Quantum and Classical Data Transmission through Completely Depolarizing Channels in a Superposition of Cyclic Orders

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Completely depolarizing channels are often regarded as the prototype of physical processes that are useless for communication: any message that passes through them along a well-defined trajectory is completely erased. When two such channels are used in a quantum superposition of two alternative orders, they become able to transmit some amount of classical information, but still no quantum information can pass through them. Here, we show that the ability to place N completely depolarizing channels in a superposition of N alternative causal orders enables a high-fidelity heralded transmission of quantum information with error vanishing as $1/N$. This phenomenon highlights a fundamental difference with the $N=2$ case, where completely depolarizing channels are unable to transmit quantum data, even when placed in a superposition of causal orders. The ability to place quantum channels in a superposition of orders also leads to an increase of the classical communication capacity with N, which we rigorously prove by deriving an exact single-letter expression. Our results highlight the more complex patterns of correlations arising from multiple causal orders, which are similar to the more complex patterns of entanglement arising in multipartite quantum systems.

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Introduction.—Shannon's information theory was initially developed under the assumption that the information carriers were classical systems [1]. At the fundamental level, however, physical systems obey the laws of quantum mechanics, which enable radically new communication protocols [2,3] and give rise to a variety of new communication capacities [4].

Traditionally, the extension of Shannon's theory to the quantum domain assumed that the configuration of the communication devices was fixed. In principle, however, quantum theory is compatible with scenarios where the communication devices are arranged in a coherent superposition of alternative configurations. For example, the available devices could act in different orders, and the choice of order could be controlled by the state of a quantum system using a primitive known as the "quantum switch" [5,6]. Similarly, the devices could be used as alternatives to one another, and the choice of which device is used for communication could be controlled by the state of a quantum system, giving rise to a superposition of alternative quantum evolutions [7–12].

The ability to superpose different configurations of communication devices can be exploited to achieve advantages over the standard model of quantum Shannon theory, where the configuration of the channels is fixed. The advantages of the superposition of orders have been shown in Refs. [13–19], while the advantages of the superposition of channels have been shown in Refs. [9–11]. At a conceptual level, these advantages can be rigorously formalized in a resource-theoretic framework, where the resources are communication devices, and the allowed operations on them include placement operations, which determine the arrangement of the communication devices in space and time [20]. Different advantages can then be understood as the result of different ways to enlarge the set of placement operations allowed by standard quantum Shannon theory. At a more practical level, new communication protocols with superpositions of configurations have been experimentally realized [21–25]. The information-theoretic advantages of the superposition of causal orders have also inspired a new line of investigation in quantum thermodynamics [26].

One of the most striking advantages of the superposition of configurations is the ability to communicate through channels that completely block information when used in a definite configuration. The prototype of such channels is the completely depolarizing channel, which outputs white noise independently of its input. Strikingly, it was shown that two completely depolarizing channels can be used for transmitting classical information when arranged in a superposition of two alternative orders [13]. On the other hand, this phenomenon is limited to the transmission of classical bits. In this Letter, we will show that when two completely depolarizing channels are combined in the quantum switch, the resulting channel cannot be used to transmit quantum data.

While the communication advantages of the quantum switch of two channels are well known, much less is known about the advantages of the quantum switch of multiple channels. Recent works [16,17] considered the amount of classical bits transmitted through N completely depolarizing channels, showing an increase of the Holevo information [27]. However, the Holevo information is only a lower bound to the actual capacity [28,29], and an increase in the Holevo information does not necessarily imply an increase in the capacity. Moreover, the increase in the capacity, while technically interesting, would only be a quantitative improvement in a task that can already be accomplished with $N = 2$ channels. A natural question is whether there exists some communication task that cannot be achieved at all by superposing the order of two channels but instead becomes possible when multiple channels are used.

