Home health care routing and scheduling problem with the consideration of outpatient services

Abstract

In China, family doctor contract service can provide not only the home health care service for the elderly or patients with mobility difficulties at their homes but also the outpatient service mainly for ordinary patients in the community care center. This paper presents a home health care routing and scheduling problem with the consideration of outpatient services. By considering the constraints about time windows, skill requirements, and working regulations, the problem is formulated as a mixed-integer nonlinear and convex programming model to minimize the total travel costs of the door-to-door service and the total waiting penalties of out-patients, and maximize the total benefit of patients' preference satisfaction. We adopt an outer-approximation method to obtain its global ε-optimal solutions for the small scale problem and develop a hybrid genetic algorithm to solve the large problem. A small instance is set up to analyze the problem properties and the performance of the outer-approximation method. The results of large scale examples show that the proposed hybrid genetic algorithm can provide high-quality solutions with short computing times.

Keywords: Home health care scheduling; Vehicle routing problem; Door-to-door service; Outpatient service; Patient preference satisfaction.

1. Introduction

Population aging is a global trend. In China, people older than 60 was 16.15 percent in 2015 and it is expected to rise to 17.17 percent by the end of 2020 (Intellectual Research Consulting Group, 2017). Moreover, in 2019, the World Health Organization predicted that, the number of people aged 60 and older would grow by 56%, from 962 million to 1.4 billion between 2017 and 2030, and would be more than 2.1 billion by 2050 (World Health Organization, 2019).

Home health care (HHC) is an industry that aims to assign doctors to serve patients at home and provide some essential services such as medical tests, wound care, psychological counseling and caring visits, etc. (Lanzarone et al., 2012; Liu et al., 2014). It is more sutiable for the elderly, patients with chronic diseases or mobility difficulties. With the population aging, the demand for medical resources is increasing, HHC service develops rapidly and becomes an efficient and professional industry to solve the pension burden in some developed countries such as France, Germany, Australia, etc. (Shi et al., 2017). However, in China, since HHC service is usually provided by the private HHC organization and the price is expensive, it develops slowly and only covers few patients in some big cities such as Shanghai, Beijing, etc. (Zhuo et al., 2015).

To better meet the demands of most patients for long-term and continuous health care service in China, family doctor contract service is developed by the government under the background of hierarchical diagnosis and treatment, and performed by the doctor team in community care center (Zhou, 2018). Doctors in the team not only provide HHC services at patients' homes (mainly for the elderly or patients with mobility difficulties), but also provide outpatient services for patients who need general medical treatment or large medical equipment at the community care center. For ease of description, we call these two service modes as door-to-door and outpatient services, respectively. Note that the patients who normally receive door-to-door services may also receive outpatient services in some days. For example, the elderly patients usually receive services at home, but they have to go to the community care center to receive outpatient service when they need large medical equipment for physical examination.

Compared with HHC service, family doctor contract service can serve more types of patients. Meanwhile, it focuses on establishing long-term and continuous care with patients, therefore, the doctor-patient matching is more important than that in the traditional outpatient service. This new service with two service modes can allocate medical resources more flexibly and its price is cheaper than the HHC service. Therefore, it is more suitable for China with a high degree of aging but a shortage of medical resources. In 2017, it has been signed by more than 30% people, the coverage rate of "key population" such as the elderly, patients with chronic diseases or mobility difficulties is as high as 60%, and it will strive to cover all people by the end of 2020 (Medical Reform Office of the State Council of China, 2016).

In family doctor contract service, the contracted patients are required to make a reservation for service mode (door-to-door or outpatient) according to their requirements before receiving service. Although these two modes of services may be performed by the same doctor team, their operations are quite different in practice. The door-to-door services require doctors to go to patients' homes to provide the service, while the outpatient services require patients to go to the community care center to receive the service. In the scheduling problem, the former requires managers to solve the home health care routing and scheduling problem (HHCRSP), which is generally modeled as an extension of the vehicle routing problem (VRP), while the latter is commonly formulated by using queuing theory. The relevant research for each mode is relatively rich, which is reviewed in Section 2. However, there is no research to consider both modes in scheduling problems simultaneously as in the situation in China. This research aims to fill the research gap.

Specifically, the contributions of our study are as follows.

First, we introduce and model a new HHCRSP with the consideration of outpatient services. Doctors are arranged for either the door-to-door or outpatient service. By considering constraints about time windows, workload, and skill requirements, our problem is to determine the service mode for each doctor, the doctor-patient matching in outpatient service, and the routes of doctors arranged for the door-to-door service. The objective is to minimize the total travel cost of the door-to-door service and the total waiting penalties of out-patients, and maximize the total benefit of patients' preference satisfaction. The problem is formulated as a mixed-integer nonlinear and convex programming model.

Second, we adopt an outer-approximation method to obtain the global ε-optimal solutions of the mixed-integer convex programming model. Considering that the decision variables in the model are integers and the tangent lines can only be generated at integer points, in the outer approximation method, we build a 0-1 integer programming model to find the optimal breakpoints among all the integer points.

Third, to solve the large problem, we propose a hybrid genetic algorithm (HGA), by embedding a tailored local search and a shake procedure, a newly designed individual representation, novel crossover and mutation operators, and a new initial population creation procedure into genetic algorithm (GA).

Finally, we analyze the sensitivity of parameters, problem properties, and the performance of the outer-approximation method, and illustrate the performance of HGA by solving the large problem.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes and formulates the proposed problem. Section 4 presents the outer approximation method for the formulation. Section 5 describes the proposed HGA. Section 6 presents the problem properties and the results of the performance of the proposed solution methods. Finally, Section 7 concludes the paper.

2. Literature review

The general HHCRSP is to establish routes for doctors to complete the door-to-door service, and it is an extension of VRP augmented by many unusual side-constraints that are specific to the HHC context (Cissé et al., 2017; Fikar & Hirsch, 2017), such as patient preference, skill requirements, etc. In this section, we focus on the latest trend of scheduling (and routing) problems and review 15 representative studies in the last five years.

Table 1 shows the most common objectives and constraints addressed in the literature. Table 2 summarizes objectives, constraints, and solution methods considered in the reviewed articles and our article. When an article deals with one of them, the symbol (✓) is marked on the corresponding cell.

Table 1

The classification of objectives and constraints in scheduling.

Objectives		Constraints	
Abbr.	Description	Abbr.	Description
TT	Time (travel, waiting, overtime etc.)	TW	Time windows
CC	Costs (travel, service, fixed, etc.)	SM	Skill requirements
WB	Workload balance	WR	Working regulations
PS	Preference satisfaction	SY	Synchronization
		UC	Uncertainty(stochastic travel time, service time)

Table 2

Objectives, constraints, and methodologies found in the reviewed articles and our article.

	Objectives				Constraints					
Article	TT	CC	WB	PS	TW	SM	WR	SY	UC	Solution method(s)
Braekers et al. (2016)		✓		✓	✓	✓	✓			Exact/metaheuristic (MDLS)
Decerle et al. (2018)	✓			✓	✓			✓		Metaheuristic (MA)
Du et al. (2017)		✓			✓	✓				Metaheuristic (GA)
Fikar & Hirsch (2015)	✓		✓		✓	✓	✓	✓		Matheuristic
Grenouilleau et al. (2019)				✓	✓	✓	✓	✓		Matheuristic
Hashemi Doulabi et al.(2020)		✓			✓			✓	✓	Exact (branch&cut)
Hiermann et al. (2015)	✓			✓	✓	✓				Metaheuristic (VNS/MA/SA/SS)
Liu et al. (2018)		✓			✓			✓		Metaheuristic (VNS)
Mısır et al. (2015)	✓			✓	✓	✓	✓			Heuristic
Mosquera et al. (2019)	✓	✓		✓	✓	✓	✓			Metaheuristic (VNS)
Nikzad et al. (2020)	✓	✓				✓	✓		✓	Matheuristic

Shi et al. (2019)		✓			✓				✓	Metaheuristic (TS/SA/VNS)
Xiao et al. (2018)		✓			✓	✓	✓	✓		Exact
Yalçındağ et al. (2016)	✓		✓				✓			Exact/metaheuristic (GA)
Yuan et al. (2015)		✓			✓	✓	✓		✓	Exact (B&P)
Our paper		✓		✓	✓	✓	✓			Exact/Metaheuristic (HGA)

B&P：Branch-and Price; (H)GA: (Hybrid) Genetic Algorithm; MA: Memetic Algorithm; MDLS: Multi-directional Local Search; SA: Simulated Annealing; SS: Scatter Search; TS: Tabu search; VNS: Variable Neighborhood Search.

As seen in Table 2, travel cost and travel time are the most common measures found in optimization objectives of HHCRSP. Due to they are closely related to the doctors’ working time, Braekers et al. (2016) further considered the overtime cost. In addition, some studies focused on maximizing patients’ or doctors’ preference satisfaction to improve service quality (Hiermann et al., 2015; Mısır et al., 2015; Braekers et al., 2016; Decerle et al., 2018; Grenouilleau et al., 2019; Mosquera et al., 2019). For example, Braekers et al. (2016) minimized patients’ convenience in terms of visit times and doctors, Mosquera et al. (2019) maximized patients’ total preference cost regarding doctors. On the other hand, most of the existing studies consider multiple objectives simultaneously, but the methodologies to handle multiple objectives vary. Most of these studies (e.g., Hiermann et al., 2015; Mısır et al., 2015; Yalçındağ et al., 2016; Decerle et al., 2018) used the weighted-sum approach to unify all objectives into a single objective. Mosquera et al. (2019) adopted a lexicographic ordering method to hand the multitude of objectives. Braekers et al. (2016) proposed a bi-objective optimization method to obtain a set of Pareto optimal solutions, which allow them to analyze the trade-off between costs and client inconvenience. In our paper, travel costs and patients’ preference satisfaction regarding skill and doctor-patient familiarity are considered for the door-to-door service, and the objectives are unified into a single objective by the weighted-sum approach.

In terms of constraints, the key characteristics of the HHCRSP such as time windows, skill requirements and working regulations are considered in most studies, while other factors (such as synchronization, uncertainty, etc.) are seldom considered. Meanwhile, the specific implementation of these constraints can be different in these reviewed articles. For example, the implemented time windows for patients to receive service can be generally divided into hard time windows (e.g., Hiermann et al., 2015; Liu et al., 2018; Mosquera et al., 2019) and soft time windows within certain range (e.g., Mısır et al., 2015; Yuan et al., 2015; Braekers et al., 2016; Decerle et al., 2018). For working regulations, most studies set a maximum working duration for doctors (e.g., Fikar & Hirsch, 2015; Braekers et al., 2016; Mosquera et al., 2019; Nikzad et al., 2020). Xiao et al. (2018) considered the flexible lunch break requirements, while Hiermann et al. (2015) set the priority working time windows for doctors. For skill requirements, most studies considered elastic matching (e.g., Fikar & Hirsch, 2015; Hiermann et al., 2015; Yuan et al., 2015; Du et al., 2017; Braekers et al., 2016; Mosquera et al., 2019); that is, doctors with a higher skill level are allowed to visit patients with lower skill level requirment to balance the overall distribution of doctors. The elastic skill matching may reduce travel related expenses, but it may also impact doctors' satisfaction if they are required to perform multiple visits at a lower qualification level. Therefore, Fikar & Hirsch (2015) further set a maximum downgrading level for each doctor. In this study, we consider hard time windows for patients to receive services, elastic matching of skill requirements, set a maximum downgrading level and a maximum working duration for doctors.

