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Acoustic rotation of non-spherical micro-objects: characterization of acoustophoresis and quantification of rotational stability

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Abstract: Acoustic radiation force and torque arising from wave scattering have the potential to perform the biocompatible, contact-free, and precise translation and rotation of micro-objects. Controllable rotation of objects mainly relies on the use of sophisticated transducer arrays, while it is limited by the spatial Rayleigh limitation and would be costly to miniaturize the array for manipulated objects on the micron scale. Here, we make use of the shape asymmetry of an object and achieve micro-object trap and rotation using a simple standing wavefield. The conformal transformation approach is employed to map an axisymmetric, non-spherical object into a sphere, allowing fast analysis for design or real-time computer-controlled manipulation. A low-cost, simple rotational tweezing system is built, which confirms the theoretical prediction that micro-particles are trapped in the pressure nodes and remotely rotated by the action of the actuator boundaries. The rotational dynamics are guided by two stabilization modes, and Lyapunov stability analysis reveals that at least one mode leads to a stable equilibrium state. We demonstrate the significance of our approach in stably and controllably rotating micron-scale objects. The robustness and versatility of the new handy design hold promise for clinical applications and microscopic research such as drug delivery and microsurgery.

Keywords: acoustic radiation force, acoustic radiation torque, controllable rotation, axisymmetric objects, stability analysis

1. Introduction

Precise manipulation of micro-objects is an essential capability for cell and microorganism studies [1,2] and biotechnology in general. Controllable rotation of micro-objects can reveal hidden structural details, which may not be possible by mere translational manipulation. Hence, it is an indispensable and effective tool to interrogate morphological phenotype [3,4] and to operate microsurgery [5] for microorganisms. Various techniques, such as magnetic [6], optical [7], and acoustic [8,9], can manipulate particles in a contactless manner. However, the magnetic method requires particles and media to be magnetically susceptible polarizable. Acoustic radiation efficiency is orders of magnitude higher than that from optical counterpart for the same operation power, indicating that for the same radiation efficiency, the optical method requires higher operation power [10]. Consequently, a high-power operation of optical manipulation increases the risk of tissue ablation of live objects, thereby physiologically deactivating microorganisms [11]. By contrast, acoustic waves are independent of the electromagnetic properties of the materials and are often a more suitable choice for the safe handling of micro-objects [12,13].

Acoustofluidic methods have demonstrated the capacity to rotate small organisms or irregular cells on-chip [4,14,15]. Steady acoustic microstreaming vortices induced by the oscillatory motion of sharp-edge structures or sidewalls change the position and the orientation of micro-objects according to the streamlines. However, acoustofluidic methods are typically designed for object manipulation in a narrow microfluidic channel since the ultrasound-induced microstreaming around a particle in liquid is particularly vigorous near a solid structure [16]. Moreover, an exquisite microchannel design is essential to achieve precise manipulations [4,15]. Additionally, as of yet, even though many efforts have been devoted to the numerical study of ubiquitous microstreaming [17,18], no analytical expressions have been developed to quantify the dynamics of rotation in these phenomena. Considering the instability of the acoustic-induced micro-vortices, their properties are susceptible to change, partially limiting their practical applications.

Acoustic radiation methods emerged as an alternative tool for effective and precise manipulation of objects [8,9]. The Gor'kov theory is commonly employed to manipulate the Rayleigh objects where the monopole and dipole scattering dominate [19]. The acoustic twin trap has demonstrated the rotation of symmetric particles by modulating the phase distribution from the wave emitting devices [8]. For the Mie non-spherical objects, the higher order scattering contributions become considerable and partially limit the application of Gor'kov theory. The partial wave expansion [20,21] or full numerical simulation [14] helps to account for the higher-order scattering contributions reflected by complex objects. A virtual vortex has been used to rotate Mie particles and control their rotational speeds independently of the trapping force [22]. Since the transducers are usually in the diameter range of 2 mm to 10 mm, the resolution of either acoustic vortex or twin trap created by the transducer array is not fine enough to manipulate cells or microorganisms. It would be costly to extend these concepts further down in scale to the smaller objects in water [23] or potentially in biological tissue phantoms. To make a rough estimate of the smallest particle size that can be manipulated, one may begin with the Rayleigh limit [24] of the transducer array and set the spatial resolution as $R = 1.22d_f\lambda/D$, where d_f , λ and D denote the focal distance, wavelength, and aperture of the array, respectively. Since scattering effects dominate the trapping properties [20,25], transducer arrays are commonly used to manipulate the

Rayleigh objects with a size parameter of $ka \ll 1$. Without loss of generality, the particle radius may be set as $a = \lambda/5$, which means that the spatial resolution should at least satisfy $R \sim a = \lambda/5$ and the aperture becomes $D \sim 6.1d_f$. Consider that the particles are typically manipulated in the far-field region, while a greater distance requires a larger aperture, we set $d_f = 5\lambda$, thereby $D \sim 30.5\lambda \sim 152.5a$. To manipulate an object of radius in the millimeter range, D may be chosen to be around 150 mm to avoid the merging of focal spots [8,9]. It would be difficult to scale down by order of magnitude to 100 μ m, which implies an array aperture of $D = O(15\text{mm})$, thus a drastically increased cost of manufacture and implementation. This approach is, therefore, not yet available for the manipulation of microscopic particles [26].

Instead of trying to perfect the existing acoustic technique of creating an acoustic swirl using a transducer array, we explore the use of geometric asymmetry that naturally exists in non-spherical objects, such as *erythrocyte* [14] and *C. elegans* [15]. The geometric asymmetry can induce rotation using a simple standing wavefield, which offers an alternative for a stable and controllable manipulation of a single non-spherical object on the micron scale. The experimental demonstration in this work has a typical radius of 150 microns, but it is shown below that the lower limit can easily go down to around 20 microns. To efficiently manipulate the object and make use of its geometric asymmetry, the wavelength of acoustic waves should be comparable to the size of the object, i.e., $ka \approx 1$. The lower limit on object radius is thus set by the upper limit in wave frequency for which viscous damping remains moderate throughout the domain of the device. For instance, for a device with the central domain of 25 mm in BAW size, the viscous attenuation of wave energy over half of its size, 25/2 mm, may be limited to a threshold value of about 10%, namely $(e^{-\beta \times 0.025/2})^2 = e^{-0.025\beta} > 1 - 0.1 = 0.9$, where $\beta \approx (\mu' + 4\mu/3)k^2/(2\rho_0c_0)$ is the viscous attenuation coefficient [27], μ' and μ are the bulk viscosity coefficient and dynamic viscosity coefficient, respectively, ρ_0 and c_0 are the fluid density and wave speed, respectively. The threshold value for β with a 10% wave energy loss is found as $\beta = (-\ln 0.9)/0.025 = 4.2 \text{ m}^{-1}$. The corresponding particle radius is $a = k^{-1} \approx \sqrt{(\mu' + 4\mu/3)/(2\beta\rho_0c_0)}$. For water with $\mu' = 2.485 \text{ mPa} \cdot \text{s}$, $\mu = 0.89 \text{ mPa} \cdot \text{s}$, $\rho_0 = 1000 \text{ kg/m}^3$, and $c_0 = 1500 \text{ m/s}$, the threshold radius for $\beta = 4.2 \text{ m}^{-1}$ is found as 17 μm . In other words, the viscous attenuation effect can be neglected if the radius of the manipulated object suspended in water is larger than around 17 microns.

The proposed device focuses on non-spherical objects. Although an analytical model for the radiation force and torque exists for simple geometries, such as spheres [28], few theories can quantify such important geometrical features, which strongly affect the scattering properties [29], thereby particle acoustophoresis [25,30]. In this sense, the non-spherical shape parameter is an additional degree of freedom for particle manipulation that is yet to be exploited. In what follows, a theory of radiation force and torque fields will be proposed for the prediction of acoustophoresis of non-spherical particles, and the analysis of the stability [31] of the corresponding acoustic tweezers. Herein, we present a rotational tweezing system capable of performing controllable rotational manipulation of non-spherical micro-objects with the help of the property of shape asymmetry. The proposed rotational tweezing system is composed of a fixed outer module and a flexible inner module, designed in such a way that the orientation of the standing bulk acoustic wave (BAW) can be manually adjusted and thus controllably rotate the particles, adapting to the variations of the wavefield. Theoretically, a general

analytical procedure is introduced to quickly estimate (i.e., ~ 2 s vs. ~ 10 mins in full three-dimensional numerical simulations) the radiation force and torque, thereby the acoustophoresis of micro-objects with different morphological characteristics. Both analytical predictions and experiments confirm that the particles will be sunk in the acoustic radiation-force potential wells, while these particles need to adapt their orientation to make as many entities as possible towards the minimum potential state, following two stabilization modes, as given in Fig. 9(c). The whole acoustophoretic processes were theoretically predicted, which were also verified by experiments. Additionally, we considered a system consisting of a radiation gain (radiation force and torque) and viscous loss (viscous drag force and torque) to analyze the stability of the non-spherical particles when they arrive at any possible equilibrium states. Based on this concept, we show that contact-free rotational manipulation of a particle with different geometries simultaneously and controllably towards any desired direction is achievable by steering the orientation of the standing wavefield. This particle-rotation concept has the potential to revolutionize future acoustic tweezers development and benefit microscale research, including cell and microorganism microsurgery.

2. Mathematical models

2.1. Theoretical model to predict the motions of complex particles

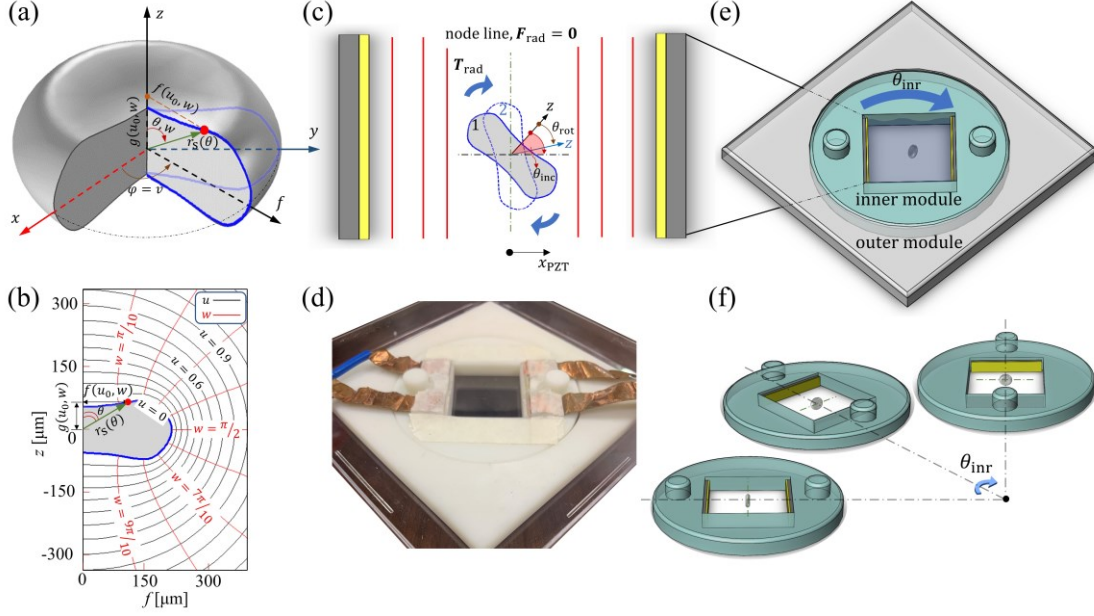


FIG. 1. Workflow from geometric asymmetry to particle rotational manipulation. (a) A special class of non-spherical particles studied here is solids of revolution formed by rotating an arbitrary profile curve around the z -axis. (b) Conformal transformation of the profile curve to a circle with radius u and polar angle w . (c) The non-spherical particle in a standing wavefield. The particle is trapped by the radiation force at the node line (central dot-dashed line) while rotated from initial position ‘1’ to final position ‘2’ by the radiation torque because of its geometric asymmetry. (d) The 3D printed rotational BAW device, and (e) its schematic diagram showing two knobs that can be rotated by hand to change the orientation of the standing wave in the square-shaped area corresponding to (c). (f) Controllably rotational manipulation of a non-spherical particle. The rotation of the inner module (θ_{inr}) will alter the orientation of the standing wavefield, thereby readjusting the final position ‘2’ and controllably rotating the particle.

