On urban co-modality: Non-cooperative and cooperative games among freight forwarder, carrier and transit operator

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Abstract

This paper models a public transit system that can serve both passengers and urban freight, i.e., urban co-modality, and investigates the system-wide impacts of co-modality on existing urban freight forwarding service, freight carrier and urban transit services. In the co-modal system, we model one transit operator that serves passengers and provides co-modal services, one freight forwarder (an intermediary) that serves freight customers and uses freight transportation services provided by a freight carrier or the transit operator, and one freight carrier that provides services to the freight forwarder. We derive the analytical conditions under which co-modal operations can improve the profits of the freight forwarder, carrier and transit operator, and the consumer surpluses of freight customers and passengers compared with the status quo without co-modality. We also analytically and numerically compare the optimal operation decisions, the three operators' profits, and the users' welfare under different games among the operators (non-cooperative and cooperative games). Our results show that when the three operators are in a non-cooperative relationship, the freight carrier might have a profit loss due to the decreased freight units allocated to the direct road channel; whereas, both the freight forwarder and the transit operator would have profit gains from the co-modality. The numerical studies further reveal that to ensure a Pareto-improving co-modal system, the operators have to reduce the freight/transit service fare and co-modal transportation price.

Keywords: Co-modality, Freight forwarder, Freight carrier, Transit operator, Cooperative game, Non-cooperative game

1. Introduction

E-commerce is one of the fast-growing industries in recent years. The surging logistics demand due to the rapid development of e-commerce market elevates the intensity and complexity of logistics activities, especially for the parcel/freight delivery within urban areas, and brings enterprises and cities new challenges. In particular, the evolving characteristics of e-commerce reshape the third-party logistics (3PL) industry, in which more third-party-forwarding logistics services emerge and have received increasing popularity in recent years (Ren et al., 2020). Compared with 3PL companies who are able to provide support for the entire supply chain, freight forwarding service providers (or freight forwarders) specialize in consolidating orders from various consignors, offer a more economic option for small-to-medium e-commerce enterprises, and help enhance the efficiency and flexibility of logistics systems (Arabzad et al., 2015). In general, freight forwarders are regarded as middlemen in the shipping process, as they possess no freight transportation asset and adopt a single or multiple carriers for freight transportation.

In the context of urban parcel delivery, road transportation is mostly adopted by the employed carriers for fulfilling the demand for freight shipping service. Due to the e-commerce boom, the surging parcel traffic generates negative externalities in terms of traffic congestion, air pollution, and fuel consumption. The concept of 'Urban Co-modality' has been developed in this context and is built upon the idea of using under-utilized capacity in public transit systems

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during non-peak hours to transport freight (Taniguchi & Thompson, 2014).[1] Given the increasing applications of urban co-modality, the planning and optimization problems in relation to co-modal systems have attracted much attention recently, such as transit-truck transportation scheduling and synchronization problem (Behiri et al., 2018; Pimentel & Alvelos, 2018), location selection of distribution centers (Zhao et al., 2018), optimization of freight delivery scheme (Cheng et al., 2018), and vehicle routing problem under co-modal networks (Trentini et al., 2012; Masson et al., 2017; Mourad et al., 2021). However, there are limited studies examining and quantifying the impact of urban co-modal systems on stakeholders with the consideration of an intermediary connecting freight customers and carriers. This study aims to bridge this gap.

Existing empirical studies suggested that co-modal services offer opportunities to reduce freight traffic and bring favorable environmental impacts, such as reducing carbon emissions and fuel consumption (Kikuta et al., 2012; De Langhe, 2017; Bruzzone et al., 2021; Zhu et al., 2023). Co-modality can potentially benefit the service users and operators, particularly public transit operators who could generate additional revenue by providing freight-on-transit services (Hu et al., 2020). Besides, substantial under-utilized transit capacity would boost the efficiency of urban logistics systems and improve the freight service quality and customer satisfaction, which will ultimately translate into greater freight demand and revenue gain for freight forwarders and carriers (Cochrane et al., 2017). In addition, the reduction in freight traffic brought by co-modal operation could potentially enhance the reliability and ridership of bus services, especially for the routes operating in city centers (Trentini & Mahléné, 2010; Arvidsson et al., 2016; Guo et al., 2020). Moreover, empirical studies (e.g., Van Duin et al., 2019) also suggested that urban co-modality would create job opportunities (e.g., handling and supervising parcels in transit systems), resulting in long-term societal benefits.

Despite the benefits of co-modal services, introducing the co-modal service, while providing an additional option, brings a new competitor to existing freight carriers, and thus may reduce the potential profit of freight carriers, which will further affect freight forwarders' operation decisions, as well as the resulting level of service for freight customers. To the best of our knowledge, strategic interactions among freight forwarders, carriers and transit operators in a co-modal system, and how these interactions will further interact with passengers and freight customers, and how the co-modal service will impact carriers' profit and operation decisions have not been analytically examined.

This study proposes an analytical framework to uncover the impact of introducing the urban co-modality on the existing urban freight forwarding services and urban transit services and to provide insights into interactions among operators and optimal operation decisions of a co-modal system. In the co-modal context, we consider a transit market with one transit operator who serves passengers, and a freight (forwarding) market with one freight forwarder who serves freight customers and may use both the direct road mode operated by one carrier and the co-modal mode provided by the transit operator. The carrier and the transit operator serve the freight units assigned to the direct road channel and the co-modal channel, respectively, and charge the freight forwarder freight transportation fares. Under such a setting, the transit market endogenously interacts with the freight market due to the cross-type flow/demand interactions on the shared freight-passenger transit service (i.e., the co-modal channel). The freight forwarder determines the freight fare charged to the customers and the modal-split strategy for collected freight units. The carrier determines the price for transporting the freight on road and the trucking capacity used to transport freight. The transit operator determines the transit fare charged to the passengers, the transit service frequency, the co-modal price for transporting freight and the transit capacity reserved for freight. Note that these operation decisions will affect levels of services for transit passengers and freight customers and govern the supply-demand equilibrium for the interacting transit and freight markets.

The aforementioned co-modal problem is relevant to existing studies on the freight modal-split or mode choice problems in the context of short- and/or long-haul intercity transportation chain. This is because the coexistence of the direct road mode and the co-modal mode in an urban setting indeed has a similar nature of those problems. The mode choice problems in the freight transportation sector have been extensively investigated in literature of supply chain management, which mostly focused on coalitions and competitions among freight transportation service providers, such as business cooperation among multiple freight operators (Saeed, 2013), competition between air and high-speed rail transportation (Tsunoda, 2018), competition between intermodal and direct freight service providers (Tamannaei et al., 2021). Due to the fact that intercity freight transportation rarely involves a mode that accommodates both

[1]Several different terminologies for co-modality have been used in the literature, such as cargo hitching, passenger-and-package sharing, freight-on-transit (FOT) services, or synergy between passenger and freight (Cochrane et al., 2017; Elbert & Rentschler, 2021).

freight and passenger flows and that generates cross-type flow interactions, existing models cannot be directly applied for co-modal systems.

Recently, Ma et al. (2022) examined the modal-split and pricing problem under an urban bi-modal freight network (road and co-modal modes) using a game-theoretical approach with a simplification on the passenger and freight demand setting, and demonstrated the potential of co-modal systems in enhancing the transit and freight operators' profits. Following that, Ma et al. (2023) further investigated the effect of co-modality on service qualities of transit and freight services in the game between a transit operator and a freight operator. However, existing studies are based on the setting where the sole freight operator collects freight, operates the direct road channel (for transporting freight), and competes/cooperates with the transit operator who operates the co-modal channel. The outsourcing arrangement (a key feature of urban logistics activities serving the e-commerce industry) among the freight forwarder, carrier and co-modal service provider (i.e., transit operator) for transporting urban freight and its impact on the existing freight and transit systems, levels of services, users' welfare and operators' profits have not been studied.

The main contributions of this paper are threefold. (i) This study is the first to model the urban co-modality considering an outsourcing arrangement among a freight forwarder, a carrier and a transit operator for urban freight transportation. How the introduction of co-modal services will directly affect the existing carrier's operation decisions and profits/benefits, and thus endogenously impact the levels of service for freight customers is established. The co-modal system equilibrium is formulated, and the properties of the interacting transit and freight markets are analyzed via the comparative statics regarding operators' operation decisions. (ii) We derive the analytical conditions for the existence of Pareto-improving co-modal operation decision combinations, under which the profits of the freight forwarder, carrier and transit operator, and the consumer surpluses of freight customers and passengers are increased or at least not decreased after introducing the co-modal service. (iii) The analytical properties of the freight forwarder's, carrier's and transit operator's optimal operation decisions under different games among the three operators are explored. Specifically, the optimal operation decisions and profits under the Nash equilibrium and Nash arbitrated solution are compared analytically and numerically, which provides understanding regarding the system-wide impacts of the co-modality solution.

The remainder of this paper is organized as follows. Section 2 presents the problem description and model formulation. Section 3 defines the Pareto-improving operation decision combinations, presents its existence conditions and demonstrates how to find a Pareto-improving operation decision combination. Section 4 formulates and analyzes the non-cooperative and cooperative games for the co-modality. Section 5 conducts numerical study and sensitivity analysis. Section 6 concludes the paper.

2. Model formulation

This section begins with the problem description and then introduces formulations in relation to the freight customers, transit passengers, freight forwarder, carrier and transit operator. The effects of operators' operation decisions on the supply-demand equilibrium on the freight side and that on the transit side (i.e., the equilibrium users' costs and demand) are examined, respectively.

2.1. Problem description

We consider a stylized intra-city freight transportation network as shown in Fig. 1a, which permits tractable analytical derivations. A freight forwarder collects freight from consignors, consolidates the freight at the origin service center (OSC), and prepares for transportation to another service center near to consignees, i.e., the destination service center (DSC). Since the freight forwarder does not have the transportation assets, it outsources the freight shipping services to other service providers or carriers. The freight forwarder chooses between the two freight transportation modes (sometimes referred to as channels later on): the direct road mode and the co-modal mode. The freight assigned to the direct road channel are transported by one carrier. While, the freight assigned to the co-modal channel are transported in a shared-used transit system (with freight and passenger flows) which is operated by one transit operator (Fig. 1b). To use the co-modal service, there will be connection trips between the OSC and the departure transit station and the destination transit station and DSC (refer to Fig. 1a). Additional operating cost incurred by the connection trips will be either shared between the freight forwarder and the transit operator or solely covered by one of them.

Considering that the first-mile trips (connecting consignors and logistics service points) and last-mile trips (connecting logistics service centers and consignees) are necessary for both channels, to simplify the analytical model, we assume that the road channel and the co-modal channel are symmetric in terms of the first- and last-mile operations, including pick-up, routing, sorting and drop-off tasks. Therefore, the first- and last-mile operating costs will not be incorporated in the model. The differences between the two channels are mainly reflected by how the collected freight units are transported between the logistics service centers. In particular, the road channel directly transports the freight between the service points without transshipment operation. Whereas, the co-modal channel involves not only the connection trips between logistics service centers and transit stations but also the mixed-used passenger-freight transit vehicles. Such a setting allows us to focus on modeling and examining how the connection trips and mixed passenger-freight transit usage impact operators' operation decisions, freight and transit service qualities and the overall system performance.

In the following, the consignors and consignees are collectively termed as the freight customers whose costs are governed by freight fare and levels of freight service. The freight forwarder, carrier and transit operator are sometimes collectively called operators.

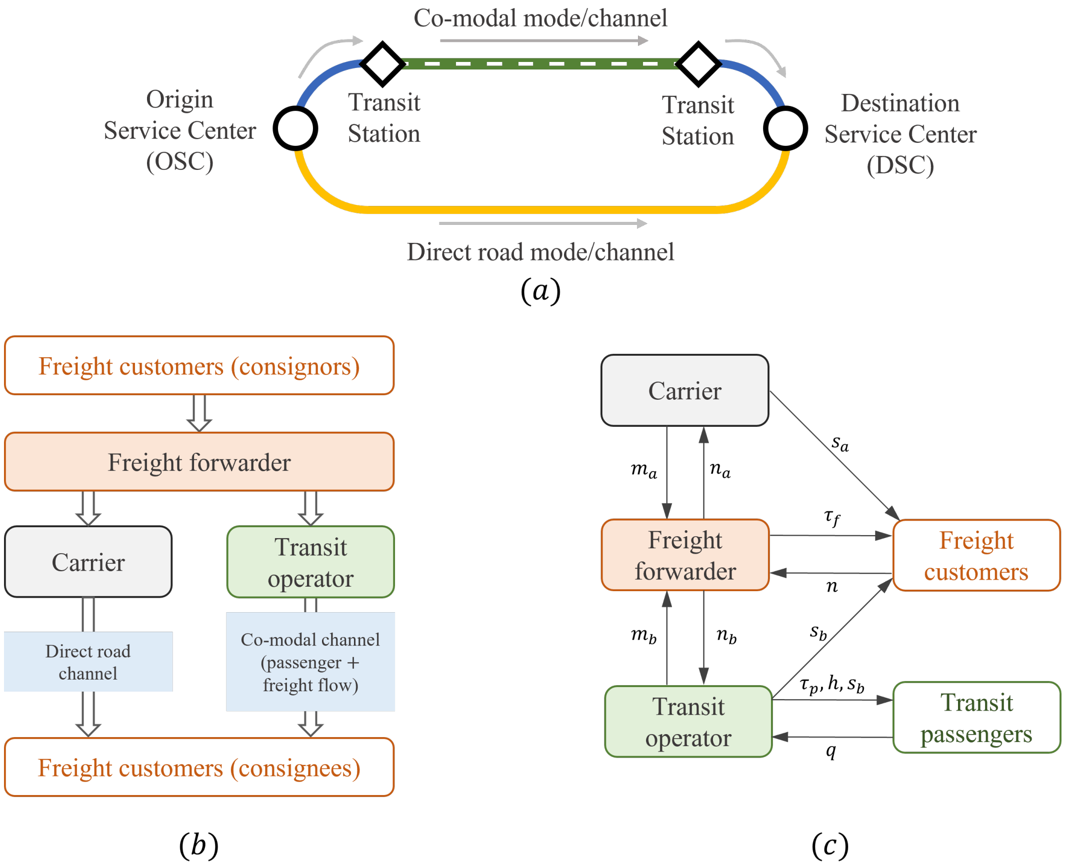

Fig. 1. Problem setting: (a) Bi-modal freight network with co-modality; (b) flow chart of the freight transportation; and (c) multi-sided interactions among operators and users.