The HHCRSP problem is NP-hard because it is an extension of VRP. The solution method for this problem can be broadly classified into three categories: exact methods, heuristic-based methods (including metaheuristics and heuristic), and matheuristics. Although exact methods (e.g., Yuan et al., 2015; Hashemi Doulabi et al., 2020) can get optimal solutions, their computation time is heavily restricted by the problem size. Therefore, most researchers prefer to adopt metaheuristics/heuristics to obtain a good solution instead of an exact solution to address their (large-size) problems. As seen from Table 2, while Mısır et al. (2015) used a hyper-heuristic to solve their problem, the main used metaheuristics are classical, including GA (Yalçındağ et al., 2016; Du et al., 2017), variable neighborhood search (Hiermann et al., 2015; Liu et al., 2018; Mosquera et al. 2019; Shi et al., 2019), tabu search (Shi et al., 2019), multi-directional local search (Braekers et al., 2016), and simulated annealing (Hiermann et al., 2015; Shi et al., 2019). To combine the advantages of the exact method and the metaheuristic, matheuristics received the least attention in the last five years (Grenouilleau et al., 2019; Nikzad et al., 2020; Fikar & Hirsch., 2015); for example, Fikar & Hirsch (2015) generated problem clusters by incorporating set partitioning and linear programming techniques to optimize start time and enable synchronization. However, the matheuristics still have some limitations in problem size compared with heuristic-based methods. In our problem, we adopt the branch and cut method embedded in CPLEX for the small scale problems. On the other hand, considering GA has many advantages such as simple structure and high search efficiency, we develop a new hybrid GA by embedding a local search into the basic GA framework to solve the large problems.

As for the scheduling research about the outpatient service, there exist many methodologies and solution techniques to reduce costs and improve service quality (Ahmadi-Javid et al., 2017). Queuing theory is one of the most common methodologies since the models require fewer data and are simple to use. Moreover, queueing models can obtain some information about activities (waiting times, utilization rates, queue times, and lengths) and provide the reference for the decision-makers. M/M/1 and M/M/s are the two most popular queuing systems for the outpatient service (Lakshmi & Appa Iyer, 2013). In an outpatient service system with s doctors, each doctor is treated as a separate service station, the service can be viewed as s independent M/M/1 queuing sequence and each patient assigned to a doctor according to the appointment (e.g., Hopkins et al., 2008; Adeleke et al., 2009; Cochran & Broyles, 2010), while in M/M/s, the system assigns the arrived patients to doctors according to the station usage (e.g., Agnihothri & Taylor, 1991; Green, 2006; de Véricourt & Jennings, 2011). M/M/s is more suitable for the case that all doctors have the same service rate or all patients have the same selection opportunity for doctors. In our problem, patients have reservation preferences and specific skill requirements, and do not have the same selection opportunity for each doctor. It is more applicable to treat each doctor's service as an independent M/M/1 queuing system.

3. Problem description and formulation

3.1. Problem description

The HHCRSP with the consideration of the outpatient service can be described as follows.

Before receiving a service, the contracted patients are required to make a reservation for service mode according to their requirements. On each day each patient can only reserve one service mode, either outpatient or door-to-door service mode. Moreover, there is a fixed number of patients each day. Then, we can define $N = \{1,2,\dots,|N|\}$ as a set of patients, $N_1 = \{1,2,\dots,|N_1|\}$ as the set of patients who have reserved the door-to-door service (home healthcare patients) and $N_2 = \{|N_1| + 1, |N_1| + 2, \dots, |N|\}$ as the set of patients who have reserved the outpatient service (out-patients). Thus, $N = N_1 \cup N_2$.

A group of doctors, $K = \{1,2, \dots, |K|\}$, has an identical hard service time window at the care center, and is required to be arranged to serve both classes of patients. Each doctor is qualified with a skill level to show the serviceability or quality. Meanwhile, each patient has a skill level requirement, which limits that the service must be performed by a doctor with the same or a higher skill level. To avoid the overwork of high-skill qualified doctors and better balance the workload of doctors, we set a maximum allowable skill level deviation E ($E \geq 0$) between the doctor skill level and the patient skill level required. Each doctor can be only assigned to one service mode at most, and has the maximum continuous working duration R.

For the door-to-door service, we consider a complete directed graph $G = (V, A)$, where V is the set of nodes and A is the set of arcs. Nodes consist of patients' homes and the care center. Each patient's home is represented by a separate node in this graph. The care center is represented by both the starting depot and ending depot, i.e., nodes 0 and $|N| + 1$, where each doctor must start at and return to, respectively. Thus, $V = \{0, |N| + 1\} \cup N_1$ and $A = \{(i, j): i \in V\backslash\{|N| + 1\}, j \in V\backslash\{0\}, i \neq j\}$. Each patient has a service duration and a hard time window for starting a service, and each doctor must start a service within the hard time window.

For the outpatient service, each doctor's working time duration is R. Although the service starting time range for patients may be defined during reservation, the incoming stream of patients can also be considered as a Poisson process. We assume each doctor's service process can be modeled as an independent M/M/1 queuing system. For doctor k, the service time is exponentially distributed with a mean of $\frac{1}{u_k}$, where u_k is the service rate. When the number of patients assigned to doctor k is t_k, the incoming stream of patients is a Poisson process with a rate of $\lambda_k = \frac{t_k}{R}$. Thus, the total waiting time of patients served by doctor k can be expressed as $T_k = \frac{\lambda_k}{u_k(u_k - \lambda_k)} \cdot t_k = \frac{u_k \cdot R^2}{u_k \cdot R - t_k} - \frac{t_k}{u_k} - R$.

In HHC, service quality and doctor-patient familiarity are two important factors affecting patient preference. Patients prefer to the doctors who have a higher skill level or served them before (Cabana & Jee, 2004; Fan et al., 2005; Sanscorrales et al., 2006). We use p_{ik} to model doctor-patient familiarity; it equals 1(0) if doctor k has served (not served) has served patient i before. We use the service skill level deviation between a doctor and a patient to denote the service quality. The larger the deviation, the better the service quality.

The problem is to determine the doctor assignments to the two modes, the doctor-patient matching in the outpatient service, and the routes of doctors for the door-to-door service starting and ending at the care center. The objectives are to minimize the total travel cost of doctors for the door-to-door service and the total waiting penalties for the outpatient service, and maximize the total benefit of patients' preference satisfaction.

3.2. Problem formulation

Sets

N_1	Set of patients who need the door-to-door service, $\{1,2,\dots,\lvert N_1\rvert\}$.
N_2	Set of out-patients, $\{\lvert N_1\rvert+1,\lvert N_1\rvert+2,\dots,\lvert N_1\rvert+\lvert N_2\rvert\}$.
N	Set of all patients, $N_1 \cup N_2$.
K	Set of doctors, $\{1,2,\dots,\lvert K\rvert\}$.
V	Set of vertices, including the patients who need the door-to-door service and the care center, $N_1 \cup \{0, \lvert N\rvert+1\}$.
A	Set of arcs, $\{(i,j): i \in V\backslash\{\lvert N\rvert+1\}, j \in V\backslash\{0\}, i \neq j\}$.

Parameters

R	Maximum continuous working duration.
E	The maximum skill level deviation.
q_i	Required skill level of patient i.
Q_k	The qualified skill level of doctor k.
p_{ik}	1 if doctor k has served patient i before; 0 otherwise.
t_{ij}	Travel time from node i to j.
c_{ij}	Travel cost from node i to j.
$[e_i, l_i]$	The hard time window at node i for starting a service.
τ_i	Service duration at node i (and $\tau_0 = \tau_{\lvert N\rvert+1} = 0$).
u_k	The average service rate of doctor k in an outpatient service.
W_1	The unit waiting penalty of outpatients.

W_2 The unit benefit of patients' preference satisfaction.

Decision variables

y_{ik} 1 if doctor k is assigned to serve patient i; 0 otherwise.

x_{ijk} 1 if doctor k travels from node i to j; 0 otherwise.

Auxiliary variables

ta_{ik} The time when doctor k arrives at node i.

t_k The number of outpatients served by doctor k.

δ_k 1 if doctor k is assigned to a door-to-door service; 0 otherwise.

Formulation

[P1]

$$\text{Min } \sum_{k\in K}\sum_{(i,j)\in A}(c_{ij}\cdot x_{ijk}) + W_1\sum_{k\in K}\left(\frac{u_k\cdot R^2}{u_k\cdot R - t_k} - \frac{t_k}{u_k} - R\right) - W_2\sum_{k\in K}\sum_{i\in N} y_{ik}\cdot(p_{ik} + Q_k - q_i) \quad (1)$$

s. t.