Acoustic waves exert acoustic radiation force and torque on objects because of the momentum transfer that arises from sound scattering [33–35]. To explore the use of torque on non-spherical shapes, we study the special class of shapes, solids of revolution, formed by rotating a profile curve around the z -axis, shown in Fig. 1(a). Apart from perfect spheres and cylinders [36,37], there is no easy analytical solutions. This simplification of the exterior shape neglects the effect of asymmetry, which is a potential degree of freedom to tune acoustophoretic processes. For example, the radiation torque for a spherical particle vanishes under the acoustic plane wavefields. However, for non-spherical particles, the shape asymmetry naturally induces an additional radiation torque that rotates the objects. In this sense, it provides a new route to rotational manipulation of the asymmetric micro-object by a commonly used plane wave system. The difficulty lies in quantifying and utilizing this degree of freedom brought by geometric asymmetry. However, it is possible to conformally map [38–40] the profile curve in the solids of revolution to circles and solves the wave equation in the mapped space; in the same manner, we deal with sound scattering by perfect spheres. The mapped quasi-spherical

coordinate system is illustrated in Fig. 1(b).

The physical space with slice zOf shown in Fig. 1(a) is related to the mapping space in Fig. 1(b) by the following

$$\begin{cases} r_s(u_0, w) = \sqrt{g^2(u_0, w) + f^2(u_0, w)} \\ \theta(u_0, w) = \cos^{-1}\left(g(u_0, w)/r_s(u_0, w)\right) \\ \varphi = v \end{cases} \quad (1)$$

Here, r_s is the projection of the spherical radial coordinate on the zOf plane, while θ and φ are the polar and azimuthal angular coordinates, respectively. Two mapping functions $f(u, w)$ and $g(u, w)$ are introduced to trigonometry prescribe of the particle surface,

$$\begin{cases} g(u, w) = c_{-1}e^u \cos(w) + \sum_{n=0}^{\infty} c_n e^{-nu} \cos(nw) \\ f(u, w) = c_{-1}e^u \sin(w) - \sum_{n=0}^{\infty} c_n e^{-nu} \sin(nw) \end{cases},$$

where u , w , and v are the new radial, polar angular, and azimuthal angular coordinates. The mapping coefficients c_n are determined by mapping the modal expansion series on particle surface as given in Supplemental Material Eq. (S7). In addition, specific procedures and solutions of the mapping coefficients for several typical geometries are provided in Supplemental Material Sec. 1.1 and Tab. S1.

In this way, the separation of variables can be used to solve the corresponding Helmholtz wave equation for the incident and scattering potential fields as given in Supplemental Material of Eqs. (S12) and (S13). The Neumann boundary condition across the scatterer surface (See Supplemental Material Sec. 1.2, Eq. (S14) and works [25,29]) yields

$$\sum_{n=0}^N a_{nm'} \Gamma_{n,u}^{n',m'} + \sum_{n=0}^N s_{nm'} a_{nm'} \Lambda_{n,u}^{n',m'} = 0, \quad (2)$$

($n' = 0, 1, \dots, N; m' = -N, \dots, 0, \dots, N$)

where $N \geq ka + 6$ [41] is the truncation number with wavenumber k and averaged radius a . The structural functions $\Gamma_{n,u}^{n',m'}$ and $\Lambda_{n,u}^{n',m'}$, depending on the mapping relations of Eq. (1), are known and given in Eq. (S15). For the standing wave propagating along x_{PZT} -direction (i.e., $(\theta_{\text{inc}}, \varphi_{\text{inc}}) = (0, 0)$), or the standing wave propagating along arbitrary $(\theta_{\text{inc}}, \varphi_{\text{inc}})$ directions while the particle located at the nodal line (i.e., the acoustic radiation-force potential well at $x_{PZT} = 0$), the beam-shape coefficients (a_{nm}) can be formulated, respectively, as

$$a_{nm} = -4\pi \sin\left(kx_{PZT} + \frac{n\pi}{2}\right) Y_n^m(0, 0)^*, \quad (3a)$$

or

$$a_{nm} = -4\pi \sin\left(\frac{n\pi}{2}\right) Y_n^m(\theta_{\text{inc}}, \varphi_{\text{inc}})^*. \quad (3b)$$

Here, $Y_n^m(\theta_{\text{inc}}, \varphi_{\text{inc}})$ is the spherical harmonic function of n -th order and m -th degree determined by the incidence angle of the incident plane wave $(\theta_{\text{inc}}, \varphi_{\text{inc}})$. Through inserting the beam-shape coefficients into Eq. (2), the scalar scattering expansion coefficients (s_{nm}) of the non-spherical particle are determined by solving the system of linear equations.

Then, the radiation force and torque due to the incident and scattered fields on a particle are (refer to Supplemental Material Sec. 2 for detailed derivations)

$$\mathbf{F}_{\text{rad}} = -\frac{\hat{p}_0^2}{4\rho_0 c_s^2 k^2} \text{Re} \sum_{n,m} a_{nm}(1 + s_{nm})(\mathbf{M} \cdot \mathbf{I}), \quad (4a)$$

$$\mathbf{T}_{\text{rad}} = -\frac{\hat{p}_0^2}{4\rho_0 c_s^2 k^3} \text{Re} \sum_{n,m} a_{nm}(1 + s_{nm})(\mathbf{N} \cdot \mathbf{J}), \quad (4b)$$

respectively, where

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} iA_{n+1}^{m+1} & -iB_{n+1}^{m+1} & iC_{n+1}^{m+1} & -iD_{n+1}^{m+1} & 0 & 0 \\ A_{n+1}^{m+1} & -B_{n+1}^{m+1} & -C_{n+1}^{m+1} & D_{n+1}^{m+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2iE_{n+1}^{m+1} & -2iF_{n+1}^{m+1} \end{bmatrix}, \\ \mathbf{I} &= [b_{n+1,m+1}^* \quad b_{n-1,m+1}^* \quad b_{n+1,m-1}^* \quad b_{n-1,m-1}^* \quad b_{n+1,m}^* \quad b_{n-1,m}^*], \\ \mathbf{N} &= \begin{bmatrix} G_n^m & G_n^{-m} & 0 \\ -iG_n^m & iG_n^{-m} & 0 \\ 0 & 0 & 2m \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} b_{n,m+1}^* \\ b_{n,m-1}^* \\ b_{n,m}^* \end{bmatrix}. \end{aligned}$$

Here, $\hat{p}_0$ is the pressure amplitude of the incident wave, k and c_s are, respectively, the wavenumber and adiabatic speed of sound in the host fluid of density ρ_0 . Symbol Re means taking the real part of the expressions, superscript $*$ means complex conjugate, and ^{T} is matrix transpose. The scattering expansion coefficients $b_{nm} \equiv a_{nm} \cdot s_{nm}$ depend on the incident plane wave (a_{nm}) and the geometric features (s_{nm}). The weighting coefficients A_n^m , B_n^m , C_n^m , D_n^m , E_n^m , F_n^m and G_n^m are specified in Eqs. (S18) and (S21).

In essence, it is possible to assume that the host fluid is at rest, where the acoustic streaming is negligible since the particles are placed far away from the solid structures [15,16]. The particles are driven by acoustic radiation force, partially impeded by the Stokes drag force [32], but eventually, gather at the pressure node (refer to theoretical predictions for different geometries in Fig. S2). Similarly, the particle rotation is driven by acoustic radiation torque, limited by the torque arising from the viscous shear stresses on its surface, denoted by $\mathbf{T}_{\text{drag}}$. Utilizing the linear relation between the drag torque, $\mathbf{T}_{\text{drag}}$, and the relative angular velocity of the particle with respect to the background host fluid, $\boldsymbol{\omega}$, we yield

$$\mathbf{T}_{\text{drag}} = -\mathbf{L} \cdot \boldsymbol{\omega}, \quad (5)$$

where $\mathbf{L}$ is a 3×3 drag coefficient matrix, depending on the geometric features of particles, which can be determined by a three-dimensional numerical simulation (Supplemental Material Sec. 3 for more information), and the estimated $\mathbf{L}$ for different geometries are listed in Tab. 1. Then, the rotational dynamics is able to describe by the moment balance between the radiation torque (Eq. (4b)) and the drag torque (Eq. (5)) as

$$I \frac{d\boldsymbol{\omega}}{dt} = \mathbf{T}_{\text{rad}} + \mathbf{T}_{\text{drag}}, \quad (6)$$

where I means the moment of inertia of the particle.

TABLE 1: Drag matrix for different particles based on Eq. (S23), in which the drag torques are computed by a surface integration of the surface traction on the body surface. The average radius of these particles is set to $a = 150 \mu\text{m}$.

Drag matrix	Sphere	Ellipsoid	Diamond	Disc
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$\mathbf{L} \times 10^{-14}$	$\begin{bmatrix} 7.18 & 0 & 0 \\ 0 & 7.18 & 0 \\ 0 & 0 & 7.18 \end{bmatrix}$	$\begin{bmatrix} 6.69 & 0 & 0 \\ 0 & 6.69 & 0 \\ 0 & 0 & 4.84 \end{bmatrix}$	$\begin{bmatrix} 7.28 & 0 & 0 \\ 0 & 7.28 & 0 \\ 0 & 0 & 7.62 \end{bmatrix}$	$\begin{bmatrix} 11.9 & 0 & 0 \\ 0 & 11.9 & 0 \\ 0 & 0 & 15.0 \end{bmatrix}$
$[\text{N} \cdot \text{s} \cdot \text{m}]$				

2.2. Numerical validation schemes of radiation force and torque

To validate the accuracy of the radiation force and torque exerted on the complex geometries described in Eq. (4), the COMSOL Multiphysics finite element models have been performed. The numerical acoustic radiation force and torque ($\mathbf{F}$ and $\mathbf{T}$) on an object could be obtained from a surface integration of the object [33-35]

$$\mathbf{F} = \int_{\mathbf{R}} \langle L \rangle d\mathbf{A}_{\mathbf{R}} - \rho_0 \int_{\mathbf{R}} d\mathbf{A}_{\mathbf{R}} \cdot \langle \mathbf{u}\mathbf{u} \rangle, \quad (7)$$

$$\mathbf{T} = -\rho_0 \int_{\mathbf{R}} \langle (d\mathbf{A}_{\mathbf{R}} \cdot \mathbf{u}) \cdot (\mathbf{r} \times \mathbf{u}) \rangle, \quad (8)$$

where the angle bracket $\langle \cdot \rangle$ denotes the time average of the variable therein. L is the acoustic Lagrange density defined as $L = \frac{1}{2} \rho_0 \mathbf{u} \cdot \mathbf{u} - \frac{1}{2\rho_0 c_s^2} p^2$, where $\rho_0 \mathbf{u} \cdot \mathbf{u}$ is the flux of momentum density.