We now discuss the multi-sided interactions among the freight forwarder, carrier, transit operator, freight customers and passengers which are depicted in Fig. 1c. The freight forwarder determines the freight fare charged to the customers τ_f and the amount of the freight to be transported by the carrier n_a and that to be transported by the transit operator n_b. The carrier determines the road transportation price (per freight unit) m_a and the trucking capacity s_a. The transit operator determines the co-modal transportation price (per freight unit) m_b and the capacity of the transit system reserved for freight transportation, i.e., freight-on-transit (FOT) or co-modal capacity, s_b. The capacities s_a

and s_b will affect the levels of service for freight customers. It follows that τ_f, s_a and s_b impact the freight demand n. Besides, the transit operator also serves passengers and determines the transit fare τ_p charged to the passengers and the transit service frequency h. The transit demand q is impacted by τ_p, h and s_b (a larger s_b means fewer spaces and more crowding for passengers). Note that we consider the freight forwarder shares the information about the freight customers' response to trucking capacity changes (i.e., the change of n with s_a) with the carrier, and also shares the information about the freight customers' response to FOT capacity changes (i.e., the change of n with s_b) with the transit operator (the freight carrier and transit operator may also observe/infer this information due to their involvement in the freight transportation process). While, other information about their operation decisions are only visible among the operators if cooperation (among the operators) exists. Subscript 'a' denotes the road channel and 'b' indicates the co-modal channel.

The main notations in this paper are summarized in Table B1 in Appendix B. Those not included in Table B1 are specified in the text.

2.2. Freight customers, freight forwarder and carrier

(Freight customers) Consider a freight customer making a request for parcel transportation directly to the freight forwarder who is an intermediary between the customers and the carrier or the transit operator. The parcels collected by the freight forwarders are then allocated to the two channels based on freight forwarder's mode-split strategies for freight; and the freight customers do not need to concern which channel is used for transporting their parcels. Instead, the customers evaluate the overall freight cost based on (i) freight fare and (ii) total service time (non-monetary cost) provided by the forwarder (sum of expected delivery time and unexpected delay). The total service time depends on levels of service of the two channels. Let n denote the freight demand which is elastic and is a strictly decreasing function of the total freight cost c_f, i.e.,

$$n = D_f(c_f) \tag{1}$$

where $D_f(c_f) \geq 0$ and $D'_f(c_f) < 0$.

The total freight cost c_f consists of three components: the freight fare τ_f, the expected delivery time cost if there is no delay t_f, and the delay cost l_f that depends on operation decision of the freight forwarder, given by

$$c_f = \tau_f + t_f + l_f(n_a, n_b, s_a, s_b) \tag{2}$$

where $l_f(\cdot)$ is a function of four variables, the freight volumes in road channel n_a and co-modal channel n_b, and the freight transportation capacity in road channel s_a and co-modal channel s_b. Without loss of generality, it is assumed that the delay function $l_f(\cdot)$ is twice differentiable with respect to n_a, n_b, s_a and s_b. It increases with n_a and n_b (e.g., a larger freight volume of a channel generates more processing delay), and decreases with s_a and s_b (e.g., a greater channel capacity means less delays for processing and freight transportation), i.e., $l'_{f,n_a} = \partial l_f/\partial n_a > 0$, $l'_{f,n_b} = \partial l_f/\partial n_b > 0$, $l'_{f,s_a} = \partial l_f/\partial s_a < 0$, $l'_{f,s_b} = \partial l_f/\partial s_b < 0$. It is assumed that given n_a and n_b, when $s_i \to +\infty$, $i \in a, b$, $l_f \to 0$. The intuition behind this is that when the transportation capacity of the two channels are extremely large, there would be no delay in freight transportation.[2] Let $w \in [0, 1]$ denote the proportion of freight units allocated to the co-modal channel (i.e., modal-split set by the freight forwarder). Thus, the freight volume in each channel is given by $n_a = (1 - w)n$ and $n_b = wn$, respectively. Note that given s_i, the trucking capacity of the direct road channel will not impact the co-modal channel's level of service, and vice versa.

The freight market's supply-demand equilibrium is defined by the demand function in Eq. (1) and the cost/supply function in Eq. (2). Since $\partial c_f/\partial n > 0$ and $D'_f(c_f) = dn/dc_f < 0$, it can be verified that the freight market has a unique supply-demand equilibrium. We further let n^* and c_f^* denote the equilibrium freight demand and freight cost,

[2]The analytical analysis in this paper does not rely on a specific function form of $l_f(\cdot)$. A future study may consider a specific delay function to characterize the case that the delay may only increase (or increase noticeably) when a certain freight volume is reached, which could be formulated by a piecewise function with a number of segments. Real-world data can be also applied to calibrate the delay function.

respectively. Based on Eqs. (1) and (2), we can obtain the following:

$$\begin{aligned}&\frac{\partial n^*}{\partial \tau_f}=\frac{D'_f}{\beta_f}<0;\ \frac{\partial n^*}{\partial s_a}=\frac{D'_f}{\beta_f}l'_{f,s_a}>0;\ \frac{\partial n^*}{\partial s_b}=\frac{D'_f}{\beta_f}l'_{f,s_b}>0;\ \frac{\partial n^*}{\partial w}=n^*(l'_{f,n_b}-l'_{f,n_a})\frac{D'_f}{\beta_f}\\&\frac{\partial c^*_f}{\partial \tau_f}=\frac{1}{D'_f}\frac{\partial n^*}{\partial \tau_f}>0;\ \frac{\partial c^*_f}{\partial s_a}=\frac{1}{D'_f}\frac{\partial n^*}{\partial s_a}<0;\ \frac{\partial c^*_f}{\partial s_b}=\frac{1}{D'_f}\frac{\partial n^*}{\partial s_b}<0;\ \frac{\partial c^*_f}{\partial w}=\frac{1}{D'_f}\frac{\partial n^*}{\partial w}\end{aligned} \tag{3}$$

where $\beta_f = 1-D'_f[l'_{f,n_a}(1-w)+l'_{f,n_b}w] > 0$. Eq. (3) says that when the freight fare τ_f increases, the equilibrium freight demand (resp. cost) decreases (resp. increases); and when the trucking capacity s_a or FOT capacity s_b increases, the freight demand (resp. cost) increases (resp. decreases). Next, the signs of $\partial n^*/\partial w$ and $\partial c^*_f/\partial w$ are governed by the sign of $(l'_{f,n_b} - l'_{f,n_a})$. For instance, when $l'_{f,n_b} > l'_{f,n_a}$, meaning the marginal increase in the non-monetary cost brought by the increase in freight volume on the co-modal channel is greater than that brought by the increase in freight volume on the direct road channel, increasing the proportion of freight transported by the co-modal mode (w) will lead to a decreased freight demand. This might be the case given a transit system with relatively small spare capacities for freight transportation, where assigning more freight units to the co-modal channel would incur additional shipping delay and thus lower the attractiveness of the freight service (i.e., freight demand). When $l'_{f,n_b} = l'_{f,n_a}$, indicating the two channels are symmetric in terms of the impact of the freight volume on the channels' levels of service, the freight forwarder's modal-split strategy for freight will pose no effect on the equilibrium freight demand, i.e., $\partial n^*/\partial w = 0$. In addition, to quantify freight customers' welfare, we further let ψ_f denote the consumer surplus of freight customers, and ψ_f can be written as:

$$\psi_f = \int_0^n D_f^{-1}(t)dt - nc_f \tag{4}$$

(Freight forwarder) The freight forwarder's profit π_f can be written as follows:

$$\pi_f = \tau_f n - m_a n_a - m_b n_b - k_f(n) - \alpha k_c(n_b) \tag{5}$$

where $\tau_f n$ is the total revenue from freight customers, m_a is the direct road transportation price per freight unit set by the carrier, $m_a n_a$ is the payment to the carrier, m_b is the co-modal transportation price per freight unit set by the transit operator, $m_b n_b$ is the payment to the transit operator, $k_f(\cdot)$ is the freight forwarder's general operating cost in relation to the freight volume (e.g., storage cost and labor cost for handling freight), $k_c(\cdot)$ is the operating cost in relation to the connection trips (between freight service centers and transit stations). In particular, $k_f(\cdot)$ is an increasing and convex function of n, and $k_c(\cdot)$ is an increasing and convex function of n_b, i.e., $k'_f = dk_f/dn > 0$, $d^2k_f/dn^2 \geq 0$, $k'_c = dk_c/dn_b > 0$ and $d^2k_c/dn_b^2 \geq 0$. It is noteworthy that in this paper, we consider different co-modal service operation scenarios, in which the connection trip cost could be either solely covered by the freight forwarder or the transit operator, or jointly shared by the two operators. The parameter $\alpha = [0, 1]$ is introduced to the freight forwarder's profit function and transit operator's profit function (to be detailed in Eq. (13)) for describing these two scenarios, where $\alpha = 1$ means the connection cost is solely covered by the freight forwarder, $\alpha = 0$ means the connection cost is solely covered by the transit operator, and the cost is shared when $\alpha \in (0, 1)$. We will examine how the value of α will impact the optimal operation decisions and system efficiency metrics in Section 4. Next, before introducing the co-modality, the freight forwarder's profit can be written as follows:

$$\pi_f^0 = \tau_f n - m_a n - k_f(n) \tag{6}$$

(Freight carrier) The profit of the freight carrier who operates the direct road channel can be written as follows:

$$\pi_a = m_a n_a - k_a(n_a, s_a) \tag{7}$$

where $m_a n_a$ is the revenue from transporting freight and $k_a(\cdot)$ is the operating cost in relation to road transportation for freight which is governed by freight volume n_a on the direct road channel and trucking capacity s_a. Specifically, $k_a(\cdot)$ is an increasing and convex function of n_a and s_a, i.e., $k'_{a,n_a} = \partial k_a/\partial n_a > 0$, $k'_{a,s_a} = \partial k_a/\partial s_a > 0$, $\partial^2 k_a/\partial n_a^2 \geq 0$ and $\partial^2 k_a/\partial s_a^2 \geq 0$. When the co-modal channel is not in place, the carrier's profit can be written as:

$$\pi_a^0 = m_a n - k_a(n, s_a) \tag{8}$$

2.3. Transit passengers and transit operator

(Transit passengers) Similar to the freight demand, the transit demand is also elastic and is a strictly decreasing function of the total transit cost c_p.

$$q = D_p(c_p) \tag{9}$$

where $D_p(c_p) > 0$ and $D'_p(c_p) < 0$.

The transit passenger cost c_p consists of three components: the transit fare τ_p, the constant in-vehicle travel time cost t_p, and the waiting time delay cost l_p that depends on passenger volume and transit level of service, i.e.,

$$c_p = \tau_p + t_p + l_p(q, h, s_b) \tag{10}$$

where the delay cost $l_p(\cdot)$ is a twice differentiable function with respect to three variables, i.e., passenger demand q, transit service frequency h, and reserved transit capacity for FOT s_b, such that $l'_{p,q} = \partial l_p/\partial q > 0$, $l'_{p,s_b} = \partial l_p/\partial s_b > 0$, $l'_{p,h} = \partial l_p/\partial h < 0$. Given q and s_b, when $h \to +\infty$, $l_p \to 0$. The intuition behind this is that if the transit service frequency can be set extremely large, passengers would experience no waiting delay.