Doctor assignment constraints:

$$0 \le t_k < u_k \cdot R \qquad \forall k \in K \qquad (2)$$

$$t_k = \sum_{i\in N_2} y_{ik} \qquad \forall k \in K \qquad (3)$$

$$y_{jk} = \sum_{i\in V\backslash\{|N|+1\}} x_{ijk} \qquad \forall j \in N_1, k \in K \qquad (4)$$

$$y_{ik} \le \delta_k \qquad \forall i \in N_1, k \in K \qquad (5)$$

$$y_{ik} \le 1 - \delta_k \qquad \forall i \in N_2, k \in K \qquad (6)$$

$$\sum_{k\in K} y_{ik} = 1 \qquad \forall i \in N \qquad (7)$$

Skill constraints:

$$q_i \le \sum_{k\in K} y_{ik}\cdot Q_k \qquad \forall i \in N \qquad (8)$$

$$q_i \ge \sum_{k\in K} y_{ik}\cdot Q_k - E \qquad \forall i \in N \qquad (9)$$

Working hours constraints:

$$ta_{|N|+1,k} - ta_{0k} \le R \qquad \forall k \in K \qquad (10)$$

Routing constraints:

$$\sum_{i\in V\backslash\{|N|+1\}} x_{ijk} - \sum_{i\in V\backslash\{0\}} x_{jik} = 0 \qquad \forall j \in N_1, k \in K \qquad (11)$$

$$\sum_{i\in N_1} x_{0ik} = \delta_k \qquad \forall k \in K \qquad (12)$$

$$\sum_{i\in N_1} x_{i,|N|+1,k} = \delta_k \qquad \forall k \in K \qquad (13)$$

Time window constraints:

$ta_{0k} \geq e_0\delta_k \qquad \forall k \in K \qquad (14)$

$ta_{|N|+1,k} \leq l_{|N|+1} \cdot \delta_k \qquad \forall k \in K \qquad (15)$

$ta_{jk} \geq max\{e_i, ta_{ik}\} + \tau_i + t_{ij} - M\left(1 - x_{ijk}\right) \qquad \forall i \in V\backslash\{|N| + 1\}, j \in V\backslash\{0\}, k \in K \qquad (16)$

$0 \leq ta_{ik} \leq l_i \cdot y_{ik} \qquad \forall i \in N_1, k \in K \qquad (17)$

Binary and definitional constraints:

$x_{ijk} \in \{0,1\} \qquad \forall(i,j) \in A, k \in K \qquad (18)$

$y_{ik} \in \{0,1\} \qquad \forall i \in N, k \in K \qquad (19)$

$\delta_k \in \{0,1\} \qquad \forall k \in K \qquad (20)$

Objective (1) is to minimize the total travel cost for the door-to-door service and the total waiting penalties of out-patients, and maximize the total benefit of patients' preference satisfaction simultaneously. Constraints (2) ensure that the number of outpatients served by a doctor cannot exceed the doctor's average service capacity in an outpatient service. Constraints (3) defines the number of outpatients served by doctor *k*. Constraints (4) show the relationship between y_{ik} and x_{ijk}. Constraints (5) and (6) ensure that each doctor can be only assigned to one service mode at most. Constraints (7) guarantee that every patient must be served by one doctor. Constraints (8) and (9) make sure that every patient can only be served by a doctor with the same or a higher skill level but not exceed the maximum skill level deviation *E*. Constraints (10) ensure that the working hours of doctors for the door-to-door service is less than the maximum working hours. Constraints (11) are the flow-conservation constraints for the doctors to serve home healthcare patients. They ensure that a doctor who visits a patient in a door-to-door service must eventually leave that patient. Constraints (12) and (13) ensure that each doctor assigned to door-to-door service must start from the care center and finally returns to the care center. Constraints (14) and (15) guarantee that the doctors are assigned to the door-to-door service within the time window of the care center. Constraints (16) ensure that the arrival times of doctors for the door-to-door service are correctly set. Constraints (17) guarantee that if a doctor visits one patient in the door-to-door service, the arrival time must not be later than the end of the patient's time window. Constraints (18) to (20) are binary constraints for decision variables.

4. An ε-global optimization method

[P1] presented in Section 3 is a mixed-integer nonlinear programming model with nonlinear constraints (16) and a nonlinear term $\frac{u_k \cdot R^2}{u_k \cdot R - t_k}$ in the objective function. In order to implement it by optimization solvers such as CPLEX, we linearized the [P1] model. Constraints (16) can be easily linearized to constraints (21) and (22) as follows:

$$ta_{jk} \geq e_i + \tau_i + t_{ij} - M(1 - x_{ijk}) \qquad \forall i \in V \backslash \{|N| + 1\}, j \in V \backslash \{0\}, k \in K \quad (21)$$

$$ta_{jk} \geq ta_{ik} + \tau_i + t_{ij} - M(1 - x_{ijk}) \qquad \forall i \in V \backslash \{|N| + 1\}, j \in V \backslash \{0\}, k \in K \quad (22)$$

On the other hand, $\frac{u_k \cdot R^2}{u_k \cdot R - t_k}$ is convex, because the value of the second derivative is positive at the domain $0 \leq t_k < u_k \cdot R$. The outer approximation method can be used to obtain global ε-optimal solutions by solving the relaxation problem.

4.1. The outer-approximation method

The outer-approximation method is one of the basic approaches to handle mixed-integer programming problems with nonlinear equality constraints or general model structure (Fletcher & Leyffer, 1994). It aims to generate a piecewise-linear function with as few pieces as possible within an approximation error of ε and has been widely used in various research fields, such as revenue management in liner shipping studies (e.g., Wang & Meng, 2012; Wang et al., 2015), electric vehicle fleet size and trip pricing problems (e.g., Xu et al., 2018), bike rebalancing problems (e.g., Li & Liu, 2021). Among them, Li & Liu (2021) formulated a 0-1 integer programming problem by minimizing the total number of breakpoints to find the optimal breakpoints among all integer points. This method shows great sensitivity to ε and can obtain fewer tangent lines compared with the existing methods by Wang & Meng (2012) and Xu et al. (2018) for a given ε.

In our study, we transform formulation [P1] by approximating the convex function $\frac{u_k \cdot R^2}{u_k \cdot R - t_k}$ in the objective function with a series of piecewise-linear functions by the outer approximation method of Li & Liu (2021). The procedure is as follows.

Step 1: Define the convex function $g(t_k) = \frac{u_k \cdot R^2}{u_k \cdot R - t_k}$ and set an approximation error ε.

Step 2: Calculate the slope $\beta(t_k) = g'(t_k) = -\frac{u_k \cdot R^2}{(u_k \cdot R - t_k)^2}$ and the intercept $\gamma(t_k) = g(t_k) - g'(t_k) \cdot t_k = \frac{u_k \cdot R^2 (u_k \cdot R - 2t_k)}{(u_k \cdot R - t_k)^2}$ of the tangent line of curve $g(t_k)$ at point t_k.

Step 3: For each $k \in K$, define $\hat{\varepsilon} = \frac{\varepsilon}{|K|}$ to allocate the total tolerance ε and the set of optimal breakpoints S_k is determined by solving formulation [B], where it is formulated using $\gamma(t_k), \beta(t_k)$, and the following notations.

Sets

C Set of points, indexed by $0, 1, \dots, \min\{\lceil u_k \cdot R - 1 \rceil, |N_2|\}$;

Decision variables

$\bar{x}_m$ = 1 if point m is selected as a breakpoint; = 0 otherwise;

$\bar{y}_{im}$ = 1 if the tangent line with breakpoint m is selected to calculate the difference between the actual value and the approximation value at point *i*; = 0 otherwise;

Formulation

[B]

$$\min \quad \sum_{m \in C} \bar{x}_m \tag{B1}$$

s. t.

$$g(t_k{}^{(i)}) - (\beta(t_k{}^{(m)}) \cdot i + \gamma(t_k{}^{(m)})) \cdot \bar{y}_{im} - M \cdot (1 - \bar{y}_{im}) \le \hat{\varepsilon} \qquad \forall i, m \in C \tag{B2}$$

$$\sum_{m \in C} \bar{y}_{im} \ge 1 \qquad \forall i \in C \tag{B3}$$

$$\bar{x}_m \ge \bar{y}_{im} \qquad \forall i, m \in C \tag{B4}$$

$$\bar{y}_{im} \in \{0,1\}, \quad \bar{x}_m \in \{0,1\} \qquad \forall i, m \in C \tag{B5}$$

The objective (B1) is to minimize the total number of breakpoints. Constraints (B2) ensure that the difference between the actual value and the approximation value is not larger than $\hat{\varepsilon}$, where M is a large positive number. Constraints (B3) indicate that at least one breakpoint is selected for each point *i*. Constraints (B4) show the relationship between $\bar{x}_m$ and $\bar{y}_{im}$. Constraints (B5) are binary constraints for the decision variables.

4.2. Relaxation problem (lower bound)

We introduce a new auxiliary variable A_k to replace $\frac{u_k \cdot R^2}{u_k \cdot R - t_k}$ in the objective function. By adding the linear relaxation constraints with breakpoint m ($\forall m \in S_k$), new formulation [P2] provides the lower bound of formulation [P1].

[P2]

$$min \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} (c_{ij} \cdot x_{ijk}) + W_1 \sum_{\boldsymbol{k} \in \boldsymbol{K}} (A_k - \frac{\boldsymbol{t_k}}{\boldsymbol{u_k}} - R) - W_2 \sum_{\boldsymbol{k} \in \boldsymbol{K}} \sum_{i \in C} y_{ik} \cdot (p_{ik} + Q_k - q_i) \tag{23}$$

Subject to constraints (2)-(15), (17)-(22) and

$$A_k \ge \beta(t_k{}^{(m)}) \cdot t_k + \gamma(t_k{}^{(m)}) \qquad \forall m \in S_k, k \in K \tag{24}$$

Let Opt and LB be the optimal objective value to [P1] and [P2], respectively. Thus, LB is a lower bound of Opt. Let $(x^*_{ijk}, y^*_{ik}, t^*_k, A^*_k)$ be the optimal solution to [P2]. An upper bound of Opt can be determined by

$$UB = \sum_{k \in K} \sum_{(i,j) \in A} (c_{ij} \cdot x^*_{ijk}) + W_1 \sum_{k \in K} (\frac{u_k \cdot R^2}{u_k \cdot R - t^*_k} - \frac{t^*_k}{u_k} - R) - W_2 \sum_{k \in K} \sum_{i \in N} y^*_{ik} \cdot (p_{ik} + Q_k - q_i) \tag{25}$$

According to the piecewise-linear approximation scheme, it follows that

$$LB \leq Opt \leq UB \leq LB + \varepsilon \tag{26}$$

5. HGA method

Although the outer approximation method can obtain the global ε-optimal solution, it is only restricted to the small-scale problem. In this section, we propose an HGA to solve the proposed problem over a large instance. This HGA uses a new individual representation and has novel developments in the key steps including initial population creation, crossover, and mutation, and embeds a tailored local search method and a shake procedure. The structure is presented in Algorithm 1.

Algorithm 1. HGA

```
Initialize population P(0), take the best individual in P(0) as the gobal best individual G_best and
initial best individual B(0).
Set iterkeep=0, iter=0.
While (iterkeep < T)
  Generate offspring population P(iter + 1) by selection, crossover, and mutation.
  Local search is applied to the best half individuals in P(iter + 1), and then copy the best individual
  in P(iter + 1) to B(iter + 1).
  If f(B(iter + 1)) ≥ f(G_best)
    iterkeep = iterkeep+1.
  Else
    G_best= B(iter + 1).
    iterkeep = 0.
  End if
  If (iterkeep =10)
    Apply the shake procedure to P(iter + 1).
  End if
  iter = iter + 1.
End while
Reture G_best.
```

In HGA, a maximum number of iterations between two improvements T is used as a stopping condition. To improve the search efficiency, a local search procedure is applied to the best half individuals after mutation. If the gobal best individual G_{best} is not improved in 10 consecutive iterations, the shake procedure is applied to maintain the diversity of the population.

