A Cell-Based Dynamic Congestion Pricing Scheme Considering Travel Distance and Time Delay

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ABSTRACT

This study introduces the dynamic congestion pricing (DCP) problem with the consideration of the actual travel distance and time delay (*i.e.*, a joint distance and time-delay toll, JDTDT) in a dynamic network, which is more equitable and effective compared with existing tolling scheme such as flat tolling. The nonlinear distance-based toll is approximated by a stepwise linear toll function and the congestion-based toll is measured by the delay inside the cordon charging area. The system dynamics can be reflected in two aspects: (*a*) travelers' path choice decisions follow the dynamic user equilibrium (DUE) principle and (*b*) the joint distance and time-delay toll takes a time-varying pattern. The dynamic traffic flow component is represented by the path-based cell transmission model (CTM). A new averaging scheme is proposed to estimate the en-route travel time for the travelers departing at the same time of each path with the output of path-based CTM. In our proposed averaging approach, two new arriving time indexes are introduced, to calculate the en-route travel time. To better depict the dynamic congestion pricing problem, a multi-period demand scheme is adopted during the entire modeling horizon. Then, a bi-level programming model for the DCP is formulated to obtain the optimal toll value. The aim of the upper level is to optimize the total system travel time, and the lower level depicts travel behaviors based on the DUE theory, which is modeled as a variational inequality problem and solved with a self-adaptive gradient projection (SAGP) algorithm. A hybrid SAGP and artificial bee colony algorithm is developed to solve the proposed bi-level model. Finally, four numerical tests are conducted to verify the proposed methodology. Results indicate that the percentage reductions of the minimum total system travel time in the dynamic JDTDT scheme are 6.28%, 4.30% and 7.45% compared to that obtained by the static joint JDTDT, the dynamic joint distance and time toll, and the dynamic pure distance toll, respectively.

KEYWORDS

Joint Distance and Time-delay Toll; Path-Based Cell Transmission Model; Dynamic User Equilibrium; Bi-Level Programming Model; Variational Inequality

1 Introduction

As one of the demand side strategies for transportation management, congestion pricing is widely recognized among economists as an effective economic measurement to ease the traffic congestion problem and improve the system performance in urban areas, and also has received more and more attention both academically and practically. Studies of congestion pricing focus on the first-best pricing as well as the second-best pricing. In the transportation network modeling and analysis, most studies consider that every link in the network is tolled as the first-best pricing scheme (e.g., Yang and Huang 1998; Sumalee and Xu 2011), while only a subset of the links in the network is tolled as the second-best pricing scheme (e.g., Liu and McDonald 1999; Verhoef 2000, 2002; Zhang et al. 2011; Liu, Meng, and Wang 2014; Di, Liu, and Ban 2016; Han, Wang, and Zhu 2017); interested readers can get a comprehensive review from Yang and Huang (2005).

One of the critical issues in congestion pricing is to determine the charging rates. Most of the actualized congestion pricing policies adopt the flat-toll method in a cordon-based toll scheme, including the pay-per-entry charge as well as the daily licensing charge, in disregard of the actual travel distance inside the cordon charging area. Consequently, this toll method may cause unfair charging problems due to undercharging for long journeys and overcharging for short ones, and the disruptive problems because it may increase the congestion level on the boundary routes immediately outside the cordon (May et al. 2008). Moreover, it is not thoroughly compelling for easing traffic congestion with the flat-toll method, because a portion of drivers may designedly utilize more road segments inside the cordon charging area to maximize the utility of their defrayed toll (Meng, Liu, and Wang 2012). This may actually increase the congestion phenomena inside the cordon area. Therefore, in order to give full play to congestion pricing in alleviating urban traffic congestion, and improve the fairness and effectiveness of congestion pricing, it is necessary to consider the travel distance (or usage) inside the cordon charging area and establish the distance-based congestion toll scheme. Meng, Liu, and Wang (2012) and Liu et al. (2017) addressed the optimal distance-based congestion pricing problem and adopted a piecewise linear function to formulate the nonlinear distance-based toll. It is worth noting that the distance-based pricing scheme will be the next generation of Electronic Road Pricing (ERP) system in Singapore (Land Transport Authority of Singapore 2013).

However, as claimed in Liu, Wang, and Meng (2014), the distance-based pricing model still has its limitation: travelers would wittingly select the shorter routes to reduce their toll in the pricing cordon, regardless this route is highly congested. In order to cope with this problem, they proposed a joint distance and time toll (JDTT) scheme. Nevertheless, there is an overlap between the distance toll and time toll in terms of the free flow travel time. For example, a traveler went through a charging cordon along with a particular route, spending 15mins and traveling for 6km inside the cordon. However, it only takes him 12mins with the same route when the network is in a free-flow state. In the part of distance-based toll, it has already contained the toll in the free-flow traffic state because the total length of the experienced route is fixed and related to the constant free-flow travel time. This overlap implies that there is an overcharge in JDTT and leads to sub-optimality. Thus, this part should be cut out. To this end, the time-based toll should be replaced by a time-delay-based toll scheme. In this paper, we extend the JDTT to the joint distance and time-delay toll (JDTDT), which is more efficient than the JDTT.

As for the congestion toll problem, most studies focus on deterministic processes in static transport networks. Nevertheless, there are significant limitations of congestion pricing based on static traffic assignment (Chiu et al. 2011; Chung et al. 2012; Dong and Mahmassani 2013). Static models only focus on travelers' static path choice decisions, and the static user equilibrium ignores the time-dependent nature of traffic flows. Moreover, the influence of the current toll on the congestion level in future is not taken into consideration in static pricing models (Wie and Tobin 1998). Furthermore, due to the inherent dynamics of a transportation system, the travel behavior of people will change as the external circumstances change. Therefore, it is important to introduce the dynamic traffic assignment (DTA) theory for the application of the JDTDT scheme. This study formulates the JDTDT problem using DTA theory.

The dynamic congestion pricing problem is usually formulated as a bi-level programming model, with the upper level of optimizing the total system performance and the lower level for modeling the dynamic path choice of each individual. The lower level problem can be formulated as a dynamic user equilibrium (DUE) problem, which is to determine the route flow pattern such that the total generalized costs incurred by travelers for each OD pair departing at any time are equal and minimal (Ran and Boyce 1996; Szeto and Lo 2004). In the DUE problem, two issues of crucial importance are flow dynamics and flow propagation constraints. In other words, how to obtain the actual path travel times from path flows is a crucial problem. Numerous studies are conducted in this field (e.g., Ban et al. 2008; Han, Piccoli, and Friesz 2015; Han, Piccoli, and Szeto 2016; Huang and Lam 2002; Long et al. 2013; Long et al. 2016; Zhan and Ukkusuri 2017). In this paper, the dynamic traffic flow component is represented by a path-based CTM (Doan and Ukkusuri 2012; Ukkusuri, Han, and Doan 2012; Doan and Ukkusuri 2015). Compared to the original CTM (Daganzo 1994, 1995), the main advantages of the path-based CTM are as follows: (*a*) cells and cell connectors can be traced in different paths, (*b*) the flows at merging and diverging links can be uniquely determined without exogenous turning ratios, and (*c*) the waiting time of each cell occupancy is no longer needed. For the sake of avoiding calculating the inverse function (Lo and Szeto 2002), a new averaging scheme is proposed to estimate the en-route travel time for the traffic departing simultaneously of each path with the output of path-based CTM. In our proposed averaging approach, two new arriving time indexes are introduced, making it more concise to calculate the en-route travel time. Besides, compared with other cell-based dynamic traffic assignment models adopting a uniform demand as an input during the entire modeling horizon (e.g., Lo and Szeto 2002; Doan and Ukkusuri 2015), a multi-period demand scheme is adopted as input to better depict the dynamic congestion pricing problem.

The DUE problem is modeled as a variational inequality (VI) problem, which is the lower level for the dynamic congestion pricing problem, while the upper level is to minimize the total system time. It is well known that the bi-level programming problem is NP-hard and cumbersome to solve (Jeroslow 1985; Gao, Wu, and Sun 2005; Rahmani, Jenelius, and Koutsopoulos 2015; Rahmani and MirHassani 2015; Kheirkhah, Navidi, and Bidgoli 2016). Therefore, a hybrid self-adaptive gradient projection (SAGP) and artificial bee colony (ABC) algorithm is proposed to solve the proposed bi-level model, with the SAGP to solve the VI problem for the lower level and ABC for solving the dynamic JDTDT problem for the upper level. Noting that the system dynamics can be reflected in two aspects: (*a*) travelers' path choice decisions follow the DUE principle and (*b*) the JDTDT is extended from the static pattern to the time-varying pattern, which can be handled by changing the toll value every discrete time interval (e.g., 30 min in the ERP system at Singapore). At the end of each time interval, the travel demand changes, and thus a new JDTDT is levied.

This paper aims to solve the dynamic congestion pricing problem taking into consideration the actual travel distance and congestion level inside the cordon charging area. The contributions of this paper are threefold. The first one is that we originally propose an integrated modeling methodology for the novel optimal joint distance and time-delay toll design problem, and extend it from static to dynamic transportation networks. The second one is that we propose a new averaging scheme to estimate the en-route travel time in the dynamic network loading process for the traffic departing simultaneously of each path with the output of path-based CTM for a general transportation network. The last one is that a hybrid self-adaptive gradient projection and artificial bee colony algorithm is developed to solve the proposed bi-level programming model.

The following sections are structured as follows: the next section introduces the distance-based toll, the congestion-based toll, the JDTDT and the path-based CTM. Section 3 presents the time-varying JDTDT and the DUE conditions; then a bi-level programming model is built for the dynamic JDTDT problem. Section 4 develops a hybrid SAGP and ABC algorithm to solve the proposed model. Section 5 presents the numerical results, and finally, Section 6 concludes this paper.

2 Problem Description

As for a strongly connected transportation network $G=(N,A)$, we use N and A denote the sets of nodes and directed links, respectively. W represents the set of all origin-destination (OD) pairs. P^w denotes the set of paths connecting an OD pair $w \in W$ and q^w denotes the travel demand between OD pair $w \in W$. The modeling period is subdivided into time intervals for departure and also charging interval for charging tolls. Other notations are summarized in alphabetical order in Table 1.

