tr}(\mu^z) = \text{tr}(\mu^a) = 0$.

$SU(2N_f)$ symmetry for $r > 0$ in the UID phase [34,40]. However, at criticality, $r = 0$, this symmetry is explicitly broken and the scalar-field fluctuations lead to different values for Δ_{VBS} and Δ_{AFM} . The scalar contributions to the AFM bilinear are shown in Fig. 5, yielding

$$\Delta_{\text{AFM}}|_{\text{GN-XY}} = 2 - \frac{8}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2). \quad (11)$$

The lowest-dimensional operator in the conformal field theory with the symmetries of the staggered dimer operator is the scalar field ϕ^a . Its scaling dimension can be computed using the Dyson equation (see Fig. 6 and Ref. [46]). The scalar contributions are shown in Fig. 5(b), yielding

$$\Delta_{\text{VBS}}|_{\text{GN-XY}} = \Delta_\phi|_{\text{GN-XY}} = 1 - \frac{4}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2). \quad (12)$$

Since $\Delta_\phi = (D - 2 + \eta_\phi)/2$, we obtain the order-parameter anomalous dimension of the pure Gross-Neveu-XY model (i.e., without the coupling to a gauge field) as

$$\eta_\phi|_{\text{GN-XY}} = 1 - \frac{8}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (13)$$

which is consistent with the previous calculation [71].

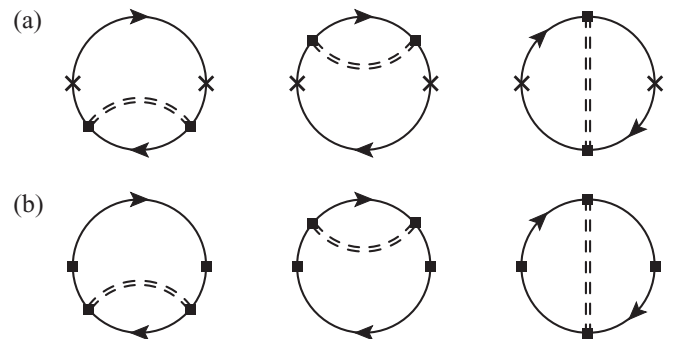


FIG. 5. Diagrams that contribute to the scaling dimensions Δ_{AFM} (a) and Δ_{VBS} (b) at $\mathcal{O}(1/N_f)$ from scalar fluctuations. Dashed lines correspond to scalar-field propagators at $\mathcal{O}(1/N_f^0)$.

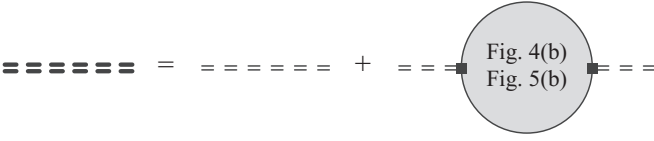


FIG. 6. Dyson equation for the scalar-field two-point correlator at $O(1/N_f)$. Thick dashed line corresponds to scalar-field propagator at $O(1/N_f)$.

Including both gauge-field and scalar-field contributions yields the scaling dimensions in the QED₃-GN-XY model as

$$\Delta_{\text{AFM}}|_{\text{QED}_3\text{-GN-XY}} = 2 - \frac{40}{3\pi^2 N_f} + O(1/N_f^2), \quad (14)$$

$$\Delta_{\text{VBS}}|_{\text{QED}_3\text{-GN-XY}} = 1 + \frac{28}{3\pi^2 N_f} + O(1/N_f^2), \quad (15)$$

with the order-parameter anomalous dimension as

$$\eta_\phi|_{\text{QED}_3\text{-GN-XY}} = 1 + \frac{56}{3\pi^2 N_f} + O(1/N_f^2). \quad (16)$$

We note that the two-point correlator of the fermion bilinear $\bar{\psi}_i(\mu^a \otimes \mathbb{1}_2 \otimes \mathbb{1}_{N_f/2})_{ij} \psi_j$ vanishes to all orders in the large- N_f calculation, which can be understood as a consequence of the fact that this operator becomes zero when integrating out ϕ in the critical large- N_f theory [46–48].² The correlation-length exponent ν can be obtained within an analogous calculation to that of Ref. [46] for the QED₃-Gross-Neveu-Ising case. We find

$$\nu^{-1}|_{\text{QED}_3\text{-GN-XY}} = 1 - \frac{80}{3\pi^2 N_f} + O(1/N_f^2), \quad (17)$$

in agreement with the recent calculation presented in Ref. [72].