5.1. Individual representation

Shi et al. (2017) proposed an encoding method by using a list table to represent an individual. In this method, every individual consists of several rows, and each row represents a route. We generalize the method to fit our problem by considering the doctor-patient assignment in the outpatient service and the routes of doctors for the door-to-door service simultaneously. In the generalized version, the first item of each row represents the doctor assigned to participate in a service; the second item represents whether the doctor is arranged for an outpatient service (denoted by 1) or a door-to-door service (denoted by 0), and the rest items are the patients to be served by the doctor in the first item. Note that the patient array order is also the sequence that patients are served if the doctor is arranged for the door-to-door service. While for the outpatient service, the patient array is unordered, and the arranged patients' services are subject to a first-come-first-served basis. Fig. 1 shows an example. There are totally 5 doctors. The first three doctors are arranged for the door-to-door service, and the last two doctors are arranged for the outpatient service. This representation can easily cover the doctor scheduling and need not decode or encode again during the algorithm implementation.

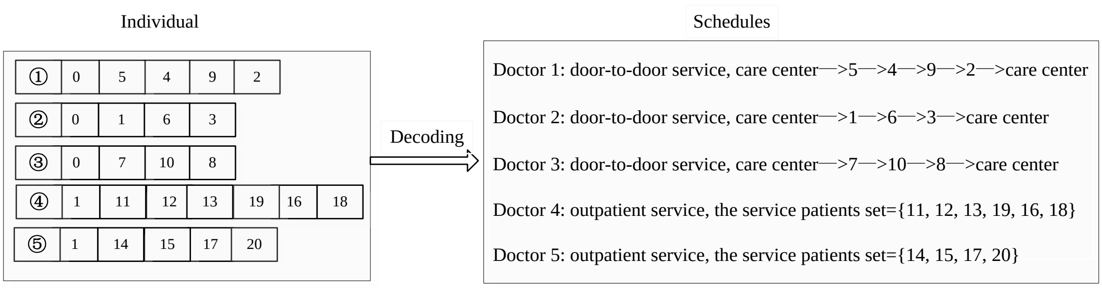

Fig. 1. The representation of an individual.

5.2. Initial population

To better maintain population diversity, we adopt two ways to generate the initial population. Half of the individuals are generated in a "random" way. We start a new row by randomly selecting an unassigned patient and a doctor available to serve this patient, and then keep inserting other patients randomly into this row without violating any constraints. If no available patients can be inserted into the current row, a new row is created. This process is repeated until all the patients are assigned. Another half of the individuals are generated in a "superior" way by adopting a tailored initialization method that is based on the insertion heuristic for vehicle routing problem with time window (VRPTW) developed by Solomon (1987). The main structure is presented in Algorithm 2.

Algorithm 2. The tailored initialization method

1: Set $N1_{not} = N_1, N2_{not} = N_2,\ N_{not} = N,\ K_{not} = K$.

2: **While** ($N_{not} \neq \emptyset$ and $K_{not} \neq \emptyset$)

Random select a patient $i \in N_{not}$.
If ($i \in N1_{not}$ && a doctor *k* who satisfies $q_i \le Q_k \le q_i + E$ exists)
Randomly select a available doctor k and construct a service route: $k \rightarrow 0 \rightarrow i$; delete i from $N1_{not}$ and N_{not}, and delete *k* from K_{not}.
While ($N1_{not} \neq \emptyset$)
Copy all patients in $N1_{not}$ who satisfy constraints to the set M.
If ($M \neq \emptyset$)
Select the best patient $u^* \in M$ and insert it into the current route by the criteria φ_1 and φ_2.
Delete u^* from both $N1_{not}$ and N_{not}.
Else
Break;
End if
End while
Else if ($i \in N2_{not}$ && a doctor *k* who satisfies $q_i \le Q_k \le q_i + E$ exists)
Randomly select a available doctor *k* and construct a service patient set *SP*={i}; delete i from $N2_{not}$ and N_{not}, and delete *k* from K_{not}.
Copy all patients in $N2_{not}$ whose skill requirements are within the range $[Q_k - E, Q_k]$ to *P*.
While ($P \neq \emptyset$ and |*SP*| < $u_k \cdot R - 1$)
Select the best insert patient $u^* \in P$ by the criterion φ_2 and add it to set *SP*.
Delete u^* from $N2_{not}$, N_{not} and *P*.
End while
Else
Break;
End if
End while
If ($N_{not} = \emptyset$)
A new individual constructed.
End if

In algorithm 2, N_{not} and K_{not} represent the unarranged patient and doctor sets, respectively. We put the unassigned patients of the door-to-door and outpatient services into the sets $N1_{not}$ and $N2_{not}$, respectively. An unassigned patient is randomly selected, and an available doctor who can serve this patient is selected to start a new row. If the doctor is arranged for a door-to-door service, we find the best feasible insertion position for each unassigned patient by satisfying all the constraints (criterion φ_1), and select the "best" patient with the maximal preference satisfaction (criterion φ_2), and then insert the "best" patient into the best insertion position. If the doctor is arranged for an outpatient service, we directly adopt the criterion φ_2 to select the "best" patient with the maximal preference satisfaction for this doctor, and add this patient to the tail of the patient array. If no available patient can be inserted into the current row, a new row is started unless all patients are assigned. The criteria φ_1 and φ_2 are defined in the following subsections.

5.2.1 Criterion φ_1

Suppose the current door-to-door service sequence is $(i_0, i_1, \ldots, i_m)$, where i_0 and i_m are the care center. For each unassigned patient u, the best feasible insertion position in the route can be found by the following formula without violating any constraint.

$$\varphi_1(i_{p^*-1}, u, i_{p^*})) = \min\left[\alpha_1\varphi_{11}\left(i_{p-1}, u, i_p\right) + \alpha_2\varphi_{12}\left(i_{p-1}, u, i_p\right)\right], p = 1, \ldots m \qquad (27)$$

where $\alpha_1 + \alpha_2 = 1, 0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1$. $\varphi_{11}\left(i_{p-1}, u, i_p\right) = c_{i_{p-1},u} + c_{u,i_p} - c_{i_{p-1},i_p}$. $\varphi_{12}\left(i_{p-1}, u, i_p\right)$ is the difference between the total waiting time of the doctor in the original route and that after inserting u in arc $\left(i_{p-1}, i_p\right)$. This criterion tries to find a position with the minimal weighted sum of travel cost and waiting time saving of the doctor arranged for the door-to-door service.

5.2.2 Criterion φ_2

The best patient u^* is selected by

$$\varphi_2(u^*, k) = \max[p_{uk} + Q_k - q_u] \qquad (28)$$

where $p_{uk} + Q_k - q_u$ is the preference satisfaction of patient u if u is assigned to be served by doctor k. This criterion tries to find the best patient with the maximal preference satisfaction for the doctor in service.

5.3. Parent selection

We adopt tournament selection, by randomly selecting two individuals and evaluating them. The better one participates in the crossover, mutation, and the local search improvement. Repeat until the number of selected individuals reaches the population size.

5.4. Crossover

We propose a new crossover operator for the door-to-door and outpatient services synchronously. The feasibility is checked during the crossover operation to guarantee that offsprings are feasible.

Randomly select two parents P1, P2 and a doctor k_1 for crossover operation. Offspring 1 is gerenated by P1 and the row for k_1 in P2, and inherits most of P1. Similarly, offspring 2 is gerenated by P2 and the row for k_1 in P1, and inherits most of P2. For easy description, we take the generation of offspring 1 as an example to describe the procedure. There are three cases in crossover operation.

Case 1: If k_1 is not arranged in P2, copy P1 to offspring 1.

Case 2: If k_1 is arranged in P2 but not arranged in P1, remove the patients served by doctor k_1 from P1, and then add the row for k_1 in P2 to P1. For example, in Fig. 2, the corresponding row of doctor 6 in P2 is R2, then remove patients 10, 9, 2 from P1, and add R2 to P1 to form offspring 1.

Case 3: If k_1 is arranged both in P1 and P2, there are four crossover strategies.

(a) If k_1 is arranged for an outpatient service in P1 and a door-to-door service in P2, copy P1 to offspring 1.

(b) If k_1 is arranged for a door-to-door service in both P1 and P2, and the corresponding rows are R1 and R2, respectively (See Fig. 3), then do the following.

Step 1. Delete the patients in R2 {10, 9, 2} from P1 (See Fig. 3(a)).

Step 2. Delete R1 from P1 and add R2 in P1 (See Fig. 3(b)).

Step 3. Reinsert the remaining patients in R1 {5, 4} into any rows of the door-to-door service in P1 (See Fig. 3(c)) without violating any constraints. If reinsertion fails, copy P1 to offspring 1.

(c) If k_1 is arranged for a door-to-door service in P1 and an outpatient service in P2, and the corresponding rows are R1 and S2, respectively (See Fig. 4), then do the following.

Step 1. Delete the patients in S2 {11, 15, 13} from P1 (See Fig. 4(a)).

Step 2. Delete R1 from P1 (See Fig. 4(b)).

Step 3. Reinsert the patients in R1 {1, 6, 3} into the rows of other door-to-door services in P2 and add S2 to P1 (See Fig. 4(c)).

(d) If k_1 is arranged for an outpatient service in both P1 and P2, and the corresponding rows are S1 and S2, respectively (See Fig. 5).

Step 1. Delete the patients in S2 {14, 13, 16} from P1 (Fig. 5(a)).

Step 2. Reinsert the patients in S2 {14, 13, 16} into S1 (Fig. 5(b)) randomly.

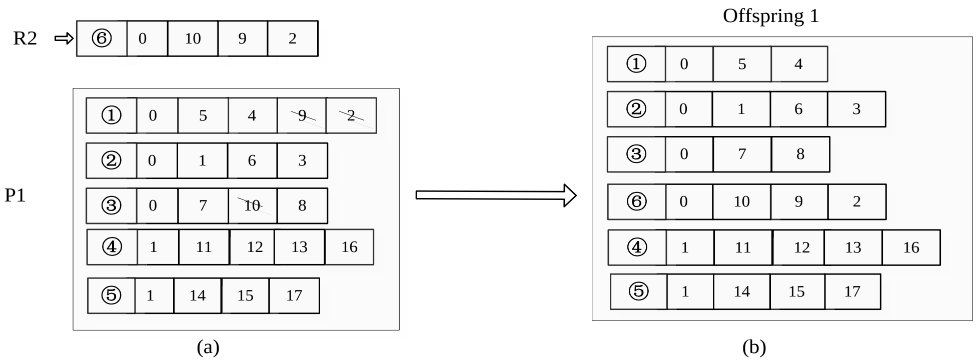

Fig. 2 The illustration in Case 2.