The acoustic velocity and the acoustic pressure are $\vec{u}$ and p , respectively. The spherical surface $\mathbf{R}$ surrounding the scattering particle is sufficiently far to involve the scatterer, and the direction of the integration element $d\mathbf{A}_{\mathbf{R}}$ is along the outer normal of the surface.

The outer normal vector of integrating surface $\mathbf{R}$ can be expressed as $d\mathbf{A}_{\mathbf{R}} = \mathbf{e}_{\mathbf{R}} dA_{\mathbf{R}}$, where $\mathbf{e}_{\mathbf{R}} = \left(\frac{x}{a_{\mathbf{R}}}, \frac{y}{a_{\mathbf{R}}}, \frac{z}{a_{\mathbf{R}}} \right)$ defined as the unit outer normal vector of spherical surface $\mathbf{R}$ with a radius of $a_{\mathbf{R}} = \sqrt{x^2 + y^2 + z^2}$. The point position on the integrating surface is denoted as (x, y, z) under the Cartesian coordinate system. Inserting $d\mathbf{A}_{\mathbf{R}} = \mathbf{e}_{\mathbf{R}} dA_{\mathbf{R}}$ into Eqs. (7) and (8), the radiation force and torque are rearranged along corresponding coordinate axes under the Cartesian coordinate system as

$$F_x = \mathbf{F} \cdot \mathbf{e}_x = \int_{\mathbf{R}} \frac{1}{4} \frac{x}{a_{\mathbf{R}}} \left[\rho_0 \text{Re}(\hat{\mathbf{u}} \cdot \hat{\mathbf{u}}^*) - \frac{1}{\rho_0 c_s^2} \text{Re}(\hat{p}\hat{p}^*) \right] dA_{\mathbf{R}} \quad (9)$$

$$- \frac{\rho_0}{2} \int_{\mathbf{R}} \left[\frac{x}{a_{\mathbf{R}}} \text{Re}(\hat{u}_x \hat{u}_x^*) + \frac{y}{a_{\mathbf{R}}} \text{Re}(\hat{u}_x \hat{u}_y^*) + \frac{z}{a_{\mathbf{R}}} \text{Re}(\hat{u}_x \hat{u}_z^*) \right] dA_{\mathbf{R}},$$

$$F_y = \mathbf{F} \cdot \mathbf{e}_y = \int_{\mathbf{R}} \frac{1}{4} \frac{y}{a_{\mathbf{R}}} \left[\rho_0 \text{Re}(\hat{\mathbf{u}} \cdot \hat{\mathbf{u}}^*) - \frac{1}{\rho_0 c_s^2} \text{Re}(\hat{p}\hat{p}^*) \right] dA_{\mathbf{R}}$$

$$- \frac{\rho_0}{2} \int_{\mathbf{R}} \left[\frac{x}{a_{\mathbf{R}}} \text{Re}(\hat{u}_y \hat{u}_x^*) + \frac{y}{a_{\mathbf{R}}} \text{Re}(\hat{u}_y \hat{u}_y^*) + \frac{z}{a_{\mathbf{R}}} \text{Re}(\hat{u}_y \hat{u}_z^*) \right] dA_{\mathbf{R}},$$

$$F_z = \mathbf{F} \cdot \mathbf{e}_z = \int_{\mathbf{R}} \frac{1}{4} \frac{z}{a_{\mathbf{R}}} \left[\rho_0 \text{Re}(\hat{\mathbf{u}} \cdot \hat{\mathbf{u}}^*) - \frac{1}{\rho_0 c_s^2} \text{Re}(\hat{p}\hat{p}^*) \right] dA_{\mathbf{R}}$$

$$- \frac{\rho_0}{2} \int_{\mathbf{R}} \left[\frac{x}{a_{\mathbf{R}}} \text{Re}(\hat{u}_z \hat{u}_x^*) + \frac{y}{a_{\mathbf{R}}} \text{Re}(\hat{u}_z \hat{u}_y^*) + \frac{z}{a_{\mathbf{R}}} \text{Re}(\hat{u}_z \hat{u}_z^*) \right] dA_{\mathbf{R}},$$

$$T_x = \mathbf{T} \cdot \mathbf{e}_x = -\frac{\rho_0}{2} \int_{\mathbf{R}} \frac{xy}{a_{\mathbf{R}}} \text{Re}(\hat{u}_x \hat{u}_z^*) + \frac{y^2 - z^2}{a_{\mathbf{R}}} \text{Re}(\hat{u}_y \hat{u}_z^*) + \frac{yz}{a_{\mathbf{R}}} \text{Re}(\hat{u}_z \hat{u}_z^*) \quad (10)$$

$$- \frac{x^2}{a_{\mathbf{R}}} \text{Re}(\hat{u}_x \hat{u}_y^*) - \frac{yz}{a_{\mathbf{R}}} \text{Re}(\hat{u}_y \hat{u}_y^*) dA_{\mathbf{R}},$$

$$\begin{aligned}
T_y = \mathbf{T} \cdot \mathbf{e}_y &= -\frac{\rho_0}{2} \int_R \frac{xz}{a_R} \text{Re}(\hat{u}_x \hat{u}_x^*) + \frac{yz}{a_R} \text{Re}(\hat{u}_x \hat{u}_y^*) + \frac{z^2 - x^2}{a_R} \text{Re}(\hat{u}_x \hat{u}_z^*) \\
&\quad - \frac{xy}{a_R} \text{Re}(\hat{u}_y \hat{u}_z^*) - \frac{xz}{a_R} \text{Re}(\hat{u}_z \hat{u}_z^*) dA_R, \\
T_z = \mathbf{T} \cdot \mathbf{e}_z &= -\frac{\rho_0}{2} \int_R \frac{x^2 - y^2}{a_R} \text{Re}(\hat{u}_x \hat{u}_y^*) + \frac{xy}{a_R} \text{Re}(\hat{u}_y \hat{u}_y^*) + \frac{xz}{a_R} \text{Re}(\hat{u}_y \hat{u}_z^*) \\
&\quad - \frac{xy}{a_R} \text{Re}(\hat{u}_x \hat{u}_x^*) - \frac{yz}{a_R} \text{Re}(\hat{u}_x \hat{u}_z^*) dA_R.
\end{aligned}$$

Based on the above equations, we can evaluate the radiation force and torque used to validate the theoretical model by inserting the acoustic pressure field and particle velocity obtained from the numerical simulations. Note that it should typically take about ten minutes (for a common personal computer) to finish a fully three-dimensional finite element computation for the acoustic pressure and particle velocity, which are then brought into Eqs. (9) and (10) for the numerical radiation force and torque exerted on the complex particles.

2.3. Lyapunov analysis in particle rotation stability

Affected by the acoustic radiation torque and the drag torque, the rotation trajectories can be predicted by the moment balance between the radiation torque (Eq. (4b)) and the drag torque (Eq. (5)), while the particles orientation will eventually stabilize in an equilibrium state where the net torque is zero. Nonetheless, the existence of equilibrium does not necessarily imply stability. Therefore, the Lyapunov stability analysis helps judge whether the current state is a stable equilibrium.

Consider that a cluster of N_p particles are being a potential equilibrium state in a host fluid. According to the Lyapunov stability theorem [31,45], the equation of motion of Eq. (6) near equilibrium is equivalent to its linear approximation (i.e., $\mathbf{T}_{\text{rad}} \approx \mathbf{K} \cdot \Delta\boldsymbol{\theta}_{\text{rot}}$) as

$$I \frac{d^2 \Delta\boldsymbol{\theta}_{\text{rot}}}{dt^2} = \mathbf{K} \cdot \Delta\boldsymbol{\theta}_{\text{rot}} - \mathbb{L} \cdot \frac{d\Delta\boldsymbol{\theta}_{\text{rot}}}{dt}, \quad (11)$$

where $\mathbf{K}$ is the radiation torque stiffness matrix, $\mathbb{L}$ is the diagonal assembled drag matrix, and $\Delta\boldsymbol{\theta}_{\text{rot}}$ are the angular disturbance of these particles from their equilibrium positions. These variables satisfy:

$$\begin{aligned}
K_{ij} &= \frac{\partial T_{\text{rad},i}}{\partial \Delta\theta_{\text{rot},j}}, & \mathbb{L} &= \text{diag}(\mathbf{L}_1, \dots, \mathbf{L}_{N_p}), \\
\mathbf{T}_{\text{rad}} &= [T_{\text{rad},x1}, T_{\text{rad},y1}, T_{\text{rad},z1}, \dots, T_{\text{rad},xN_p}, T_{\text{rad},yN_p}, T_{\text{rad},zN_p}]^T, \\
\Delta\boldsymbol{\theta}_{\text{rot}} &= [\Delta\theta_{\text{rot},x1}, \Delta\theta_{\text{rot},y1}, \Delta\theta_{\text{rot},z1}, \dots, \Delta\theta_{\text{rot},xN_p}, \Delta\theta_{\text{rot},yN_p}, \Delta\theta_{\text{rot},zN_p}]^T.
\end{aligned}$$

With the help of similarity transformation, we decouple Eq. (11) to a set of second-order constant coefficient differential equations using eigenmatrix $\mathbf{V} = [\mathbf{V}_1, \dots, \mathbf{V}_{N_p}]$, where $\mathbf{V}_i$ is the i -th eigenvector of $\mathbf{K}$, corresponding to the i -th eigenvalue $K_i = (\mathbf{V}^{-1}\mathbf{K}\mathbf{V})_{ii}$:

$$I \frac{d^2 \vartheta_i}{dt^2} + D_i \frac{d\vartheta_i}{dt} - K_i \vartheta_i = 0, \quad (12)$$

where $D_i = (\mathbf{V}^{-1}\mathbb{L}\mathbf{V})_{ii} > 0$ represents the system damping for the i -th eigenvalue, $\boldsymbol{\vartheta} = \mathbf{V}^{-1} \cdot \Delta\boldsymbol{\theta}_{\text{rot}}$ or $\Delta\boldsymbol{\theta}_{\text{rot}} = \mathbf{V} \cdot \boldsymbol{\vartheta}$. By solving Eq. (12), we analyze the stability characteristics of particles in current

equilibrium positions. In the following, we discuss several stability categories that depend upon the nature of $K_i = \text{Re}(K_i) + \text{Im}(K_i) \cdot i$.