The transit market's supply-demand equilibrium is defined by the demand function in Eq. (9) and the cost/supply function in Eq. (10). Since $\partial c_p/\partial q > 0$ and $D'_p(c_p) = dq/dc_p < 0$, it can be verified that the transit market also has a unique supply-demand equilibrium. Denote q^* and c_p^* the equilibrium passenger demand and transit cost, respectively. Based on Eqs. (9) and (10), we can obtain the following:

$$\begin{aligned} &\frac{\partial q^*}{\partial \tau_p} = \frac{D'_p}{\beta_p} < 0; \ \frac{\partial q^*}{\partial h} = \frac{D'_p}{\beta_p} l'_{p,h} > 0; \ \frac{\partial q^*}{\partial s_b} = \frac{D'_p}{\beta_p} l'_{p,s_b} < 0 \\ &\frac{\partial c_p^*}{\partial \tau_p} = \frac{1}{D'_p}\frac{\partial q^*}{\partial h} > 0; \ \frac{\partial c_p^*}{\partial s_a} = \frac{1}{D'_p}\frac{\partial q^*}{\partial h} < 0; \ \frac{\partial c_p^*}{\partial s_b} = \frac{1}{D'_p}\frac{\partial q^*}{\partial s_b} > 0 \end{aligned} \tag{11}$$

where $\beta_p = 1 - D'_p l'_{p,q} > 0$. Eq. (11) says that when the transit fare τ_p or FOT capacity s_b increases, the equilibrium passenger demand (resp. transit cost) decreases (resp. increases); and when the transit frequency increases, the equilibrium passenger demand (resp. transit cost) increases (resp. decreases). Furthermore, we let ψ_p denote the consumer surplus of passengers, and ψ_p can be written as:

$$\psi_p = \int_0^q D_p^{-1}(t)dt - qc_p \tag{12}$$

(Transit operator) The transit operator's profit π_b can be written as follows:

$$\pi_b = \tau_p q + m_b n_b - k_p(q, h) - k_b(n_b) - (1 - \alpha)k_c(n_b) \tag{13}$$

where $\tau_p q$ is the revenue from passengers, $m_b n_b$ is the revenue from providing the co-modal service, $k_p(q, h)$ is the operating cost of the transit service, $k_b(\cdot)$ is the operating cost in relation to freight transportation within the transit system, and $k_c(\cdot)$, as discussed earlier, is the connection trip cost. In particular, $k_p(\cdot)$ is an increasing and convex function of q and h, i.e., $k'_{p,q} = \partial k_p/\partial q$, $k'_{p,h} = \partial k_p/\partial h$, $\partial^2 k_p/\partial q^2 \geq 0$, and $\partial^2 k_p/\partial h^2 \geq 0$. $k_b(\cdot)$ is an increasing and convex function of $(1 - w)n$, i.e., $k'_b = dk_b/dn_b > 0$ and $d^2 k_b/dn_b^2 > 0$. When there is no co-modality, the transit operator's profit is written as follows:

$$\pi_b^0 = \tau_p q - k_p(q, h) \tag{14}$$

3. The Pareto-improving co-modal system

This section investigates operation decision combination(s) that will generate a Pareto-improving co-modal system, under which the profits of the freight forwarder, carrier and transit operator, and the consumer surpluses of the freight customers and passengers are increased or at least not decreased compared with the status quo without the co-modality. Such an operation decision combination is termed as Profit-and-Consumer-surplus-based Pareto-improving (PCPI) operation decision combination. In particular, Section 3.1 gives a formal definition of the PCPI operation decision combination, and derives the sufficient condition that guarantees the existence of such a decision combination. Section 3.2 introduces an optimization problem that helps find one PCPI operation decision combination, and the analytical properties of such a solution will also be examined and discussed.

3.1. Pareto-improving operation decisions

The formal definition of the PCPI operation decision combination is as follows:

Definition 1. *Given an operation decision combination at the status quo* $(\tilde{\tau}_f, \tilde{m}_a, \tilde{s}_a, \tilde{\tau}_p, \tilde{h})$ *without the co-modal service, the operation decision combination* $(\tau_f, w, m_a, s_a, \tau_p, h, m_b, s_b)$ *(where* $nw > 0$ *and* $s_b > 0$*) under the co-modal service is defined as a "Profit-and-Consumer-surplus-based Pareto-improving" (PCPI) operation decision combination (with respect to the status quo) if it fulfills the following five inequalities where at least one inequality strictly holds:*

$$\pi_f \geq \tilde{\pi}_f,\ \pi_a \geq \tilde{\pi}_a,\ \pi_b \geq \tilde{\pi}_b,\ \psi_f \geq \tilde{\psi}_f,\ \psi_p \geq \tilde{\psi}_p \tag{15}$$

where '~' denotes the variables/quantities at the status quo without co-modality.

Remark 1. *A PCPI operation decision combination is able to improve the social surplus,* $SS = \pi_f + \pi_a + \pi_b + \psi_f + \psi_p$*, i.e., the sum of the freight forwarder's, carrier's and transit operator's profits in Eqs. (5), (7) and (13) and freight customers' and transit passengers' consumer surpluses in Eqs. (4) and (12) against the status quo.*

Following Definition 1, the condition for the existence of PCPI operation decisions can be derived, and the result is presented in the following proposition.

Proposition 1. *Given an operation decision at the status quo without the co-modal service, i.e.,* $(\tilde{\tau}_f, \tilde{m}_a, \tilde{s}_a, \tilde{\tau}_p, \tilde{h})$*, there exists at least one PCPI operation decision if the following Eq. (16) hold:*

$$k'_{a,n_a}(\tilde{n}, \tilde{s}_a) > k'_b(0) + k'_c(0) + k'_{p,h}(\tilde{q}, \tilde{h}) \frac{l'_{p,s_b}(\tilde{q}, \tilde{h}, 0)}{l'_{p,h}(\tilde{q}, \tilde{h}, 0)} \frac{l'_{f,n_b}(\tilde{n}, 0, \tilde{s}_a, 0) - l'_{f,n_a}(\tilde{n}, 0, \tilde{s}_a, 0)}{l'_{f,s_b}(\tilde{n}, 0, \tilde{s}_a, 0)} \tag{16a}$$

$$l'_{f,n_b}(\tilde{n}, 0, \tilde{s}_a, 0) > l'_{f,n_a}(\tilde{n}, 0, \tilde{s}_a, 0) \tag{16b}$$

$$\tilde{\pi}_a > 0 \tag{16c}$$

Proof. To prove Proposition 1, it suffices to show that, under conditions Eqs. (16), there exists at least one combination of operation decisions that gives a PCPI outcome. We will first specify one decision combination, and then prove that this decision combination can lead to a system equilibrium that exists and is a PCPI outcome. The full proof is presented in Appendix A. □

Proposition 1 identifies the sufficient conditions under which there exists at least one PCPI operation decision combination. From the perspective of co-modal operation decisions, the conditions in Eqs. (16a)-(16c) are exogenous, which depend on the status quo (without co-modality) and the delay and cost function characteristics at the status quo. Thus, the evaluation of these conditions is independent of the interactions among operators in the co-modal system.

We now interpret the three conditions in Proposition 1. The three conditions prescribe the system conditions given the status quo operation decisions $(\tilde{\tau}_f, \tilde{m}_a, \tilde{s}_a, \tilde{\tau}_p, \tilde{h})$, where co-modality is not introduced, i.e., $w = 0, s_b = 0$. Firstly, the condition Eq. (16a) requires that, at the status quo, the marginal operating cost of the direct road channel k'_a is greater than that of the co-modal channel (i.e., the sum of freight transportation cost of the co-modal channel and transit operating cost due to an increased service frequency) jointly shared by the forwarder and transit operator, i.e., $k'_b + k'_c + k'_{p,h} \frac{l'_{p,s_b}}{l'_{p,h}} \frac{l'_{f,n_b} - l'_{f,n_a}}{l'_{f,s_b}}$. Secondly, the condition Eq. (16b) requires that, with respect to the status quo, the marginal freight delay cost brought by the marginal increase in freight volume on the co-modal channel is greater than that on the road channel. This would be the case in practice because the co-modal transportation is regarded as the intermodal freight transportation in the urban context, which involves the transshipment operations, loading/unloading operations and additional handling arrangements within transit stations. These operations might cause additional freight transportation time/delay. Thus, given a marginal increase in freight units transported by the co-modal channel, one may expect a greater increment in freight customers' average non-monetary cost (in comparison with the direct road channel without transshipment operations). Thirdly, the condition Eq. (16c) means that when the co-modality is not in place, the carrier earns a positive profit.

3.2. *Optimization problem for solving a PCPI operation decision*

This subsection first introduces an optimization problem that aims to find one PCPI operation decision and then discusses the analytical properties of such a solution. The solution to the optimization problem in Eq. (17) is a PCPI operation decision.

$$\max V(\tau_f, w, m_a, s_a, \tau_p, h, m_b, s_b) = (\pi_f - \tilde{\pi}_f)(\pi_a - \tilde{\pi}_a)(\pi_b - \tilde{\pi}_b)(\psi_f - \tilde{\psi}_f)(\psi_p - \tilde{\psi}_p) \tag{17}$$

subject to (i) $\pi_f \geq \tilde{\pi}_f$; $\pi_a \geq \tilde{\pi}_a$; $\pi_b \geq \tilde{\pi}_b$; $\psi_f \geq \tilde{\psi}_f$ and $\psi_p \geq \tilde{\psi}_p$; (ii) the supply-demand equilibria of freight and transit markets defined by Eqs. (1)-(2) and Eqs. (9)-(10); and (iii) constraints on the decision variables. It is noteworthy that the objective function in (17) is symmetric in terms of the five efficiency metrics, i.e., the profit increments and consumer surplus increments. Hence, we label the solution to problem (17) as 'PCPI-S', where 'S' indicates the 'symmetric' objective function.

We consider the interior solution at which the three operators' profits and freight customers' and transit passengers' consumer surpluses are all strictly increased, i.e., $\pi_f > \tilde{\pi}_f$, $\pi_a > \tilde{\pi}_a$, $\pi_b > \tilde{\pi}_b$, $\psi_f > \tilde{\psi}_f$ and $\psi_p > \tilde{\psi}_p$. By further letting $\gamma_f = \frac{\pi_f - \tilde{\pi}_f}{(\pi_f - \tilde{\pi}_f) + (\psi_f - \tilde{\psi}_f)}$, $1 - \gamma_f = \frac{\psi_f - \tilde{\psi}_f}{(\pi_f - \tilde{\pi}_f) + (\psi_f - \tilde{\psi}_f)}$, $\gamma_p = \frac{\pi_b - \tilde{\pi}_b}{(\pi_b - \tilde{\pi}_b) + (\psi_p - \tilde{\psi}_p)}$, and $1 - \gamma_p = \frac{\psi_p - \tilde{\psi}_p}{(\pi_b - \tilde{\pi}_b) + (\psi_p - \tilde{\psi}_p)}$, where $\gamma_f \in (0, 1)$ and $\gamma_p \in (0, 1)$, the first-order conditions (FOCs) can be derived and written as follows:

$$\left(\tau_f \frac{\partial n^*}{\partial \tau_f} + n^* - K' \frac{\partial n^*}{\partial \tau_f}\right)(1 - \gamma_f) - \left(n^* \frac{\partial c_f^*}{\partial \tau_f}\right)\gamma_f = 0 \tag{18a}$$

$$\left[\tau_f \frac{\partial n^*}{\partial w} - K' \frac{\partial n^*}{\partial w} - n^*(k_b' + k_c' - k_{a,n_a}')\right](1 - \gamma_f) - \left(n^* \frac{\partial c_f^*}{\partial w}\right)\gamma_f = 0 \tag{18b}$$

$$\left(\tau_f \frac{\partial n^*}{\partial s_a} - K' \frac{\partial n^*}{\partial s_a} - k_{a,n_a}'\right)(1 - \gamma_f) - \left(n^* \frac{\partial c_f^*}{\partial s_a}\right)\gamma_f = 0 \tag{18c}$$

$$\left(\tau_f \frac{\partial n^*}{\partial s_b} - K' \frac{\partial n^*}{\partial s_b} + \tau_p \frac{\partial q^*}{\partial s_b} - k_{p,q}' \frac{\partial q^*}{\partial s_b}\right) - \left(n^* \frac{\partial c_f^*}{\partial s_b}\right)\frac{\gamma_f}{1 - \gamma_f} - \left(q^* \frac{\partial c_p^*}{\partial s_b}\right)\frac{\gamma_p}{1 - \gamma_p} = 0 \tag{18d}$$

$$\left(\tau_p \frac{\partial q^*}{\partial \tau_p} + q^* - k_{p,q}' \frac{\partial q}{\partial \tau_p}\right)(1 - \gamma_p) - \left(q^* \frac{\partial c_p^*}{\partial \tau_p}\right)\gamma_p = 0 \tag{18e}$$

$$\left(\tau_p \frac{\partial q^*}{\partial h} - k_{p,q}' \frac{\partial q^*}{\partial h} - k_{p,h}'\right)(1 - \gamma_p) - \left(q^* \frac{\partial c_p^*}{\partial h}\right)\gamma_p = 0 \tag{18f}$$

$$(n^* - n^* w)\left[(\pi_f - \tilde{\pi}_f) - (\pi_a - \tilde{\pi}_a)\right] = 0 \tag{18g}$$

$$(n^* w)\left[(\pi_f - \tilde{\pi}_f) - (\pi_b - \tilde{\pi}_b)\right] = 0 \tag{18h}$$

where $K' = k_f' + (1 - w)k_{a,n_a}' + w(k_b' + k_c')$ is the marginal operating cost of the freight transportation service jointly shared by the three operators due to the marginal increase in the freight demand. Specifically, K' is the sum of (i) marginal operating costs of the direct road mode $(1 - w)k_{a,n_a}'$, (ii) the marginal operating cost of the co-modal mode $w(k_b' + k_c')$, and (iii) the freight forwarder's marginal general operating cost that is not related to freight transportation k_f'. Eqs. (18a)-(18f) indicate that when there is a marginal increase in τ_f, w, s_a, s_b, τ_p and h, the weighted sum of the marginal effect on the sum of the three operators' profits and that on the consumer surpluses of passengers and freight customers should be equal to zero. Eqs. (18g) and (18h) state that at the PCPI-S solution, the amount of profit gains (against the status quo) received by the freight forwarder, carrier and transit operator are identical. This is because, the objective function in Eq. (17) is symmetric as mentioned earlier. Based on Eqs. (18g) and (18h), the weight γ_f can then be further written as $\gamma_f = \frac{\pi_f-\tilde{\pi}_f}{(\pi_f-\tilde{\pi}_f)+(\psi_f-\tilde{\psi}_f)} = \frac{\pi_a-\tilde{\pi}_a}{(\pi_f-\tilde{\pi}_f)+(\psi_f-\tilde{\psi}_f)} = \frac{\pi_b-\tilde{\pi}_b}{(\pi_f-\tilde{\pi}_f)+(\psi_f-\tilde{\psi}_f)}$. To summarize, the PCPI-S operation decision determined by Eq. (18) balances the trades-off among profits of the three operators and consumer surpluses, in which the five system efficiency metrics are able to be improved simultaneously against the status quo.