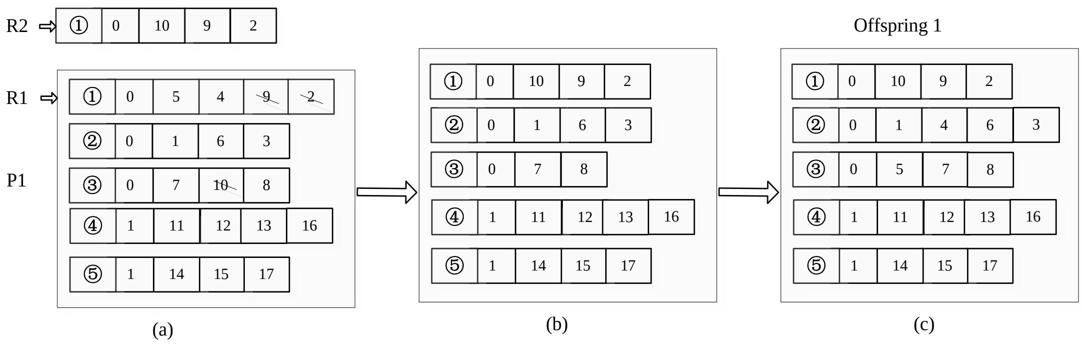

Fig. 3 The illustration in Case 3(b).

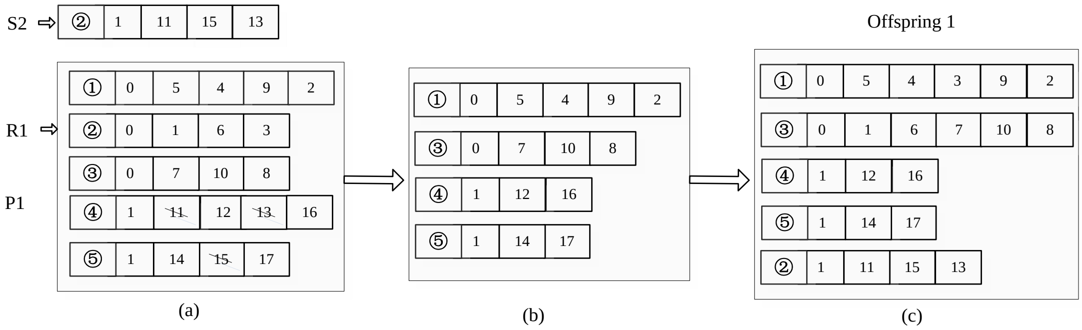

Fig. 4 The illustration in Case 3(c).

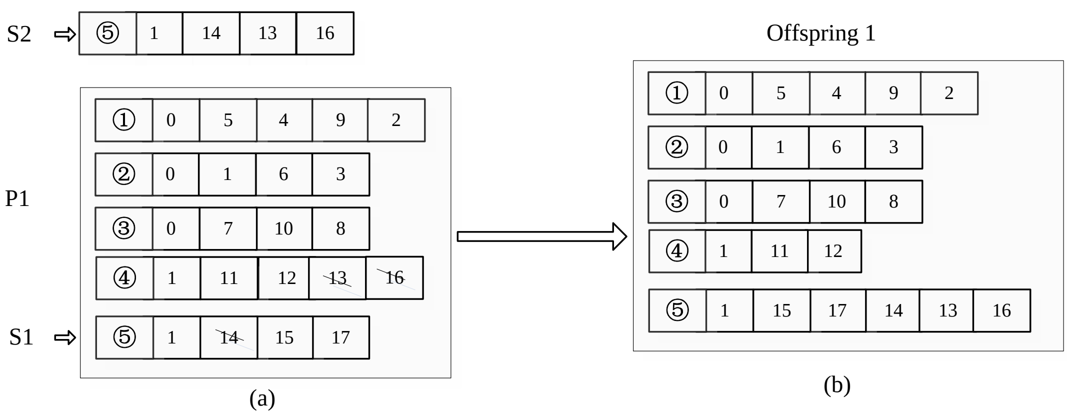

Fig. 5 The illustration in Case 3(d).

5.5. Mutation

Mutation operators are used to maintaining the diversity of the population from one generation to the next. Considering the problem characteristic, we design six different mutation operators for the doctor-patient assignment of the outpatient and door-to-door services and the routes of doctors for the door-to-door service. Operators (1)-(4) are specifically for the routes, while operators (5)-(6) are applicable to doctor-patient assignments of both the outpatient and door-to-door services:

(1) Intra two-node exchange: randomly select two different patients and then swap them.

(2) Intra 2-opt: randomly select two patients and then reverse the sub route between them.

(3) Intra single-node relocation: randomly delete one patient, then reinsert this patient into the route.

(4) Inter two-point swap: randomly select two patients from different routes, take the selected patients as cut points. Then cut each route into two pieces and swaps the two parts to form two new routes.

(5) Inter two-node exchange: randomly select two patients using the same service mode from different rows and then swap them.

(6) Inter single-node relocation: randomly delete one patient from a row and then insert this patient into another row.

For each time, we randomly choose one operator from (1)-(6) for the door-to-door service and one operator from (5)-(6) for the outpatient service. The new individual replaces the old one if it is feasible.

5.6. Local search

The local search operator is applied to the best half individuals in each generation to improve search efficiency. In the problem, we adopt nine local search moves. The first six moves are the same as the mutation operators described in Section 5.5, moves (7)-(8) are specifically for routes, move (9) is the doctor-patient matching operator.

(7) Multiple node relocation: randomly delete a set of patients DP with $|DP| \geq 2$. Then reinsert the patients in set DP into the individual.

(8) Swapping two subsequences: randomly select two independent subsequences from different routes, and swap them.

(9) Swapping two doctors: randomly select a route, and swap the related doctor with another doctor who has the same service skill.

The procedure is shown in Algorithm 3.

Algorithm 3. Local search	
1:	Put moves (1)-(8) into a move set.
2:	Select the first move from the move set.
3:	If the selected move belongs to moves (1)-(4) or (7)-(8), the move is carried on the rows for the door-to-door service. If the selected move belongs to moves (5)-(6), the move is carried on the rows for the outpatient service.
4	If the selected move can yield an improvement, adopt this move repeatedly until no improvement is yielded after 20 times, and then go to step 7.
5:	Remove the selected move from the set.
6:	If the set is empty, go to step 7. Otherwise, go to step 2.
7:	Move (9) repeatedly adopted until no improvement is yielded after T_{local} times.
8:	Replace the former individual with the new one.

5.7. Shake procedure

To prevent the algorithm from falling into a local optimum as the solution diversity decreases, the shake procedure is applied if the gobal best individual G_{best} is not improved in 10 consecutive iterations: Randomly select individuals from the population with a shake probability P_s, then each selected individual is perturbed by adopting one of the nine moves for the local search procedure in Section 5.6. The original individual will be replaced by the newly generated individual only if the new one is feasible.

6. Computational results

We conducted computational experiments to illustrate the problem properties and tested the efficiency of the proposed method. A small instance setting is shown in Section 6.1, which is used in Section 6.2 - Section 6.5. All exact solutions were obtained by adopting branch and cut method embedded in IBM-ILOG CPLEX 12.6.3. The HGA was coded in C#. All experiments were performed on a computer equipped with an Intel Core i7-2600U CPU 3.4GHz PC with a 16GB RAM.

6.1. Small example setting

In this small instance, there are 6 doctors, 10 out-patients, and 8 home healthcare patients. This size is good enough to illustrate the problem properties and to obtain the exact solution in a short time. The home healthcare center coordinate is (4, 13) and its time window is [0min, 540min]. Doctors' maximum continuous working duration is 480min. The values of the travel cost and travel time are set to equal the rounded integer of the distance between the two patients. The other doctors' and patients' information are shown in Table A.1~Table A.3.

6.2. Sensitivity analysis

We solved the example in Section 6.1 to optimality with different combinations of parameters (E, W_1, W_2), and analyzed the effect of these parameters on the operational strategy. For E, we consider three cases: no skill level deviation ($E = 0$), at most one skill-level deviation ($E = 1$) and at most two skill-level deviations ($E = 2$). For W_1 and W_2, we consider different parameter combinations. When $W_1 = 5$, W_2 varies from [1,50] with an increment of 0.1. When $W_2 = 5$, W_1 varies from [1,50] with an increment of 0.1. To guarantee that the resultant solution is globally optimal, we set the allocated tolerance $\hat{\varepsilon} = 0$. Table 3 gives the computation results of travel cost, waiting time of out-patients, and the overall preference satisfaction of patients with different parameter combinations. Fig. 6~Fig. 11 show the trends of travel cost, the total waiting penalties of out-patients, and the total benefit of patients' preference satisfaction with different parameter combinations.

Table 3

Computation results under different parameter combinations.

(a) E = 0

W_1	W_2	Travel cost ($)	Waiting time of out-patients (min)	Overall preference satisfaction of patients
1-50	5	663	31.0	11
5	1-50	663	31.0	11

(b) E = 1

W_1	W_2	Travel cost ($)	Waiting time of out-patients (min)	Overall preference satisfaction of patients
1-1.1	5	635	54.0	20
1.2-2.5	5	635	32.0	15
2.6-50	5	635	26.2	12
5	1-9.7	635	26.2	12
5	9.8-21.9	635	32.0	15
5	22-28	635	54.0	20
5	28.1-50	663	54.0	21

(c) E = 2

W_1	W_2	Travel cost ($)	Waiting time of out-patients (min)	Overall preference satisfaction of patients
1-1.1	5	589	43.6	20
1.2-1.3	5	589	39.5	19
1.4-2.4	5	589	32.0	17
2.5-50	5	589	26.2	14
5	1-9.7	589	26.2	14
5	9.8-18.6	589	32.0	17
5	18.7-20.8	589	39.5	19
5	20.9-41	589	43.6	20
5	41.1-47.9	589	51.8	21
5	48-50	589	71.0	23

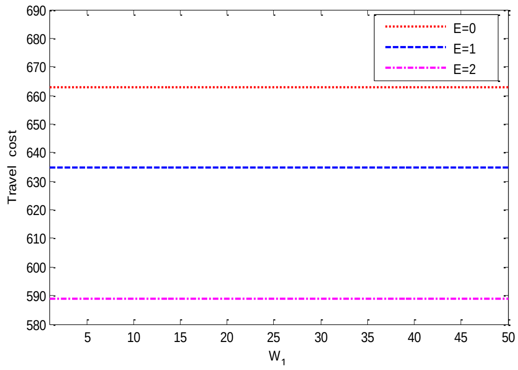

Fig. 6 The relation between travel cost and W_1 when $W_2 = 5$.