2.3.1. Real eigenvalues

Here, $K_i = \text{Re}(K_i)$ with $\text{Im}(K_i) = 0$, there are three scenarios for the solutions, depending on the discriminant $\chi = D_i^2 + 4IK_i$. In all three cases, the general solution depends on two constants C_1 and C_2 , determined by the initial conditions. If $\chi > 0$ or $K_i > \frac{-D_i^2}{4I}$, the characteristic polynomial has two distinct real roots. In this case, the general solution is

$$\vartheta_i = C_1 \exp \left[\left(\frac{-D_i + \sqrt{D_i^2 + 4IK_i}}{2I} \right) t \right] + C_2 \exp \left[\left(\frac{-D_i - \sqrt{D_i^2 + 4IK_i}}{2I} \right) t \right]. \quad (13)$$

To make the angular disturbance converge with time, the coefficients before time t of Eq. (13) must be negative, which yields $K_i < 0$. If $K_i > 0$, the disturbance is diverging with time and thus corresponds to an unstable equilibrium. If $\chi = 0$ or $K_i = \frac{-D_i^2}{4I}$, the characteristic polynomial has a double root, and the general solution is

$$\vartheta_i = (C_1 + C_2 t) \exp \left[\left(\frac{-D_i}{2I} \right) t \right]. \quad (14)$$

In this case, the coefficients $\frac{-D_i}{2I}$ is negative, meaning that the angular disturbance always converges with time. If $\chi < 0$ or $K_i < \frac{-D_i^2}{4I}$, the characteristic polynomial has two complex conjugate roots, and the general solution is

$$\vartheta_i = \left[C_1 \cos \left(\frac{\sqrt{-4IK_i - D_i^2}}{2I} t \right) + C_2 \sin \left(\frac{\sqrt{-4IK_i - D_i^2}}{2I} t \right) \right] \exp \left[\left(\frac{-D_i}{2I} \right) t \right]. \quad (15)$$

In this case, the coefficients $\frac{-D_i}{2I}$ is negative, and the angular disturbance will converge with time.

Considering the above three different solutions, we conclude that for $K_i \geq 0$, a minor angular disturbance will induce instability. For $K_i < 0$, the particles will maintain stability.

2.3.2. Complex eigenvalues

By contrast, if $K_i = \text{Re}(K_i) + \text{Im}(K_i) \cdot i$ with $\text{Im}(K_i) \neq 0$. There are two cases for the solutions, depending on the imaginary part of the eigenvalue K_i . If $\text{Im}(K_i) > 0$, the general solution is

$$\begin{aligned} \vartheta_i = C_1 \exp \left\{ \left[-\frac{D_i}{2I} + \sqrt{\frac{\text{Im}(K_i)}{2I} \left(\sqrt{\lambda_0^2 + 1} - \lambda_0 \right)} + i \sqrt{\frac{\text{Im}(K_i)}{2I \left(\sqrt{\lambda_0^2 + 1} - \lambda_0 \right)}} \right] t \right\} \\ + C_2 \exp \left\{ \left[-\frac{D_i}{2I} - \sqrt{\frac{\text{Im}(K_i)}{2I} \left(\sqrt{\lambda_0^2 + 1} - \lambda_0 \right)} - i \sqrt{\frac{\text{Im}(K_i)}{2I \left(\sqrt{\lambda_0^2 + 1} - \lambda_0 \right)}} \right] t \right\}, \end{aligned} \quad (16)$$

where $\lambda_0 = -\frac{4I \cdot \text{Re}(K_i) + D_i^2}{4I \cdot \text{Im}(K_i)}$. By contrast, if $\text{Im}(K_i) < 0$, the general solution becomes

$$\begin{aligned} \vartheta_i = C_1 \exp \left\{ \left[-\frac{D_i}{2I} + \sqrt{\frac{-\text{Im}(K_i)}{2I} \left(\sqrt{\lambda_0^2 + 1} + \lambda_0 \right)} - i \sqrt{\frac{-\text{Im}(K_i)}{2I \left(\sqrt{\lambda_0^2 + 1} + \lambda_0 \right)}} \right] t \right\} \\ + C_2 \exp \left\{ \left[-\frac{D_i}{2I} - \sqrt{\frac{-\text{Im}(K_i)}{2I} \left(\sqrt{\lambda_0^2 + 1} + \lambda_0 \right)} - i \sqrt{\frac{-\text{Im}(K_i)}{2I \left(\sqrt{\lambda_0^2 + 1} + \lambda_0 \right)}} \right] t \right\}. \end{aligned} \quad (17)$$

In order to ensure that the disturbance converges with time, the eigenvalues K_i should ensure that the coefficients $-\frac{D_i}{2I} + \sqrt{\left(\sqrt{\lambda_0^2 + 1} - \lambda_0 \right) \frac{\text{Im}(K_i)}{2I}}$ in Eq. (16), and the coefficients $-\frac{D_i}{2I} + \sqrt{\left(\sqrt{\lambda_0^2 + 1} + \lambda_0 \right) \frac{-\text{Im}(K_i)}{2I}}$ in Eq. (17) are negative. These requirements lead to $\text{Re}(K_i) < -\frac{I}{D_i^2} \text{Im}(K_i)^2$ for a stable equilibrium; otherwise, $\text{Re}(K_i) \geq -\frac{I}{D_i^2} \text{Im}(K_i)^2$ corresponds to an unstable equilibrium.

2.3.3. Stability conditions

Overall considering the situations in Sections 2.3.1 and 2.3.2, we conclude the conditions for the stable equilibrium and the unstable equilibrium as

$$\begin{cases} \text{Stable equilibrium: } \text{Re}(K_i) < -\frac{I}{D_i^2} \text{Im}(K_i)^2 \\ \text{Unstable equilibrium: } \text{Re}(K_i) \geq -\frac{I}{D_i^2} \text{Im}(K_i)^2 \end{cases}. \quad (18)$$

When the eigenvalues K_i satisfy the condition of the stable equilibrium, a minor disturbance will be weakened with time increasing, and the particles will be attracted toward these stable equilibrium positions, thereby maintaining the stability. By contrast, if the eigenvalues K_i satisfy the condition of the unstable equilibrium, even a minor angular disturbance will induce instability. The corresponding equilibrium state is unstable even though the net torque is zero, and the particles can escape from the unstable equilibrium positions at a small cost.

3. Design and characteristics of acoustic rotational tweezer

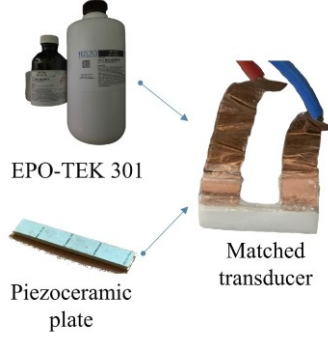


FIG. 2. Design of the ultrasound matched transducer (UMT). The transducer is composed of a piezoceramic plate and the Al_2O_3 powder epoxy composites applied to its front face. The electrodes of the piezoceramic plate are connected with copper foils, which are welded to the electrical wires to transmit power.

In this study, a pair of ultrasound matched transducers (UMTs) are used to produce the standing BAW that includes an ultrasonic piezoceramic plate (25 mm × 4 mm × 1 mm, APC850, APC International, Ltd.) and a matching layer made of mixing of Al_2O_3 in an epoxy matrix (EPO-TEK 301, Epoxy Technology, Inc.) [42,43] as illustrated in Fig. 2. To weaken the dispensable reflections, the matching layer is designed to be a layer with a thickness that is an odd multiple of a quarter wavelength of incoming waves to minimize the reflections of the incoming acoustic energy [44]. Moreover, this layer is able to prevent unnecessary contact between the transducers and the host fluid, which may cause a short circuit and damage the experimental equipment. The resonance frequency of UMTs is measured and calibrated by a vibration measurement system. A UMT is driven by a function generator (continuous sinusoidal signal, AFG3022C, Tektronix, Inc.), and the vibrating frequencies and the physical displacements are captured by a laser vibrometer (NLV-2500, Polytec, Ltd.). These values are then transformed into continuous sinusoidal voltage signals and visualized and recorded by a digital oscilloscope (DS1102E, RIGOL Technologies, Inc.). At the resonance frequency, the physical displacement of the transducer should be maximum and thus highest transformed voltage signal recorded in the digital oscilloscope. The experimental devices and schematic diagram of the measuring system are shown in Fig. 3. In order to determine the resonance frequency of the transducers, we measure the voltage amplitudes over a wide range of exciting frequencies (1 MHz to 3 MHz). For each frequency group, we measure the voltage signals eight times to increase the reliability. The measuring resonance characteristic curves for two selected UMTs are shown in Fig. 4. Specifically, the resonance frequency of the transducers is found at 2.04 MHz, which is reasonably considered as the excitation frequency for later acoustically-manipulating experiments. Here, we employ an excitation frequency of about 2 MHz to drive the transducers. Therefore, the thickness of the matching layer is set as the third quarter of the wavelength of incoming waves, 1.0 mm (with the speed of sound being 2670 m/s for the matching layers [42]).

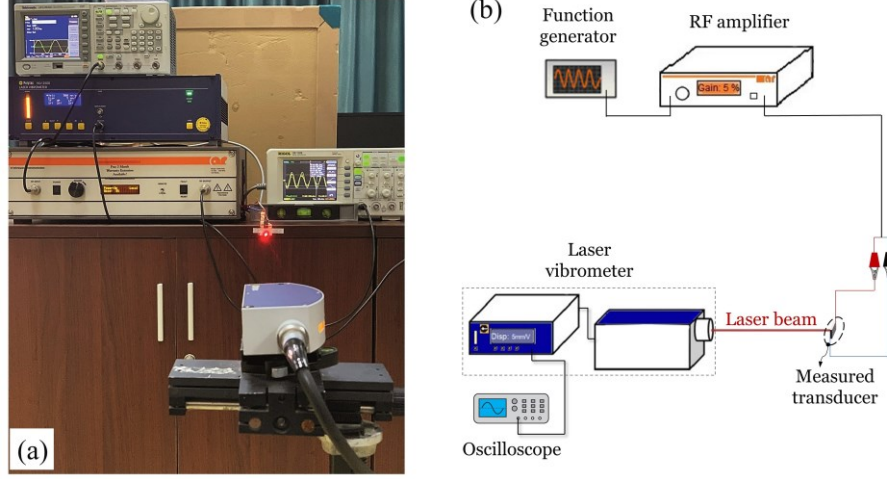


FIG. 3. The vibration measurement system used to confirm the resonance frequency of the UMTs. A vibration measurement system that includes a function generator, a RF amplifier, a laser vibrometer, and the measured transducer. The measured data is record by a digital oscilloscope. (a) Photograph of the vibration measurement system. (b) Schematic diagram of the vibration measurement system.