By substituting the partial derivatives of equilibrium freight and passenger demand with respect to τ_f, s_a, s_b, w, τ_p and h shown in Eqs. (3) and (11) into Eq. (18), and solving them simultaneously, we have:

$$\tau_f^{\text{PCPI-S}} = K' + n^*[l'_{f,n_a}(1-w) + l'_{f,n_b}w] - \frac{1-2\gamma_f}{1-\gamma_f}\frac{n^*}{D'_f} \tag{19a}$$

$$w^{\text{PCPI-S}} : k'_{a,n_a} + n^* l'_{f,n_a} = k'_b + k'_c + n^* l'_{f,n_b} \tag{19b}$$

$$s_a^{\text{PCPI-S}} : k'_{a,n_a} = -n^* l'_{f,s_a} \tag{19c}$$

$$\tau_p^{\text{PCPI-S}} = k'_{p,q} + q^* l'_{p,q} - \frac{1-2\gamma_p}{1-\gamma_p}\frac{q^*}{D'_p} \tag{19d}$$

$$h^{\text{PCPI-S}} : k'_{p,h} = -q^* l'_{p,h} \tag{19e}$$

$$s_b^{\text{PCPI-S}} : -n^* l'_{f,s_b} = q^* l'_{p,s_b} \tag{19f}$$

$$m_a^{\text{PCPI-S}} : \pi_f^{\text{PCPI-S}} - \tilde{\pi}_f = \pi_a^{\text{PCPI-S}} - \tilde{\pi}_a \tag{19g}$$

$$m_b^{\text{PCPI-S}} : \pi_f^{\text{PCPI-S}} - \tilde{\pi}_f = \pi_b^{\text{PCPI-S}} - \tilde{\pi}_b \tag{19h}$$

As can be seen from Eq. (19a), the optimal freight fare charged to the freight customers at PCPI-S solution, $\tau_f^{\text{PCPI-S}}$, consists of three terms: (i) the marginal operating cost of the freight transportation service shared by the three operators K'; (ii) the non-monetary costs of all freight customers n^* due to the marginal increase in freight demand, i.e., $n^*[l'_{f,n_a}(1-w) + l'_{f,n_b}w]$; (iii) the scaled monopoly markup $-n^*/D'_f > 0$ with a coefficient of $\frac{1-2\gamma_f}{1-\gamma_f}$. According to the definition of γ_f discussed earlier, $\gamma_f > 0.5$ means that the profit gain of the three operators (identical amount of profit gain among the operators) is larger than the welfare gain of the freight customers (against the status quo). $\gamma_f > 0.5$ also suggests that in the first-order conditions in Eq. (18), the terms related to profit are less weighted in comparison with the terms related to the consumer surplus of the freight customers. This further reveals that a Pareto-improving outcome is established in the notion that those with a greater absolute improvement will be less weighted. Besides, given $\gamma_f > 0.5$, it follows that $\frac{1-2\gamma_f}{1-\gamma_f} < 0$, which follows that the optimal strategy tends to favor more the freight customers than the operators by deducting part of the monopoly markup. In contrast to the above, when $\gamma_f < 0.5$, $\frac{1-2\gamma_f}{1-\gamma_f} > 0$, i.e., the optimal freight fare includes part of the monopoly markup. $\gamma_f = 0.5$ is a neutral case where $\frac{1-2\gamma_f}{1-\gamma_f} = 0$. One can find that the terms in the formula of the optimal transit fare $\tau_p^{\text{PCPI-S}}$ in Eq. (19d) are comparable to those in the optimal freight fare, where the detailed similar observations are omitted.

We now turn to the optimal modal-split strategy w under the PCPI-S solution. As can be seen from Eq. (19b), the optimal modal-split strategy for freight under the PCPI-S is determined in the way that the marginal cost induced by the marginal increase in the freight volume on the direct road channel ($k'_{a,n_a} + n^* l'_{f,n_a}$) balances that induced by the marginal increase in the freight volume on the co-modal channel ($k'_b + k'_c + n^* l'_{f,n_b}$).

Eq. (19c) states that at the optimal trucking capacity under the PCPI-S, $s_a^{\text{PCPI-S}}$, the marginal operating cost of the direct road channel k'_{a,n_a} balances the non-monetary cost saving of all customers n^* due to the a marginal increase in the trucking capacity $-n^* l'_{f,s_a}$. The interpretations of Eqs. (19e) and (19f) are similar to that of Eq. (19c), which are omitted. Finally, Eqs. (19g) and (19h) repeat those in Eqs. (18g) and (18h), which indicate that at the PCPI-S road/co-modal transportation price (for one freight unit), the three operators' profit gains (against the status quo) are identical.

Remark 2. *Based on the first-order optimality conditions in Eq. (18), by assuming constant k'_f, k'_{a,n_a}, k'_{a,s_a}, k'_b, k'_c, $k'_{p,q}$, and $k'_{p,h}$ and applying the point elasticity, the profits of the freight forwarder, carrier and transit operator can be derived and are as follows:*

$$\pi_f^{PCPI\text{-}S} = \frac{1}{3}\Pi^{PCPI\text{-}S} + \frac{1}{3}(2\tilde{\pi}_f - \tilde{\pi}_a - \tilde{\pi}_b) \tag{20a}$$

$$\pi_a^{PCPI\text{-}S} = \frac{1}{3}\Pi^{PCPI\text{-}S} + \frac{1}{3}(2\tilde{\pi}_a - \tilde{\pi}_f - \tilde{\pi}_b) \tag{20b}$$

$$\pi_b^{PCPI\text{-}S} = \frac{1}{3}\Pi^{PCPI\text{-}S} + \frac{1}{3}(2\tilde{\pi}_b - \tilde{\pi}_f - \tilde{\pi}_a) \tag{20c}$$

where

$$\Pi^{PCPI\text{-}S} = \underbrace{\frac{n}{\sigma^n_{\tau_f} + 1}\left[(\sigma^n_{s_a} - 1)K' + \frac{n^*}{D'_f}\frac{\gamma_f}{1-\gamma_f}(\sigma^n_{\tau_f} + \sigma^n_{s_a})\right]}_{\text{income from freight service}} + \underbrace{\frac{q}{\sigma^q_{\tau_p} + 1}\left[(\sigma^q_h - 1)k'_{p,q} + \frac{q^*}{D'_p}\frac{\gamma_p}{1-\gamma_p}(\sigma^q_{\tau_p} + \sigma^q_h)\right]}_{\text{income from transit service}}$$

and σ^y_x is the elasticity of y with respect to x, i.e., $\sigma^y_x = \frac{\partial y}{\partial x}\frac{x}{y}$.

The point elasticity formulae are applied for deriving the semi-explicit solution to the optimal operation decisions and the operators' profits. As can be seen from Eq. (20), the three operators' profits brought by the PCPI-S operation decision are governed by the profits of these operators under the status quo $(\tilde{\pi}_f, \tilde{\pi}_a, \tilde{\pi}_b)$, and the total income from freight service and the transit service $\Pi^{\text{PCPI-S}}$.

4. Non-cooperative and cooperative games

This section presents the non-cooperative and cooperative games among the freight forwarder, carrier and transit operator. Fig. 2 shows the three game-theoretical models to be formulated and examined, which are:

- ONC (Fig. 2a): Operators' Optimal operation decisions under No Co-modality (ONC). We consider that without co-modality, the freight forwarder and carrier are under a non-cooperative relationship and maximize their profits independently. Meanwhile, the transit operator maximizes its benefit, i.e., the sum of profit and passengers' consumer surplus.
- NE (Fig. 2b): The non-cooperative simultaneous game among the three operators. The solution to this game is Nash equilibrium (NE).
- NAS (Fig. 2c): The cooperative Nash arbitration scheme (NAS) among the three operators (or the Nash bargaining game, Nash 1950)

The optimal operation decisions and operators' profits under the three models are derived, analyzed and compared.

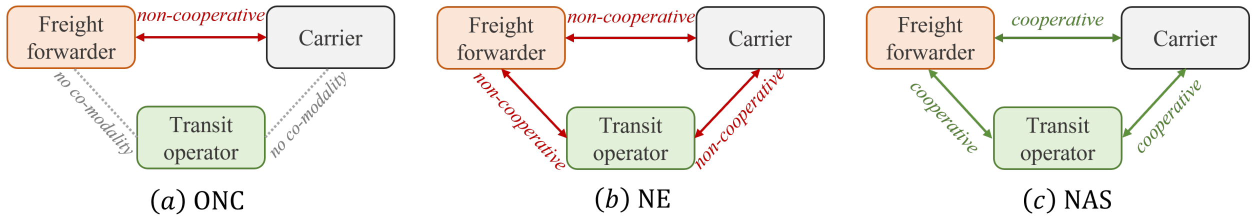

Fig. 2. Summary of game-theoretical models

4.1. *Optimal operation under no co-modality (ONC)*

4.1.1. *Transit operator under ONC*

Before the introduction of the co-modal service, the freight market and the transit market are independent. The transit operator's optimal operation decision (transit fare and service frequency) under no co-modality is the solution to the benefit maximization problem max $z_b^0(\tau_p, h) = \tau_p q - k_p + \psi_b$ which is subject to the supply-demand equilibrium of the transit's side that is defined by Eqs. (9)-(10), and the constraints on the decisions variables, i.e., $0 \leq \tau_p \leq T_p$ and $0 \leq h \leq H$, where T_p is the price bound set by transportation authorities or local governments, H is the maximum service frequency (that depends on infrastructure and operational issues).

With the consideration of interior solutions to the transit operator's welfare maximization problem, the following first-order conditions (FOCs) can be derived:

$$\tau_p \frac{\partial q^*}{\partial \tau_p} + q - k'_{p,q} \frac{\partial q^*}{\partial \tau_p} - q^* \frac{\partial c_p^*}{\partial \tau_p} = 0 \tag{21a}$$

$$\tau_p \frac{\partial q^*}{\partial h} - k'_{p,q} \frac{\partial q^*}{\partial h} - k'_{p,h} - q^* \frac{\partial c_p^*}{\partial h} = 0 \tag{21b}$$

In Eq. (21a), given a marginal increase in the transit fare, the first term on the left-hand side $\tau_p \frac{\partial q^*}{\partial \tau_p}$ is the marginal decrease in the revenue due to the decreased passenger demand (induced by the marginal increase in the transit fare); the second term q^* is the additional transit fare collected from the q^* passengers due to the increased fare; the third term $-k'_{p,q} \frac{\partial q^*}{\partial \tau_p}$ is the marginal saving on the operating cost induced by the decreased demand; and the fourth term $q^* \frac{\partial c_p^*}{\partial \tau_p}$ is the marginal change in the consumer surplus of passengers.

In Eq. (21b), given a marginal increase in the transit service frequency, the first term on the left-hand side $\tau_p \frac{\partial q^*}{\partial h}$ is the marginal revenue increase due to the increased transit demand (brought by the increased service frequency); the second and third terms $-k'_{p,q} \frac{\partial q^*}{\partial h} - k'_{p,h}$ are the marginal operating cost; and the fourth term $-q^* \frac{\partial c_p^*}{\partial h}$ is the marginal change in consumer surplus of passengers.

By substituting the partial derivatives of equilibrium passenger demand with respect to τ_p and h shown in Eq. (11) into Eq. (21), and solving them simultaneously, it gives:

$$\tau_p^{\mathrm{ONC}} = k'_{p,q} + q^* l'_{p,q}; \quad h^{\mathrm{ONC}} : k'_{p,h} = -q l'_{p,h} \tag{22}$$

As can be seen from Eq. (22), τ_p^{ONC} consists of two terms: (i) the marginal operating cost $k'_{p,q}$ and (ii) the non-monetary cost of all transit passengers q^* brought by the marginal increase in passenger demand $q^* l'_{p,q}$. Next from the optimality condition of h^{ONC}, it is clear that at the optimal transit service frequency h^{ONC}, the marginal operating cost due to increased service frequency $k'_{p,h}$ offsets the non-monetary saving of all transit passengers q^* due to the marginal increase in the service frequency $-q^* l'_{p,h}$.

Remark 3. *Based on the first-order optimality conditions in Eq. (21), by assuming constant $k'_{p,q}$ and $k'_{p,h}$, and applying the point elasticity, the profit of the transit operator can be derived and is as follows:*

$$\pi_b^{ONC} = \frac{q^*}{\sigma_{\tau_p}^q + 1} \left[k'_{p,q}(\sigma_h^q - 1) + \frac{q^*}{D'_p}(\sigma_q^{\tau_p} + \sigma_h^q) \right] \tag{23}$$

The optimal operation decisions of the transit operator under ONC in Eq. (22) could be considered as the operation decision at the status quo, i.e., $\tilde{\tau}_p = \tau_p^{\mathrm{ONC}}$ and $\tilde{h} = h^{\mathrm{ONC}}$. The profit of the transit operator under ONC in Eq. (23) will be further compared with that under the co-modality in Section 4.2 in order to obtain additional insights into the impact of the co-modality on operators' profits.

4.1.2. Freight forwarder and carrier under ONC

We now examine optimal operation decision of the freight forwarder and the carrier under ONC. We consider that under ONC, the freight forwarder and the carrier are in a non-cooperative relationship, where they optimize their operation decisions independently and simultaneously (i.e., Nash equilibrium, NE). One can obtain the optimal operation decisions of the two operators under ONC by solving the problems $\max \pi_f^0(\tau_f) = \tau_f n - m_a n - k_f$ and $\max \pi_a^0(m_a, s_a) = m_a n - k_a$ simultaneously, where both problems are subject to supply-demand equilibrium of the freight side defined by Eqs. (1)-(2), and the constraints on the decisions variables, i.e., $0 \leq \tau_f \leq T_f$, $0 \leq m_a \leq M_a$, and $\underline{S}_a \leq s_a \leq \bar{S}_a$. T_f and M_a are the price bounds that are subject to government regulations or local pricing policies, $\underline{S}_a > 0$ is the minimum trucking capacity that ensures the freight transportation operation, $\bar{S}_a$ is the maximum amount of trucking capacity that could be acquired by the carrier. Note that the constraints on the decision variables ensures the compactness of the feasible domain and thus guarantees the existence of the Nash equilibrium.