Table 1: Notations (in alphabetical order)

Notation	Definition
d	index of charging intervals for charging a time-varying toll
$f_{p,t}^w$	flow during time interval t on path p of OD pair w
$h_{p,t}$	departure rate during time interval t on path p
i	cell index
(i,j)	indices for links
l	travel distance inside the cordon charging area
p	$p \in P^w$

Notation	Definition
q^w	total demand between OD pair w
r	origin cell
s	sink cell
t	index for time intervals
w	$w \in W$
$x^i_{p,t}$	occupancy of cell i at the beginning of time interval t of path p
$\bar{x}^i_t$	$\bar{x}^i_t = \sum_p x^i_{p,t}$
$\tilde{x}^{i,j}_t$	aggregated occupancy of diverging cell i at the beginning of time interval t which will go to cell j
x^i_t	occupancy of cell i at the beginning of time interval t
$y^{i,j}_{p,t}$	flow on path p from cell i to cell j at the beginning of time interval t
$y^{i,j}_t$	flow from cell i to cell j at the beginning of time interval t
C_D	set of diverging cells
C_M	set of merging cells
C_R	set of source cells
C_S	set of sink cells
C_O	set of ordinary cells
E_D	set of diverging links
E_M	set of merging links
E_O	set of ordinary links
N^i	jam density of cell i
P^w	set of paths between OD pair $w \in W$
Q^i	maximum flow out of cell i
T	maximum time horizon
T_d	d th charging interval of time-varying tolls, $T_d \in T$
W	set of all OD pairs
α	value-of-time
β	positive pricing rate with the time spent in the congestion
δ^i_p(or δ^j_p, δ^k_p)	if cell i (or j,k) is on path p, then δ^i_p(or $\delta^j_p, \delta^k_p) = 1$; otherwise δ^i_p(or $\delta^j_p, \delta^k_p) = 0$
$\eta^w_{p,t}$	travel time of path p connecting OD pair w for flow departing during time interval t
θ_1	predetermined weight for distance-based toll
θ_2	predetermined weight for congestion-based toll
$\lambda^r_{p,t}$	cumulative traffic on path p departing from cell r at the beginning of time interval t

Notation	Definition
$\lambda^{s}_{p,\omega}$	cumulative traffic on path p arriving at cell s at the beginning of time interval t
μ	an infinitesimal number ($\mu > 0$)
ν	ratio of the backward speed to the free-flow speed
π^{w}_{t}	minimum cost during time interval t of OD pair w
$\tau(l,\beta)$	joint distance and time-delay toll function
$\tau_{d}(l,\beta)$	time-varying joint distance and time-delay toll function for the charging interval d
$\varphi(\Delta t)$	congestion-based toll function
$\phi(l)$	distance-based toll function
ω_1	minimum time index value which fulfills $\lambda^{s}_{p,\omega_1} \geq \lambda^{r}_{p,t-1}$
ω_2	minimum time index value which fulfills $\lambda^{s}_{p,\omega_2} \geq \lambda^{r}_{p,t}$
Γ^{-1}_{i}	set of predecessors of cell i
Γ_{i}	set of successors of cell i
Δt	delay
$\psi^{w}_{p,t}$	generalized travel cost of the flow on path p between OD pair w departing during time interval t
Ω	set of feasible vectors of path flows

2.1 Distance-based toll

As shown in the left of Figure 1, the distance-based toll function can be described as a continuous nonlinear function. However, it is difficult to analytically deduce and solve this type of function. A general solution method is to approximate the nonlinear function with a stepwise linear function according to the travel distance l in the cordon. Assuming that the minimal and maximal length inside the cordon charging area are l_0 and l_K, we can divide the travel distance into K equal intervals and the distance-toll function of each interval can be expressed by the two endpoints as shown in Figure 1. This piecewise linear approximation method follows that in Meng, Liu, and Wang (2012).

Suppose that the vertexes of travel distances are $l = (l_0, l_1, \cdots, l_k, \cdots, l_K)^T$, and the corresponding toll values are $\phi = \left(\phi_0, \phi_1, \cdots, \phi_k, \cdots, \phi_K\right)^T$. Let l^{w}_{p} be the distance length of path $p \in P^w$ inside the cordon pricing area. Suppose l^{w}_{p} lies in the k th interval of the distance toll function. Then we can approximate the distance-toll of path $p \in P^w$ inside the cordon pricing area by the following function:

$$\phi^{w}_{p} = \phi(l^{w}_{p}) = \phi_{k-1} + \frac{l^{w}_{p} - l_{k-1}}{l_k - l_{k-1}}\left(\phi_k - \phi_{k-1}\right) \quad (1)$$

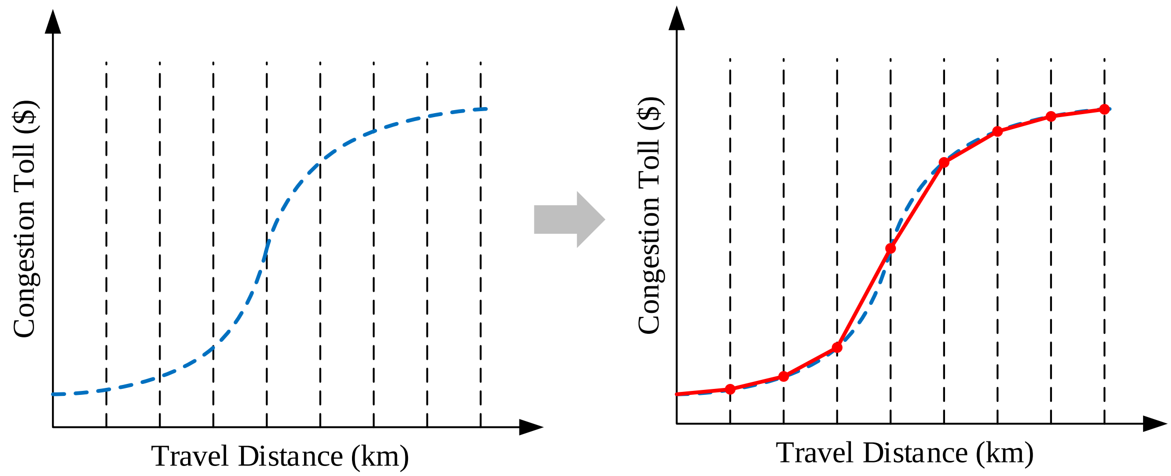


Figure 1: Stepwise linear toll function.

2.2 Time-delay-based toll

As mentioned before, the distance-based pricing model still has its limitation because travelers would wittingly select their shortest routes to reduce their toll, regardless of the congestion level of their routes. This may violate the intention of encouraging detour under congestion pricing. Liu, Wang, and Meng (2014) adopted a JDTT scheme to deal with this problem. However, there is an overlap between the distance-based toll and time-based toll in terms of the free flow travel time, which implies an overcharge. Here, we propose a JDTDT scheme to handle the overcharge problem. Note that the difference between the actual travel time and the free flow travel time is the delay, which indicates the congestion level in the network. The congestion-toll φ_p^w of path $p \in P^w$ between OD pair w is

$$\varphi_p^w = \varphi(\Delta t_p^w) = \beta \cdot \Delta t_p^w \tag{2}$$

where φ_p^w is assumed to be proportional to the delay Δt_p^w of path $p \in P^w$ inside the cordon charging area and β is the charging rate with the time spent in the congestion. Hence, the congestion-based toll problem becomes to determine the optimal β inside the cordon pricing area. It is worth noting that the calculated delay in this manuscript is actually a 'future' delay, rather than a 'current' delay. The delay is calculated by the difference between the en-route travel time inside the charging cordon and the free flow travel time, and the en-route travel time is calculated by the difference between the time leaving outside the charging cordon and the time entering the charging cordon. Thus, the delay is a real one, not a predicted one, and the time-delay-based toll will be charged based on the real delay experienced by the travelers.

2.3 Joint distance and time-delay toll (JDTDT)

Based on the distance-toll and congestion-toll proposed above, we can formulate the JDTDT function τ, which is expressed as a weighted sum of the distance-toll ϕ and the time-delay-toll φ, namely

$$\tau_p^w = \tau(l_p^w, \beta) = \theta_1 \phi(l_p^w) + \theta_2 \varphi(\Delta t_p^w) \tag{3}$$

where θ_1 and θ_2 are the predetermined weights of distance-toll and congestion-toll. It is evident that the JDTDT problem is uniquely determined by the nonlinear distance-toll function and the congestion-toll charging rate β. When the weight of distance-toll θ_1 equals zero, then the JDTDT reduces to a pure congestion-toll scheme; similarly, when the weight of congestion-toll θ_2 or the congestion-toll charging rate β equals zero, the JDTDT reduces to a pure distance-toll scheme. Consequently, the proposed JDTDT in this paper is a generalized version of the congestion pricing scheme, which includes the pure distance-toll scheme as well as the congestion-toll scheme. In Section 5, we will compare different kinds of toll schemes and evaluate the performance of each toll scheme.

2.4 Path-based cell transmission model

Cell transmission model (CTM) is initially proposed by Daganzo (1994, 1995) to investigate the dynamic traffic assignment problem. It has been verified that CTM is capable of capturing the traffic dynamics (e.g., the queue spillback, queue formulation, and queue dissipation) (Lo and Szeto 2002). After discretizing the road segments into cells and time into intervals, then the CTM can be formulated as follows:

$$x_{t+1}^{i} = x_{t}^{i} + y_{t}^{i-1,i} - y_{t}^{i,i+1} \tag{4}$$

$$y_{t}^{i,i+1} = \min\left\{x_{t}^{i}, Q^{i}, Q^{i+1}, \gamma \cdot \left[N^{i+1} - x_{t}^{i+1}\right]\right\} \tag{5}$$

where x_t^i is the occupancy of cell i at the beginning of t, $y_t^{i,i+1}$ is the flow from the upstream cell i to the downstream cell $i+1$ at the beginning of t, Q^i is the maximum flow out of cell i, N^{i+1} is the jam density of downstream cell $i+1$, and γ is the ratio of the backward shockwave speed to the forward speed (i.e., free-flow speed). Note that the cell length usually adopts the free flow travel distance in one time interval.