Here, we answer the question in the affirmative, providing a concrete example of a communication task that can only achieved when $N > 2$ causal orders are superposed. We consider N completely depolarizing channels combined in a superposition of N causal orders related to each other by cyclic permutations. We show that a high-fidelity heralded transmission of quantum bits can be achieved with error vanishing as $1/N$. Our finding is in stark contrast to the impossibility of quantum data transmission through $N = 2$ completely depolarizing channels and highlights a genuinely new feature arising from $N > 2$ channels in alternative causal orders. The high-fidelity heralded transmission of quantum data is also potentially relevant for the task of entanglement distribution in quantum networks [30] and for the task of private classical communication [31,32].

In addition to establishing the possibility of heralded quantum communication, we analytically determine the classical communication capacity of N completely depolarizing channels in a superposition of N causal orders, and we demonstrate that the capacity increases monotonically with N. To this purpose, we establish a connection between the quantum switch of completely depolarizing channels and the universal quantum NOT gate [33–36]. We then use this connection to prove a single-letter formula for the classical capacity. Our result demonstrates that increasing the number of "useless" channels leads to an increase in the number of bits that can be reliably transmitted. To the best of our knowledge, this is the first rigorous demonstration of a task where the benefit of the superposition of causal orders grows monotonically with the number of configurations that are superposed.

Communication devices in a quantum superposition of alternative orders.—A communication device transmitting a quantum system is described by a quantum channel, that is, a completely positive trace preserving linear map $\mathcal{C}$ transforming linear operators on the system's Hilbert space $\mathcal{H}$. Any such map can be written in the Kraus representation $\mathcal{C}(\rho) = \sum_i C_i \rho C_i^\dagger$, where the Kraus operators $\{C_i\}$ satisfy $\sum_i C_i^\dagger C_i = I$.

Here, we consider the application of N channels in a coherent superposition of different alternative orders. The superposition of orders is constructed using the quantum switch [5,6], a higher-order operation that transforms two quantum channels into a new quantum channel in which the two input channels are executed in one of two alternative orders depending on the state of control qubit, called the "order qubit." Here, we adopt the original definition of the quantum switch [5], where the two channels act in two subsequent time steps, possibly allowing for intermediate operations. Mathematically, the quantum switch transforms two input quantum channels $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$ into the output channel

$$\mathcal{S}[\mathcal{C}^{(1)}, \mathcal{C}^{(2)}](\cdot) = \sum_{j_1, j_2} W_{j_1 j_2} \cdot W_{j_1 j_2}^\dagger, \qquad (1)$$

whose Kraus operators W_{ij} are defined as

$$W_{ij} \coloneqq |0\rangle\langle 0| \otimes C_{j_1}^{(1)} \otimes C_{j_2}^{(2)} + |1\rangle\langle 1| \otimes C_{j_2}^{(2)} \otimes C_{j_1}^{(1)}, \qquad (2)$$

where the three systems in the tensor product on the right-hand side are the order qubit, the input system in the first time step, and the input system in the second time step. Here, $\{C_{j_1}^{(1)}\}$ and $\{C_{j_2}^{(2)}\}$ are Kraus operators for channels $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$, respectively. Note that, while the individual Kraus operators W_{ij} depend on the choice of Kraus representation for $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$, the overall quantum channel $\mathcal{S}[\mathcal{C}^{(1)}, \mathcal{C}^{(2)}]$ depends only on the channels $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$ themselves, making the quantum switch a well-defined operation on quantum channels [6,37].

It is worth stressing that, while the order of the two processes $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$ inside the quantum switch is indefinite, the channel $\mathcal{S}[\mathcal{C}^{(1)}, \mathcal{C}^{(2)}]$ produced by the quantum switch has a well-defined causal structure: the input of the first time slot is provided first, followed by the output of