With the consideration of interior solutions, where possible, the following first-order conditions (FOCs) can be derived:

$$\tau_f \frac{\partial n^*}{\partial \tau_f} + n^* - m_a \frac{\partial n^*}{\partial \tau_f} - k'_f \frac{\partial n^*}{\partial \tau_f} = 0 \tag{24a}$$

$$m_a^{\mathrm{ONC}} = M_a \tag{24b}$$

$$m_a \frac{\partial n^*}{\partial s_a} - k'_{a,n_a} \frac{\partial n^*}{\partial s_a} - k'_{a,s_a} = 0 \tag{24c}$$

In Eq. (24a), given the marginal increase in the freight fare, the first term on the left-hand side $\tau_f \frac{\partial n^*}{\partial \tau_f}$ is the marginal decrease in the revenue due to the decreased freight demand induced by the marginal increase in the freight fare; the second term n^* is the additional freight fare collected from the n^* customers due to the increased fare; the third term $-m_a \frac{\partial n^*}{\partial \tau_f}$ is the marginal saving on the road transportation fee induced by the decreased demand; and the fourth term $-k'_f \frac{\partial n^*}{\partial \tau_f}$ is the marginal saving on the freight forwarder's operating cost due to the decreased demand.

Eq. (24b) gives the formula of the optimal pricing strategy of the road mode under ONC. It is clear that m_a^{ONC} falls on the upper bound M_a. This is because the road transportation price does not have direct or indirect effect on the freight demand under ONC. A more detailed explanation is as follows. Firstly, the road transportation fee has no direct effect on the freight customers' cost due to the fact that the fee is not paid directly by freight customers. Whereas, the fee is covered by the freight forwarder (i.e., intermediary between the customers and the carrier). Next, under the regime ONC, the freight forwarder and carrier maximize their profit simultaneously. This is to say that the carrier's road fee does not impact the freight forwarder's pricing decision or the freight customers' cost. Thus, there is no indirect effect of road fare on freight cost. Since the road transportation price imposes no effect on the freight cost (and thus freight demand), it can be set at the allowed maximum under ONC.

In Eq. (24c), given a marginal increase in the trucking capacity, the first term on the left-hand side $m_a \frac{\partial n^*}{\partial s_a}$ is the marginal revenue increase due to the increased freight demand (brought by the increased trucking capacity); the second and third terms $-k'_{a,n_a} \frac{\partial n^*}{\partial s_a} - k'_{a,s_a}$ is the marginal operating cost.

By substituting the partial derivatives of equilibrium freight demand with respect to τ_f and s_a shown in Eq. (3) into Eq. (24), and solving them simultaneously, it gives:

$$\tau_f^{\mathrm{ONC}} = M_a + k'_f + n^* l'_{f,n_a} - \frac{n^*}{D'_f};\ m_a^{\mathrm{ONC}} = M_a;\ s_a^{\mathrm{ONC}} : (M_a - k'_{a,n_a}) \frac{\partial n^*}{\partial s_a} = k'_{a,s_a} \tag{25}$$

As can be seen from Eq. (25), the optimal freight fare under ONC with non-cooperative relationship between the freight forwarder and the carrier, i.e., τ_f^{ONC}, consists of four terms: (i) the marginal road transportation cost M_a, (ii) the marginal general operating cost k'_f, (iii) the non-monetary cost of all customers n^* brought by a marginal increase in freight volume l'_{f,n_a}, and (iv) the monopoly markup $-\frac{n^*}{D'_f}$. The expressions/conditions for m_a^{ONC} and s_a^{ONC} repeat those in Eqs. (24b) and (24c). Based on the optimality condition for s_a^{ONC} in Eq. (25), we can obtain the following remark.

Remark 4. *Given an interior solution of the optimal trucking capacity under ONC, i.e., s_a^{ONC}, the carrier's marginal operating cost associated with a marginal increase in the freight volume is always smaller than the upper bound of the direct road price M_a, i.e., $M_a - k'_{a,n_a} > 0$.*

Proof. Based on the optimality condition for s_a^{ONC} where $(M_a - k'_{a,n_a})\frac{\partial n^*}{\partial s_a} = k'_{a,s_a}$, since $k'_{a,s_a} > 0$ and $\frac{\partial n^*}{\partial s_a} > 0$, one can readily obtain $M_a - k'_{a,n_a} > 0$. □

Remark 5. *Based on the first-order optimality conditions in Eq. (25), by assuming constant k'_f, k'_{a,n_a} and k'_{a,s_a}, and applying the point elasticity, the profits of the freight forwarder and carrier under ONC can be derived and are as follows:*

$$\pi_f^{ONC} = \frac{-1}{\sigma_{\tau_f}^n + 1}(M_a + k'_f)n^* > 0;\ \pi_a^{ONC} = (M_a - k'_{a,n_a})(1 - \sigma_{s_a}^n)n^* \tag{26}$$

Based on Eq. (24a), one can verify that $\sigma_{\tau_f}^n < -1$ as $\tau_f^{\mathrm{ONC}} > M_a + k'_f$. It follows that the freight forwarder's profit is always positive, i.e., $\pi_f^{\mathrm{ONC}} > 0$. Next, based on the expression of π_a^{ONC}, it is clear that considering an interior s_a^{ONC} with $M_a - k'_{a,n_a} > 0$ (see Remark 4), the sign of carrier profit under ONC is only governed by the value of $\sigma_{s_a}^n$. Then, the carrier earns a positive profit if $0 < \sigma_{s_a}^n < 1$ (i.e., one percent increase in trucking capacity yields less than one percent increase in freight demand), a negative profit if $\sigma_n^{s_a} > 1$ (i.e., one percent increase in trucking capacity yields more than one percent increase in freight demand), and a zero profit if $\sigma_n^{s_a} = 1$.

4.2. Non-cooperative game in the co-modality

We first consider the Nash equilibrium among the freight forwarder, carrier and transit operator, where the three operators maximize their profits/benefits independently and simultaneously. We let 'NE' denote this case. The freight forwarder solves $\max \pi_f(\tau_f, w)$ which is subject to supply-demand equilibrium of the freight side defined by Eqs. (1)-(2), and the constraints on the decisions variables, i.e., $0 \le \tau_f \le T_f$, $0 \le w \le 1$. The carrier solves $\max \pi_a(m_a, s_a)$ which is also subject to supply-demand equilibrium of the freight side defined by Eqs. (1)-(2), and the constraints on the decisions variables, i.e., $0 \le m_a \le M_a$, and $\underline{S}_a \le s_a \le \bar{S}_a$. The transit operator solves $\max z_b(\tau_p, h, m_b, s_b) = \pi_b + \psi_p$ which is subject to the supply-demand equilibria of freight and transit market defined by Eqs. (1)-(2) and Eqs. (9)-(10), and the constraints on the decisions variables, i.e., $0 \le \tau_p \le T_p$, $0 \le h \le H$, $0 \le m_b \le M_b$ and $0 \le s_b \le v_t$, where $v_t = v_t(q, v)$ is the remaining capacity on the public transit system which is governed by the passenger demand q and the total capacity in the transit system v.

With the consideration of interior solutions where possible, optimality conditions for the Nash equilibrium (NE) are derived and displayed below:

$$\tau_f \frac{\partial n^*}{\partial \tau_f} + n^* - (1-w)m_a \frac{\partial n^*}{\partial \tau_f} - m_b w \frac{\partial n^*}{\partial \tau_f} - k'_f \frac{\partial n^*}{\partial \tau_f} - \alpha w k'_c \frac{\partial n^*}{\partial \tau_f} = 0 \tag{27a}$$

$$\tau_f \frac{\partial n^*}{\partial w} + m_a n^* - (1-w)m_a \frac{\partial n^*}{\partial w} - m_b w \frac{\partial n^*}{\partial w} - m_b n^* - k'_f \frac{\partial n^*}{\partial w} - \alpha k'_c (n + w \frac{\partial n^*}{\partial w}) = 0 \tag{27b}$$

$$m_a^{\mathrm{NE}} = M_a \tag{27c}$$

$$m_a(1-w)\frac{\partial n^*}{\partial s_a} - k'_{a,n_a}(1-w)\frac{\partial n^*}{\partial s_a} - k'_{a,s_a} = 0 \tag{27d}$$

$$\tau_p \frac{\partial q^*}{\partial \tau_p} + q^* - k'_{p,q} \frac{\partial q^*}{\partial \tau_p} - \frac{\partial c_p^*}{\partial \tau_p} q^* = 0 \tag{27e}$$

$$\tau_p \frac{\partial q^*}{\partial h} - k'_{p,q} \frac{\partial q}{\partial h} - k'_{p,h} - \frac{\partial c_p}{\partial h} q^* = 0 \tag{27f}$$

$$m_b^{\text{NE}} = M_b \tag{27g}$$

$$\tau_p \frac{\partial q^*}{\partial s_b} + m_b w \frac{\partial n^*}{\partial s_b} - k'_p \frac{\partial q^*}{\partial s_b} - w k'_b \frac{\partial n^*}{\partial s_b} - (1-\alpha) w k'_c \frac{\partial n^*}{\partial s_b} - \frac{\partial c_p^*}{\partial s_b} q^* = 0 \tag{27h}$$

The FOCs/expressions for the optimal decision strategies under NE with respect to τ_f, m_a, s_f, τ_p and h are comparable to those in Eq. (21) and (24), and thus the detailed description is omitted. Regarding m_b in Eq. (27g), it is clear that the co-modal fee under NE is also set at the allowed maximum, where the corresponding reasoning is similar to that for Eq. (24b). It is noteworthy that the exact solutions under NE will be further governed by the values of w and s_b.

We now focus on explaining the FOCs for w and s_b. In Eq. (27b), given a marginal increase in the proportion of freight units assigned to the co-modal channel w, the first term on the left-hand side $\tau_f \frac{\partial n^*}{\partial w}$ is the marginal revenue brought by a marginal variation in modal-split strategy for freight; the second to fifth terms $m_a n^* - (1-w) m_a \frac{\partial n^*}{\partial w} - m_b w \frac{\partial n^*}{\partial w} - m_b n^*$ are the marginal change in the transportation fee (i.e., road plus co-modal fees) due to a marginal increase in w; the sixth term $-k'_f \frac{\partial n^*}{\partial w}$ is the marginal operating cost of the freight forwarder; and the seventh and eighth terms $-\alpha k'_c (n + w \frac{\partial n^*}{\partial w})$ describe the marginal operating cost in relation to the connection trips.

In Eq. (27h), given a marginal increase in the freight-on-transit (FOT) capacity reserved for freight, the first term on the left-hand side $\tau_p \frac{\partial q^*}{\partial s_b}$ is the marginal revenue loss induced by the decreased passenger demand due to co-modal operation; the second term $m_b w \frac{\partial n^*}{\partial s_b}$ is the marginal revenue increase from the co-modal service brought by increased freight volume on the co-modal channel (i.e., $w \frac{\partial n^*}{\partial s_b}$); the third term is $-k'_p \frac{\partial q^*}{\partial s_b}$ is the marginal saving in transit service operating cost due to the decreased transit demand; the fourth and fifth terms $-w k'_b \frac{\partial n^*}{\partial s_b} - (1-\alpha) w k'_c \frac{\partial n^*}{\partial s_b}$ together describe the marginal co-modal transportation operating cost of the transit operator.

By substituting the partial derivatives of equilibrium freight and passenger demand with respect to τ_f, s_a, s_b, w, τ_p and h shown in Eqs. (3) and (11) into Eq. (27), and solving them simultaneously, it gives:

$$\begin{aligned}
&\tau_f^{\text{NE}} = (1-w^{\text{NE}})M_a + w^{\text{NE}} M_b + k'_f + \alpha w^{\text{NE}} k'_c + n^* \left[l'_{f,n_a}(1-w^{\text{NE}}) + l'_{f,n_b} w^{\text{NE}} \right] - \frac{n^*}{D'_f} \\
&w^{\text{NE}} : M_a + n^* l'_{f,n_a} = M_b + n^* l'_{f,n_b} + \alpha k'_c \\
&m_a^{\text{NE}} = M_a \\
&s_a^{\text{NE}} : (1-w^{\text{NE}})[M_a - k'_{a,n_a}] \frac{\partial n^*}{\partial s_a} = k'_{a,s_a} \\
&\tau_p^{\text{NE}} = k'_{p,q} + q^* l'_{p,q}; \\
&h^{\text{NE}} : k'_{p,h} = -q^* l'_{p,h} \\
&m_b^{\text{NE}} = M_b \\
&s_b^{\text{NE}} : w^{\text{NE}}[M_b - k'_b - (1-\alpha) k'_c] \frac{\partial n^*}{\partial s_b} = q^* l'_{p,s_b}
\end{aligned} \tag{28}$$

As can be seen from Eq. (28), the formula for τ_f^{NE} is also comparable to that for τ_f^{ONC} as it also consists of three components: (i) the marginal operating cost, i.e., $(1-w^{\text{NE}})M_a + w^{\text{NE}} M_b + k'_f + \alpha w^{\text{NE}} k'_c$; (ii) the non-monetary cost of all customers n^* brought by the marginal increase in freight volume on the road channel and that on the co-modal channel $n^*\left[l'_{f,n_a}(1-w^{\mathrm{NE}})+l'_{f,n_b}w^{\mathrm{NE}}\right]$; and (iii) the monopoly markup $-\frac{n^*}{D'_f}$. The optimal condition with respect to w^{NE} says that the modal-split strategy for freight should be set in the way that the marginal cost of using the direct road channel $M_a + n^* l'_{f,n_a}$ balances the marginal cost of using the co-modal channel $M_b + n^* l'_{f,n_b} + \alpha k'_c$.