In order to make contact with the $4 - \epsilon$ expansion result discussed in the next section, it is instructive to generalize Eqs. (16) and (17) to general space-time dimension $2 < D < 4$. Using the integration formulas derived in Refs. [45,71], we obtain

$$\eta_\phi|_{\text{QED}_3\text{-GN-XY}} = 4 - D + \frac{4(D^2 - 2)\Gamma(D - 1)}{D^2(D - 2)\Gamma(-D/2)\Gamma(D/2)^3} \times \frac{1}{N_f} + O(1/N_f^2) \quad (18)$$

and

$$\nu^{-1}|_{\text{QED}_3\text{-GN-XY}} = D - 2 + \frac{2(D^2 - 2D + 2)\Gamma(D)}{D(D - 2)\Gamma(1 - D/2)\Gamma(D/2)^3} \times \frac{1}{N_f} + O(1/N_f^2), \quad (19)$$

in agreement with Ref. [73]. Further expanding the above equations in $\epsilon = 4 - D$ leads to

$$\eta_\phi|_{\text{QED}_3\text{-GN-XY}} = \epsilon + \frac{7\epsilon}{2N_f} - \frac{9\epsilon^2}{8N_f} + O(\epsilon^3, 1/N_f^2), \quad (20)$$

$$\nu^{-1}|_{\text{QED}_3\text{-GN-XY}} = 2 - \epsilon - \frac{15\epsilon}{2N_f} + \frac{41\epsilon^2}{8N_f} + O(\epsilon^3, 1/N_f^2), \quad (21)$$

²We are grateful to J. Maciejko for pointing this out to us.

which, as we show below, agrees with the leading-order result that we obtain within the $4 - \epsilon$ expansion. This constitutes another nontrivial cross-check of our calculations.

The above analysis neglects the compactness of the U(1) gauge field in our lattice model. A compact gauge field admits instanton events, which correspond to insertion of magnetic flux in a localized region of space-time. In fact, the monopole operators, which perform this flux insertion, are relevant when the fermions are gapped out, leading to confinement of charges in the VBS phase [15,16,35,39]. In the gapless UID phase, the scaling dimension $\Delta_{q=1/2}$ of the least irrelevant monopole operator with minimal magnetic charge $q = 1/2$ (corresponding to 2π flux insertion) can be computed by employing the operator-state correspondence, which implies that $\Delta_{q=1/2}$ is given by the ground-state energy of the corresponding theory on a sphere in the presence of a magnetic monopole [74]. For pure QED₃, this computation gives

$$\Delta_{q=1/2}|_{\text{QED}_3} = c_{\text{QED}_3} 2N_f + O(1/N_f^0) \quad (22)$$

with $c_{\text{QED}_3} = 0.265 \dots$. (The correction at order $O(1/N_f^0)$ has been computed in [75]). We note that $\Delta_{q=1/2} \propto N_f$ in the large- N_f limit. The technical reason for this property is that the fermions can be integrated out in this limit, such that $\Delta_{q=1/2}$ is simply the ground-state energy of N_f free Dirac fermions in the presence of a classical background gauge field. At large enough N_f , monopoles are therefore irrelevant and the ground state of QED₃ hence describes a stable UID phase with conformal invariance.³ The same argument applies to the QED₃-GN-XY model. Hence,

$$\Delta_{q=1/2}|_{\text{QED}_3\text{-GN-XY}} = c_{\text{QED}_3\text{-GN-XY}} 2N_f + O(1/N_f^0). \quad (23)$$

The constant $c_{\text{QED}_3\text{-GN-XY}}$ can be computed along the lines of Refs. [50,51]. In fact, this calculation turns out to be agnostic towards the number of scalar boson components,⁴ such that the leading-order monopole scaling dimension of the QED₃-GN-XY model agrees with that of the QED₃-Gross-Neveu-Ising model, $c_{\text{QED}_3\text{-GN-XY}} = c_{\text{QED}_3\text{-GN-Ising}} = 0.195 \dots$ [50]. In particular, $\Delta_{q=1/2} > D = 3$ for large enough N_f , such that all monopole operators are irrelevant and the critical theory indeed describes a stable RG fixed point with a unique infrared relevant direction (corresponding to the tuning parameter of the continuous transition).