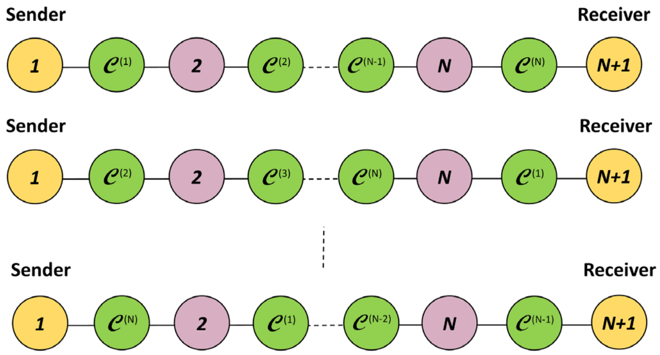

FIG. 1. Communication through N channels in a superposition of N cyclic orders. A sender, located at node 1 of a quantum communication network, sends messages to a receiver, located at node $N+1$, through a sequence of intermediate nodes, labeled as $2, \ldots, N$. The intermediate nodes are connected by N quantum channels $\mathcal{C}^{(1)}, \ldots, \mathcal{C}^{(N)}$, which have been placed in one of N configurations related by cyclic permutations, as shown in the graphic. The choice of configuration is controlled by a quantum system in a coherent superposition.

the first time slot, the input of the second time slot, and, finally, the output of the second time slot. Accordingly, a communication protocol using the channel $\mathcal{S}[\mathcal{C}^{(1)}, \mathcal{C}^{(2)}]$ will have a well-defined causal structure: first, the sender inputs a state in the first time slot, then the first time slot is connected to the second with some intermediate operation, and finally the receiver collects the output of the second time slot.

When $N > 2$ channels are available, the quantum switch operation [Eq. (1)] can be applied to each pair of channels, thus generating all possible permutations of their orders [38]. In a resource theory of communication, the quantum switch can be viewed as an operation performed by a communication provider that places the available communication devices between the sender and receiver [20]. Here, we consider a placement of the N devices in a network with $N-1$ intermediate nodes, as illustrated in Fig. 1. Again, note that the causal structure of the process generated by the quantum switch is well-defined, even though the N channels inside the quantum switch act in an indefinite order. As a consequence, the causal structure of the communication protocol in the network of Fig. 1 is well-defined: first, the sender inputs the state in the first node, then the first intermediate party receives the output at the second node and sends it to the third node, and so on until, finally, the receiver receives the output at the last node.

We will assume that the order qubits are inaccessible to the sender and are initialized by the communication provider in a fixed state before the beginning of the communication protocol. Also, we will take the intermediate nodes in Fig. 1 to contain identity operations, so that the effective channel available to the sender and receiver becomes

$$\mathcal{C}_{\text{eff}}(\rho) = \sum_{\pi,\pi' \in \mathsf{S}} \omega_{\pi,\pi'} |\pi\rangle\langle\pi'| \otimes \mathcal{C}_{\pi\pi'}(\rho), \tag{3}$$

where S is a set of permutations, ω is the state of the order qubits (with matrix elements $\omega_{\pi,\pi'}$ and support in a subspace spanned by an orthonormal basis $\{|\pi\rangle\}_{\pi\in\mathsf{S}}$ labeled by permutations in S), and

$$\mathcal{C}_{\pi\pi'}(\rho) \coloneqq \sum_{j_1,\ldots,j_N} C^{\pi(1)\cdots\pi(N)}_{j_{\pi(1)},\ldots,j_{\pi(N)}} \rho C^{\pi'(1)\cdots\pi'(N)\dagger}_{j_{\pi'(1)},\ldots,j_{\pi'(N)}} \tag{4}$$

with the notation $C^{i_1\cdots i_N}_{j_{i_1}\ldots j_{i_N}} \coloneqq C^{(i_1)}_{j_{i_1}} \cdots C^{(i_N)}_{j_{i_N}}$, where $\{C^{(i)}_{j_i}\}$ are Kraus operators for channel $\mathcal{C}^{(i)}$.

Heralded quantum communication through completely depolarizing channels.—When the configuration of the channels is fixed, the completely depolarizing channel $\mathcal{D}(\cdot) \coloneqq I/d\mathrm{Tr}[\cdot]$ is the prototype of a useless channel: since its output is independent of the input, this channel does not permit the transmission of any data, be it classical or quantum.