Next, the optimality conditions for s_a^{NE} and s_b^{NE} in Eq. (28) balances the trades-off between operators' marginal revenue in concern and the marginal cost due to increased freight volume on the channel in concern. The optimality conditions for m_a^{NE} and m_b^{NE} simply repeat those in Eqs. (27c) and (27g). Similar to Remark 4, when an interior s_a^{NE} and an interior s_b^{NE} are considered, one can also derive that $M_a > k'_{a,n_a}$ and $M_b > k'_b + (1-\alpha)k'_c$ under NE. Finally, the optimality conditions for τ_p^{NE} and h^{NE} are comparable to those under ONC in Eqs. (21a) and (21b).

We now examine how the upper bound of the direct road transportation price (per freight unit) M_a, that of the co-modal price M_b, and the value of α jointly affect the freight forwarder's modal-split strategy for freight. The result is summarized in Remark 6.

Remark 6. *After introducing the co-modality, under NE, both the direct road mode and the co-modal mode will be used if $M_a + n^* l'_{f,n_a} = M_b + n^* l'_{f,n_b} + \alpha k'_c$; the co-modal mode will not be used if $M_a + n^* l'_{f,n_a} < M_b + n^* l'_{f,n_b} + \alpha k'_c$; and the direct road mode will not be used if $M_a + n^* l'_{f,n_a} > M_b + n^* l'_{f,n_b} + \alpha k'_c$.*

Proof. The first part of the remark repeats the interior optimality condition for w^{NE} in Eq. (28). The remainder of the remark can be derived based on the KKT conditions for the Nash equilibrium problem by considering the corner solutions $w^{\mathrm{NE}} = 0$ and $w^{\mathrm{NE}} = 1$, respectively, of which the details are omitted. □

We now discuss the two inequalities in Remark 6. Firstly, $M_a + n^* l'_{f,n_a} < M_b + n^* l'_{f,n_b} + \alpha k'_c$ renders an outcome where no freight would be assigned to the co-modal mode. This could be the case when $\alpha = 1$, which means if the freight forwarder is responsible for the additional connection trip cost, the co-modal mode will be a less favorable mode and thus not be utilized. Besides, a greater value of M_b and/or a greater l'_{f,n_b} (meaning the co-modal channel's levels of freight transportation service deteriorates faster when the freight volume on the co-modal channel increases) will also lead to such an outcome. Secondly, similar to the above, given a greater M_a and l'_{f,n_a}, the inequality $M_a + n^* l'_{f,n_a} > M_b + n^* l'_{f,n_b} + \alpha k'_c$ will be more likely to hold, yielding the outcome where no freight would be assigned to the direct road mode.

Remark 7. *Based on the first-order optimality conditions in Eq. (27), by assuming constant k'_f, k'_{a,n_a}, k'_{a,s_a}, k'_b, k'_c, $k'_{p,q}$ and $k'_{p,h}$, and applying the point elasticity, the profits of the freight forwarder, carrier and transit operator can be derived and are as follows:*

$$\begin{aligned}
\pi_f^{NE} &= n^* \frac{-1}{\sigma^n_{\tau_f}+1}\left[(1-w^{NE})M_a + M_b w^{NE} + k'_f + \alpha w^{NE} k'_c\right] > 0 \\
\pi_a^{NE} &= n^*(1-\sigma^n_{s_a})(M_a - k'_{a,n_a})(1-w^{NE}) \\
\pi_b^{NE} &= \underbrace{\frac{q^*}{\sigma^q_{\tau_p}+1}\left[k'_{p,q}(\sigma^q_h - 1) + \frac{q^*}{D'_p}(\sigma^{\tau_p}_q + \sigma^q_h)\right]}_{\text{income from transit service}} + \underbrace{n^* w^{NE}[M_b - k'_b - (1-\alpha)k'_c]}_{\text{income from co-modal service} > 0}
\end{aligned} \tag{29}$$

Similar to Remark 5, based on Eq. (27a), one can also verify that $\sigma^n_{\tau_f} < -1$, which leads to $\pi_f^{\mathrm{NE}} > 0$. The formula of π_a^{NE} is also comparable with π_a^{ONC} in Eq. (26), except an additional term $(1 - w^{\mathrm{NE}})$ in π_a^{NE}. Regarding the formula of π_b^{ONC}, the term describing the income from transit service under NE is comparable with that under ONC in Eq. (23) but the exact value is subject to s_b^{NE}. Next, considering an interior s_b^{ONC} with $M_b - k'_b - (1-\alpha)k'_c > 0$ (as discussed in the above), the term describing the income from the co-modal service in the formula of π_b^{NE} is positive. This validates that the introduction of co-modality generates additional revenue gain for the transit operator, which is consistent with the findings in the existing empirical studies (see e.g., Hu et al. 2020).

We now discuss the impact of the co-modality on the carrier's profit. It is evident that apart from $\sigma^n_{s_a}$, the value of π_a^{NE} is particularly relevant to the optimal modal-split strategy for freight under NE, i.e., w^{NE}. Suppose that under both ONC and NE, $0 < \sigma^n_{s_a} < 1$, leading to $\pi_a^{\mathrm{ONC}} > 0$ and $\pi_a^{\mathrm{NE}} > 0$. If the interior optimal $w^{\mathrm{NE}} \to 1$, $\pi_a^{\mathrm{NE}} \to 0$. It follows that $\pi_a^{\mathrm{ONC}} > \pi_a^{\mathrm{NE}}$, indicating the carrier might be worse-off (compared with the status quo without co-modality) when the co-modality is introduced. By contrast, given $\sigma^n_{s_a} > 1$ under both ONC and NE, we have $\pi_a^{\mathrm{ONC}} < 0$ and $\pi_a^{\mathrm{NE}} < 0$.

Again, if the interior optimal $w^{\text{NE}} \to 1$, $\pi_a^{\text{NE}} \to 0$. It then follows that $\pi_a^{\text{ONC}} < \pi_a^{\text{NE}}$, meaning the carrier would have less profit loss and thus benefit from the co-modality. We will further compare the profits of the carriers under ONC and NE via numerical studies in Section 5.2.

4.3. Cooperative game in the co-modality

This subsection presents the cooperative Nash arbitration scheme (or Nash bargaining game, Nash 1950) among the three operators, where they jointly optimize the 8 decisions variables $(\tau_f, w, m_a, s_a, \tau_p, h, m_b, s_b)$ to maximize their payoffs against a disagreement point, such as ONC or NE. The following analysis is based on a disagreement of the ONC, i.e., $(\pi_f^{\text{ONC}}, \pi_a^{\text{ONC}}, z_b^{\text{ONC}})$. The solution to the following optimization problem is a Nash arbitrated solution (NAS):

$$\max U(\tau_f, w, m_a, s_a, \tau_p, h, m_b, s_b) = (\pi_f - \pi_f^{\text{ONC}})(\pi_a - \pi_a^{\text{ONC}})(\pi_b - \pi_b^{\text{ONC}}) \tag{30}$$

subject to (i) $\pi_f \geq \pi_f^{\text{ONC}}$; $\pi_a \geq \pi_a^{\text{ONC}}$; and $\pi_b \geq \pi_b^{\text{ONC}}$; (ii) the supply-demand equilibria of freight and transit markets defined by Eqs. (1)-(2) and Eqs. (9)-(10); and (iii) constraints on the decision variables.

By considering the interior solutions at which the three operators' profits are all strictly increased with the existence of freight units on the co-modal channel, i.e., $\pi_f > \pi_f^{\text{ONC}}$, $\pi_a > \pi_a^{\text{ONC}}$, $\pi_b > \pi_b^{\text{ONC}}$ and $n^* w > 0$, the FOCs for the Nash arbitrated solution can be derived as follows:

$$\tau_f \frac{\partial n^*}{\partial \tau_f} + n^* - K' \frac{\partial n^*}{\partial \tau_f} = 0 \tag{31a}$$

$$\tau_f \frac{\partial n^*}{\partial w} - K' \frac{\partial n^*}{\partial w} - n^* k_b' - n^* k_c' + n^* k_{a,n_a}' = 0 \tag{31b}$$

$$\tau_f \frac{\partial n^*}{\partial s_a} - K' \frac{\partial n^*}{\partial s_a} - k_{a,s_a}' = 0 \tag{31c}$$

$$\tau_f \frac{\partial n^*}{\partial s_b} - K' \frac{\partial n^*}{\partial s_b} + \tau_p \frac{\partial q^*}{\partial s_b} - k_{p,q}' \frac{\partial q^*}{\partial s_b} - q^* \frac{\partial c_p^*}{\partial s_b} = 0 \tag{31d}$$

$$\tau_p \frac{\partial q^*}{\partial \tau_p} + q^* - k_{p,q}' \frac{\partial q^*}{\partial \tau_p} - q^* \frac{\partial c_p^*}{\partial \tau_p} = 0 \tag{31e}$$

$$\tau_p \frac{\partial q^*}{\partial h} - k_{p,q}' \frac{\partial q^*}{\partial h} - k_{p,h}' - q^* \frac{\partial c_p^*}{\partial h} = 0 \tag{31f}$$

$$(n^* - n^* w)\left[(\pi_f - \pi_f^{\text{ONC}}) - (\pi_a - \pi_a^{\text{ONC}})\right] = 0 \tag{31g}$$

$$(n^* w)\left[(\pi_f - \pi_f^{\text{ONC}}) - (z_b - z_b^{\text{ONC}})\right] = 0 \tag{31h}$$

Note that $K' = k_f' + (1 - w)k_{a,n_a}' + w(k_b' + k_c')$ is the marginal cost of the freight transportation service jointly shared by the three operators induced by the marginal increase in the freight demand. In particular, K' is the sum of (i) marginal operating cost of the direct road mode $(1 - w)k_{a,n_a}'$, (ii) the marginal operating cost of the co-modal mode $w(k_b' + k_c')$, and (iii) the freight forwarder's marginal general operating cost k_f'. The presence of K' in the FOCs for the Nash arbitrated solution reflects the cooperative relationship among the three operators, because it emphasizes that the optimal operation decisions under the NAS is based on the evaluation of the marginal effect of the freight demand change on the three operators' joint operating costs.

We now further discuss the FOCs in Eq. (31). In Eq. (31a), given the marginal increase in the freight fare, the interpretations on the first and second terms on the left-hand side, i.e., $\tau_f \frac{\partial n^*}{\partial \tau_f}$ and n^* are identical to those in Eq. (24a) of Section 4.1, which are omitted; the third term $-K' \frac{\partial n^*}{\partial \tau_f}$ is the marginal saving on the freight transportation service jointly shared by the three operators due to the decreased demand induced by the marginal increase in the freight fare.

In Eq. (31b), given a marginal increase in the proportion of freight units assigned to the co-modal channel w, the first term on the left-hand side $\tau_f \frac{\partial n^*}{\partial w}$ is the marginal revenue, the second term $-K' \frac{\partial n^*}{\partial w}$ is the marginal change in the operating cost jointly shared by the three operators; the third and fourth terms $-n^* k'_b - n^* k'_c$ are the marginal co-modal operating cost; and the fifth term $n^* k'_{a,n_a}$ is the marginal saving on operating cost of the direct road transportation.

Next, both Eq. (31c) and Eq. (31d) says that given a marginal increase in the trucking/FOT capacity s_a/s_b, the marginal increase in revenue from freight customers should offset the marginal decrease in benefit jointly shared by the three operators. The FOCs for the Nash arbitrated solution with respect to τ_p and h are also comparable to those in Eqs. (21), and thus the detailed description is omitted. Regarding Eqs. (31g) and (31h), given an interior $n^* w$, we should have $\pi_f - \pi_f^{\mathrm{ONC}} = \pi_a - \pi_a^{\mathrm{ONC}} = z_b - z_b^{\mathrm{ONC}}$, meaning the three operator's payoff gains brought by the co-modality and business cooperation (against the status quo) will be identical.

By substituting the partial derivatives of equilibrium freight and passenger demand with respect to τ_f, s_a, s_b, w, τ_p and h into FOCs at NAS in Eq. (31), and solving them simultaneously, it gives:

$$\tau_f^{\mathrm{NAS}} = K' + n^*[l'_{f,n_a}(1 - w^{\mathrm{NAS}}) + l'_{f,n_b} w^{\mathrm{NAS}}] - \frac{n^*}{D'_f} \tag{32a}$$

$$w^{\mathrm{NAS}} : k'_{a,n_a} + n^* l'_{f,n_a} = k'_b + k'_c + n^* l'_{f,n_b} \tag{32b}$$

$$s_a^{\mathrm{NAS}} : k'_{a,n_a} = -n^* l'_{f,s_a} \tag{32c}$$

$$\tau_p^{\mathrm{NAS}} = k'_{p,q} + q^* l'_{p,q} \tag{32d}$$

$$h^{\mathrm{NAS}} : k'_{p,h} = -q^* l'_{p,h} \tag{32e}$$

$$s_b^{\mathrm{NAS}} : n^* l'_{f,s_b} = -q^* l'_{p,s_b} \tag{32f}$$

$$m_a^{\mathrm{NAS}} : \pi_f^{\mathrm{NAS}} - \pi_f^{\mathrm{ONC}} = \pi_a^{\mathrm{NAS}} - \pi_a^{\mathrm{ONC}} \tag{32g}$$

$$m_b^{\mathrm{NAS}} : \pi_f^{\mathrm{NAS}} - \pi_f^{\mathrm{ONC}} = \pi_b^{\mathrm{NAS}} - \pi_b^{\mathrm{ONC}} \tag{32h}$$

One can find that the formulae or optimality conditions for τ_f^{NAS}, s_a^{NAS}, τ_p^{NAS}, h^{NAS} and s_b^{NAS} are comparable to those under NE in Eq. (28). Regarding the optimality condition with respect to w^{NAS}, it indicates that the modal-split strategy for freight that is jointly determined by the three operators through cooperation should be set in the way that the marginal cost of using the direct road mode balances the marginal cost of using the co-modal mode.