The emerging picture resembles the situation at the deconfined Néel-VBS critical point in quantum magnets, which is described by the noncompact CP¹ model [3,76] (see Fig. 7). In the gapless UID phase (Néel phase in the case of quantum magnets), which is described by QED₃ (symmetry-broken phase of the noncompact CP¹ model), monopoles are irrelevant in the large- N_f limit, and we can neglect the compactness of the U(1) gauge field. The same applies to the quantum critical point, described by the QED₃-GN-XY model (noncompact CP¹ model) at criticality. However, when the fermions

³For small N_f , the scaling dimension $\Delta_{q=1/2}$ may drop below $D = 3$, such that monopoles would become relevant. The critical value of N_f , below which this happens, cannot be computed in a controlled way within the $1/N_f$ expansion.

⁴We thank W. Witczak-Krempa for pointing this out to us.

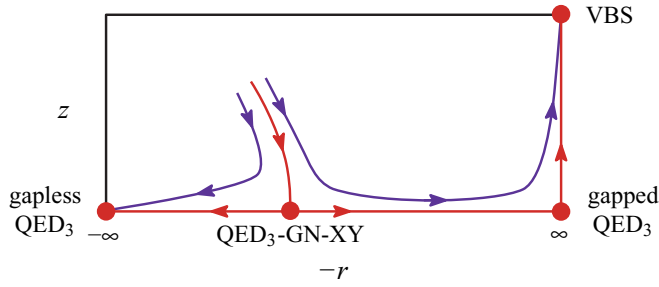


FIG. 7. Schematic RG flow in the space spanned by tuning parameter r and monopole fugacity z for large N_f . The theory defined by Eq. (5) describes the line $z = 0$. Both in the U1D phase, described by the infrared stable fixed point denoted as gapless QED₃, and at the QED₃-GN-XY quantum critical point, monopole operators are (dangerously) irrelevant. However, when the fermions acquire a mass gap, the fugacity z grows towards the infrared, indicating a proliferation of monopoles and confinement. The corresponding fixed point, denoted as gapped QED₃, is unstable towards the formation of VBS order.

acquire a mass gap in the ordered phase of the QED₃-GN-XY model (corresponding to the paramagnetic phase of the non-compact CP¹ model), monopoles become relevant, indicating confinement and VBS order [15,16,35,39,77]. As a consequence, on the VBS side of the transition, two independent diverging length scales are present, just as in the deconfined Néel-VBS critical point in quantum magnets [3]. In addition to the correlation length ξ associated with the flow of the tuning parameter, a longer length scale ξ' that is associated with the proliferation of monopoles exists. Observation of ξ' in a QMC simulation is in principle possible, but requires significantly larger lattices [78]. We leave this for future work.

B. RG analysis in $D = 4 - \epsilon$ dimensions

We now demonstrate the existence of the quantum critical QED₃-GN-XY fixed point beyond the $1/N_f$ expansion within a standard RG analysis. The QED₃-GN-XY theory can be straightforwardly generalized to general space-time dimension $2 < D < 4$ by replacing the $2N_f$ flavors two-component Dirac fermions by N_f flavors of four-component fermions [79]. In agreement with the QED₃-Gross-Neveu-Ising case [42–44], all couplings become simultaneously marginal in four space-time dimensions. This allows one to set up an ϵ expansion around four space-time dimensions in the same way as in the standard Wilson-Fisher ϕ^4 theory. Integrating over the D -dimensional momentum shell $\Lambda/b < \sqrt{\omega^2 + \vec{q}^2} < \Lambda$ with cutoff Λ and $b > 1$ causes the couplings to flow according to the equations