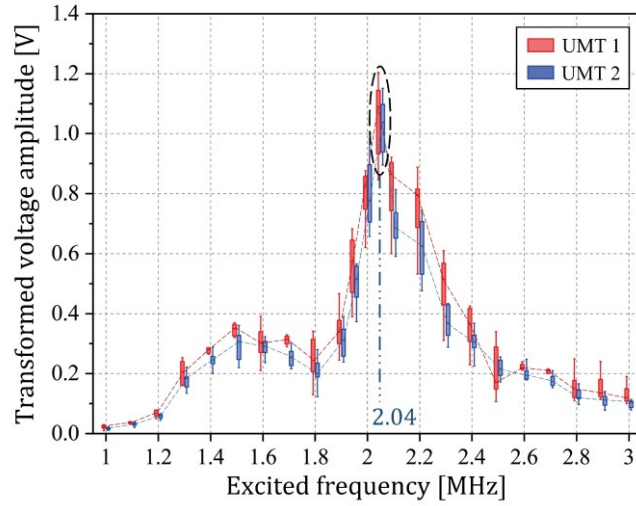


FIG. 4. Determinization of the resonance frequency of the UMTs. The resonance patterns of the ultrasound matched transducers (UMTs) used in generating bulk acoustic standing wave. The transformed voltage amplitude of different UMTs across a range of excitation frequencies. The input voltage from the function generator and the power gain percentage set in amplifier are $1 V_{pp}$ and 5%. Data is collected from eight separate measurements per group and the measured voltage amplitudes are plotted with the mean, interquartile range and 95% confidence intervals. The blue and red dashed curves connect the mean values of each group. The resonance frequency of both UMTs are located at near 2.04 MHz.

To explore the rotational dynamics experimentally, we used a rotational BAW device consisting of a fixed outer module and a rotatable inner module (Fig. 1(d), see Fig. 5 for more details). The 3D-printed cylindrical inner module (White resin V4, Formlabs, Inc.) has a height of 5 mm, a radius of 30 mm, and a rectangular cavity of $28 \times 26 \times 5 \text{ mm}^3$ in its center. Two UMTs are flanked on the two interior parallel walls. The outer module is a 3D-printed cuboid of $80 \times 80 \times 5 \text{ mm}^3$ in size, which is fixed

on the observation platform of a metallurgical microscope (BA310MET, Motic Electric Group Co., Ltd.). The inner module is embedded in the outer module, precisely allowing smooth rotation. When the inner module is turned by hand on the two knobs, illustrated in Fig. 1(e), the standing wave pattern formed between the two UMTs changes direction. As revealed in later sections, non-spherical particles will rotate as a result to cater for the variations of the standing wavefield, as illustrated in Fig. 1(f).

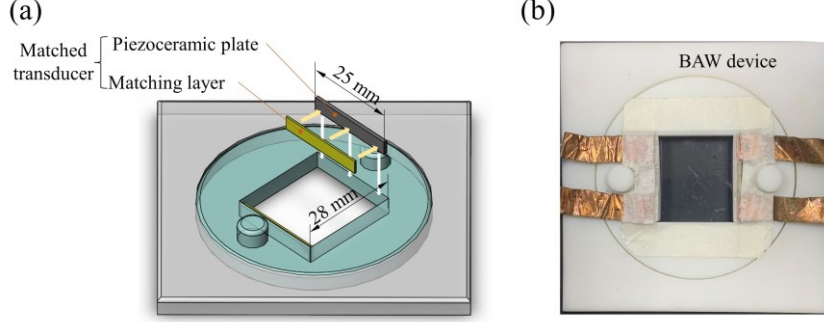


FIG. 5. Design of the BAW platform. (a) The schematic diagram of the BAW device that involves four matched transducers. (b) Photograph of the matched transducer (piezoceramic plate with epoxy matching layer as shown in Fig. 2) and the BAW device used in experiments.

A schematic diagram of the experimental apparatus used in this study is given in Fig. 6. First, a continuous sinusoidal signal ($f = 2.04$ MHz) is initially generated using a function generator (AFG3022C, Tektronix, Inc.), after which this signal is amplified by an RF power amplifier (150A100D, Amplifier Research, Corp.). Then, the amplified signal is supplied to the UMTs, wired in parallel to deliver their signals in phase. The cavity of the BAW device is filled with deionized water (mass density of $\rho_0 = 1000$ kg/m³ and adiabatic speed of sound of $c_s = 1481$ m/s) as a medium to form an acoustic standing wavefield. Subsequently, an irregular particle (ellipsoid or disc for an averaged radius of $a = 150$ μ m) made of polystyrene (PS) material with a mass density of $\rho_p = 1050$ kg/m³ and an adiabatic speed of sound of $c_p = 2340$ m/s is slowly injected into the BAW device with deionized water. The manipulated polystyrene particle is free suspended before supplying an electric signal to the transducers. We first adjust the microscope to visualize the particles better. Then, we dynamically capture the irregular particles using a $5\times$ objective lens on a metallurgical microscope (BA310MET, Motic Electric Group Co., Ltd.) and image by a color microscope camera (PRO205CCD, Motic Electric Group Co., Ltd.) when the particle is being transported or rotated after powering on the transducers. When a continuous electrical signal supplies to excite the UMTs, a pair of counterpropagating BAWs are generated, forming a standing wavefield with pressure node lines perpendicular to the direction of wave propagation (i.e., x_{PZT} -direction in Fig. 1(c)). The interference of counterpropagating BAWs will create a standing wavefield with pressure node lines perpendicular to the wave propagation direction (i.e., x_{PZT} -direction in Fig. 1(c)). The resonance frequency of UMTs is measured (~ 2.04 MHz) and used to maximize the radiated acoustic pressure (see Fig. 4). The transducers are then connected in parallel through wires and driven by a single function generator and amplifier.

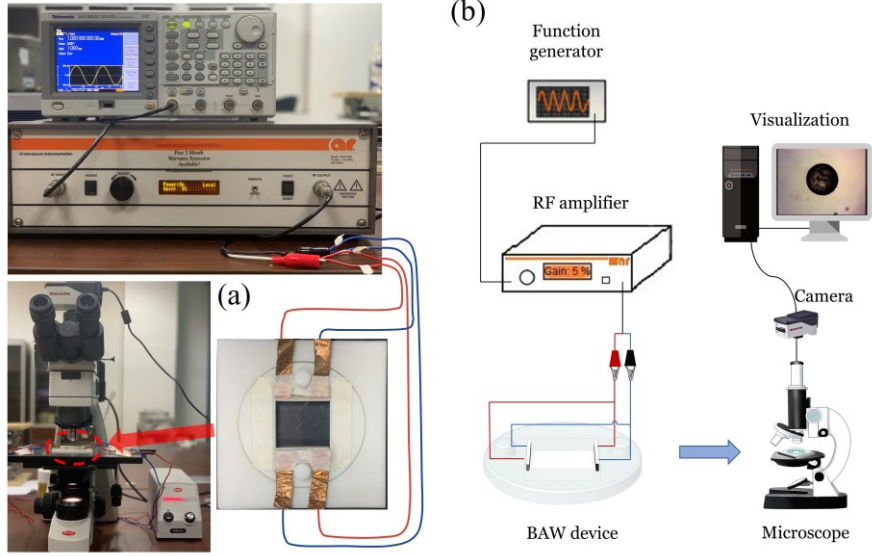


FIG. 6. The experimental setup for particle trapping and rotational manipulation. The system includes a function generator, a RF amplifier, the self-design BAW device, and the visualization devices (microscope, camera and computer). (a) Photograph of the acoustically manipulating experimental setup. (b) Schematic diagram of the corresponding experiment setup. Note that the excitation frequency is 2.04 MHz.

The radiation force traps the particle at node lines, where the force vanishes. Meanwhile, the particle will be rotated from initial position ‘1’ to final position ‘2’ by the radiation torque induced by the geometric asymmetry, as demonstrated in Fig. 1(c). In order to quantify the rotation, we define the angle between the symmetric axis (i.e., z -axis in black) of the particle when it is at initial position ‘1’ and the horizontal line (i.e., x_{PZT} -axis) as the incident angle, θ_{inc} , which also represents the orientation of the standing wavefield; it ranges from 0 to π , corresponding to $+x_{\text{PZT}}$ -axis and $-x_{\text{PZT}}$ -axis respectively. Additionally, the angle at which the particle rotates from the initial position ‘1’ to the final position ‘2’ is defined as the rotation angle of the particle, θ_{rot} . Consistently, the angle that the inner module rotates is defined as the rotation angle of the inner module, θ_{inr} (Figs. 1(e) and 1(f)). θ_{rot} and θ_{inr} are positive when the particle and the inner module rotate clockwise.

4. Results and discussion

4.1. Dynamic predictions of the complex particle under a standing wavefield

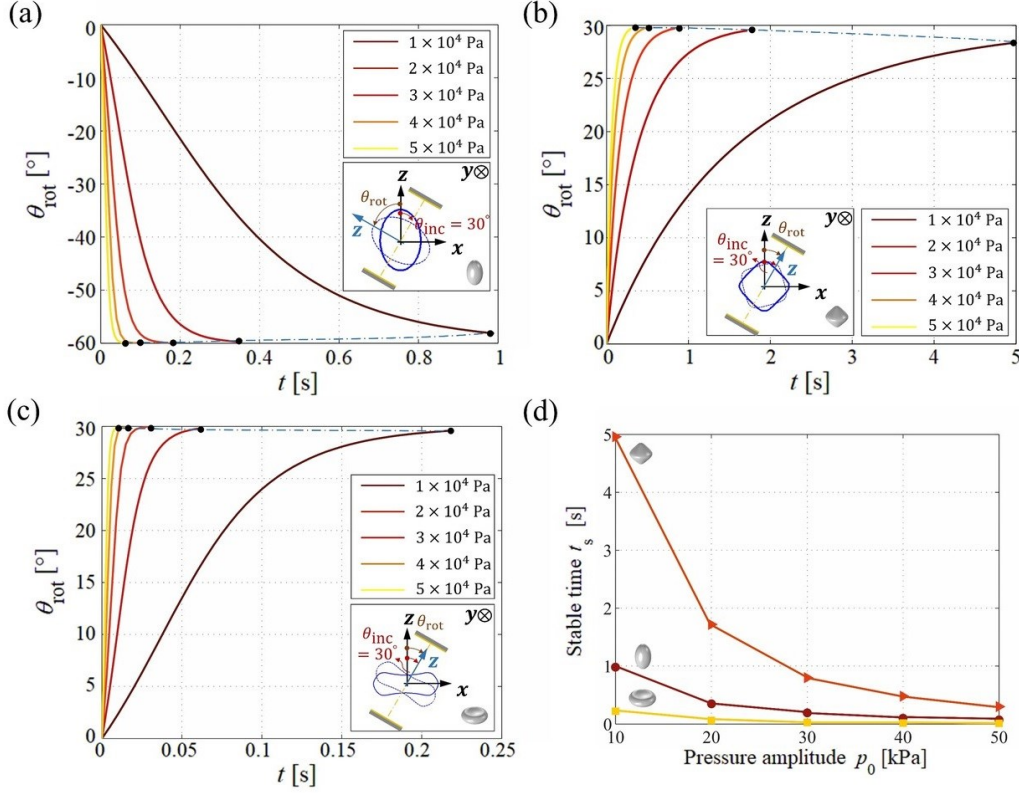


FIG. 7. Rotational dynamics under different pressure amplitudes. Theoretical predictions of the rotational dynamics depending on the pressure amplitude ranging from 1×10^4 Pa to 5×10^4 Pa for (a) ellipsoid, (b) diamond, and (c) disc. The additional schematic diagrams depict the initial (blue solid profiles) and the final (blue dashed profiles) positions of the corresponding particles. The black points in sub-figures (a), (b), and (c) represent the stable states, while the stable information is collected in sub-figure (d). In all cases, the incidence angle is set to $\theta_{\text{inc}} = 30^\circ$ and the average radius of these particles is set to $a = 150 \mu\text{m}$.