Eqs. (4)-(5) provide a fundamental principle of CTM for a series of cells connected together. To make this model appropriate for general transportation networks with multiple OD pairs, the following features are necessary: (*a*) generalization for merge and diverge junctions, (*b*) modeling the traffic for each OD pair and (*c*) keeping the first-in-first-out (FIFO) characteristic. In order to avoid the exogenous turning ratios in the original CTM (Daganzo, 1994, 1995), Ukkusuri, Han, and Doan (2012) and Doan and Ukkusuri (2015) proposed a path-based CTM to determine the traffic flows at the merging and diverging cells. Besides, it is also complicated to keep track of the waiting time of each cell occupancy in the original CTM. In this paper, we use the path-based CTM to model the cell update as well as flow propagation process. In the path-based CTM, the cell occupancies and flows in cells and links are modeled in terms of path. Compared to the original CTM, the main advantages of the path-based CTM are (*a*) cells as well as cell connectors are modeled in terms of path, (*b*) the flows at merging and diverging links are modeled without exogenous turning ratios, and (*c*) the waiting time of each cell occupancy is no longer needed explicitly. Figure 2 depicts different types of cells and links in the cell representation networks. For each type of cell and link, path-based CTM can be expressed as follows:

Initialization

$$x_{p,0}^{i} = 0 \quad \forall i \in C, p \in P^{w}, P^{w} \in P \tag{6}$$

$$y_{p,0}^{i,j} = 0 \quad \forall (i,j) \in E, p \in P^{w}, P^{w} \in P \tag{7}$$

Source cells

$$x_{p,t}^{i} = \delta_{p}^{i} \cdot (h_{p,t-1} + x_{p,t-1}^{i} - y_{p,t-1}^{i,j}) \quad \forall i \in C_{R}, j \in \Gamma_{i}, t = 1, \cdots, T \tag{8}$$

Ordinal cells

$$x_{p,t}^{i} = \delta_{p}^{i} \cdot (x_{p,t-1}^{i} + y_{p,t-1}^{k,i} - y_{p,t-1}^{i,j}) \quad \forall i \in C_{O}, k \in \Gamma_{i}^{-1}, j = \Gamma_{i}, t = 1, \cdots, T \tag{9}$$

Merging and diverging cells

$$x_{p,t}^{i} = \delta_{p}^{k}\delta_{p}^{i}\delta_{p}^{j} \cdot (x_{p,t-1}^{i} + y_{p,t-1}^{k,i} - y_{p,t-1}^{i,j}) \quad \forall i \in C_{M} \cup C_{D}, k \in \Gamma_{i}^{-1}, j = \Gamma_{i}, t = 1, \cdots, T \tag{10}$$

Sink cells

$$x_{p,t}^{i} = \delta_{p}^{i} \cdot (x_{p,t-1}^{i} + y_{p,t-1}^{k,i}) \quad \forall i \in C_{S}, k \in \Gamma_{i}^{-1}, t = 1, \cdots, T \tag{11}$$

Ordinary links

$$y_{p,t}^{i,j} = \delta_{p}^{i} \cdot \left[\min\left(\bar{x}_{t}^{i}, Q^{i}, Q^{j}, \gamma\left(N^{j} - \bar{x}_{t}^{j}\right)\right) \cdot \frac{x_{p,t}^{i}}{\bar{x}_{t}^{i} + \mu} \right] \quad \forall (i,j) \in E_{O}, j \in \Gamma_{i}, t = 1, \cdots, T \tag{12}$$

Diverging links

$$y_{p,t}^{i,j} = \delta_{p}^{i}\delta_{p}^{j} \cdot \left[\min\left(\tilde{x}_{t}^{i,j}, Q^{j}, \gamma\left(N^{j} - \bar{x}_{t}^{j}\right)\right) \cdot \min\left(1, \frac{Q^{i}}{\sum_{j' \in \Gamma_{i}} \left(\min\left(\tilde{x}_{t}^{i,j'}, Q^{j'}, \gamma\left(N^{j'} - \bar{x}_{t}^{j'}\right)\right)\right) + \mu} \right) \right] \cdot \frac{x_{p,t}^{i}}{\tilde{x}_{t}^{i,j} + \mu}$$
$$\forall i \in C_{D}, j \in \Gamma_{i}, t = 1, \cdots, T \tag{13}$$

Merging links

$$y_{p,t}^{k,i} = \delta_{p}^{k}\delta_{p}^{i} \cdot \left[\min\left(Q^{k}, \bar{x}_{t}^{k}\right) \cdot \min\left(1, \frac{\min\left(Q^{i}, \gamma\left(N^{i} - \bar{x}_{t}^{i}\right)\right)}{\sum_{k' \in \Gamma_{i}^{-1}} \left(\min\left(Q^{k'}, \bar{x}_{t}^{k'}\right)\right) + \mu} \right) \right] \cdot \frac{x_{p,t}^{k}}{\bar{x}_{t}^{k} + \mu} \tag{14}$$
$$\forall i \in C_{M}, k \in \Gamma_{i}^{-1}, t = 1, \cdots, T$$

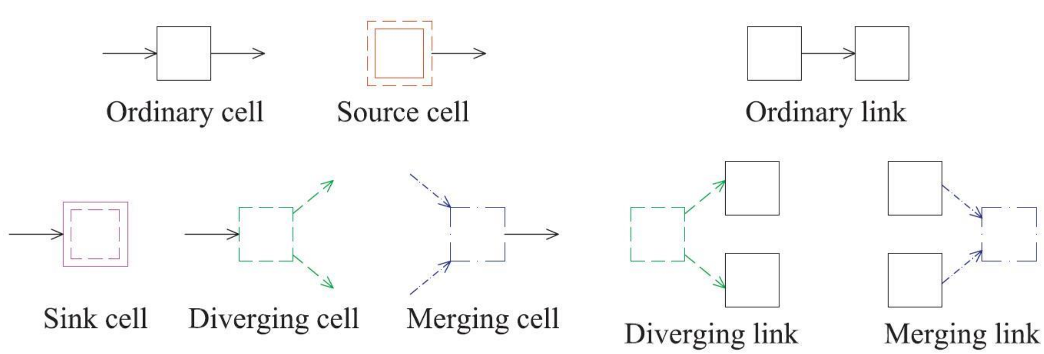


Figure 2: Different types of cells and links in cell representation networks.

Eqs. (6) and (7) assume that the initial cell occupancies and outflows equal to zero. Actually, it is also possible to start the CTM loading by other nonzero values in terms of the actual traffic conditions. Eqs. (8)-(11) depict the path-based (disaggregate) cell updating procedure, and Eqs. (12)-(14) are the path-based (disaggregate) flow propagation constraints. Noting that the turning ratio in the path-based CTM is not exogenous, but uniquely determined by the supply and demand of upstream and downstream cells. The details can be found in Ukkusuri, Han, and Doan (2012) and are not repeated here.

To avoid the discontinuity in the flow propagation process, Lebacque and Khoshyaran (2005) claimed that the node flow solutions should respect the invariance principle. This principle states that when the flows are under supply/demand constraints, the node flow solutions should be invariant to the increases in the demand/supply. Those solutions violating the invariance principle may result in unrealistic dynamics on the links, i.e., the likelihood of waves propagating in the wrong directions. Most of the traffic flow models for merging and diverging junctions in the literature (e.g., the exogenous ratio distribution in Daganzo 1995, the demand proportional distribution in Jin and Zhang 2003, the capacity proportional distribution in Ni and Leonard 2005, just to name a few) do not satisfy the invariance principle, while only a few studies in recent tend to respect the invariance principle (Lebacque and Khoshyaran 2005; Corthout et al. 2012; Flötteröd and Rohde 2011; Jin 2010; Tampère et al. 2011; Jabari 2016).

To ensure the invariance principle, the distribution of supply must be independent of the ratio of the demands (Tampère et al. 2011). As for the merging and diverging cells in the path-based CTM, it does not exist the distribution problem because it is impossible for different incoming (or outgoing) cells of one particular receiving (or sending) cell that are in the same path. Besides, as claimed in Tampère et al. (2011), the invariance principle for supply in the diverging links is automatically satisfied because the solution is derived by distributing the supply rather than the demand. Thus, only the merging links need to be considered for the invariance principle in the path-based CTM. It is obvious that when we calculate $y_{p,t}^{k,i}$ of the merging links, the endogenous ratio of $\frac{\min\left(Q^k, \bar{x}_t^k\right)}{\sum_{k' \in \Gamma_i^{-1}} \left(\min\left(Q^{k'}, \bar{x}_t^{k'}\right)\right) + \mu}$ is demand-dependent, thus may violate the invariance principle. However, the main focus of this paper is on the joint distance and time-delay based dynamic congestion pricing, rather than the first-order node model of the dynamic network loading problem. In the future research, it is necessary to investigate the node models of the dynamic network loading process taking into considerations of the invariance principle and then study the dynamic congestion pricing problem with this more realistic dynamic network loading process.

3 Mathematical Model

In order to formulate the cell-based dynamic JDTDT problem, we first introduce the time-varying JDTDT problem, and then the DUE conditions and the VI formulation for the lower level problem is introduced. Finally, a bi-level programming model of this problem is proposed in this section.

3.1 Time-varying JDTDT

In Section 2, we have described the JDTDT scheme for static networks; here we will extend it to a time-varying scheme. It is impractical to change the toll value at every moment because travelers cannot respond to such frequent change of the toll value. An alternative is to change the toll value every half hour, which is a comparatively reasonable time interval for travelers to make responses to the change and currently adopted in the ERP system of Singapore. At the end of each time interval, the travel demand changes, and a new toll value is levied. Note that we adopt a multi-period demand scheme as an input to better depict the dynamic congestion pricing problem in this paper.