$$\frac{de^2}{d \ln b} = \epsilon e^2 - \frac{4}{3} N_f e^4, \quad (24)$$

$$\frac{dg^2}{d \ln b} = \epsilon g^2 - (N_f + 1)g^4 + 6e^2 g^2, \quad (25)$$

$$\frac{d\lambda}{d \ln b} = \epsilon \lambda - 2N_f g^2 (\lambda - g^2) - 5\lambda^2, \quad (26)$$

where we have rescaled $(e^2, g^2, \lambda) \mapsto (e^2, g^2, \lambda)/(4\pi^2 \Lambda^\epsilon)$ and assumed that the theory is tuned to the critical point. Note

that these flow equations agree with those of the continuum description of the Kekulé transition on the honeycomb lattice in the presence of a U(1) gauge field [80]. They are also equivalent to those of Ref. [50] upon setting their N_b to 2. The flow admits an infrared stable RG fixed point at

$$(e_*^2, g_*^2, \lambda_*) = \left(\frac{3}{4N_f}, \frac{2N_f + 9}{2N_f(N_f + 1)}, \frac{C(N_f) - 8 - N_f}{10(N_f + 1)} \right) \epsilon + \mathcal{O}(\epsilon^2), \quad (27)$$

with $C(N_f) = \sqrt{N_f^2 + 56N_f + 424 + 810/N_f}$.

The wave function renormalization function of the two-component scalar field reads

$$\gamma_\phi(g^2) = N_f g^2. \quad (28)$$

This expression is formally identical to the corresponding equation in the nongauged Gross-Neveu-XY model, because the XY scalar field is not charged. At the fixed point, we obtain the order parameter anomalous dimension as

$$\eta_\phi = \gamma_\phi(g_*^2) = \frac{2N_f + 9}{2(N_f + 1)} \epsilon + \mathcal{O}(\epsilon^2). \quad (29)$$

Further expanding this result for large N_f leads to

$$\eta_\phi = \epsilon + \frac{7\epsilon}{2N_f} - \frac{7\epsilon}{2N_f^2} + \mathcal{O}(\epsilon^2, 1/N_f^3), \quad (30)$$

in agreement with Eq. (20). The correlation-length exponent ν can be obtained from the flow of the tuning parameter,

$$\frac{dr}{d \ln b} = (2 - 2\lambda - N_f g^2)r + 2\lambda - 2N_f g^2, \quad (31)$$

implying

$$\nu^{-1} = 2 - \frac{2C(N_f) + 8N_f + 29}{10(N_f + 1)} \epsilon + \mathcal{O}(\epsilon^2). \quad (32)$$

To next-to-leading order in $1/N_f$, we obtain

$$\nu^{-1} = 2 - \epsilon - \frac{15\epsilon}{2N_f} + \frac{87\epsilon}{2N_f^2} + \mathcal{O}(\epsilon^2, 1/N_f^3), \quad (33)$$

in agreement with Eq. (21), as announced above.

IV. EVIDENCE FOR QED₃-GN-XY UNIVERSALITY

We now discuss three possible universality classes that could in principle describe the continuous U1D-VBS transition observed in the lattice model. Among these three scenarios, we will see that the numerical data is consistent only with the QED₃-GN-XY universality class.

The transition between the U1D disordered phase and the VBS ordered phase is detected via a real two-component order parameter. Hence, if the U1D-VBS transition were a standard quantum transition within the Landau paradigm, quantum-to-classical mapping would apply, and one would expect the transition to be in the classical three-dimensional XY universality class. Within Scenario 1 (Scen1), we thus assume the critical exponents [81,82]

$$\text{Scen1: } \nu^{-1}|_{XY} = 1.48864(22), \quad \eta_\phi|_{XY} = 0.0385(7). \quad (34)$$