Now, suppose that N completely depolarizing channels are combined by the quantum switch, generating the effective channel $\mathcal{C}_{\text{eff}}$ in Eq. (3). In the following, we will take S to be the set of cyclic permutations π, mapping the index a into the index $\pi(a) = (a+k) \bmod N$ for some given $k \in \{0, \ldots, N-1\}$, and we will set $\omega = |e_0\rangle\langle e_0|$, with $|e_0\rangle = \sum_\pi |\pi\rangle/\sqrt{N}$.

A convenient Kraus representation of the completely depolarizing channel is a uniform mixture of an orthogonal unitary basis $\{U_i\}_{i=1}^{d^2}$, namely $\mathcal{D}(\rho) = \sum_{i=1}^{d^2} U_i \rho U_i^\dagger/d^2$, where d is the dimension of the system and $\mathrm{Tr}[U_i^\dagger U_j] = d\delta_{i,j}$. Using this representation, we derive the relations

$$\mathcal{C}_{\pi\pi}(\rho) = \frac{I}{d} \quad \text{and} \quad \mathcal{C}_{\pi\pi'}(\rho) = \frac{\rho}{d^2} \quad \forall\, \pi \neq \pi' \tag{5}$$

(see the Supplemental Material [39]). Inserting these relations into Eq. (3) yields the expression

$$\begin{aligned}\mathcal{C}_{\text{eff}}(\rho) &= \frac{I}{N} \otimes \frac{I}{d} + \sum_{\pi\neq\pi'} |\pi\rangle\langle\pi'| \otimes \frac{\rho}{Nd^2} \\ &= \frac{I}{N} \otimes \frac{I}{d} + (N|e_0\rangle\langle e_0| - I) \otimes \frac{\rho}{Nd^2},\end{aligned} \tag{6}$$

the second equality following from the relations $N|e_0\rangle\langle e_0| = \sum_{\pi,\pi'} |\pi\rangle\langle\pi'|$ and $I = \sum_\pi |\pi\rangle\langle\pi|$. Rearranging the terms in Eq. (6), we rewrite the effective channel as

$$\mathcal{C}_{\text{eff}}(\rho) = (1-p)\rho_0 \otimes \mathcal{E}_0(\rho) + p\rho_1 \otimes \mathcal{E}_1(\rho), \tag{7}$$

where $\rho_0 \coloneqq |e_0\rangle\langle e_0|$ and $\rho_1 \coloneqq (I - |e_0\rangle\langle e_0|)/(N-1)$ are orthogonal states of the control system, $\mathcal{E}_0$ and $\mathcal{E}_1$ are the quantum channels defined by

$$\mathcal{E}_0(\rho) := \frac{N-1}{N-1+d^2}\rho + \frac{d^2}{N-1+d^2}\frac{I}{d}, \tag{8}$$

and

$$\mathcal{E}_1(\rho) := \frac{d^2}{d^2-1}\frac{I}{d} - \frac{1}{d^2-1}\rho, \tag{9}$$

respectively, and $p := (N-1)(d^2-1)/(Nd^2)$. Two alternative ways to generate the channel $\mathcal{C}_{\rm eff}$ from depolarizing channels in a superposition cyclic orders are discussed in the Supplemental Material [39].

Equation (7) shows that the effective channel $\mathcal{C}_{\rm eff}$ is a mixture of two channels $\mathcal{E}_0$ and $\mathcal{E}_1$, flagged by two orthogonal states of the order qubits. By measuring the order qubits, it is then possible to herald the occurrence of the channels $\mathcal{E}_0$ and $\mathcal{E}_1$.

The channel $\mathcal{E}_1$ is independent of N. For $d = 2$, it is the universal NOT channel introduced by Bužek, Hillery, and Werner [33] and experimentally realized in a series of works [33–36]. The universal NOT gate is known to be an entanglement-breaking channel [40], or equivalently, a "measure-and-reprepare" channel, which can be realized by measuring the input and preparing an output state depending on the measurement outcome [63]. Since entanglement-breaking channels have zero quantum capacity [41], no quantum information can be transmitted through the channel $\mathcal{E}_1$. For $d > 2$, the channel $\mathcal{E}_1$ is a generalization of the universal NOT and can be characterized as the channel that minimizes the fidelity between a generic input state $|\psi\rangle$ and the corresponding output state $\mathcal{E}_1(|\psi\rangle\langle\psi|)$. In the Supplemental Material [39], we show that $\mathcal{E}_1$ is entanglement-breaking and therefore unable to transmit any quantum data.