For simplicity, we only consider the morning commute traffic in this study. Hence, the whole modeling period can be set from 7:30 am to 9:30 am, which means a total of 120 minutes for the charging time length. As claimed before, the toll value changes every half hour, so there will be four charging intervals and thus four different toll patterns during the entire charging duration. For the d th subinterval, the JDTDT can be expressed as:

$$\tau_{p,d}^{w} = \tau_d(l_p^w, \beta) = \theta_1 \cdot \phi_d(l_p^w) + \theta_2 \cdot \varphi_d(\Delta t_p^w) \tag{15}$$

Note that the specific time when vehicles arriving at the charging cordon can be determined according to the travel time from the origin to the charging cordon, thus we can calculate the correct charging interval d as follows:

$$d = \left\lceil \frac{(t + t') \cdot \ell}{30} \right\rceil \tag{16}$$

where $\lceil \cdot \rceil$ is the smallest integer greater than or equal to the number in the brackets, t' and ℓ are the travel time from the origin to the charging cordon and the time interval length (unit of minute) in the CTM, respectively.

3.2 Dynamic user equilibrium problem

The equilibrium conditions of an ideal DUE state can be stated as follows: the total generalized costs incurred by travelers for each OD pair departing simultaneously are equal and minimal. Mathematically, it can be formulated as

$$f_{p,t}^{w}(\tau_{p,d}^{w}) \cdot \left(\psi_{p,t}^{w} - \pi_t^w\right) = 0, \qquad \forall w \in W, p \in P^w, t \in T \tag{17}$$

$$\psi_{p,t}^{w} - \pi_t^w \geq 0, \qquad \forall w \in W, p \in P^w, t \in T \tag{18}$$

where

$$\psi_{p,t}^{w} = \alpha \cdot \eta_{p,t}^{w} + \tau_{p,d}^{w}, \qquad \forall w \in W, p \in P^w, t \in T \tag{19}$$

Note that the path flow $f_{p,t}^w$ is a function of the toll value $\tau_{p,d}^w$. Thus, the DUE problem is to find a feasible $\mathbf{f} = \left\{ \mathbf{f} \,\middle|\, f_{p,t}^w, \forall w \in W, \forall p \in P^w, \forall t \in T \right\}$ which satisfies Eqs. (17)-(18) and the demand-flow incidence relationship as well as non-negativity constraints:

$$\sum_{p\in P^w} f_{p,t}^w(\tau_{p,d}^w) = q_t^w \qquad \forall w\in W, t\in T \tag{20}$$

$$\mathbf{f}(\boldsymbol{\tau}) \geq \mathbf{0},\ \mathbf{u} \geq \mathbf{0} \tag{21}$$

where $\mathbf{u}$ is a column vector of π_t^w, i.e., $\mathbf{u} = \left\{\mathbf{u} \middle| \pi_t^w, \forall w\in W, \forall t\in T\right\}$.

The DUE problem introduced in Eqs. (17)-(21) is equivalent to a finite dimensional VI problem:

$$\left(\mathbf{f}(\boldsymbol{\tau}) - \mathbf{f}^*(\boldsymbol{\tau})\right)^T \cdot \boldsymbol{\Psi}^* \geq 0 \qquad \forall \mathbf{f}(\boldsymbol{\tau}) \in \Omega \tag{22}$$

where the superscript * represents the optimal solution, $\boldsymbol{\Psi}$ is the column vector of $\psi_{p,t}^w$, i.e., $\boldsymbol{\Psi} = \left\{\boldsymbol{\Psi} \middle| \psi_{p,t}^w, \forall w\in W, \forall p\in P^w, \forall t\in T\right\}$, and Ω is the set of feasible vectors of path flows, which fulfills the demand-flow incidence relationship in Eq. (20) and non-negativity condition in Eq. (21).

Ran and Boyce (1996) demonstrated the equivalence between the DUE problem (17)-(21) and the VI problem (22), and Lo and Szeto (2002) discussed the existence as well as the uniqueness of the solution of this proposed VI problem. Those interested in the detailed proofs may refer to their studies.

A vital issue of solving the DUE problem in Eq. (22) is to dynamically model the travel times as a unique mapping function of path flows. As for traditional static traffic assignment problems, the link performance function (e.g., Bureau of Public Roads, known as the BPR function) is widely adopted to describe travel times from traffic volumes. However, the static BPR-type function can only express the steady-state link travel time as a mapping function of the traffic volume on that link, without consideration of the oversaturation, queue spillback or peak spreading. By encapsulating the CTM in DUE, Lo and Szeto (2002) proposed an averaging scheme to calculate the actual path travel time from the output of CTM so that the whole traffic departing simultaneously has one uniquely determined average en-route travel time (AERTT) $\eta_{p,t}^w$. However, this method needs to calculate an inverse function to obtain the AERTT. Based on the path-based CTM, a more concise approach to obtain the AERTT is proposed in this paper.

According to the CTM discussed before, we know that the output of CTM is the cell occupancies of traffic at each time interval. Then, the cumulative traffic departing from the origin cell *r* on path *p* at the beginning of time interval *t* is the sum of the cumulative traffic departing from cell *r* on path *p* at the beginning of time interval $t-1$ and the outflow of cell *r* on path *p* during time interval *t*, and the cumulative traffic arriving at the sink cell *s* on path *p* at the beginning of time interval ω is the sum of the cumulative traffic arriving at cell *s* on path *p* at the beginning time interval $\omega-1$ and the inflow of cell *s* on path *p* during time interval ω, namely,

$$\lambda_{p,t}^r = \lambda_{p,t-1}^r + y_{p,t}^{r,j} \qquad \forall r\in C_R, j\in \Gamma_i, t=1,\cdots,T \tag{23}$$

$$\lambda_{p,\omega}^s = \lambda_{p,\omega-1}^s + y_{p,\omega}^{k,s} \qquad \forall s\in C_S, k\in \Gamma_s^{-1}, \omega=1,\cdots,T \tag{24}$$

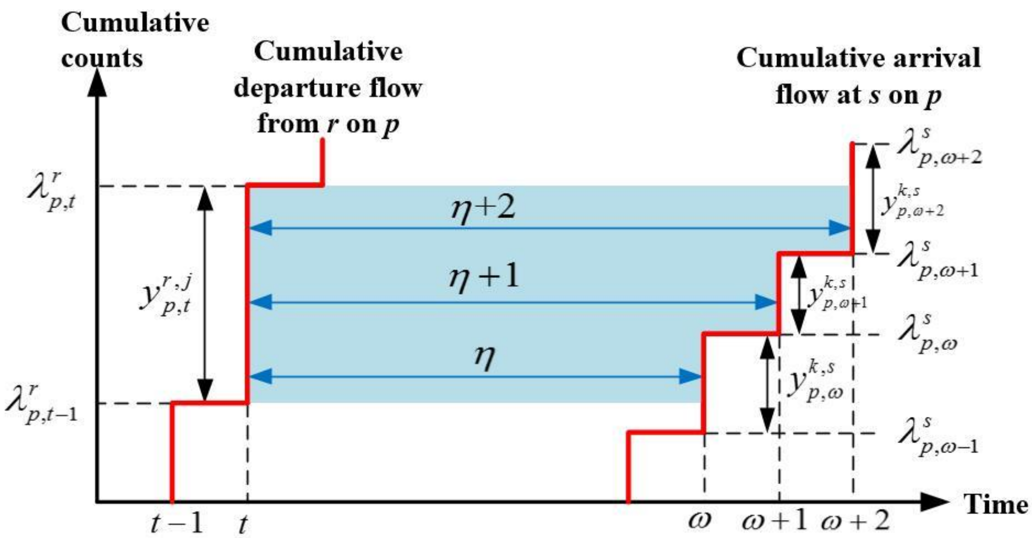


Figure 3: The cumulative vehicle counts.

Figure 3 depicts the cumulative vehicle count curves $\lambda^r_{p,t}$ and $\lambda^s_{p,\omega}$ on path p between the origin node r and destination node s (say OD pair w). In reality, the entire $y^{r,j}_{p,t}$ departing from cell r on path p during time interval t may not arrive at the destination s at the same time when time is discretized. This is illustrated in Figure 3, with the abuse use of notations that t also means the beginning of a time interval (similar for ω). As shown in Figure 3, the fastest travelers have an en-route travel time of η, while the slowest travelers have an en-route travel time of $\eta+2$. In this paper, we also adopt the AERTT to represent the actual en-route travel time of the whole traffic departing simultaneously on this path. Firstly, we introduce two arriving time indexes ω_1 and ω_2, where ω_1 is the minimum time index value which fulfills $\lambda^s_{p,\omega_1} \ge \lambda^r_{p,t-1}$, and ω_2 is the minimum time index value which fulfills $\lambda^s_{p,\omega_2} \ge \lambda^r_{p,t}$. Then, the AERTT can be expressed as follows:

$$\eta^w_{p,t} = \begin{cases} \omega_1 - t \quad if \ \omega_2 - \omega_1 = 0 \\ \dfrac{(\omega_1 - t)\cdot\left(\lambda^s_{p,\omega_1} - \lambda^r_{p,t-1}\right) + (\omega_2 - t)\cdot\left(\lambda^r_{p,t} - \lambda^s_{p,\omega_1}\right)}{y^{r,j}_{p,t} + \mu} \quad if \ \omega_2 - \omega_1 = 1 \\ \dfrac{(\omega_1 - t)\cdot\left(\lambda^s_{p,\omega_1} - \lambda^r_{p,t-1}\right) + \sum_{n=1}^{\omega_2-\omega_1-1}\left[(\omega_1 + n - t)\cdot y^{k,s}_{p,\omega_1+n}\right] + (\omega_2 - t)\cdot\left(\lambda^r_{p,t} - \lambda^s_{p,\omega_2\text{-}1}\right)}{y^{r,j}_{p,t} + \mu} \\ \qquad if \ \omega_2 - \omega_1 > 1 \end{cases} \tag{25}$$

3.3 Bi-level programming model

The objective of the dynamic congestion pricing problem in this paper is to determine the time-varying JDTDT which satisfies the DUE principle by optimizing the total system performance. This can be mathematically formulated as a bi-level programming model, the upper level of which is to optimize the total system performance and the lower level is to achieve a DUE state, which can be expressed as a VI formulation. The details of this bi-level model are given as follows:

Upper level

$$\underset{\boldsymbol{\tau}(\phi,l,\beta)}{Min} \sum_{t\in T} \sum_{p\in R^w} \sum_{w\in W} \eta_{p,t}^w \cdot f_{p,t}^w(\tau_{p,d}^w) \tag{26}$$

where $\boldsymbol{\tau}$ is a nonnegative toll vector for Eq. (15), which is a combination of the distance-based toll ($\phi_0 \le \phi_d \le \phi_K$) and the congestion-based toll ($\varphi_d \ge 0$).