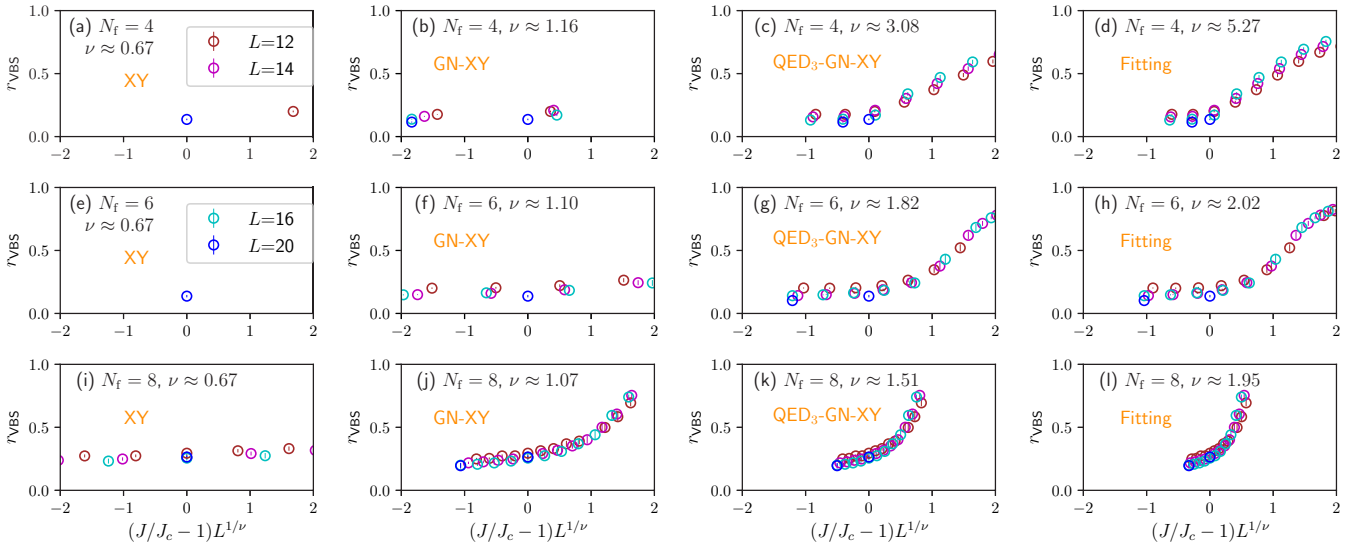


FIG. 8. Data collapses of VBS correlation ratios according to Eq. (43) within the three different scenarios (first three columns) and using fitted values (last column) for $N_f = 4$ (top row), $N_f = 6$ (middle row), and $N_f = 8$ (bottom row). (a),(e),(i) Scenario 1, assuming standard XY universality. (b),(f),(j) Scenario 2, assuming pure Gross-Neveu-XY universality. (c),(g),(k) Scenario 3, assuming QED₃-GN-XY universality. (d),(h),(l) Using fitted values as denoted in the insets.

However, it is well known that the presence of gapless fermions at the transition can change the universality class [83]. Previous studies in systems with Dirac fermions (but without a fluctuating gauge field) coupled to an O(2) order parameter have put forward the Gross-Neveu-XY universality class [56,84–90]. This describes our Scenario 2 (Scen2), with exponents

$$\text{Scen2: } \nu^{-1}|_{\text{GN-XY}} = 1 - \frac{16}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (35)$$

$$\eta_\phi|_{\text{GN-XY}} = 1 - \frac{8}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (36)$$

$$\Delta_{\text{VBS}}|_{\text{GN-XY}} = 1 - \frac{4}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (37)$$

$$\Delta_{\text{AFM}}|_{\text{GN-XY}} = 2 - \frac{8}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2) \quad (38)$$

(cf. Sec. III). We note in particular that $\Delta_{\text{VBS}}|_{\text{GN-XY}} < 1$. Below, we will show that this result is inconsistent with our numerical data, which we ascribe to the presence of gapless gauge-field excitations at the transition. We will demonstrate that the data is instead consistent, within error bars, with our theoretical predictions for the QED₃-GN-XY universality class. The universal exponents within this scenario, dubbed Scenario 3 (Scen3), have been computed in Sec. III:

$$\text{Scen3: } \nu^{-1}|_{\text{QED}_3\text{-GN-XY}} = 1 - \frac{80}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (39)$$

$$\eta_\phi|_{\text{QED}_3\text{-GN-XY}} = 1 + \frac{56}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (40)$$

$$\Delta_{\text{VBS}}|_{\text{QED}_3\text{-GN-XY}} = 1 + \frac{28}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2), \quad (41)$$

$$\Delta_{\text{AFM}}|_{\text{QED}_3\text{-GN-XY}} = 2 - \frac{40}{3\pi^2 N_f} + \mathcal{O}(1/N_f^2). \quad (42)$$