The time-dependent rotational dynamics, driven by radiation torque and impeded by viscous drag torque, is solved with a time step of $\Delta t = 0.01 \sim 0.001$ s. Figures 7(a), 7(b), and 7(c) present the theoretical predictions of the different irregular particles under different pressure amplitudes over time. The incidence angle is set to $\theta_{\text{inc}} = 30^\circ$. We start the predictions from $t = 0$ s with a time interval of Δt and end the calculations when the errors of the rotational angle, θ_{rot} , among two adjacent time steps is less than 1%. The period in stabilizing the geometries is defined as stabilization time, t_s . The theoretical stable time under different pressure amplitudes for various particles is concluded in Fig. 7(d). In the stable states, the symmetry axis (z-axis) of the ellipsoid is orthogonal to the orientation of the standing wave (counter-clockwise rotation, $\theta_{\text{rot}} = -60^\circ$), while the symmetry axis (z-axis) of diamond and disc is parallel to the orientation of the standing wave (clockwise rotation, $\theta_{\text{rot}} = 30^\circ$), though more stabilization time is required to stabilize the particles for smaller pressure amplitudes (Supplemental Movie SM1).

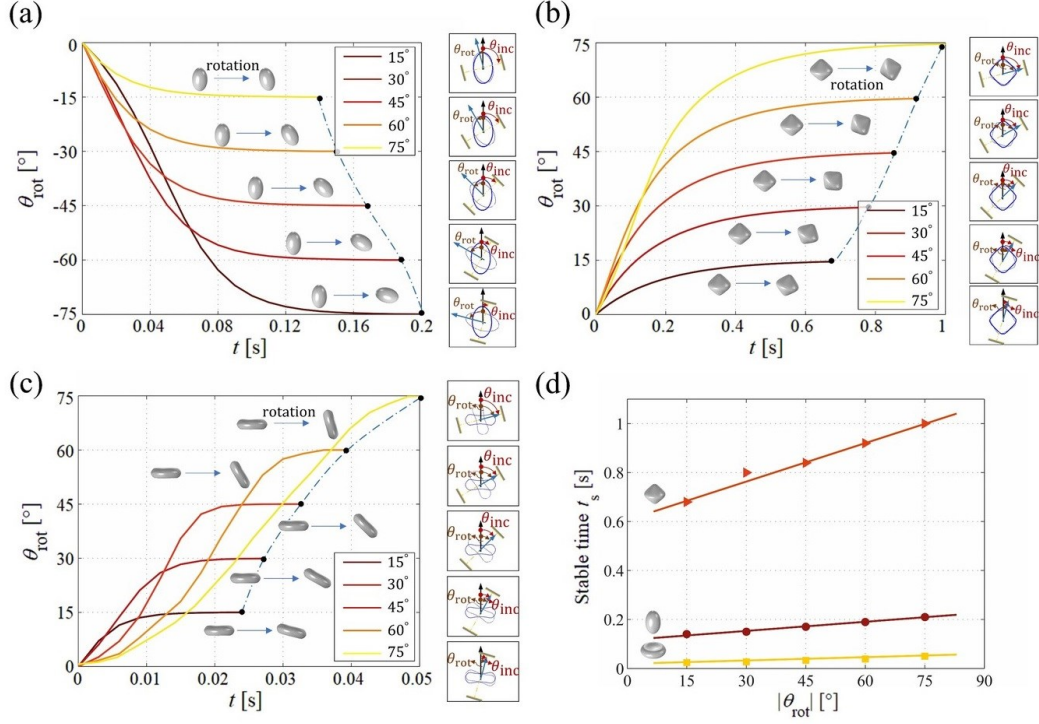


FIG. 8. Rotational dynamics under different incidence angles. Theoretical predictions of the rotational dynamics depending on the incidence angle range from $\theta_{\text{inc}} = 15^\circ$ to 75° for (a) ellipsoid, (b) diamond, and (c) disc. The additional schematic diagrams depict the initial (blue solid profiles) and the final (blue dashed profiles) positions of the corresponding particles. The black points in sub-figures (a), (b), and (c) represent the stable states. Sub-figure (d) gives the required stable time for different particles depending on rotation. In all cases, the pressure amplitude is set to 3×10^4 Pa and the average radius of these particles are set to $a = 150 \mu\text{m}$.

Similarly, Figs. 8(a), 8(b), and 8(c) present the theoretical predictions of the different irregular particles under different incidence angles, θ_{inc} , over time. The pressure amplitude is set to $\hat{p}_0 = 3 \times 10^4$ Pa. Figure 8(d) describes a linear relationship between the stable time and the magnitude of the rotational angle. It should be emphasized that the radiation torque drives the ellipsoid (z-axis) orthogonal to the orientation of the standing wave (orthogonal mode, $\theta_{\text{rot}} = \theta_{\text{inc}} - 90^\circ$), and makes the diamond and disc (z-axis) rotate parallel to the orientation of the standing wave (parallel mode, $\theta_{\text{rot}} = \theta_{\text{inc}}$). Furthermore, the experimental validation is available in Supplemental Movie SM2.

4.2. Two stabilization modes and acoustophoretic processes

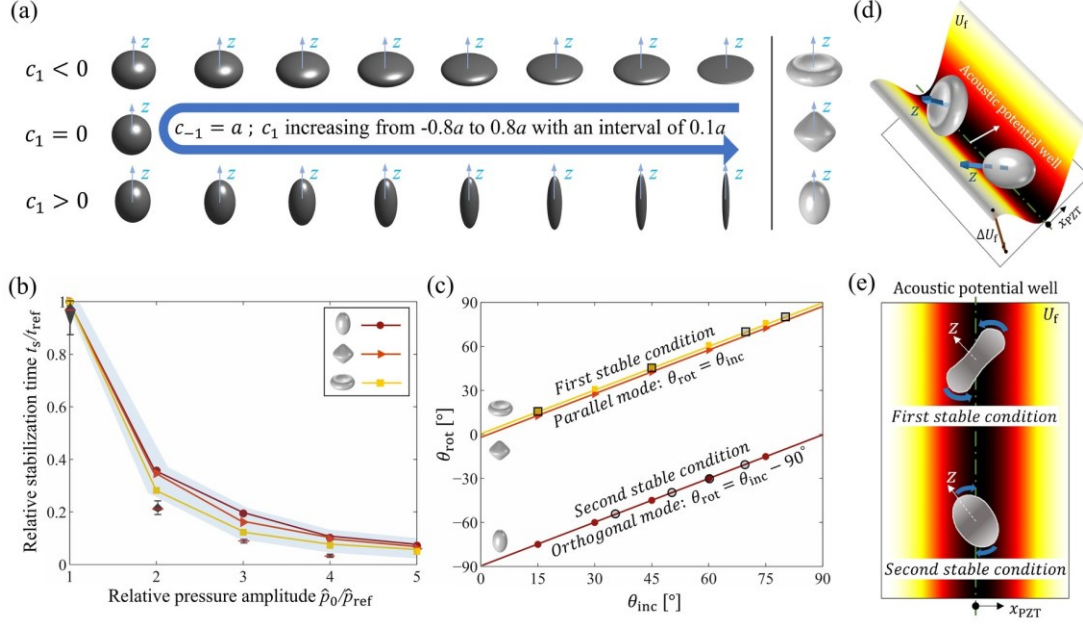


FIG. 9. Theoretical and experimental results of the stabilization time and two stabilization modes. (a) Quantification of the geometric features of a class of ellipsoidal particles using the first-order mapping coefficient c_1 . More complex geometries can be generally classified into $c_1 \in (-a, 0)$, $c_1 = 0$, and $c_1 \in (0, a)$. (b) Theoretical predictions (solid symbols) and experimental measurements (the error bars) of the stabilization time as a function of the incident pressure amplitude. The experimental data is collected from five separate measurements per group, and plotted with the mean, interquartile range, and 95% confidence intervals. The corresponding dynamic processes are given in Fig. 7. (c) Theoretical predictions (solid symbols) and experimental validations (transparent symbols) of the relationship between the rotation angle (θ_{rot}) and the incident angle (θ_{inc}), with corresponding dynamic processes given in Fig. 8. (d)-(e) The acoustic radiation-force potential field in the propagation plane within a half wavelength (ranging from $-\lambda/4$ to $\lambda/4$ centered at $x_{PZT} = 0$). The particles should fall into the potential well in a stable state and maintain the overall minimum potential energy by rotating the non-spherical particles, following two stabilization modes.

To analyze the effect of geometric asymmetry on particle rotation, we use the first-order mapping coefficient c_1 , while the higher-order mapping coefficients cannot be neglected for accurately calculating the radiation force and torque in Eq. (4). As demonstrated in Fig. 9(a), the mapping coefficients (i.e., $c_{-1} = a$ and $c_1 \in (-a, a)$, where a is the averaged radius of non-spherical particles) is used to quantify the geometric features of a class of ellipsoidal particles. Without loss of the generality, the effect of geometric asymmetry of the non-ellipsoidal particles is basically consistent with corresponding ellipsoidal particles with the same c_1 . The c_1 of ellipsoid, diamond, and disc discussed later are equal to $a/5$, 0 , and $-a/2$, respectively, which indicates that the effect of geometric asymmetry is close to the ellipsoidal particles with $c_1 = a/5$, $c_1 = 0$, and $c_1 = -a/2$. Note that the other non-ellipsoidal geometric features for diamond and disc are described by the higher-order mapping coefficients c_3 and c_5 , which may weaken or strengthen the radiation force and torque, therefore decelerating or accelerating the acoustophoretic and stabilizing processes. But these hardly change the end status of rotation. Specific mapping coefficients for several typical geometries discussed in this paper are given in Tab. S1. It can be found that the geometric features can be divided

into two major sub-groups with $c_1 \in (-a, 0)$ and $c_1 \in (0, a)$ in Fig. 9(a), indicating that the particles may be rotated and follow two stabilization modes.

The rotation process caused by the geometric asymmetry should eventually stabilize at the stable positions where the radiation torque vanishes (refer to Eq. (6)). The time required for a particle to rotate and reach the stable mode is analyzed in Fig. 9(b) as a function of the amplitude of the driving sound pressure. Both the sound pressure and the rotation time are presented in ratios to a reference case with subscripts _{ref}. The results indicate that a higher-pressure amplitude corresponds to a shorter period to stabilize a particle (i.e., stabilization time), following a parabolic pattern for different geometries. This is consistent with the parabolic relationship between the radiation torque and the pressure amplitude, according to Eq. (4b). A number of experimental measurements (the error bars in Fig. 9(b)) are provided; there is a complete agreement between the experiments and the predictions. Predictions of the dynamic rotation trajectory for various geometries under different pressure amplitudes (or $\hat{p}_0$) are shown in Fig. 7. Furthermore, we recorded the dynamic rotation process of an ellipsoid ($t_{\text{ref}} \sim 30$ s) under different pressure amplitudes in Supplemental Movie SM1. By increasing the pressure amplitude ($\hat{p}_0$), the particles can stabilize as rapidly as ~ 1 sec, a time scale particularly convenient for manipulation by the human hand. Figure 9(c) clarifies the rotational routes of different particles; these results tell that at different incident angles (θ_{inc}), the corresponding rotation angles (θ_{rot}) of the trapped particles experience before being stabilized. The results definite the effect of asymmetry that the symmetric axis (z-axis) of the stabilized particles is either parallel (first stable condition, $\theta_{\text{rot}} = \theta_{\text{inc}}$) or orthogonal (second stable condition, $\theta_{\text{rot}} = \theta_{\text{inc}} - 90^\circ$) to the orientation of the standing wave (x_{PZT} -axis). Dynamic predictions of the rotation trajectory for various geometries at different incident angles (θ_{inc}) are shown in Fig. 8, while a verified experimental video that demonstrates the rotation process of an ellipsoid (corresponding to black circle marks in Fig. 9(c)) and a disc (corresponding to black square marks in Fig. 9(c)) is provided in Supplemental Movie SM2.