Lower level

The DUE problem is formulated by a VI problem (22).

4 Solution Algorithms

It is well known that the bi-level programming problem is NP-hard and cumbersome to solve, see Jeroslow (1985), etc. A hybrid SAGP and ABC algorithm is developed to solve the proposed bi-level model, with SAGP to solve the VI problem of the lower level and ABC to solve the time-varying JDTDT problem of the upper level.

4.1 Self-adaptive gradient projection algorithm

There are many solution algorithms for solving VI problems, we choose the SAGP algorithm because it can automatically calculate and obtain an appropriate step size based on the results of previous iterations, and relieve the computational burden due to the projection process on a nonnegative orthant (Chen, Zhou, and Xu 2012). Generally, the SAGP algorithm is used for solving static traffic equilibrium problems, and in this paper we will modify and extend this algorithm to solve the DUE problem. The procedure of the SAGP algorithm is summarized below:

Step 0: Set $\vartheta \in (0,1)$, $u \in [0.5,1]$, $\varepsilon > 0$, $\rho_{\max} > 0$, $\rho_0 > 0$ and $\mathbf{f}_0 \in \Omega$; set $\gamma_0 = \rho_0$ and $k = 0$.

Step 1: Find the smallest nonnegative integer l_k such that $\rho_{k+1} = \gamma_k \cdot u^{l_k}$ and update the non-shortest path flows:

$$\tilde{f}_{p,k}^w = \max\left[0, f_{p,k}^w - \rho_{k+1} \cdot F_{p,k}^w\right], \qquad \forall p \in P^w, p \ne \bar{p}_k^w, w \in W \tag{27}$$

which satisfies the following constraint:

$$(2-\vartheta)\rho_{k+1}(\tilde{\mathbf{f}}_k - \tilde{\mathbf{f}}_{k+1})^T - \rho_{k+1}^2 \left\|\mathbf{F}_k - \mathbf{F}_{k+1}\right\|^2 \ge \max\left\{\frac{(\rho_{k+1})^2 - (\rho_k)^2}{(\rho_k)^2}\left\|e(\tilde{\mathbf{f}}_k, \rho_k)\right\|^2, 0\right\} \tag{28}$$

where $\bar{p}_k^w$ is the shortest path between OD pair w in the kth iteration; $F_{p,k}^w = \eta_{p,k}^w - \eta_{\bar{p},k}^w$, $\mathbf{F}_k$ is the vector of $(\cdots, F_{p,k}^w, \cdots)^T$; $\tilde{\mathbf{f}}_k$ is the vector of $(\cdots, \tilde{f}_{p,k}^w, \cdots)^T$; and $e(\tilde{\mathbf{f}}_k, \rho_k) = \tilde{\mathbf{f}}_k - P_{R_+^{|P|\cdot|R|\cdot|S|}}[\tilde{\mathbf{f}}_k - \rho_k \mathbf{F}(\tilde{\mathbf{f}}_k)]$. Then, update the shortest path flows:

$$f_{\bar{p}_{w,k+1}}^w = q^w - \sum_{\substack{p\in P^w \\ p\ne \bar{p}_{w,k}}} \tilde{f}_{p,k+1}^w, \qquad \forall w \in W \tag{29}$$

and set $f^{w}_{p,k+1} = \tilde{f}^{w}_{p,k+1}, \forall p \in P^{w}, p \neq \bar{p}_{w,k}, w \in W$

Step 2: If the inequality condition of (30) is fulfilled, then select $\gamma_{k+1} = \min\left\{\frac{\rho_{k+1}}{u}, \rho_{\max}\right\}$; otherwise $\gamma_{k+1} = \rho_{k+1}$.

$$\begin{aligned} &0.5\rho_{k+1}(\tilde{\mathbf{f}}_k - \tilde{\mathbf{f}}_{k+1})^T(\mathbf{F}_k - \mathbf{F}_{k+1}) - (\rho_k)^2 \| \mathbf{F}_k - \mathbf{F}_{k+1} \|^2 \\ &\geq \max\left\{\frac{(\rho_{k+1})^2 - (\rho_k)^2}{(\rho_k)^2} \| e(\tilde{\mathbf{f}}_k, \rho_k) \|^2, 0\right\} \end{aligned} \quad (30)$$

Step 3: If a predetermined convergence criterion is satisfied, then stop with $\mathbf{f}_{k+1}$ as the final solution; otherwise, set $k = k+1$ and go to step 1.

4.2 Artificial bee colony algorithm

The ABC algorithm was proposed by Karaboga (2005) for solving unimodal and multi-modal numerical optimization problems. Recent years, the ABC algorithm has attracted more and more attention in transportation studies (e.g., Chen et al. 2015; Huang et al. 2016; Szeto, Wu, and Ho 2011). Unlike the existing evolutionary algorithms such as the particle swarm optimization algorithm and the genetic algorithm, the local search mechanism in the ABC algorithm is much better, and this can enhance the quality of solutions. The procedure of the ABC algorithm is shown below:

Step 1: Set the colony size N_c, the number of employed bees N_e, the number of onlookers $N_o = N_c = N_e$; set the counter *limit*; set the iteration counter $I = 1$, and its maximum value $I_{\max}$.

Step 2: Generate randomly the initial solutions (i.e., food sources), and calculate the fitness for every employed bee. Initialize *limit* as zero.

Step 3: Conduct a neighborhood search according to the current solution, and evaluate the fitness of the newly generated neighbor solution. If the neighbor solution is better, then substitute the current solution with the newly generated neighbor solution, and reset *limit* as zero; otherwise, do not change the current solution but increase *limit* by one.

Step 4: Each onlooker chooses a solution in terms of the roulette wheel selection method. More specifically, generate a random number R which is uniformly distributed between $[0,1)$; if the chosen probability is larger than R, then the onlooker will conduct a neighborhood search to find a neighbor solution and evaluate its fitness. If the neighbor solution is better, then substitute the current solution with the neighbor solution and reset *limit* as zero; otherwise, do not change the current solution but increase *limit* by one.

Step 5: According to the current solutions, choose the one with the highest fitness. If there exists one solution which cannot improve its fitness within *limit* iterations, and it is not the best solution with the highest fitness, then the corresponding employed bee will become a scout and conduct a neighborhood search again. Then, generate randomly a new solution and reset its *limit* to zero.

Step 6: Set $I = I+1$. If $I < I_{\max}$, then return to Step 3; otherwise, terminate the algorithm and output the best solution.

5 Numerical Examples

We conduct four numerical tests here to assess the proposed methodology. These four tests include: *a*) the dynamic JDTDT; *b*) the static JDTDT; *c*) the dynamic JDTT; and *d*) the dynamic pure distance toll. Based on these four tests, we have three comparisons, i.e., comparing the dynamic JDTDT with the other three toll schemes, and to verify the effectiveness of the proposed dynamic JDTDT scheme in this paper.

5.1 The dynamic JDTDT

As shown in Figure 4, the test network in this paper is similar to the Nguyen-Dupius network, which has been used in the study of Szeto and Lo (2004) to solve the DUE problem. It has 13 nodes and 19 links. The dashed line indicates a cordon charging area. The traffic demand and path information of the numerical network are tabulated in Table 2. The cell representation consisting of 63 cells is consistent with that used by Szeto and Lo (2004) except for the bottleneck in the network. It should be noted that the minimum and maximum length inside the cordon charging area is 3.2 km and 5.6 km, respectively, and the range is 2.4 km. Therefore, we assume that the piecewise linear toll function has 3 linear charging intervals with 4 vertices, and the length of each interval is 0.8 km.

The numerical experiment is coded in Matlab R2016a running on a laptop with Inter(R) Core(TM) i7-5500U CPU @2.40GHz, 2.39GHz and 8.00G RAM, and the detailed input parameters include:

- Free flow speed: 48 km/h
- Backward shock-wave speed: 18 km/h
- Jam density: 125 vehicles/km
- Flow capacity: 1800 vehicles/h/lane
- Number of lanes: 2
- Each time interval length: 1 min
- Modeling horizon: 2 hours in total, and 30 min (or 30 time steps) of each sub-period
- The length of each cell: 0.8 km

Other parameters: $\alpha = 1.0$, $\theta_1 = 0.6$, $\theta_2 = 0.4$, $\phi_{\min} = 1.0$, $\phi_{\max} = 3.0$, $\vartheta = 0.2$, $u = 0.6$, $\varepsilon = 0.001$, $\rho_0 = 1.0$, $N_c = 40$, $N_e = 20$, $limit = 2$, $I_{\max} = 500$