Note that now $\Delta_{\text{VBS}}|_{\text{QED}_3\text{-GN-XY}} > 1$.

In the vicinity of the quantum critical point at coupling $J = J_c$, the dimensionless correlation ratio r_{VBS} should obey the finite-size scaling law

$$r_{\text{VBS}}(J, L) = \mathcal{F}[(J/J_c - 1)L^{1/\nu}] \quad (43)$$

with scaling function $\mathcal{F}(x)$ and linear system size L . Figure 8 shows the correlation ratios r_{VBS} as a function of the rescaled coordinate $x := (J/J_c - 1)L^{1/\nu}$ for $N_f = 4, 6, 8$, using the predictions for $1/\nu$ within the three scenarios proposed above, as well as using fitted values. Due to the limited system sizes that are available within tenable simulation times, the data collapses are not perfect in all cases. However, for all flavor numbers checked, the data collapses using the standard XY prediction are significantly worse than for the other two scenarios. This rules out Scenario 1, revealing that the transition evades the usual quantum-to-classical mapping. The differences between the qualities of the data collapses in Scenarios 2 and 3 are less significant, in particular for $N_f = 8$. The reason for this behavior is that the critical exponents in these two scenarios approach the same limiting value for large N_f , $\lim_{N_f \rightarrow \infty} (1/\nu) = 1$. Nevertheless, for $N_f = 4$ and 6, we note that the spread of values of r_{VBS} within the ranges of x shown is significantly larger within Scenario 3 than for the other two scenarios. We interpret this as first evidence for QED₃-GN-XY universality (Scenario 3).

To clearly rule out the pure Gross-Neveu-XY universality class (Scenario 2), we have performed fits to the dimer $C_D(r)$ and spin $C_S(r)$ correlation functions, assuming power-law behaviors

$$C_D(r) \propto \frac{1}{r^{2\Delta_{\text{VBS}}}}, \quad C_S(r) \propto \frac{1}{r^{\Delta_{\text{AFM}}}}, \quad (44)$$

with exponents Δ_{VBS} and Δ_{AFM} . The fits are shown as straight lines in the log-log plots of Fig. 3, and we plot the extracted exponents in Fig. 9 in comparison with the theoretical predictions for Gross-Neveu-XY universality (Scenario

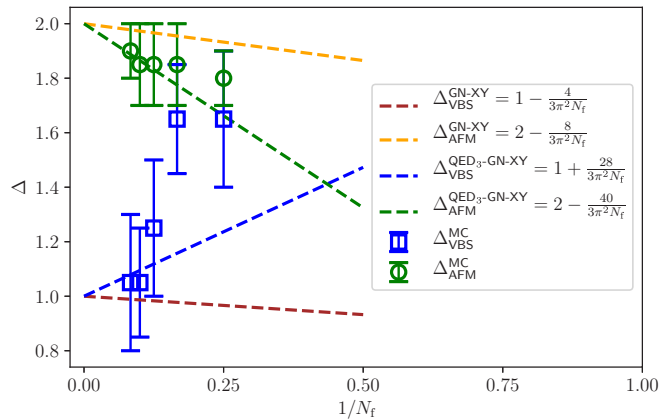


FIG. 9. Scaling dimensions Δ_{AFM} and Δ_{VBS} as extracted from the QMC data of Fig. 3, in comparison to the predictions of the Gross-Neveu-XY and QED₃-GN-XY models, respectively. The fact that Δ_{VBS} is above 1 is consistent only with the QED₃-GN-XY universality class (Scenario 3).