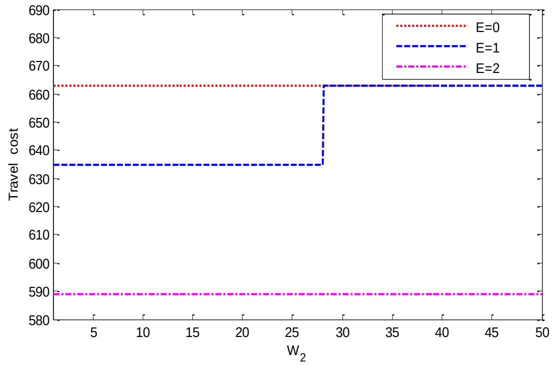

Fig. 7 The relation between travel cost and W_2 when $W_1 = 5$.

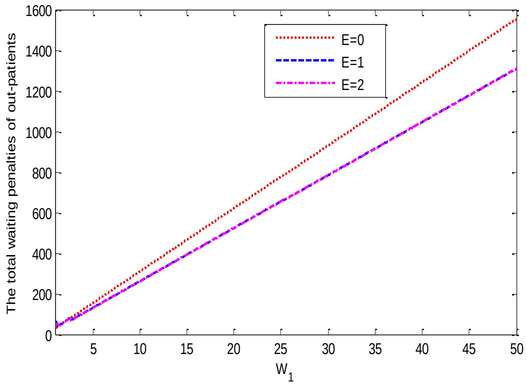

Fig. 8 The relation between the total waiting penalties of out-patients and W_1 when $W_2 = 5$.

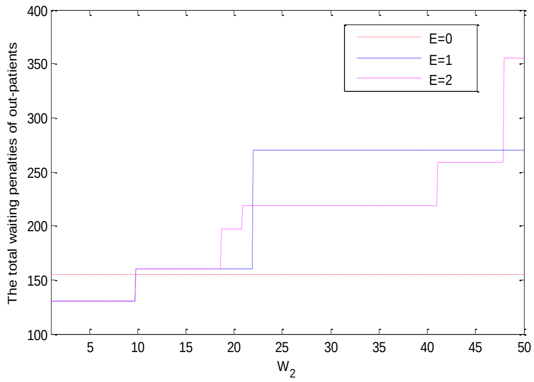

Fig. 9 The relation between the total waiting penalties of out-patients and W_2 when $W_1 = 5$.

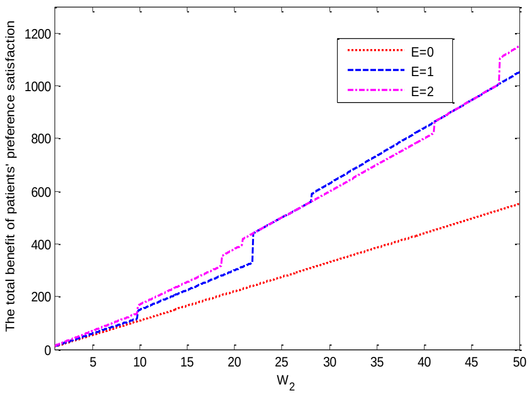

Fig. 10 The relation between the total benefit of patients' preference satisfaction and W_2 when $W_1 = 5$.

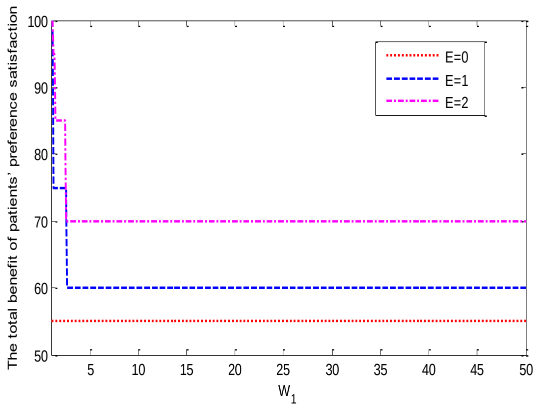

Fig. 11 The relation between the total benefit of patients' preference satisfaction and W_1 when $W_2 = 5$.

6.2.1. Sensitivity analysis of the maximum skill level deviation E

From Table 3, it is observed that for a smaller *E*, the travel cost, waiting time of out-patients and overall preference satisfaction of patients fluctuate less with the changes of W_1 and W_2. This is mainly due to the fact that a smaller *E* value leads to stricter doctor-patient skill matching and hence the resultant solutions have less variability. On the other hand, for specific values of W_1 and W_2, it is easily observed that the travel cost of the optimal route for the door-to-door service decreases with the growth of *E* in Fig. 6 and Fig. 7, because a larger *E* value leads to more feasible home healthcare patient matching to each doctor, which results in an optimal routing plan with a lower travel cost.

From Table 3, Fig. 8 and Fig. 9, we can see that the effect of *E* toward the waiting time and the total waiting penalties of out-patients is also obvious. When *E* = 0, strict doctor-patient skill matching and other constraints make the solution unchanged, which leads to the situation that the total waiting time of out-patients does not change with W_1 and W_2. With *E* increasing, doctor-patient matching and the number of out-patients served by each doctor may change, which may ultimately lead to the change in waiting time and total waiting penalties for the out-patient service.

As seen from Fig. 10 and Fig. 11, a larger *E* value leads to a higher total benefit of patients' preference satisfaction in most cases. We can explain it by the fact that a larger *E* value corresponds to more flexible matching opportunities and the patient prefers the doctor with a better skill, which eventually increases the overall preference satisfaction of patients as well. On the other hand, as there is a trade-off between the different parts of the objective, some special cases such as the overall preference satisfaction of patients for a larger E being smaller also exist (as seen from Fig. 11, when W_2 is between 28.1 and 41, the total benefit of patients' preference satisfaction when E = 2 is less than that when E = 1).

6.2.2. Sensitivity analysis of the unit waiting penalty of out-patients W_1

When $W_2 = 5$, for a specific E value, W_1 affects the waiting time of out-patients directly, and the variation of travel cost and the overall preference satisfaction of patients have some regularities. As expected, from Fig. 6, it is observed that the travel cost remains the same with W_1 increasing due to the fact that the constraints (such as time windows, skill matching, etc.) make the single route for the door-to-door service remain the same.

From Table 3 and Fig. 8, we can see that when E = 0, with the growth of W_1, the waiting time of out-patients remains 31.0 (min) due to strict doctor-patient matching, and the total waiting penalties increases linearly with a slope of 31.0 accordingly. When E = 1 and E = 2, the waiting time of out-patients decreases but the total waiting penalties of outpatients increase with W_1 increasing in range [1, 2.6] (E = 1) and [1, 2.5] (E = 2). When W_1 ≥2.6 (E = 1) and $W_1 \geq$ 2.5 (E = 2), the waiting time of out-patients remains 26.2 (min), and the total waiting penalties of outpatients increase linearly with a slope of 26.2 accordingly. This can be explained by the fact that the doctor-patient matching reaches the state with the least waiting time of out-patients, and does not change anymore.

With W_1 increasing, the contribution or the importance of the overall preference satisfaction of patients to the objective value decreases. As a result, when E = 1 and E = 2, the overall preference satisfaction of patients decreases promptly and then constantly equals 12 (E = 1) or 14 (E = 2) when W_1 ≥ 2.6 or $W_1 \geq$ 2.5 as shown in Table 3. It can be explained by the fact that the doctor-patient matching changes to reduce the overall preference satisfaction of patients so as to reduce the waiting time of outpatients if possible as W_1 increases. However, when E = 0, the overall preference satisfaction of patients equals 11 and remains the same with W_1 increasing because for the strict skill matching, doctor-patient matching is difficult to change.

6.2.3. Sensitivity analysis of the unit benefit of patients' preference satisfaction W_2

From Table 3 and Fig. 7, it is observed that W_2 affects the overall preference satisfaction of patients directly for specific values of W_1 and E. However, the travel cost seldom changes with the increase of W_2. Only when E = 1, W_1 = 5 and W_2 increases from 28 to 28.1, the travel cost changes from 635($) to 663($). This is because doctor-patient matching changes to increase the overall preference satisfaction of patients, which results in a long travel route for doctors who involve in the door-to-door service.

From Table 3 and Fig. 11, we can see that, with W_2 increasing, the overall preference satisfaction of patients equals 11 when E = 0 due to the strict doctor-patient matching, and the total benefit of patients' preference satisfaction increases linearly with a slope of 11 accordingly. When E = 1 and E = 2, the overall preference satisfaction of patients increases with the growth of W_2 which leads to the increasing trends in the total benefit of patients' preference satisfaction. In contrast, with W_2 increasing, the waiting time of out-patients remains the same when E = 0 but increases in a stepwise manner when E = 1 and E = 2 as shown in Table 3.

6.3. The trade-off between the outpatient and door-to-door services

In this section, we analyze the trade-off of the scheduling between the outpatient and door-to-door services. For ease of description, we split the objective function in [P2] into f_1 and f_2, which represent the performance measure for the door-to-door service (the difference between travel cost and the total benefit of preference satisfaction of home healthcare patients) and the outpatient service (the difference between the total waiting penalties of out-patients and the total benefit of preference satisfaction of out-patients) respectively, shown as follows.

$$f_1 = \sum_{k\in K} \sum_{(i,j)\in A} (c_{ij} \cdot x_{ijk}) - W_2 \sum_{k\in K} \sum_{i\in N_1} y_{ik} \cdot (p_{ik} + Q_k - q_i) \tag{29}$$

$$f_2 = W_1 \sum_{k\in K} (A_k - \frac{t_k}{u_k} - R) - W_2 \sum_{k\in K} \sum_{i\in N_2} y_{ik} \cdot (p_{ik} + Q_k - q_i) \tag{30}$$

We introduce a new objective function $f_3 = f_1 + W_3 \cdot f_2$, where W_3 is used to show the ratio of the two objectives. We also adopt the instance in Section 6.1 to minimize f_3, and set $W_1 = 5, W_2 = 5$ and E = 1. W_3 varies from [0,2] with an increment of 0.01. Table 4 shows the computation results of f_1, f_2, travel cost, the total waiting penalties of out-patient, the total benefit of preference satisfaction of all patients, home healthcare patients, and out-patients, and the doctors arranged in the outpatient service with the corresponding number of outpatients served shown in bracket regarding the different values of W_3.

Table 4

Computation results regarding different W_3.