Table 2: Dynamic traffic demand and paths for Nguyen-Dupius network

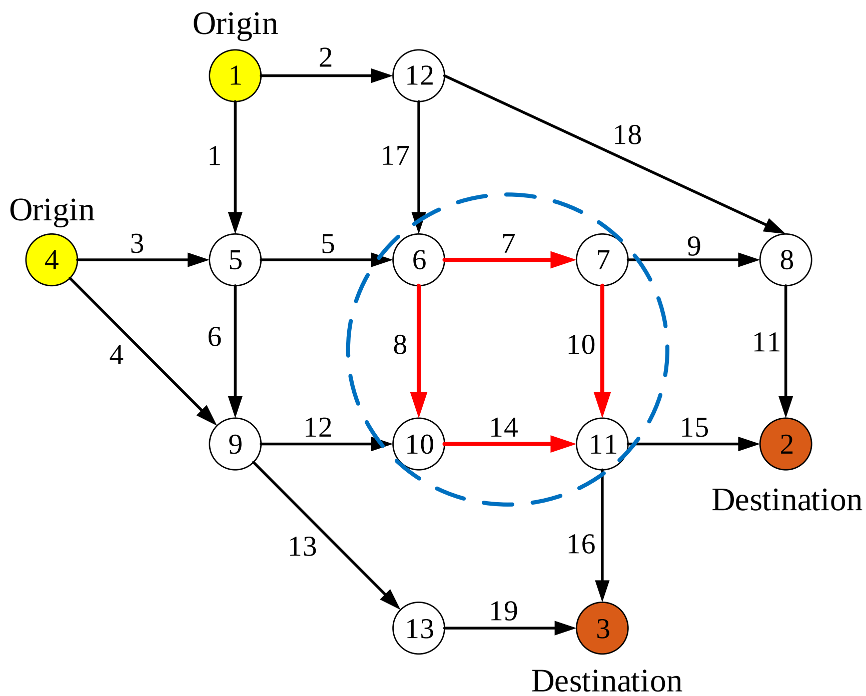
OD	Demand per time interval				Path No.	Node sequence
	07:30-08:00	08:00-08:30	08:30-09:00	09:00-09:30		
(1, 2)	40	32	26	20	1	1-12-8-2
					2	1-5-6-7-8-2
					3	1-5-6-7-11-2
					4	1-5-6-10-11-2
					5	1-5-9-10-11-2
					6	1-12-6-7-8-2
					7	1-12-6-7-11-2

					8	1-12-6-10-11-2
(1, 3)	70	60	48	36	9	1-5-9-13-3
					10	1-5-6-7-11-3
					11	1-5-6-10-11-3
					12	1-5-9-10-11-3
					13	1-12-6-7-11-3
					14	1-12-6-10-11-3
(4, 2)	64	52	40	30	15	4-9-10-11-2
					16	4-5-6-7-8-2
					17	4-5-6-7-11-2
					18	4-5-6-10-11-2
					19	4-5-9-10-11-2
(4, 3)	64	52	40	30	20	4-9-13-3
					21	4-9-10-11-3
					22	4-5-9-13-3
					23	4-5-6-7-11-3
					24	4-5-6-10-11-3
					25	4-5-9-10-11-3

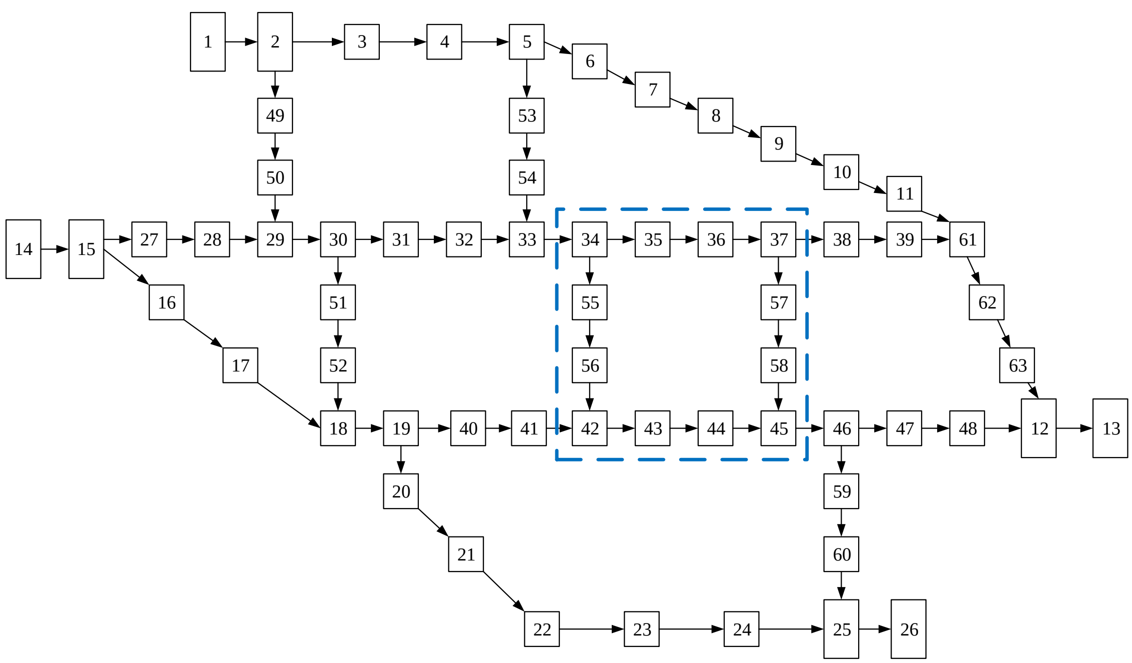
Table 3: The objective value of different toll functions

Toll Pattern	Value of β	Obj*	Reduction of the Obj*
Dynamic JDTDT	0.00	482,119	7.45%
	0.20	476,773	6.26%
	0.40	462,247	3.03%
	0.60	**448,666**	**-**
	0.80	459,677	2.45%
	0.99	478,602	6.67%
Static JDTDT	0.00	483,621	7.79%
	0.20	478,514	6.65%
	0.40	**476,837**	**6.28%**
	0.60	477,423	6.41%
	0.80	479,224	6.81%
	0.99	480,149	7.02%
Dynamic JDTT	0.00	482,119	7.45%
	0.20	473,832	5.61%
	0.40	469,793	4.71%
	0.60	**467,968**	**4.30%**
	0.80	472,821	5.38%
	0.99	478,787	6.71%

Note: the last column is the reduction rate of the total system travel time, and this rate is calculated in terms of the difference between each Obj* and 448,666 divide by each Obj*.



(a) Nguyen-Dupuis network with toll cordon



(b) Cell representation of Nguyen-Dupuis network

Figure 4: Network structure.

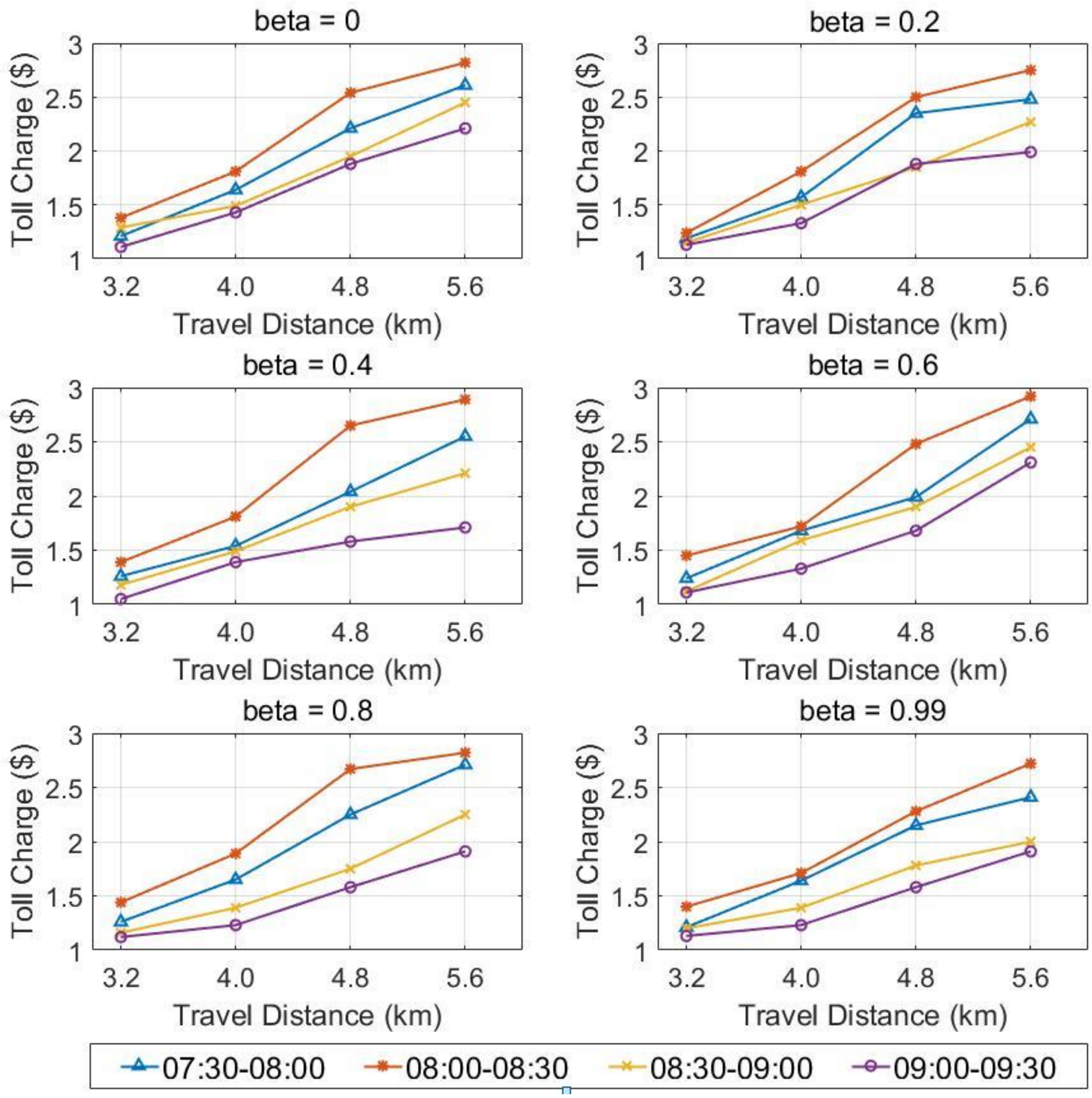


Figure 5: The optimal distance toll function with different values of β.