2) and QED₃-GN-XY universality (Scenario 3). While the errors are large, the overall behavior is significantly closer to the QED₃-GN-XY predictions, in particular for larger flavor numbers $N_f \geq 8$. We also note that we find $\Delta_{\text{VBS}} > 1$ for all considered values of N_f . This rules out Scenario 2, suggesting that the transition observed in the simulation is indeed described by a new universality class in which the fluctuating U(1) gauge field plays a significant role.

V. CONCLUSIONS

In conclusion, we have provided evidence for the existence of an unconventional quantum critical point between a deconfined phase, characterized by gapless fermionic degrees of freedom coupled to a U(1) gauge field, and a conventionally ordered phase with spontaneously broken XY symmetry, in which the fermions acquire a band gap and the gauge field confines. The critical point can hence be understood as an example of a continuous confinement transition in 2 + 1 space-time dimensions.

Using a finite-size scaling analysis of our QMC data, we have demonstrated that this transition evades the usual quantum-to-classical mapping. We have also shown that the numerical data is inconsistent with the pure Gross-Neveu-XY universality class. Instead, the data is consistent with our predictions for a novel universality class, dubbed QED₃-GN-XY, which includes besides the fluctuating fermionic degrees of freedom also the effects of the coupling to the U(1) gauge field.

We have shown that the corresponding QED₃-GN-XY field theory can be studied within a large- N_f expansion in fixed dimension, or alternatively within an ϵ expansion in $D = 4 - \epsilon$ space-time dimensions. Using these expansion schemes, we have computed estimates for the characteristic exponents that describe the QED₃-GN-XY universality class, including the order-parameter anomalous dimension η_ϕ and the correlation-length exponent ν , as well as the scaling dimensions of the spin and dimer correlation functions. The accuracy of these estimates increases for larger flavor number, and should be

reasonable for our simulations of the cases $N_f = 6$ and 8. (e.g., in the pure Gross-Neveu-XY universality class, the accuracy of the leading-order $1/N_f$ results is expected to be of the order of $\lesssim 3\%$ for $N_f = 8$ [56,59]). For $N_f = 4$, the corrections may be more significant. This could be one of the reasons for the reduced quality of the data collapse in this case. Improving on this would require one to extend our analytical calculations to the next order in the large- N_f expansion (e.g., along the lines of the calculation in Ref. [45] for the QED₃-Gross-Neveu-Ising case) and/or to a higher order in the $4 - \epsilon$ expansion (e.g., along the lines of Refs. [43,44]). On the numerical side, refined quantitative predictions for the critical exponents and universal scaling dimensions require simulations on significantly larger lattices. Besides an improved characterization of the universality class, such simulations could also facilitate a search of the predicted emergent second length scale ξ' , associated with the proliferation of monopoles on the VBS side of the transition.

Note added. Recently, a related preprint appeared [91], which extends the calculation of the critical exponents of the QED₃-GN-XY universality class to $O(1/N_f^2)$ in the large- N_f expansion and to $O(\epsilon^4)$ in the $4 - \epsilon$ expansion. Their leading-order results agree with ours.

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APPENDIX: MEASUREMENT OF CORRELATION FUNCTIONS

We measure connected correlation functions in real space. The spin correlator is defined as

$$C_S(r) = \sum_{\alpha\beta} \langle S_\beta^\alpha(r) S_\alpha^\beta(0) \rangle - \sum_{\alpha\beta} \langle S_\beta^\alpha(r) \rangle \langle S_\alpha^\beta(0) \rangle, \quad (\text{A1})$$

and the dimer correlator is

$$C_D(r) = \langle D(r)D(0) \rangle - \langle D(r) \rangle \langle D(0) \rangle. \quad (\text{A2})$$

For the spin correlator, the background is zero, $\langle S_\beta^\alpha(r) \rangle = 0$, which is a consequence of the unbroken $SU(2)$ spin symmetry. However, for the dimer correlator, the background may be finite, $\langle D(r) \rangle \neq 0$, and must be measured numerically. Near the

quantum critical point and for large distances r , the difference between the two terms in the right-hand side of Eq. (A2) may become significantly smaller than their individual magnitudes, such that the dimer correlator has a larger uncertainty.

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