Physically, there is an acoustic radiation-force potential field U_f , defined as $\mathbf{F}_{\text{rad}} = -\nabla U_f$ analogously to Gor'kov's theory. The acoustic radiation-force potential field U_f as a function of position x_{PZT} can be derived by inserting $a_{nm} = -4\pi\sin(kx_{\text{PZT}} + n\pi/2)Y_n^m(0,0)^*$ given in Eq. (3a) into Eq. (4a). Consider that $a_{nm} \cdot a_{n+1,m} \sim -\sin(2kx_{\text{PZT}})$ and $a_{nm} \cdot a_{n-1,m} \sim -\sin(2kx_{\text{PZT}})$, the radiation force field follows $\mathbf{F}_{\text{rad}} \sim -\hat{p}_0^2 \sin(2kx_{\text{PZT}})$; integration of the radiation force field over x_{PZT} found the acoustic radiation-force potential as $U_f \sim -\hat{p}_0^2 \cos(2kx_{\text{PZT}})$, where the potential well of $\min(U_f)$ is found at $x_{\text{PZT}} = 0$, as illustrated in Fig. 9(d). Correspondingly, the potential barrier should satisfy $\Delta U_f \sim 2\hat{p}_0^2$, which indicates that higher sound pressure amplitude prevents particles transition among adjacent potential wells, serving a more stable acoustic tweezer. The particles will sink to the acoustic radiation-force potential wells where minimum energy is required to maintain the current state. In order to arrive at a minimum potential state, they should rearrange their attitude to make as many entities as possible towards the potential wells, as depicted in Fig. 9(e).

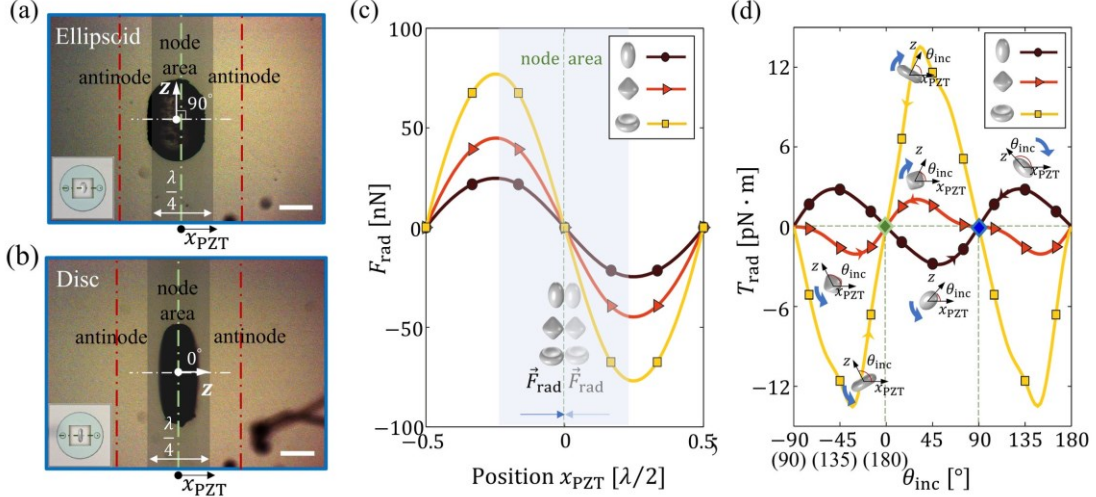


FIG. 10. Non-spherical particles trapping and their rotational dynamics. Experimental images show the stabilized positions of (a) an ellipsoid and (b) a disc after supplying power to the UMTs in the BAW device. Both ellipsoid and disc are of the averaged radius at $a \approx 150 \mu\text{m}$. (c) The theoretical predictions (solid lines, Eq. (4a)) and numerical simulations (symbols) of the radiation force along x_{PZT} direction. The radiation force gathers the particles at the pressure node region or the acoustic radiation-force potential well ($x_{\text{PZT}} = 0$). (d) In the potential well ($x_{\text{PZT}} = 0$), the theoretical predictions (solid lines, Eq. (4b)) and numerical simulations (symbols) of the radiation torque. The radiation torque dynamically rotates the particles (θ_{rot}) towards the stabilized positions, following the modes described in Fig. 9(c). Scale bar = $200 \mu\text{m}$.

Specifically, particle rotational dynamics is demonstrated in the BAW device integrated in a metallurgical microscope (see Fig. 6) to illustrate the fact that our approach can be transposed to standard microbiology experiments. As one can see, upon activation of the driving signal, the polystyrene (PS) ellipsoid (Fig. 10(a), average radius $a \approx 150 \mu\text{m}$ with size parameter $ka \approx 1.3$) and the PS disc (Fig. 10(b), average radius $a \approx 150 \mu\text{m}$ with size parameter $ka \approx 1.3$) are both instantaneously trapped in the potential wells by the radiation force, which is consistent with the theoretically predicted patterns in Fig. 10(c). Note that the radiation force gathers the non-spherical particles towards the node region regardless of the differences in the incident angle and the particle geometry (see Fig. S2). Meanwhile, in the acoustic radiation-force potential well ($x_{\text{PZT}} = 0$), the radiation torque rotates the irregular particles (specifically, $T_{\text{rad}} > 0$ for clockwise rotation, $\theta_{\text{rot}} > 0$; $T_{\text{rad}} < 0$ for counter-clockwise rotation, $\theta_{\text{rot}} < 0$), and finally stabilizes them at corresponding stable positions as depicted in Fig. 10(d), satisfying two stable conditions described in Fig. 9(c). In Supplemental Movie SM2, experimental videos are provided to demonstrate the stabilization processes in different inclined angles. As shown in Fig. 10(a), the ellipsoid is stabilized at the second stable position (the orthogonal mode), where the z -axis is orthogonal to the x_{PZT} -axis. In this equilibrium moment, $\theta_{\text{inc}} = 90^\circ$ and $T_{\text{rad}} = 0$, which corresponds to the stable state highlighted by a blue rhombic symbol in Fig. 10(d). By contrast, the disc turns to the first stable position (the parallel mode) with $\theta_{\text{inc}} = 0$ and $T_{\text{rad}} = 0$, as shown in Fig. 10(b); this stable state is marked by a green rhombic symbol in Fig. 10(d). Note that the analytical and experimental results for the disc are consistent with a previous experimental study for the red blood cells [46]. Furthermore, a numerical scheme is presented to validate the predicted radiation force and torque based on Eqs. (9) and (10). These calculations show

an estimation of the force and torque of the order of, respectively, 50 pN and 3~12 pN · m under pressure amplitude $\hat{p}_0 = 3 \times 10^4$ Pa. The complete agreement of these results confirms the ability of the developed theory to describe experimentally observed effects.

4.3. Stability analysis for particle rotation manipulation

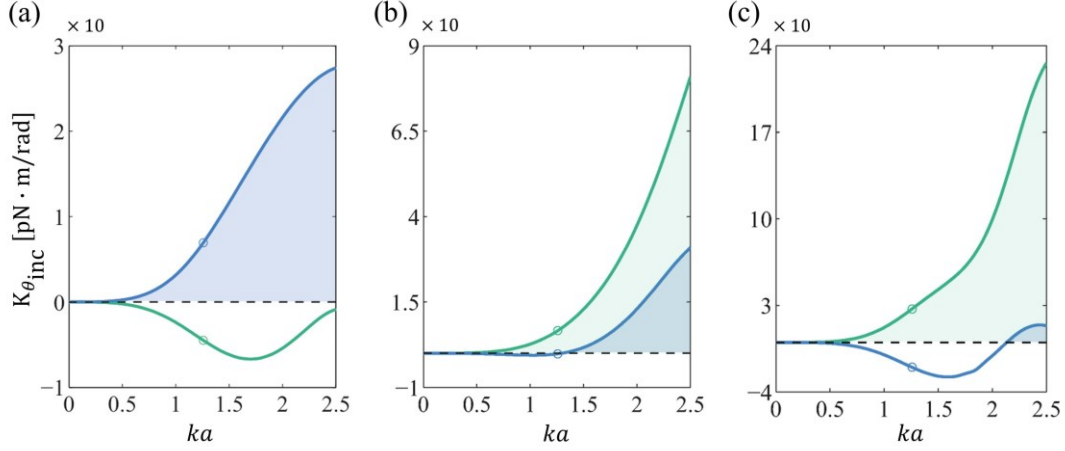


FIG. 11. Lyapunov stability analysis. The rotational stability analysis in the two potential equilibrium positions depends on the size parameter for (a) the ellipsoid, (b) the diamond, and (c) the disc, respectively. The green and blue lines indicate that the particles located in the equilibrium states, $T_{\text{rad}} = 0$, marked by green ($\theta_{\text{inc}} = 0^\circ$) and blue ($\theta_{\text{inc}} = 90^\circ$) rhombic symbols in Fig. 10(d), respectively. A positive eigenvalue $K_{\theta_{\text{inc}}}$ corresponds to a stable equilibrium state, while the negative regions represent the unstable equilibrium states. The highlighted circles here correspond to the averaged radius ($a \approx 150 \mu\text{m}$) of particles used in experiments of Fig. 10(a) and 10(b), and theoretical and numerical calculations in Figs. 10(c) and 10(d). Scale bar = $200 \mu\text{m}$.

The radiation torque drives particles towards a potential equilibrium position where they experience zero net torque. Nonetheless, the existence of equilibrium does not necessarily imply stability. Upon a minor perturbation, particles will escape from an unstable equilibrium state, and are attracted to a stable equilibrium state. Hence, a rigorous stability analysis is essential to distinguish the stable equilibrium states from the unstable ones.