Figure 5 depicts the optimal toll functions with different values of β and it is obvious that the distance tolls are nonlinear increasing functions of travel distance, which are consistent with the assumption. The second period has the highest charge mainly due to the congestion level in the cordon area. From the experiment results of the dynamic JDTDT in Table 3, we can see that the optimal one is $\beta = 0.6$, and the corresponding distance toll

$$\phi = \begin{bmatrix} 1.24 & 1.68 & 1.97 & 2.68 \\ 1.44 & 1.72 & 2.49 & 2.90 \\ 1.12 & 1.59 & 1.91 & 2.44 \\ 1.11 & 1.30 & 1.67 & 2.30 \end{bmatrix}$$

, with each row containing the vertexes of the piecewise linear toll function for each sub-period among the whole modeling horizon from 07:30 to 09:30. The convergence process of the dynamic JDTDT with $\beta = 0.60$ is shown in Figure 6. It is clear that the objective value converged in a stepwise form within less than 200 iterations.

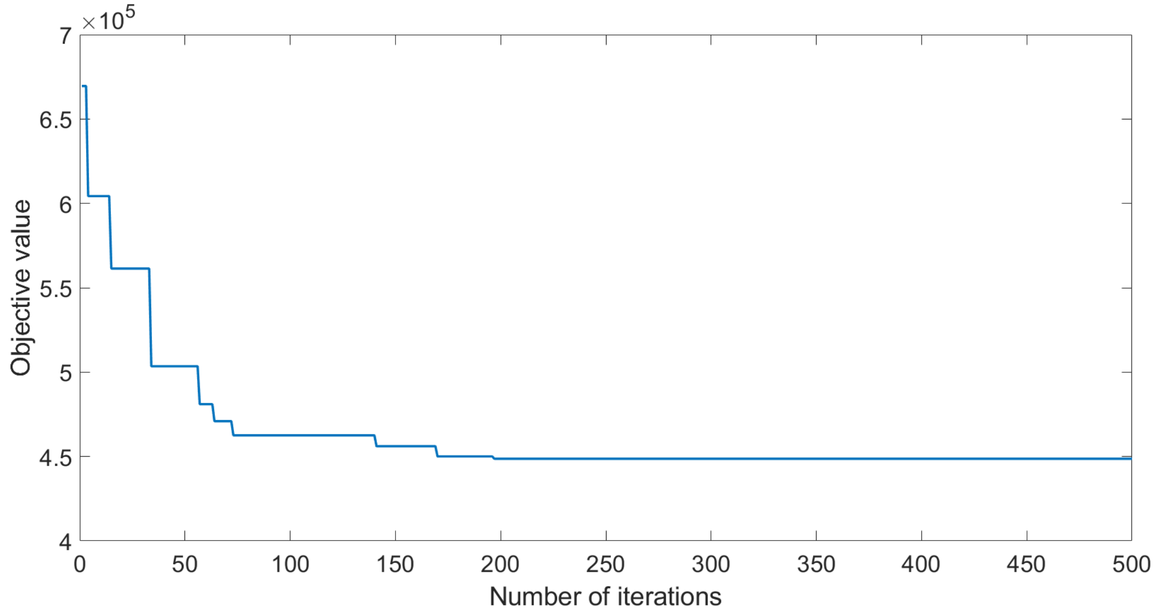


Figure 6: The convergence process of the dynamic JDTDT with $\beta = 0.60$

It should be noted that the link flows of the entire network are zero when it is 07:30 in this case. However, this assumption is obviously unrealistic. A more realistic traffic demand should be input and the experiment should be tested in the future.

5.2 Comparisons

To illustrate the superiorities of the dynamic JDTDT scheme, three controlled experiments (i.e., the static JDTDT, the dynamic JDTT, and the pure distance toll) are conducted here. The network structure in these three controlled experiments are the same as the dynamic JDTDT shown in Figure 4(a). As for the static toll pattern, the input demand in Table 4 is aggregated by the dynamic demand during the study period from 07:30 to 09:30, and the typical BPR (Bureau of Public Roads) type function is adopted to calculate the link travel times.

Table 4: Aggregate demand during the study period from 07:30 to 09:30

OD	Demand (veh)	OD	Demand (veh)
(1, 2)	3540	(4, 2)	5580
(1, 3)	6420	(4, 3)	5580

(1) The dynamic vs. static JDTDT

From the results of the static JDTDT in Table 3, it is obvious that the optimal value of β in a static JDTDT scheme is 0.40, which is different from the dynamic toll pattern. We can also see that the overall system performance of the transportation network under the dynamic toll pattern is superior to that under the static one, which is expected because the dynamic toll pattern based on the dynamic network modeling can better capture the system dynamics. Quantitatively, the total system travel time decreases by 6.28% when the JDTDT scheme is implemented from a static pattern to a dynamic pattern. This percentage reduction of the total system travel time may not be significant, but when the dynamic JDTDT is implemented for a large-scale urban area, the total value of the total system travel time may decrease signally.

(2) The dynamic JDTDT vs. JDTT

The difference between the JDTDT and the JDTT is whether the travel time or delay inside the cordon charging area is adopted in the toll scheme. Other parameters of the dynamic JDTT scheme in the cell-based DTA and solution algorithms are the same as the dynamic JDTDT scheme. From the results of the dynamic JDTT shown in Table 3, we can see that the optimal value of β is also 0.60, which is the same as the optimal value in the dynamic JDTDT scheme. Compared to the dynamic JDTT scheme, the optimal value of the total system travel time decreases by 4.30% in the dynamic JDTDT scheme, indicating that the dynamic JDTDT is more efficient than the dynamic JDTT. This result also confirms the assumption of the overcharging problem in the JDTT scheme.

(3) The JDTDT vs. pure distance toll

Note that when $\beta = 0$ for either dynamic or static toll pattern, it becomes an entirely nonlinear distance toll scheme. An interesting phenomenon can be found for both dynamic and static toll patterns, i.e., the objective value of the pure distance-toll scheme (when $\beta = 0$) is inferior to all the other cases, indicating that introducing the congestion-toll together with distance-toll leads to a better performance for the system. This phenomenon is in line with the real life of travelers who prefer to pay less for their trips inside the cordon charging area. When the pure distance-toll scheme is implemented, most travelers prefer to choose the shorter route(s) in the cordon to reduce their toll for the trip, regardless the route(s) is/are highly congested. However, after combining the congestion-toll with the distance-toll, the congestion effect will be incorporated into the payment of travelers. Thus, travelers reconsider the route choice under the JDTDT pattern. This clearly shows that the JDTDT encourages detour behaviours compared to a pure distance-toll for either the dynamic or static toll pattern, which is consistent with the principle of JDTDT. Similarly, we can obtain the reduction rate of the total system travel time in the dynamic JDTDT is 7.45% compared to the dynamic pure distance toll scheme.

6 Conclusions

This paper addressed a dynamic congestion pricing problem considering travelers' actual travel distance and congestion level in the network. The proposed dynamic JDTDT scheme is more equitable than the traditional flat toll schemes, and more effective than the static JDTDT, dynamic JDTT, and the dynamic pure distance toll. The dynamic optimal toll design problem is formulated as a bi-level programming model, the upper level of which is to optimize the total system performance, and the lower level is a DUE problem. The path-based CTM is adopted in this paper to formulate the dynamic traffic flow propagation process, and a new averaging scheme is proposed to estimate the en-route travel time for the traffic departing simultaneously of each path. Then the DUE problem can be expressed as a VI. A hybrid SAGP and ABC algorithm is developed to solve this bi-level model. The validity of the proposed methodology is verified with four numerical tests. More specifically, based on the results of the comparison, we find that the reduction rates of the minimum total system travel time in the dynamic JDTDT scheme are 6.28%, 4.30% and 7.45% compared to the static JDTDT, the dynamic JDTT and the dynamic pure distance toll, respectively. These percentage reductions may not be significant, but when the dynamic JDTDT is implemented for a large-scale urban area, the total volume of the total system travel time may decrease signally.

This paper is an initial study of the cell-based dynamic congestion pricing problem. The traffic demand is assumed piecewise fixed and the path choice decisions are deterministic in this paper. For future researches, the demand can be extended to elastic and stochastic dynamics of travelers' decision behaviors can be captured, and the toll framework can be extended to a day-to-day dynamic setting. The optimal second-best toll can be approximated with a linear programming model based on the concept of toll set (see Hearn and Ramana 1998; Lawphongpanich and Hearn 2004; Chen, Zhou, and List 2011). The congestion pricing policy can be also integrated with public transit design and network design. In addition, in the cell-based model, one link is divided to multiple cells, which largely increase the computing time for large urban transportation networks. This computing issue may be solved by the following two approaches: (*a*) parallel computing algorithms can be used, which are suitable for the cell-based dynamic congestion pricing problems due to the good property of path-based CTM; (*b*) the link transmission model, which is another discrete version of the macro-simulation approach, can be introduced to this dynamic congestion pricing problem to reduce the computational cost. In the future work, we will evaluate the proposed dynamic JDTDT scheme in mesoscopic dynamic traffic simulation packages such as DTALite (Zhou and Taylor 2014; Xiong, Zhou, and Zhang 2018).

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References

Ban, X., H. X. Liu, M. C. Ferris, and B. Ran. 2008. "A Link-Node Complementarity Model and Solution Algorithm for Dynamic User Equilibria with Exact Flow Propagations." *Transportation Research Part B* 42 (9): 823–842.

Chen, A., Z. Zhou, and X. Xu. 2012. "A Self-Adaptive Gradient Projection Algorithm for the Nonadditive Traffic Equilibrium Problem." *Computers and Operations Research* 39 (2): 127–138.

Chen, J., Z. Liu, S. Zhu, and W. Wang. 2015. "Design of Limited-Stop Bus Service with Capacity Constraint and Stochastic Travel Time." *Transportation Research Part E* 83: 1–15.

Chen, X., X. Zhou, and G. F. List. 2011. "Using time-varying tolls to optimize truck arrivals at ports." *Transportation Research Part E* 47 (6): 965-982.

Chiu, Y. C., J. Bottom, M. Mahut, A. Paz, R. Balakrishna, T. Waller, and J. Hicks. 2011. *Dynamic Traffic Assignment: A Primer*. *Transportation Network Modeling Committee*.