W_3	f_1	f_2	Travel cost ($)	Total waiting penalties of outpatients ($)	The total benefit of preference satisfaction of all patients ($)	The total benefit of preference satisfaction of home healthcare patients ($)	The total benefit of preference satisfaction of outpatients ($)	The doctors arranged in the outpatient service
0	585	263.43	635	303.43	90	50	40	1 (5), 3 (5)
0.01-0.13	585	219.83	635	269.83	100	50	50	3 (2), 6 (3), 1 (5)
0.14-0.55	600	110.46	635	135.46	60	35	25	1 (2), 5 (3), 3 (5)
0.56-2.00	605	100.75	635	130.75	60	30	30	1 (2), 5 (3), 6 (5)

When W_3 = 0, the outpatient service is not considered in f_3. As a result, f_1 reaches its minimum value of 585 and f_2 equals a maximum value of 263.43. Since travel cost is determined by the constraints such as time window, skill requirements, and working regulation, it does not change with W_3. However, the total benefit of patients' preference satisfaction of home healthcare patients obtains a maximum value of 50 when W_3 = 0. It is because most of the doctors with higher skill levels or more familiar to home healthcare patients are arranged to the door-to-door service. In this case, only two doctors (doctors 1 and 3) are arranged to the outpatient service, and the total waiting penalties of out-patients reaches the maximum value.

When W_3 > 0, the contribution of f_2 to f_3 increases and that of f_1 to f_3 decreases with W_3 increasing. Minimizing f_3 results in f_1 increasing and f_2 decreasing as W_3 is increasing. In f_1, since travel cost remains the same, the total benefit of preference satisfaction of home healthcare patients decreases with W_3 increasing. In f_2, the total waiting penalties of out-patients decrease with W_3 increasing, although the schedule may not be favorable to the out-patients (the total benefit of preference satisfaction of out-patients decreases, e.g., the value of the total benefit of preference satisfaction of out-patients when W_3 in an interval of [0.01,0.13] is larger than that when W_3 in interval [0.14, 2]). Another observation is that when W_3 changes from any value in the interval [0.14, 0.55] to any value in [0.56, 2.00], the service modes of doctors 3 and 6 arranged are exchanged, and the total waiting penalties of out-patient decreases. Combining with the fact that the service rate of doctor 3 is lower than that of doctor 6, we can conclude that arranging the doctor with higher service rates to the outpatient service can reduce the total waiting penalties of out-patients.

6.4. Effect of different allocated tolerances $\hat{\varepsilon}$ toward the computational results

To test the effect of the allocated tolerance $\hat{\varepsilon}$ in the ε-global optimization method on the computational results of the number of tangent lines, the optimal objective value in [P2], the computation time, and the relative gap regarding the different values of $\hat{\varepsilon}$, we also adopt the example described in Section 6.1. We set $W_1 = 5$, $W_2 = 5$, $\hat{\varepsilon}$ varies from [0,11] with an increment of 1. Table 5 shows the computational results when E = 1.

Table 5

Computation results regarding different $\hat{\varepsilon}$.

$\hat{\varepsilon}$	Number of tangent lines	Optimal objective value	CPU time(s)	Gap%	$\hat{\varepsilon}$	Number of tangent lines	Optimal objective value	CPU time(s)	Gap%
0	66	705.755	16.082	0	6	28	617.148	15.754	12.555
1	52	701.931	15.896	0.542	7	24	606.462	16.112	14.069
2	43	689.382	15.348	2.320	8	24	616.475	15.946	12.650
3	39	668.378	15.387	5.296	9	23	602.313	16.250	14.657
4	31	638.771	15.801	9.491	10	21	599.853	16.096	15.006
5	30	642.416	15.584	8.975	11	21	592.474	16.022	16.051

From Table 5, it is observed that with $\hat{\varepsilon}$ increasing, the number of tangent lines decreases and the problem is more relaxed, which leads to a lower optimal objective value in most cases. However, there are still some special cases for some adjacent $\hat{\varepsilon}$. This can be explained by the fact that although the number of tangent lines decreases or remains the same with $\hat{\varepsilon}$ increasing, the feasible region made by tangent lines may change slightly. Therefore, it is possible that a larger value of $\hat{\varepsilon}$ can obtain a larger optimal objective. For example, the optimal objective value when $\hat{\varepsilon}$ = 5 (8) is larger than that when $\hat{\varepsilon}$ = 4 (7). When $\hat{\varepsilon}$ increases by 2 or more, the feasible region made by tangent lines enlarges. In this case, the optimal objective value with larger $\hat{\varepsilon}$ is smaller. The relative gap (= (global optimal objective value-optimal objective value)/global optimal objective value) has a similar variation to the optimal objective value and the gap is as high as 16.051% when $\hat{\varepsilon}$ = 11. In term of computation time, there is no obvious change with different $\hat{\varepsilon}$. This is because the number of tangents that can be selected is 66 in our instance, which is quite small for the outer-approximation method.

6.5. Managerial implications

To highlight the significance of considering the outpatient services in HHC, we construct four different scenarios based on the example described in Section 6.1 respectively: (1) Scenario 1: changing the service mode of 10 outpatients to door-to-door service; (2) Scenario

2: changing the service mode of 10 outpatients to door-to-door service, and adding 3 new home healthcare patients; (3) Scenario 3: adding 3 new home healthcare patients; (4) Scenario 4: adding 3 new outpatients. Table A.4 provides the information about the relevant patients in the four scenarios, in which patient 9 to 18 are the original outpatients and patients 19 to 21 are newly added patients. We set $W_1 = 5, W_2 = 5$ and E = 1. Table 6 shows the travel cost, the total waiting penalties of out-patient, the total benefit of preference satisfaction of all patients, the total cost (the objective value) and the average cost of patients (= the objective value/the total number of patients) in each scenario.

Table 6

The computation results of each instance.

Scenarios	The number of home healthcare patients	The number of outpatients	Travel cost ($)	Total waiting penalties of outpatients ($)	The total benefit of preference satisfaction of all patients ($)	The total cost ($)	The average cost of patients ($)
Original example	8	10	635	130.75	60	705.75	39.21
Scenario 1	18	0	1334	0	100	1234	68.56
Scenario 2	21	0	—	—	—	—	—
Scenario 3	11	10	707	290.91	105	892.91	41.52
Scenario 4	8	13	635	245.76	70	810.76	38.61

In Scenario 1, without considering outpatient service, we can see that the values of the travel cost, objective value and the average cost of patients are larger than those in the original example. It indicates that the combination of door-to-door service with outpatient service can reduce the total cost compared with HHC service. Meanwhile, the total benefit of preference satisfaction of all patients in Scenario 1 is also larger than that in the original example. It can be explained by the fact that in HHC service, the outpatients' waiting penalties is not incorporated into the objective, and doctors can match the patients better as for the skill and the care continuity.

In Scenario 2, when adding 3 new home healthcare patients in Scenario 1, we can see that 6 doctors cannot complete the door-to-door services of 21 home healthcare patients. However, by considering outpatient services in HHC service, the doctors can complete the services of 21 patients despite that the 3 additional added patients are home healthcare patients (Scenario 3) or outpatients (Scenario 4). The integration of the two service modes can help doctors serve more patients. As expected, there are 21 patients in Scenario 3 and 4, with a larger proportion of the outpatients (Scenario 4), the objective value, the average cost of patients and the total waiting penalties of outpatients are lower than those with a smaller proportion of the outpatients (Scenario 3). Specially, Scenario 4 has more outpatients but lower total waiting penalties, this is mainly because that the schedule changes and more doctors with higher service rates are assigned to outpatient services.

Overall, compared with HHC service, the combination of door-to-door and outpatient services can make the doctors allocation more flexible. It can serve more patients with lower operating costs. As the primary care of hierarchical diagnosis and treatment, the combination of door-to-door and outpatient services can play a positive role in balancing the medical resources in the community care center. It is also suitable for the countries with a shortage of medical resources but a high aging degree.

6.6. Effectiveness of the hybrid genetic algorithm

6.6.1. The construction method of instances

Since no benchmark exists for the problem, we generated instances based on Solomon's VRPTW benchmark (Solomon, 1987) and combined our problem characteristics. The details are described as follows.

Solomon's instances are classified into three classes that differ by the geographical distribution of customers, which are clustered in the C type, randomly located in the R type, and semi-clustered in the RC type. For all instances in the C, R, and RC types, we consider the first $|N_1|$+1 customers as care center and home healthcare patients. To simplify the problem, the travel cost c_{ij} and travel time t_{ij} are equal to the rounded integer of the distance between node i and node j. The service duration of each home healthcare patient τ_i is uniformly distributed in [30min, 90min]. The depot (care center) has a time window [0min, 540min]. The center and width of the time window for home healthcare patient *i* are uniformly distributed in $[e_0 + t_{0i}, l_0 - t_{i0} - \tau_i]$ and [60min, 120min] respectively. All time-related parameters are rounded into integers. For all patients, the skill requirements are divided into three levels: 1, 2, 3, and the classes of patients contribute to 50%, 30%, and 20% of the total, respectively.

For each doctor, the maximum continuous working duration R = 480 min. Doctors' skill qualifications are also divided into three levels: 1, 2, 3, and each class of doctors occupies 1/3 of the total number of doctors. The average service rate u_k is uniformly distributed in $[\frac{1}{30}, \frac{1}{10}]$. If a doctor's skill qualification is not less than one patient's skill requirement, we randomly generated 1 or 0 to indicate whether the doctor has served this patient before or not; otherwise, we only use 0 to indicate that the doctor has not served this patient before.

6.6.2 Parameter tuning

To maximize the performance of HGA, the numerical parameters of HGA listed in Table 7 were tuned with the heuristic algorithm EVOCA (Riff & Montero, 2013), which is simple and can realize the automation of the algorithm design and parameter setting process. EVOCA requires as few parameters as possible and provides an easy setting for the definition of the input data. In our parameter tuning processes, HGA was regarded as the target algorithm, 8 instances with different sizes in Section 6.6.3 were randomly selected as the training set. The quality measure used for assessing the parameter configurations of HGA was defined by the negative number of obtained objective value of each instance. If more than one parameter configurations were able to obtain the optimal, the evaluation value was equal to the negative number of the average objective value. The training stopping condition for EVOCA was set to 10000 iterations in each experiment.

Table 7

Numerical parameter list

Name	Symbol	Type	Range	Precision
Crossover probability	P_c	Real	[0.8, 1.0]	0.01
Mutation probability	P_m	Real	[0.0, 0.1]	0.01
Shake probability	P_s	Real	[0.5, 1.0]	0.01
Population size	NP	Integer	[30, 100]	1
Maximum number of iterations between two improvements T	T	Integer	[300, 400]	1

We ran EVOCA 5 times with different seeds. Table 8 gives the parameter calibrations obained by adopting EVOCA with different seeds. From Table 8, we can see that the value of P_c (= 1) in configuration 2 is the same as that in configuration 1 and 3, and P_m (= 0.1) in configuration 2 is the same as that in configuration 5. Configuration 2 integrates the good features of parameters in the five obtained configurations. Moreover, configuration 2 with larger T can help the HGA obtain better solution compared with the other configurations.

Therefore, in the subsequent experiments, the HGA adopts configuration 2 as the actual parameter values.

Table 8

The parameter calibrations using different seeds.