The channel $\mathcal{E}_0$, instead, is a depolarizing channel, with probability of depolarization equal to $d^2/(N+d^2-1)$. Remarkably, this probability vanishes as d^2/N in the large N limit, enabling a perfect transmission of quantum data. It is also remarkable that the probability of high-fidelity transmission does not vanish in the large N limit: such a probability remains larger than $1/d^2$ for every value of N. For qubits, this means that the state of the target system has a probability of at least 25% of reaching the receiver with an error smaller than $4/N$.

The heralded high-fidelity transmission of quantum information could be exploited for the distribution of entanglement in quantum networks [30], which in turn serves as a primitive for distributed quantum computation [64]. Our results could also be useful for cryptographic purposes such as private classical communication [31,32] or the generation of secret keys via the BB84 [2] or E91 protocols [3]. A discussion of these applications is provided in the Supplemental Material [39].

For finite N, it is possible to show that channel $\mathcal{E}_0$ has a nonzero quantum capacity for all values of N larger than a given finite value $N_0 > 2$. For example, for $d = 2$ and $N > 13$, it is possible to show that the probability of depolarization is less than $1/4$, which guarantees that the depolarizing channel $\mathcal{E}_0$ has a nonzero quantum capacity [4]. In turn, the nonzero quantum capacity of channel $\mathcal{E}_0$ ensures that the overall channel $\mathcal{C}_{\rm eff}$ has a nonzero quantum capacity assisted by two-way classical communication [42], as shown in the Supplemental Material [39]. In the Supplemental Material we also show that quantum data transmission with the assistance of two-way classical communication is possible through the quantum switch of N cyclic permutations if and only if $N \geq d+2$.

The possibility of quantum information transmission is a fundamental difference between the bipartite and the multipartite quantum switch in a way that is somewhat reminiscent of the difference between bipartite and multipartite entanglement. For $N = 2$, we prove that no superposition of causal orders permits the transmission of quantum bits through completely depolarizing channels under the natural assumption that the sender does not use the control system to establish entanglement with the receiver (see the Supplemental Material [39] for the details).

All the results presented so far concerned the superposition of completely depolarizing channels. A natural question is whether any of our conclusions would change if we were to consider partially depolarizing channels. In particular, one could ask whether the quantum switch could enable the transmission of quantum information using $N = 2$ partially depolarizing channels that individually have zero quantum capacity. In the Supplemental Material, we answer the question in the negative, showing that the quantum capacity of each depolarizing channel is a bottleneck for the amount of quantum information one can send through the quantum switch. An interesting open question is whether the use of partially depolarizing channels could reduce the number of channels needed to achieve quantum data transmission starting from channels with zero capacity. More broadly, the study of quantum communication with partially depolarizing channels in an indefinite causal order remains an interesting problem for future research.

Enhanced transmission of classical information.—We now quantify the amount of classical bits transmittable through N depolarizing channels in a superposition of N alternative orders. By the Holevo-Schumacher-Westmoreland theorem [28,29], the classical capacity of a generic noisy channel $\mathcal{N}$ is given by $C(\mathcal{N}) = \lim_{n\to\infty}\chi(\mathcal{N}^{\otimes n})/n$, where χ is the Holevo information [27], defined as $\chi(\mathcal{N}) = \max_{(\rho_x, p_x)_{x\in\mathsf{X}}} S[\sum_x p_x \mathcal{N}(\rho_x)] - \sum_x p_x S[\mathcal{C}(\rho_x)]$, $(\rho_x, p_x)_{\in\mathsf{X}}$ being an arbitrary ensemble of quantum states, and $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$ being the von Neumann entropy. In the Supplemental Material [39], we prove that the Holevo information of the effective channel $\mathcal{C}_{\rm eff}$ is additive, and therefore the classical capacity has the single-letter formula