Here, we consider the rotational stability of a single particle that mainly rotates on the wave propagating plane as illustrated in Fig. 1(c). Consider that in any equilibrium state, most of the entities of the particle should first sink in the acoustic radiation-force potential well (Figs. 9(d) and 9(e)), the radiation torque perpendicular to the wave propagating plane is the most significant, i.e., $\mathbf{T}_{\text{rad}} \rightarrow T_{\text{rad},y}$. Hence, a three-dimensional rotational problem then simplifies to a one-dimensional rotational problem. The stability formula reduces

$$I \frac{d^2 \Delta \theta_{\text{rot}}}{dt^2} + L_{22} \frac{d \Delta \theta_{\text{rot}}}{dt} - K_y \Delta \theta_{\text{rot}} = 0. \quad (19)$$

In the Lyapunov sense (refer to Eq. (18)), the stability is fully governed by the eigenvalues K_i (or here $K_y = \frac{\partial T_{\text{rad}}}{\partial \Delta \theta_{\text{rot}}}$) by

$$\text{Re}(K_i) < -\frac{I}{D_i^2} \text{Im}(K_i)^2 \text{ or } \text{Re}(K_y) < -\frac{I}{L_{22}^2} \text{Im}(K_y)^2. \quad (20)$$

It can be found from this stable condition that sufficiently strong damping (D_i or L_{22}) can remove the excess momentum energy, thus preventing energy accumulation, and render the particle stable easier,

i.e., a wider feasible domain of Eq. (20) with the same moment of inertia (I). Here, the stiffness K_y is a real number, and thus a stable equilibrium should satisfy $K_y < 0$. In the acoustic radiation-force potential well, the radiation torque is a function of the incidence angle θ_{inc} (refer to Eqs. (4b) and (3b)), we connect θ_{inc} with the stability by introducing $K_{\theta_{\text{inc}}} = \frac{\partial T_{\text{rad}}}{\partial \Delta \theta_{\text{inc}}}$. Based on the definitions, a positive net torque will increase θ_{rot} , leading to $\Delta \theta_{\text{rot}} > 0$ and particle clockwise rotation, while this tends to decrease the incidence angle θ_{inc} , resulting in $\Delta \theta_{\text{inc}} < 0$. Hence, $K_y < 0$ is equivalent to $K_{\theta_{\text{inc}}} > 0$. In this sense, a stable equilibrium not only requires the net torque to be zero but also $K_{\theta_{\text{inc}}} > 0$. This is consistent with the experimental and the theoretical results that the disc stabilizes at $\theta_{\text{inc}} = 0$, while the ellipsoid is at $\theta_{\text{inc}} = 90^\circ$ in Figs. 10(a), 10(b), and 10(d).

Figures 11(a) to 11(c) plot $K_{\theta_{\text{inc}}}$ against the particle size parameter (ka) in two potential equilibrium states (i.e., $T_{\text{rad}} + T_{\text{drag}} = 0$), $\theta_{\text{inc}} = 0$ (green line) and $\theta_{\text{inc}} = 90^\circ$ (blue line) corresponding to the positions marked by the green and the blue rhombic symbols in Fig. 10(d). When $K_{\theta_{\text{inc}}} > 0$, the particles maintain the stability and ignore the minor disturbance. By contrast, if $K_{\theta_{\text{inc}}} < 0$, a slight disturbance will induce instability, and the particles will escape from these unstable equilibrium states. Additionally, it is possible to switch an unstable equilibrium state to a stable equilibrium state by increasing the particle size. Physically, they are too large to sink into the acoustic radiation-force wells completely, and thus the stability boundary becomes blurred.

4.4. Desirable and controllable rotation of complex particles

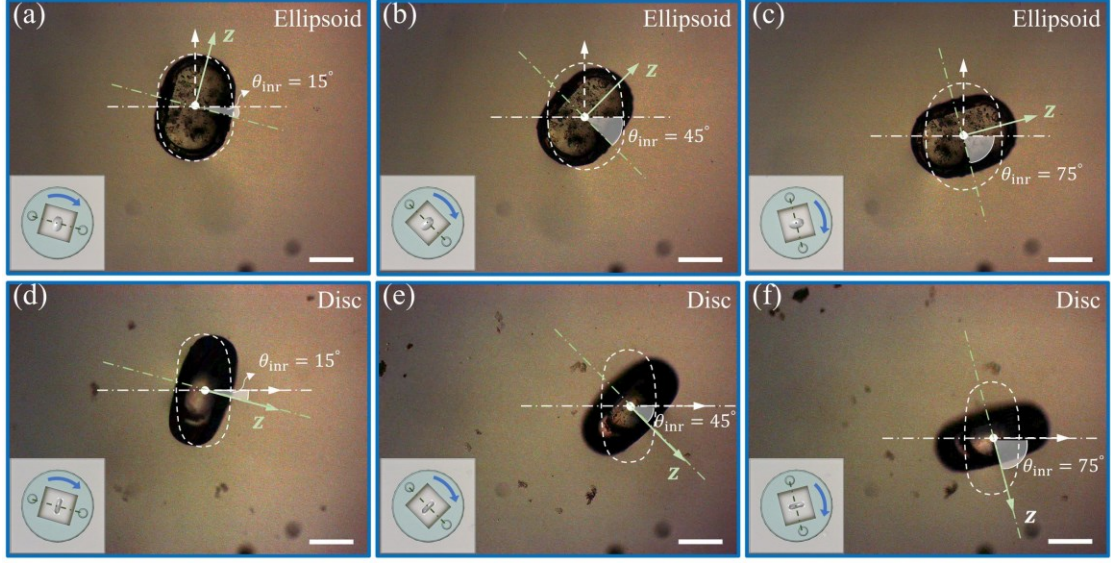


FIG. 12. Acoustic lock and rotation of various particles. Controllable rotation of (a)-(c) a PS ellipsoid and (d)-(f) a PS disc by manually rotating the inner module of the BAW device at (a), (d) $\theta_{\text{inr}} = 15^\circ$, (b), (e) $\theta_{\text{inr}} = 45^\circ$, and (c), (f) $\theta_{\text{inr}} = 75^\circ$. The white and green dash-dot lines represent the initial and final orientation of the standing wavefield, respectively. The white dashed curves depict the initial states of corresponding particles. Experimental videos can be found in Supplemental Movies SM3 and SM4. Scale bar = 200 μm .

One of the key abilities enabled by the rotational tweezer is the capture and rotation of particles in desirable directions. As an illustration, Fig. 12 shows experimental images where an ellipsoidal particle ($ka \approx 1.3$) and a disc particle ($ka \approx 1.3$) are rotated at any desirable directions by manually controlling the rotation angle of the inner module (θ_{inr}) of the BAW device (See Fig. 1(e)). The rotation of the inner module will alter the orientation of the standing wavefield, thereby rotating the particles to cater for these changes. In this process, the particles remain in a stable equilibrium position where the radiation torque is zero and the corresponding eigenvalues are positive, as shown in Figs. 10(d), and 11(a) to 11(c). Note that for the size parameter of $ka \approx 1.3$, although the rotational routes of these particles are guided by two stabilization modes (Fig. 9(c)), only one leads to a stable equilibrium state. This feature allows us to rotate the particles arbitrarily by steering the orientation of the standing BAW wavefield. The rotation rate of the wavefield should be limited by the stabilization rate of the particle (Fig. 9(b)). Typically, for the cases of $\hat{p}_0/\hat{p}_{\text{ref}} = 4$, the stabilization time $t_s \sim 1\text{s}$ corresponds to the stabilization rate at $\pi/2$ rad/s, indicating that the mechanical rotation rate of the inner module should be smaller than $\pi/2$ rad/s for a stably rotational manipulation; otherwise, the external disturbance caused by the rotation of the inner module cannot be absorbed, breaking the stable equilibrium state. More information on experiments of stable and controllable rotation of the particles, corresponding to Figs. 12(a) to 12(f), are provided in Supplemental Movies SM3 and SM4.

It is however should be emphasized that these results are based on the size parameter of $ka \approx 1.3$ with an operating frequency of $f = 2.04$ MHz and averaged radius of $a \approx 150$ μm . In this way, particle size is smaller than half of the wavelength, i.e., the width of the potential wells ($2a < \lambda/2$,

Figs. 10(a) and 10(b)). Under such a condition, particles can be stably trapped at the nodes, and effective manipulation is realized; this can be clearly seen in experiments. Practically, smaller or larger particles can be selectively manipulated by increasing or decreasing the excitation frequency. Note also that minor mechanical vibrations caused by the manual rotation of the inner module may break the potential barriers (ΔU_f) and make the particle jumps from one potential well to the adjacent ones, while the particle immediately becomes parallel or orthogonal to the orientation of the standing wavefield at a new position (smaller than 1s at $\hat{p}_0/\hat{p}_{\text{ref}} = 4$, refer to Supplemental Movie SM4). Consider that the potential barrier (ΔU_f) is proportional to $\hat{p}_0^2$, this kind of jumping phenomenon can theoretically be suppressed by appropriately increasing the pressure amplitude without causing significant acoustic streaming.

5. Conclusion

In this study, we first derived an explicit analytical method to estimate the radiation force and torque acting on an irregular particle, considering the significance of the geometrical features. This model allows rapid evaluation of the acoustic radiation force and torque generated by traveling or standing wavefields with high accuracy, which is validated by numerical counterparts (see Figs. 10(c) and 10(d)). Combining with the Stokes drag theorem, it can be used as an efficient tool to predict the dynamics of irregular particles in real-time (see Figs. 7 and 8). The theories can be conveniently extended to other systems as well, including, but not limited to, optics and electromagnetics.

We further present a low-cost and simple rotational BAW device that can steer the orientation of the standing wavefield and thus achieve controllable particle rotational manipulation by manually rotating the inner module of the BAW device (see Fig. 5). Changes in the orientation of the wavefield introduce a geometric asymmetry for the non-spherical objects, which naturally brings an additional radiation torque, rotating the objects. Through proof-of-concept experiments, we demonstrated that the particle of different shapes would be trapped at the pressure nodes while continuously and locally adjusting their orientation toward the equilibrium states. Theoretical and experimental results prove that as the acoustic radiation force and torque become stronger for larger input power, the dynamic process that particles fall into equilibrium states will be realized faster (smaller than 1 s is achievable here, seeing Fig. 9(b)). Here, the geometric features are quantified using the mapping coefficients and classified into two major groups (Fig. 9(a)), corresponding to two possible stabilization modes (Fig. 9(c)), which make the particles rotate to sink as many geometric entities as possible towards the acoustic radiation-force potential wells (depicted in Figs. 9(d) and 9(e)). Specifically, the symmetric axis of the axisymmetric particles should be parallel or orthogonal to the orientation of the standing BAW wavefield. After getting into an existing equilibrium state, the radiation torque and the accompanying Stokes drag torque become zero (i.e., the net torque is zero), thereby achieving a local equilibrium (Fig. 10(d)). Nonetheless, the existence of equilibrium does not necessarily imply stability. Hence, the Lyapunov stability theorem is employed to find a stable equilibrium state (Figs. 11(a) to 11(c)). The results indicate that only one of two stabilization modes for the axisymmetric particles drives them to a stable equilibrium state where the overall potential energy is minimal. To exhibit its powerful ability in rotational manipulation, we report the experimental realization of controllable, stable, and precise particle rotation towards any arbitrary directions by manual rotation of the BAW device (Fig. 12).

Our work provides a controllable and stable alternative for dynamic rotational manipulation of micro-objects theoretically and experimentally. Considering its low requirements, we envision that this concept will have various acoustic-wave-based applications.

Supplemental information

See the Supplemental Material for derivations of Eqs. (2), (4), and (5), evaluation of the drag coefficient matrix, and the experimental videos that show the behavior of different particles in the situations represented in Figs. 9, 10, and 12, which additionally includes Refs. [47].

Conflict of Interest

The authors declare that they have no competing interests.

Data and materials availability

All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplemental Materials.

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