We now discuss the implications of Eqs. (32g) and (32h). Note that in general, the Nash arbitrated solution will not always yield an equal payoff gain (against the disagreement point) for the agents. The reasons why the operators have the equal payoff gains are as follows. Firstly, in Section 2.2, we mentioned that τ_f, w, s_a and s_b are the four decision variables that shape freight's supply-demand equilibrium (see Eq. (3)). From the optimality conditions of these four variables under NAS, i.e., Eqs. (32a), (32b), (32c) and (32f), m_a and m_b are absent, indicating that neither direct road transportation nor co-modal transportation price governs the optimal τ_f, w, s_a or s_b under NAS. Thus, the m_a (resp. m_b) has no effect on the freight demand under NAS, and the total road (resp. co-modal) transportation fee $m_a n_a$ (resp. $m_b n_b$) transferred from freight forwarder to carrier (resp. to transit operator) only linearly increases with m_a (resp. m_b). With such a property, m_a and m_b effectively constitute a side payment between freight forwarder and carrier/transit operator, which results in the equal payoff gains among the three operators under NAS.

Remark 8. *Based on the first-order optimality conditions in Eq. (31), by assuming constant k'_f, k'_{a,n_a}, k'_{a,s_a}, k'_b, k'_c, $k'_{p,q}$ and $k'_{p,h}$, and applying the point elasticity, the profits of the freight forwarder, carrier and transit operator can be derived and are as follows:*

$$\pi_f^{NAS} = \frac{1}{3}\Pi^{NAS} + \frac{1}{3}(2\pi_f^{ONC} - \pi_a^{ONC} - z_b^{ONC}) \tag{33a}$$

$$\pi_a^{NAS} = \frac{1}{3}\Pi^{NAS} + \frac{1}{3}(2\pi_a^{ONC} - \pi_f^{ONC} - z_b^{ONC}) \tag{33b}$$

$$\pi_b^{NAS} = \frac{1}{3}\Pi^{NAS} + \frac{1}{3}(2z_b^{ONC} - \pi_f^{ONC} - \pi_a^{ONC}) \tag{33c}$$

where

$$\Pi^{NAS} = \underbrace{n^* \frac{(\sigma^n_{s_a} - 1)}{\sigma^n_{\tau_f} + 1} K'}_{\text{income from freight service}} + \underbrace{\frac{q^*}{\sigma^{q^*}_{\tau_p} + 1}\left[(\sigma^q_h - 1)k'_{p,q} + \frac{q^*}{D'_p}(\sigma^q_{\tau_p} + \sigma^q_h)\right]}_{\text{income from transit service}}$$

In Eq. (33), the three operator's profit under NAS depend on the payoffs associated with the disagreement point $(\pi_f^{\mathrm{ONC}}, \pi_a^{\mathrm{ONC}}, z_b^{\mathrm{ONC}})$ and the total income from both freight service and transit service Π^{NAS}. The formula of Π^{NAS} consists of two terms. The first term describes the income from the freight service, which is comparable with that under NE, where its sign is dependent on $\sigma^n_{s_a}$ due to $\sigma^n_{\tau_f} < -1$ (derived based on Eq. (31a)). The second term describes the income from the transit service, which is also comparable with that under ONC, NE, where its exact value is subject to the optimal s_b^{NAS}. Note that α (reflecting the scenarios of the co-modal operation in relation to the connection trip costs) is absent in the formulae of profits, thanks to the collaborative and joint decision making under which the income/cost (especially for the connection trip cost) from both freight service and transit service are shared among the three operators.

5. Numerical studies

This section presents numerical examples to further illustrate the game-theoretical models and analysis in this paper. Section 5.1 summarizes the numerical setting. Section 5.2 compares the optimal operation decisions, profits, consumer surpluses, levels of service for freight customers and transit passengers under different scenarios/games. Section 5.3 carries out sensitivity analysis.

5.1. Numerical settings

The numerical study is established based on the co-modal operation in Sydney where the freight units are shipped from Lidcombe to Sydney central business district using the Inner West and Leppington Line (T2) Train Line of New South Wales, Australia. Tables 1 and 2 summarize the function specifications (that are assumed) and values of parameters (with sources of numerical setting), respectively.

We now briefly discuss the functional forms applied in this section. Firstly, the generalized cost function of the transit service and that of the freight service both have a monetary term (i.e., service fare) and a non-monetary term that quantifies the levels of services. Specifically, for the transit service, the non-monetary cost is formulated to capture passengers' total travel time cost (consisting of boarding and alighting delay) and passengers' disutility due to in-vehicle crowding brought by passengers and freight. A distinct parameter θ describing the value of crowding is assigned to the term $\left[\omega + b_0 e^{b_1[(q+\frac{\phi s_b}{d})/h - v]}\right] \cdot t_2$ to quantify the cost of in-vehicle crowding. It is worth noting that the expression of the in-vehicle crowding cost in c_p is in line with the transit users' discomfort function developed by De Palma et al. (2015) with minor modifications. For the freight service, the non-monetary cost is governed by the total delivery time and the freight customers' value of delivery time η, in which the value of η is from Hsiao (2009) and converted into the Australian dollar (AU$).

Table 1
Function specifications

Function	Specification
Generalized cost of the freight service	$c_f = \tau_f + \eta \left[x_0 + x_1 \frac{n}{s_a+s_b} + y_1(e^{\frac{n_b}{y_2}} - 1) + z_1(e^{\frac{n_a}{z_2}} - 1) \right]$
Generalized cost of the transit service	$c_p = \tau_p + \rho \left[t_0 + \frac{1}{2h} + t_1 \left(\frac{q}{vh - \frac{\phi s_b}{d}} \right) \right] + \theta \left[\omega + b_0 e^{b_1[(q+\frac{\phi s_b}{d})/h - v]} \right] \cdot t_2$
Demand function of the freight service	$n = n_0 e^{-\varepsilon_f c_f}$
Demand function of the transit service	$q = q_0 e^{-\varepsilon_p c_p}$
Freight forwarder's operating cost	$k_f = f_1 n$
Carrier's operating cost	$k_a = a_1 n_a + a_2 n_a^2 + a_3 s_a + a_4 s_a^2$
Transit operator's operating cost of the co-modal service	$k_b = u_1 n_b + u_2 n_b^2$
Operating cost of the public transit service	$k_p = k_1 q + k_2 h$
Connection trip cost	$k_c = p_1 n_b + p_2 n_b^2$

5.2. Results under different games

This subsection compares the results of different games modeled in this study. The optimal operation decisions, profits of the three operators, consumer surpluses and costs of freight customers and passengers, and the social surplus (i.e., the sum of the three operators' profits in Eqs. (5), (7) and (13) and consumer surpluses of freight customers and passengers in Eqs. (4) and (12)) under 'optimal operation under no co-modality' (ONC), Nash equilibrium (NE), cooperation among the three operators (NAS), and one PCPI solution (i.e., PCPI-S) analyzed in Section 3.2 are summarized on Table 3 and Table 4. Besides, Table 3 and Table 4 also compare the solutions to the models under three different co-modal operation scenarios: (i) $\alpha = 0$ (the transit operator bears the cost), (ii) $\alpha = 0.5$ (both freight forwarder and transit operator evenly split the cost) and (iii) $\alpha = 1$ (the freight forwarder operator bears the cost). The profits and consumer surpluses with respect to the three operation scenarios are further visualized on the radar charts in Fig. 3. Note that the ONC scenario is regarded as the benchmark for all other models/games; and the ONC scenario is also chosen as the disagreement point of the regime NAS.

(Comparison among models) We start from comparing different model solutions under $\alpha = 0$, where the main observations are summarized as follows.

(i) When the co-modality is introduced, under NE, the total capacity for freight transportation ($s_a + s_b$) is increased considerably (compared with ONC) with 2535 units due to the significant amount of FOT capacity (s_b) offered by the transit operator, which leads to an increased freight demand. The substantial amount of transit capacity (s_b) for freight transportation also makes the co-modal channel a desirable freight shipping option, where the freight forwarder assigned around 49% of the collected freight units to the co-modal channel. Since the co-modal mode has attracted more freight, a decreased demand for freight transportation on the direct road mode occurs; and to counter the decreased revenue induced by the decreased shipping demand (i.e., freight volume on the direct road channel), the carrier sets optimal trucking capacity (s_a) at the minimum level (i.e., lower bound) with $s_a = 200$ units and keeps the pricing strategy unchanged ($m_a = 10$AU\$). However, such an adjustment in operation decisions for coping with the (freight shipping) market entry of the transit operator does not secure the carrier against profit losses (compared with ONC), i.e., $\pi_a^{\text{NE}} < \pi_a^{\text{ONC}}$. Besides, when the co-modality exists, the passengers' consumer surplus (ψ_p) is also decreased because of the higher transit fare (τ_p) set by the transit operator for compensating the additional costs incurred by the co-modal operation. By contrast, the freight forwarder and the transit operator both benefit from the co-modality (greater π_f^{NE} and z_b^{NE}) when it is compared with ONC.

(ii) When the cooperation among the freight forwarder, carrier and transit operator exists, i.e., NAS, the total capacity of freight transportation ($s_a + s_b$) is increased tremendously with 9615 units, where most of the capacity is contributed by the transit operator when it is compared with the carrier who still sets the trucking capacity at the minimum level $s_a^{\text{NAS}} = 200$. It follows that the freight demand is increased (induced by greater transportation capacity), while the passenger demand is decreased (induced by fewer spaces for passengers). In comparison with NE, under NAS, the proportion of freight units on the co-modal channel is smaller, with $w^{\text{NAS}} = 0.1956$. This is because in order to prevent the carrier from being worse-off due to the presence of the co-modal channel while ensuring that the carrier would benefit from the introduction of the co-modality, the modal-split strategy is jointly set by the three operators in the way that the co-modal service is not over-utilized. Next, compared with NE, NAS yields

Table 2
Summary of numerical settings

Variable	Description	Value
a_1	Cost of road transportation for one freight unit	0.3 AU$/unit
a_2	Coefficient in carrier's cost function	5×10^{-4} AU$/unit^2
a_3	Cost of expanding one unit of trucking capacity	2.2 AU$/unit
a_4	Coefficient in carrier's cost function	8×10^{-4} AU$/unit^2
b_0	Coefficient in the in-vehicle crowding cost function	0.06
b_1	Coefficient in the in-vehicle crowding cost function	0.005
d	Operating duration of co-modality synergy [i]	3 hours
ε_f	Coefficient in freight operator's demand function	0.06
ε_p	Coefficient in transit operator's demand function	0.02
η	Value of delivery time (homogeneous)	1.55 AU$/day
f_1	Operating cost for one customer	0.1 AU$/customer
k_1	Operating cost for one passenger	0.1 AU$/passenger
k_2	Operating cost for one train service	1500 AU$/train service
M_a	Upper bound of the direct road price m_a	10 AU$/unit
M_b	Upper bound of the co-modal price m_b	10 AU$/unit
n_0	Potential demand of freight services	10000 customers
ω	Coefficient in the in-vehicle crowding cost function	0.24
p_1	Connection trip cost for one freight unit	0.75 AU$/unit
p_2	Coefficient in connection trip cost function	1.1×10^{-3} AU$/unit^2
q_0	Potential demand of transit services [ii]	6000 passengers
ϕ	Passenger-freight unit converting coefficient (homogeneous)	0.2 passenger/freight unit
ρ	Passengers' value of time (homogeneous) [iii]	24.8 AU$/hour
$\underline{S}_a$	Lower bound of trucking capacity s_a	200 units
t_0	Total in-vehicle travel time between train stations	0.375 hours
t_1	Coefficient of boarding/alighting delay	0.05 hours
t_2	In-vehicle travel time of train	0.25 hours
t_f	Expected delivery time cost if there is no delay (= ηx_0)	1.55×10^{-2} AU$
t_p	In-vehicle travel time cost (= ρt_0)	9.3 AU$
θ	Passengers' value of crowding (homogeneous)	12.4 AU$/hour
u_1	Operating cost for a unit of freight transported by train	0.1 AU$/unit
u_2	Coefficient in the transit operator's operating cost of the co-modal service	1.1×10^{-3} AU$/unit^2
v	Capacity of a train [iv]	1200 passengers
x_0	Coefficient in the freight customers' cost function	0.01
x_1	Coefficient in the freight customers' cost function	0.2
y_1	Coefficient in the freight customers' cost function	0.01
y_2	Coefficient in the freight customers' cost function	830
z_1	Coefficient in the freight customers' cost function	0.01
z_2	Coefficient in the freight customers' cost function	850

Notes: [i] The freight-on-transit (co-modal) operation duration is set based on the length of the Sydney non-peak hours (10am - 3pm). [ii] The specifications regarding Sydney T2 transit services is based on General Transit Feed Specification data and smart transit card data (`https://opendata.transport.nsw.gov.au/dataset/timetables-complete-gtfs`). [iii] The transit passengers' value of time is extracted from the Economic Parameter Values report published by Transport for NSW (TfNSW) (`https://opendata.transport.nsw.gov.au/dataset/timetables-complete-gtfs`). [iv] The capacity of train services is based on the seating specification of Waratah trains serving the Sydney T2 line (`https://www.railway-technology.com`).

a larger social surplus, which highlights the business cooperation enhances the overall system performance.