Configuration	#1	#2	#3	#4	#5
P_c	1.0	1.0	1.0	0.8	0.97
P_m	0.07	0.1	0.06	0.03	0.1
P_s	0.82	0.76	0.83	0.87	0.88
NP	83	77	93	95	62
T	330	385	356	380	369

6.6.3. The performance of HGA

To illustrate the performance of HGA, we randomly generated 24 instances with different sizes according to the method in Section 6.6.1, and compare the results obtained by HGA and cut method embedded in CPLEX for the relaxed model [P2]. We set the maximum skill level deviation E = 1, the unit waiting penalty in the outpatient service W_1 = 5, the unit benefit of patients' preference satisfaction W_2 = 5, and the allocated tolerance $\hat{\varepsilon}$ = 0. Table 9 shows the running times (CPU) in seconds, the upper bounds (UBs), the lower bounds (LBs), and the gaps obtained by CPLEX. CPLEX was terminated after an optimal solution was found or 6 h (21600s) limit was reached. Table 9 gives the average and worse objective values obtained by HGA in 10 runs, as well as the gaps (%) representing the deviation of the average and worse objective values from the LB. It also shows the average running times (CPU) in seconds and the standard deviation of gaps obtained by HGA.

Table 9

Comparison of the performance of the exact method and the HGA.

Instance				Exact method				HGA					
Type	$\lvert K\rvert$	$\lvert N_1\rvert$	$\lvert N_2\rvert$	Obj.			CPU (s)	Ave Obj.		Worse Obj.		CPU (s)	Standard
				UB	LB	Gap%		Value	Gap%	Value	Gap%		deviation of gaps
R1	6	10	30	826.40	826.40	0.00	13.26	826.40	0.00	826.40	0.00	0.74	0.00
C1	6	10	30	1848.16	1848.16	0.00	17.78	1848.16	0.00	1848.16	0.00	0.81	0.00
RC1	6	10	30	737.67	737.67	0.00	13.99	739.32	0.22	743.19	0.74	0.61	0.00
R1	8	20	40	1705.27	1705.27	0.00	74.25	1710.12	0.28	1720.39	0.88	1.38	0.00
C1	8	20	40	1308.65	1308.65	0.00	92.34	1311.64	0.23	1362.37	3.94	1.19	0.01
RC1	8	20	40	2297.13	2297.13	0.00	128.27	2303.13	0.26	2405.57	4.51	1.42	0.00
R1	10	30	60	3862.05	3862.05	0.00	7008.35	3889.25	0.70	3937.29	1.91	3.84	0.01
C1	10	30	60	2821.59	1871.83	33.66	21600.00	1927.48	2.89	1963.88	4.69	3.86	0.02

RC1	10	30	60	3279.84	3279.84	0.00	1438.58	3334.27	1.63	3395.27	3.40	3.88	0.00
R1	12	40	80	6468.44	5264.95	18.61	21600	5476.86	3.87	5662.08	7.01	10.06	0.02
C1	12	40	80	4421.23	3185.80	27.94	21600	3302.65	3.54	3400.18	6.30	12.88	0.04
RC1	12	40	80	---	25428.97	---	21600	27030.07	5.92	27821.24	8.60	16.65	0.02
R1	15	50	100	---	4096.74	---	21600	4320.24	5.17	4522.87	9.42	41.47	0.03
C1	15	50	100	---	2417.39	---	21600	2558.02	5.50	2669.44	9.44	13.55	0.03
RC1	15	50	100	9864.44	5561.21	43.62	21600	5943.28	6.43	6288.28	11.56	62.46	0.05
R1	18	60	120	---	5554.83	---	21600	5941.47	6.51	6226.83	10.79	123.40	0.04
C1	18	60	120	---	3697.18	---	21600	3928.52	5.89	4275.28	13.52	29.35	0.04
RC1	18	60	120	---	9105.82	---	21600	9972.42	8.69	10317.02	11.74	236.62	0.05
R1	20	75	150	---	12800.62	---	21600	13972.34	8.39	14678.62	12.79	491.53	0.06
C1	20	75	150	---	6907.85	---	21600	7582.84	8.90	7965.85	13.28	201.54	0.05
RC1	20	75	150	---	9594.96	---	21600	10606.91	9.54	10856.48	11.62	332.43	0.04
R1	30	100	200	---	---	---	21600	15782.38	---	16421.56	---	526.18	---
C1	30	100	200	---	---	---	21600	13390.88	---	14652.81	---	528.44	---
RC1	30	100	200	---	---	---	21600	18801.28	---	19539.33	---	597.90	---

In Table 9, we can see that when $|K| \leq 10$, CPLEX can obtain the optimal solution except for the eighth instance (C1 type and $|K| = 10, |N_1| = 30,\ |N_2| = 60$) within 21600s. In contrast, the HGA takes less time and gets a nearly optimal solution with a average objective gap of less than 2.89% and a worse objective gap of less than 4.69%. With the problem size increasing, it is hard to find the optimal solution and can only obtain feasible solutions or LBs within 21600s by CPLEX. When $|K|$ increased to 15, only the fifth instance (RC1 type and $|K| = 15, |N_1| = 50,\ |N_2| = 100$) can obtain a feasible solution by CPLEX and the gap is as high as 43.62%. However, the HGA can obtain a better feasible solution with smaller gaps of the average and worse objective values, and the corresponding runtime is less than 62.46s. When $18 \leq |K| \leq 20$, only the LBs are obtained by CPLEX but the HGA can obtain a better solution with a average objective gap of less than 9.54% and a worse objective gap of less than 13.52%. When $|\mathrm{K}| \geq 30$, CPLEX cannot solve the problem within 21600s, while the HGA can obtain a feasible solution less than 600s. Moreover, although the standard deviation of the gaps are increases with the problem size on the whole, it is never greater than 0.06.

Overall, the above experiments illustrate the limitation of the exact method with CPLEX. HGA can obtain a better solution in a shorter time and has a stronger stability, which has the advantage of solving this problem.

7. Conclusions

By taking the family doctor contract services in China as a background, this paper presents a home health care routing and scheduling problem with the consideration of the outpatient service. The problem is formulated as a mixed-integer nonlinear and convex programming model to minimize the total travel cost, the total waiting penalties of out-patients, and maximize the total benefit of patients' preference satisfaction. We used an outer-approximation method to obtain its global ε-optimal solutions and developed HGA to solve the large size problem. To analyze the sensitivity of parameters, problem properties, and the performance of the outer-approximation method, a small instance was set up, and the results demonstrate the following: First, the solution is affected by the maximum skill level deviation E, the unit waiting penalty of outpatients W_1 and the unit benefit of patients' preference satisfaction W_2. With E increasing, the influence of W_1 and W_2 on solution will be greater, and this could make operation management more flexible. Second, with the weight for each service mode increasing, the resultant scheme will arrange doctors with higher skill levels or more familiar to patients to the corresponding mode. Managers can balance the two modes by adjusting the weight for each mode. Third, the allocated tolerance $\hat{\varepsilon}$ is sensitive to the optimal objective value in the outer approximation method. Fourth, compared with home health care service of only door-to-door services, the combination of door-to-door and outpatient services can serve more patients with lower operating costs. To illustrate the performance of the proposed HGA, 24 instances with up to 30 doctors, 100 home healthcare patients, and 200 out-patients were tested. The results show the HGA can yield high-quality solutions within short computing time, and can solve much larger size problems than the branch and cut method.

This paper is the first to consider the home health care routing and scheduling problem that combines the door-to-door and outpatient services. The research has a great practical significance for the development and improvement of family doctor contract services in China. In future research, we can extend the problem to consider multi-period scheduling and some uncertain factors (such as spontaneous patient requests, stochastic traveling time, etc.).

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Appendix A

See Tables A.1~ Table A.4.

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Table A.1

Doctors' information.

Doctor k	Q_k	u_k	Doctor k	Q_k	u_k
1	3	0.0380	4	2	0.0354
2	3	0.0356	5	2	0.0442
3	1	0.0797	6	1	0.0837

Table A.2

Home healthcare patients' information.

Patient i	Coordinates	$[e_i, l_i]$	τ_i (min)	q_i	$p_{ik}(k = 1,2,\ldots,6)$
1	(80, 2)	[139, 239]	52	3	[1 1 0 0 0 0]
2	(92, 73)	[107, 190]	89	2	[1 1 0 1 0 0]
3	(48, 57)	[63, 152]	32	1	[0 0 1 0 1 1]
4	(23, 45)	[173, 279]	83	1	[0 0 1 1 1 1]
5	(96, 54)	[225, 339]	84	3	[0 0 0 0 0 0]
6	(52, 23)	[333, 413]	77	1	[0 0 0 1 1 1]
7	(48, 62)	[290, 361]	35	2	[1 0 0 1 0 0]
8	(67, 39)	[69, 131]	45	1	[0 0 1 0 1 1]

Table A.3

Out-patients' information.

Patient i	q_i	$p_{ik}(k = 1,2,\ldots,6)$	Patient i	q_i	$p_{ik}(k = 1,2,\ldots,6)$
9	3	[0 0 0 0 0 0]	14	2	[0 0 0 0 0 0]
10	2	[1 0 0 1 1 0]	15	1	[0 0 0 0 1 1]
11	2	[1 1 0 1 1 0]	16	1	[0 0 1 1 1 1]
12	1	[0 1 1 0 1 0]	17	2	[0 0 0 1 0 0]
13	1	[0 0 0 1 0 1]	18	1	[1 0 0 0 0 0]

Table A.4

Information about relevant patients in the four scenarios.

Patient i	coordinates	$[e_i, l_i]$	τ_i (min)	q_i	$p_{ik}(k = 1,2,\ldots,6)$
9	[63, 9]	[298, 403]	35	3	[0 0 0 0 0 0]
10	[27, 54]	[139, 233]	79	2	[1 0 0 1 1 0]
11	[95, 96]	[124, 171]	71	2	[1 1 0 1 1 0]
12	[15, 97]	[202, 308]	49	1	[0 1 1 0 1 0]
13	[95, 48]	[304, 355]	87	1	[0 0 0 1 0 1]
14	[80, 14]	[234, 322]	32	2	[0 0 0 0 0 0]
15	[42, 91]	[87, 130]	56	1	[0 0 0 0 1 1]
16	[79, 95]	[112, 207]	32	1	[0 0 1 1 1 1]
17	[65,3]	[122, 213]	75	2	[0 0 0 1 0 0]
18	[84, 93]	[114, 200]	47	1	[1 0 0 0 0 0]
19	[67, 75]	[123, 222]	41	2	[0 0 1 0 0 1]
20	[74, 39]	[251, 355]	59	1	[0 0 0 1 1 0]
21	[65, 17]	[191, 256]	56	2	[1 0 0 1 0 0]