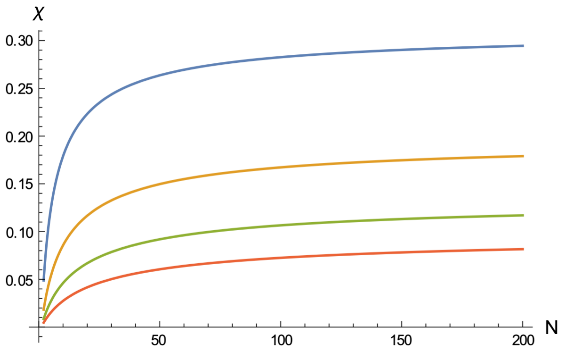

FIG. 2. Classical capacity of the effective channel $\mathcal{C}_{\rm eff}$, plotted with respect to N for message systems of dimension $d = 2, 3, 4$, and 5.

$C(\mathcal{C}_{\rm eff}) = \chi(\mathcal{C}_{\rm eff})$, for which we provide an exact expression.

The classical capacity is plotted in Fig. 2 for different values of N and d. The capacity increases monotonically with N, rigorously demonstrating the benefit of increasing the number of alternative orders. In the Supplemental Material [39], we provide an asymptotic expression for the capacity in the large N limit, showing that it decreases with d, tending to zero for $d \to \infty$. For $N = 2$, the decrease with d was observed for the Holevo information [13], although it was not known whether the actual channel capacity was also decreasing.

Conclusions.—We demonstrated a communication advantage of the superposition of multiple causal orders by showing a communication task that cannot be achieved by superposing two orders but becomes possible when the number of orders is sufficiently large. Specifically, we demonstrated that the placement of N completely depolarizing channels in a superposition of N cyclic orders enables a high-fidelity heralded transmission of quantum information with error vanishing as $1/N$. For finite N, we found that a nonzero quantum capacity assisted by two-way classical communication can be achieved with N qubit depolarizing channels whenever $N \geq 4$.

The possibility of quantum data transmission through completely depolarizing channels highlights a fundamental difference with the $N = 2$ scenario, where no quantum information can pass through completely depolarizing channels. A recent experiment [65] on the superposition of $N = 4$ channels suggests that an experimental demonstration of nonzero quantum capacity assisted by two-way classical communication could be achieved in the near future. Most importantly, our results motivate an investigation of the operational features of the different types of quantum superpositions arising when multiple causal orders are superposed.

It is intriguing to imagine that the distinction between the superposition of two and multiple causal orders could mirror the distinction between bipartite and multipartite entanglement, whose study has led to the discovery of a wealth of new quantum information protocols. In this respect, our result indicates that the superposition of multiple causal orders is a genuinely new resource that is not reducible to the superposition of $N = 2$ causal orders, just as genuine multipartite entanglement is not reducible to bipartite entanglement. We hope that our work will stimulate future explorations of the analogy between superpositions of causal orders and multipartite entanglement, thereby leading to a deeper understanding of the interplay between causality and quantum physics.

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Note added.—After the completion of this work, we became aware of Ref. [66], which independently derived the Holevo information of N completely depolarizing channels in a superposition of N cyclic permutations.

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[39] See Supplemental Material, which includes Refs. [2,3,5,6,11,13,22–25,31,32,40–62], at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.190502. The Supplemental Material contains the derivation of Eq. (5), two possible realizations of the effective channel $\mathcal{C}_{\rm eff}$, a generalization of the universal NOT to $d > 2$, a discussion of potential cryptographic applications, a protocol for two-way assisted quantum communication using channel $\mathcal{C}_{\rm eff}$, a proof that the two-way assisted capacity of $\mathcal{C}_{\rm eff}$ is nonzero if and only if $N > d + 1$, a proof that the quantum switch of $N = 2$ completely depolarizing channels is entanglement-breaking, a proof that the quantum switch cannot activate the quantum capacity of $N = 2$ partially depolarizing channels, and the derivation of a single-letter formula for the classical capacity of the effective channel.
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