Chung, B. D., T. Yao, T. L. Friesz, and H. Liu. 2012. "Dynamic Congestion Pricing with Demand Uncertainty: A Robust Optimization Approach." *Transportation Research Part B* 46 (10): 1504–1518.

Corthout, R., G. Flötteröd, F. Viti, and C. M. Tampère. 2012. "Non-unique flows in macroscopic first-order intersection models." *Transportation Research Part B* 46 (3): 343-359.

Daganzo, C. F. 1994. "The Cell Transmission Model: A Dynamic Representation of Highway Traffic Consistent with the Hydrodynamic Theory." *Transportation Research Part B* 28 (4): 269–287.

Daganzo, C. F. 1995. "The Cell Transmission Model, Part II: Network Traffic." *Transportation Research Part B* 29 (2): 79–93.

Di, X., H. X. Liu, and X. Ban. 2016. "Second Best Toll Pricing within the Framework of Bounded Rationality." *Transportation Research Part B* 83: 74–90.

Doan, K., and S. V. Ukkusuri. 2012. "On the Holding-Back Problem in the Cell Transmission Based Dynamic Traffic Assignment Models." *Transportation Research Part B* 46 (9): 1218–1238.

Doan, K., and S. V. Ukkusuri. 2015. "Dynamic System Optimal Model for Multi-OD Traffic Networks with an Advanced Spatial Queuing Model." *Transportation Research Part C* 51: 41–65.

Dong, J., and H. Mahmassani. 2013. "Improving network traffic flow reliability through dynamic anticipatory tolls." *Transportmetrica B* 1(3): 226–236.

Flötteröd, G., and J. Rohde. 2011. "Operational macroscopic modeling of complex urban road intersections." *Transportation Research Part B* 45 (6): 903-922.

Gao, Z., J. Wu, and H. Sun. 2005. "Solution Algorithm for the Bi-Level Discrete Network Design Problem." *Transportation Research Part B* 39 (6): 479–495.

Han, K., B. Piccoli, and T. L. Friesz. 2016. "Continuity of the Path Delay Operator for Dynamic Network Loading with Spillback." *Transportation Research Part B* 92: 211–233.

Han, K., B. Piccoli, and W. Y. Szeto. 2016. "Continuous-time link-based kinematic wave model: formulation, solution existence, and well-posedness." *Transportmetrica B* 4 (3): 187-222.

Han, L., D. Z. W. Wang, and C. Zhu. 2017. "The Discrete-Time Second-Best Dynamic Road Pricing Scheme." In *Transportation Research Procedia*, 23: 322–340.

Hearn, D. W., and M. V. Ramana. 1998. "Solving congestion toll pricing models." In *Equilibrium and Advanced Transportation Modeling*, 109-124.

Huang, D., Z. Liu, P. Liu, and J. Chen. 2016. "Optimal Transit Fare and Service Frequency of a Nonlinear Origin-Destination Based Fare Structure." *Transportation Research Part E* 96: 1–19.

Huang, H. J., and W. H. K. Lam. 2002. "Modeling and Solving the Dynamic User Equilibrium Route and Departure Time Choice Problem in Network with Queues." *Transportation Research Part B* 36 (3): 253–273.

Jabari, S. E. 2016. "Node modeling for congested urban road networks." *Transportation Research Part B* 91: 229-249.

Jeroslow, R. G. 1985. "The Polynomial Hierarchy and a Simple Model for Competitive Analysis." *Mathematical Programming* 32 (2): 146–164.

Jin, W. L. 2012. "Continuous kinematic wave models of merging traffic flow." *Transportation Research Part B* 44 (8): 1084-1103.
Jin, W. L., and H. M. Zhang. 2003. "On the distribution schemes for determining flows through a merge." *Transportation Research Part B* 37 (6): 521-540.
Karaboga, D. 2005. *An Idea Based on Honey Bee Swarm for Numerical Optimization*.
Kheirkhah, A., H. Navidi, and M. M. Bidgoli. 2016. "A Bi-Level Network Interdiction Model for Solving the Hazmat Routing Problem." *International Journal of Production Research* 54 (2): 459–471.
Lebacque, J. P., and M. M. Khoshyaran. 2005. "First-order macroscopic traffic flow models: Intersection modeling, network modeling." In the 16th International Symposium on Transportation and Traffic Theory, University of Maryland, College Park.
Lawphongpanich, S., and D. W. Hearn. 2004. "An MPEC approach to second-best toll pricing." *Mathematical Programming* 101 (1): 33-55.
Land Transport Authority of Singapore. 2013. *Land Transport Master Plan 2013*. Singapore Land Transport Authority.
Liu, L. N., and J. F. McDonald. 1999. "Economic Efficiency of Second-Best Congestion Pricing Schemes in Urban Highway Systems." *Transportation Research Part B* 33 (3): 157–188.
Liu, Z., Q. Meng, and S. Wang. 2014. "Variational Inequality Model for Cordon-Based Congestion Pricing under Side Constrained Stochastic User Equilibrium Conditions." *Transportmetrica A* 10 (8): 693–704.
Liu, Z., S. Wang, and Q. Meng. 2014. "Optimal Joint Distance and Time Toll for Cordon-Based Congestion Pricing." *Transportation Research Part B* 69: 81–97.
Liu, Z., S. Wang, B. Zhou, and Q. Cheng. 2017. "Robust Optimization of Distance-Based Tolls in a Network Considering Stochastic Day to Day Dynamics." *Transportation Research Part C* 79: 58–72.
Lo, H. K., and W. Y. Szeto. 2002. "A Cell-Based Variational Inequality Formulation of the Dynamic User Optimal Assignment Problem." *Transportation Research Part B* 36 (5): 421–443.
Long, J., H. J. Huang, Z. Gao, and W. Y. Szeto. 2013. "An Intersection-Movement-Based Dynamic User Optimal Route Choice Problem." *Operations Research* 61 (5): 1134–1147.
Long, J., W. Y. Szeto, Z. Gao, H. J. Huang, and Q. Shi. 2016. "The Nonlinear Equation System Approach to Solving Dynamic User Optimal Simultaneous Route and Departure Time Choice Problems." *Transportation Research Part B* 83: 179–206.
May, A. D., S. P. Shepherd, A. Sumalee, and A. Koh. 2008. "Design Tools for Road Pricing Cordons." In *Road Congestion Pricing in Europe: Implications for the United States,* 138. Northampton: Edward Elgar Publishing.
Meng, Q., Z. Liu, and S. Wang. 2012. "Optimal Distance Tolls under Congestion Pricing and Continuously Distributed Value of Time." *Transportation Research Part E* 48 (5): 937–957.
Ni, D., and J. D. Leonard II. 2005. "A simplified kinematic wave model at a merge bottleneck." *Applied Mathematical Modelling 29*(11): 1054-1072.
Rahmani, A., and S. A. MirHassani. 2015. "Lagrangean Relaxation-Based Algorithm for Bi-Level Problems." *Optimization Methods and Software* 30 (1): 1–14.

Rahmani, M., E. Jenelius, and H. N. Koutsopoulos. 2015. "Non-Parametric Estimation of Route Travel Time Distributions from Low-Frequency Floating Car Data." *Transportation Research Part C* 58: 343–362.

Ran, B., and D. Boyce. 1996. *Modeling Dynamic Transportation Networks*. *An Intelligent Transportation System Oriented Approach*. Heidelberg: Springer-Verlag.

Sumalee, A., and W. Xu. 2011. "First-Best Marginal Cost Toll for a Traffic Network with Stochastic Demand." *Transportation Research Part B* 45 (1): 41–59.

Szeto, W. Y., and H. K. Lo. 2004. "A Cell-Based Simultaneous Route and Departure Time Choice Model with Elastic Demand." *Transportation Research Part B* 38 (7): 593–612.

Szeto, W. Y., Y. Wu, and S. C. Ho. 2011. "An Artificial Bee Colony Algorithm for the Capacitated Vehicle Routing Problem." *European Journal of Operational Research* 215 (1): 126–135.

Tampère, C. M., R. Corthout, D. Cattrysse, and L. H. Immers. 2011. "A generic class of first order node models for dynamic macroscopic simulation of traffic flows." *Transportation Research Part B* 45 (1): 289-309.

Ukkusuri, S. V., L. Han, and K. Doan. 2012. "Dynamic User Equilibrium with a Path Based Cell Transmission Model for General Traffic Networks." *Transportation Research Part B* 46 (10): 1657–1684.

Verhoef, E. T. 2000. "Second-Best Congestion Pricing in General Networks" *Tinbergen Institute Discussion Paper TI 2000-084/3*, Amsterdam.

Verhoef, E. T. 2002. "Second-Best Congestion Pricing in General Networks. Heuristic Algorithms for Finding Second-Best Optimal Toll Levels and Toll Points." *Transportation Research Part B* 36 (8): 707–729.

Wie, B. W., and R. L. Tobin. 1998. "Dynamic Congestion Pricing Models for General Traffic Networks." *Transportation Research Part B* 32 (5): 313–327.

Xiong, C., X. Zhou, & L. Zhang. 2018. "AgBM-DTALite: An integrated modelling system of agent-based travel behaviour and transportation network dynamics." *Travel Behaviour and Society* 12: 141-150.

Yang, H., and H. J. Huang. 1998. "Principle of Marginal-Cost Pricing: How Does It Work in a General Road Network?" *Transportation Research Part A* 32 (1): 45–54.

Yang, H., and H. J. Huang. 2005. *Mathematical and Economic Theory of Road Pricing*. Elsevier.

Zhan, X., and S. V. Ukkusuri. 2017. "Multiclass, simultaneous route and departure time choice dynamic traffic assignment with an embedded spatial queuing model." *Transportmetrica B*, in press.

Zhang, X., H. M. Zhang, H. J. Huang, L. Sun, and T. Q. Tang. 2011. "Competitive, Cooperative and Stackelberg Congestion Pricing for Multiple Regions in Transportation Networks." *Transportmetrica* 7 (4): 297–320.

Zhou, X., and J. Taylor. 2014. "DTALite: A queue-based mesoscopic traffic simulator for fast model evaluation and calibration." *Cogent Engineering* 1 (1): 961345.