(iii) Neither NE nor NAS generates a PCPI outcome. In particular, under NE, the carrier is worse off; and under NAS, the passengers' consumer surplus is decreased (compared with ONC). Based on the solution to the PCPI-S model, it can be seen that the co-modal mode is adopted by the freight forwarder with $w^{\text{PCPI-S}} = 0.2092$, with an

Table 3 Game result: optimal operation decisions and equilibrium freight and passenger demands

Game/Model	α	τ_f	w	m_a	s_a ($\times10^2$)	τ_p	h	m_b	s_b ($\times10^3$)	$s_a + s_b$ ($\times10^3$)	n ($\times10^3$)	q ($\times10^3$)
ONC	-	28.3845	-	10.0000	3.6796	0.7851	7.3143	-	-	0.3680	1.6648	4.6091
Game/Model	α	τ_f	w	m_a	s_a ($\times10^2$)	τ_p	h	m_b	s_b ($\times10^3$)	$s_a + s_b$ ($\times10^3$)	n ($\times10^3$)	q ($\times10^3$)
NE	0.0	27.1150	0.4889	10.0000	2.0000	0.7957	7.3562	10.0000	2.3354	2.5354	1.9300	4.6082
NAS	0.0	20.3111	0.1956	8.8882	2.0000	0.8291	7.4950	7.8041	9.4157	9.6157	2.8977	4.6055
PCPI-S	0.0	18.8103	0.2092	8.4388	2.0000	0.7915	7.5163	7.1043	10.2313	10.4313	3.1599	4.6091
Game/Model	α	τ_f	w	m_a	s_a ($\times10^2$)	τ_p	h	m_b	s_b ($\times10^3$)	$s_a + s_b$ ($\times10^3$)	n ($\times10^3$)	q ($\times10^3$)
NE	0.5	27.6951	0.4626	10.0000	2.0000	0.7968	7.3602	10.0000	2.5563	2.7563	1.8673	4.6081
NAS	0.5	20.3111	0.1956	8.8882	2.0000	0.8291	7.4950	7.1174	9.4157	9.6157	2.8977	4.6055
PCPI-S	0.5	18.8103	0.2092	8.4388	2.0000	0.7915	7.5163	6.3658	10.2313	10.4313	3.1599	4.6091
Game/Model	α	τ_f	w	m_a	s_a ($\times10^2$)	τ_p	h	m_b	s_b ($\times10^3$)	$s_a + s_b$ ($\times10^3$)	n ($\times10^3$)	q ($\times10^3$)
NE	1.0	28.1444	0.4343	10.0000	2.0000	0.7973	7.3624	10.0000	2.6767	2.8767	1.8194	4.6081
NAS	1.0	20.3111	0.1956	8.8882	2.0000	0.8291	7.4950	6.4306	9.4157	9.6157	2.8977	4.6055
PCPI-S	1.0	18.8103	0.2092	8.4388	2.0000	0.7915	7.5163	5.6273	10.2313	10.4313	3.1599	4.6091

Table 4 Game result: costs, profits, consumer surpluses and social surplus

Game/Model	α	c_f	c_p	l_f	l_p	π_f ($\times10^4$)	π_a ($\times10^4$)	z_b ($\times10^5$)	π_b ($\times10^3$)	ψ_f ($\times10^5$)	ψ_p ($\times10^5$)	Social Surplus ($\times10^5$)
ONC	-	29.8814	13.1863	1.4970	12.4012	3.0440	1.3845	2.2264	-7.8139	2.7747	2.3046	2.9467
Game/Model	α	c_f	c_p	l_f	l_p	π_f ($\times10^4$)	π_a ($\times10^4$)	z_b ($\times10^5$)	π_b ($\times10^3$)	ψ_f ($\times10^5$)	ψ_p ($\times10^5$)	Social Surplus ($\times10^5$)
NE	0.0	27.4177	13.1961	0.3028	12.4004	3.2839	0.8610	2.2926	-1.1534	3.2167	2.3041	3.0287
NAS	0.0	20.6447	13.2253	0.3337	12.3962	3.3424	1.6829	2.2563	-4.6499	4.8294	2.3028	3.2417
PCPI-S	0.0	19.2008	13.1863	0.3905	12.3948	3.3339	1.6744	2.2554	-4.9147	5.2665	2.3046	3.2829
Game/Model	α	c_f	c_p	l_f	l_p	π_f ($\times10^4$)	π_a ($\times10^4$)	z_b ($\times10^5$)	π_b ($\times10^3$)	ψ_f ($\times10^5$)	ψ_p ($\times10^5$)	Social Surplus ($\times10^5$)
NE	0.5	27.9684	13.1970	0.2734	12.4003	3.2120	0.8757	2.2957	-0.8324	3.1121	2.3041	3.0157
NAS	0.5	20.6447	13.2253	0.3337	12.3962	3.3424	1.6829	2.2563	-4.6499	4.8294	2.3028	3.2417
PCPI-S	0.5	19.2008	13.1863	0.3905	12.3948	3.3339	1.6744	2.2554	-4.9147	5.2665	2.3046	3.2829
Game/Model	α	c_f	c_p	l_f	l_p	π_f ($\times10^4$)	π_a ($\times10^4$)	z_b ($\times10^5$)	π_b ($\times10^3$)	ψ_f ($\times10^5$)	ψ_p ($\times10^5$)	Social Surplus ($\times10^5$)
NE	1.0	28.4017	13.1975	0.2572	12.4002	3.1550	0.8982	2.2971	-0.6953	3.0323	2.3040	3.0056
NAS	1.0	20.6447	13.2253	0.3337	12.3962	3.3424	1.6829	2.2563	-4.6499	4.8294	2.3028	3.2417
PCPI-S	1.0	19.2008	13.1863	0.3905	12.3948	3.3339	1.6744	2.2554	-4.9147	5.2665	2.3046	3.2829

improvement in the three operators' profit and the consumer surplus of the freight customers, and an unchanged consumer surplus of the passengers (against status quo: ONC). By observing the optimal operation decisions under PCPI-S, it is clear that the freight and transit fares and the road and co-modal transportation prices are further reduced. Such a reduction in transit fare ensures the passengers' consumer surplus will at least remain unchanged when the co-modality is introduced. This observation implies that to produce a PCPI outcome, the operators have to compromise on their profits by lowering the freight/transit fare or the road/co-modal transportation fee.

(iv) Under NE, NAS and PCPI-S, the non-monetary costs of the freight customers and the passengers (l_f and l_p) are reduced compared with ONC. This indicates the co-modality holds the potential to enhance levels of service for both passengers and freight customers. Furthermore, by comparing the optimal operation decisions under all regimes, it is evident that when FOT capacity s_b is greater, the corresponding transit service frequency h is also higher. This observation indicates that the transit operator increases the transit service frequency to compensate the deteriorated levels of transit service due to more spaces occupied by freight and fewer spaces for passengers.

(Comparison among operation scenarios) Based on Tables 3 and 4, we now compare the game results under different operation scenarios (i.e., value of α), where the main observations are summarized as follows. Firstly, the NAS and PCPI-S solutions are not governed by the value of α, as these two models optimize the total profits of the operators or the total system payoff. Only NE solution is impacted by the value of α. Secondly, under NE, a smaller α (indicating the transit operator bears more connection trip cost than the freight forwarder) associates with greater w. This is because, when using the co-modal mode is less expensive for the freight forwarder, it will assign more freight units to the co-modality. By contrast, a higher α means that the freight forwarder bears more connection trip costs than the transit operator. This is to say that using the co-modal transportation is more expensive for the freight forwarder; and thus the freight forwarder not only assigns a smaller proportion of freight units to the co-modal channel (i.e., a smaller w) but also raises the freight fare τ_f to compensate the additional operating cost. It follows a decrease in freight forwarder's profit. At the same time, the transit operator offers additional FOT capacity (s_b) so that it could attract more freight and gain additional revenue from the co-modal service, as less cost (in relation to connection trips) is required to operate the co-modal service for the transit operator. In addition, since the co-modal mode is less favorable due to a greater α, a larger proportion of freight assigned to the direct freight channel brings additional

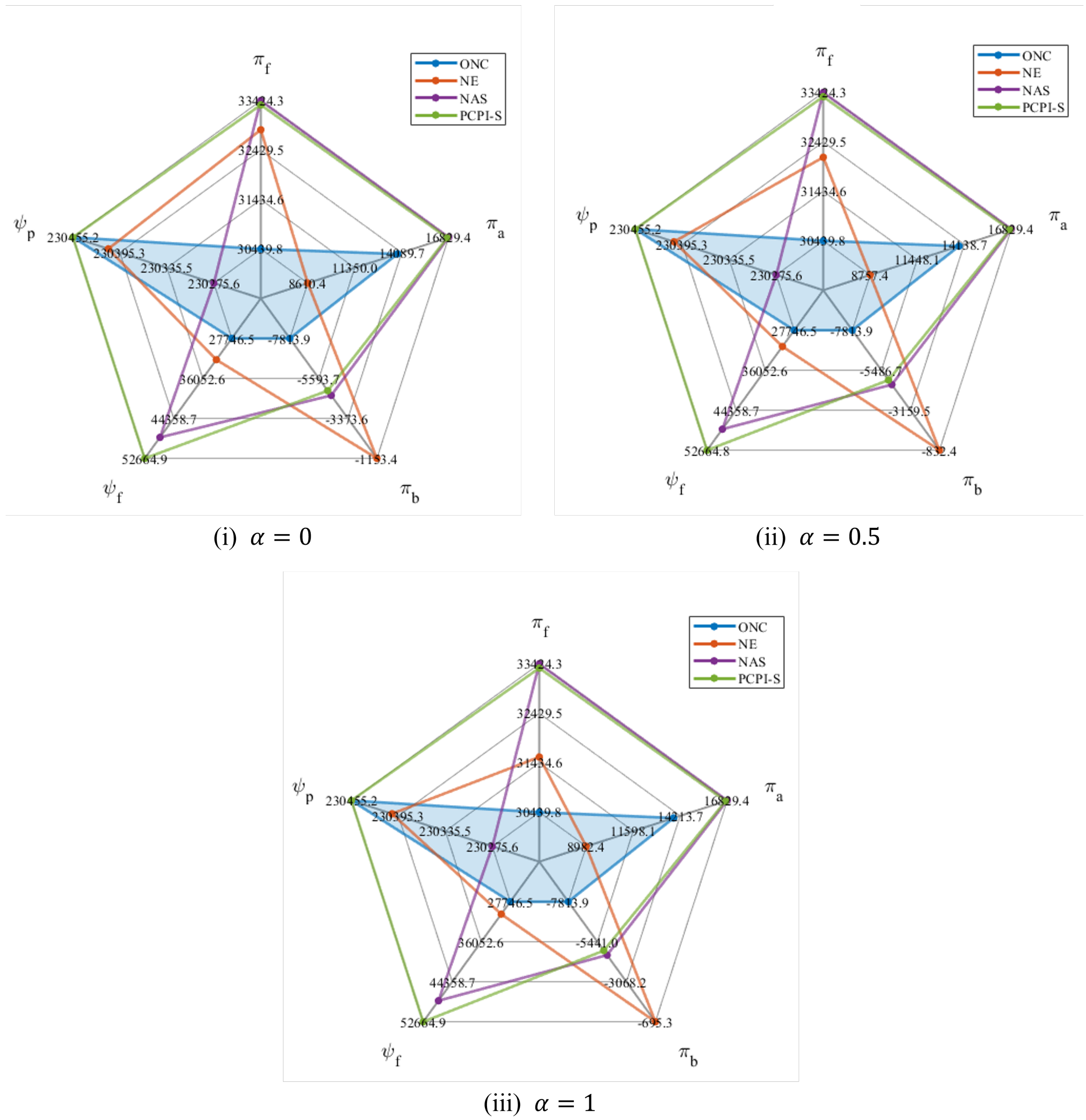

Fig. 3. Game results under different business structures. (a) $\alpha = 0$, (b) $\alpha = 0.5$ and (c) $\alpha = 1$

revenue for the carrier; and this justifies why a larger π_a^{NE} always associates with a larger α.

5.3. Sensitivity analysis

This subsection presents the numerical sensitivity analysis. We vary the value of the parameter ε_f in the freight demand function and that of the parameter ε_p in the passenger demand function, respectively, while remaining other setting in Table 2 unchanged. We consider the operation scenario where the freight forwarder and the transit operator evenly split the connection trip cost, i.e., $\alpha = 0.5$, as the results are still comparable even if different values of α are applied. Figs. 4-7 visualize the numerical sensitivity analysis regarding ε_f and ε_p. Note that for better visualizing the changes of variables, the y-axes on Figs. 4-7 are set as the percentage change of variable in concern (against the variable under the case with the lowest ε_i, i.e., $\varepsilon_f = 0.02$ when ε_f is varied, and $\varepsilon_p = 0.01$ when ε_p is varied).

(Freight customers' sensitivity to freight cost, ε_f**)** A greater ε_f (> 0) means that the freight demand decreases more sharply with respect to the generalized freight cost (i.e., demand is more sensitivity to cost), and on average, the freight customers' willingness to pay is less. We start from discussing the effect of ε_f under the ONC. Firstly, regarding τ_f in Fig. 4a, given a less favorable freight demand condition (i.e., a greater ε_f), the freight forwarder reduces the freight fare (τ_f) for the purpose of attracting customers. Despite the fare reduction, the freight forwarder will still end up with a smaller freight demand n^* (refer to Fig. 5f), i.e., the impact of a less favorable demand function/condition is unable to be recovered through the fare reduction. It follows that freight forwarder will have a decreased profit π_f (Fig. 5a), and the consumer surplus ψ_f (Fig. 5d) will also decrease, which is consistent with the smaller freight demand n^*.

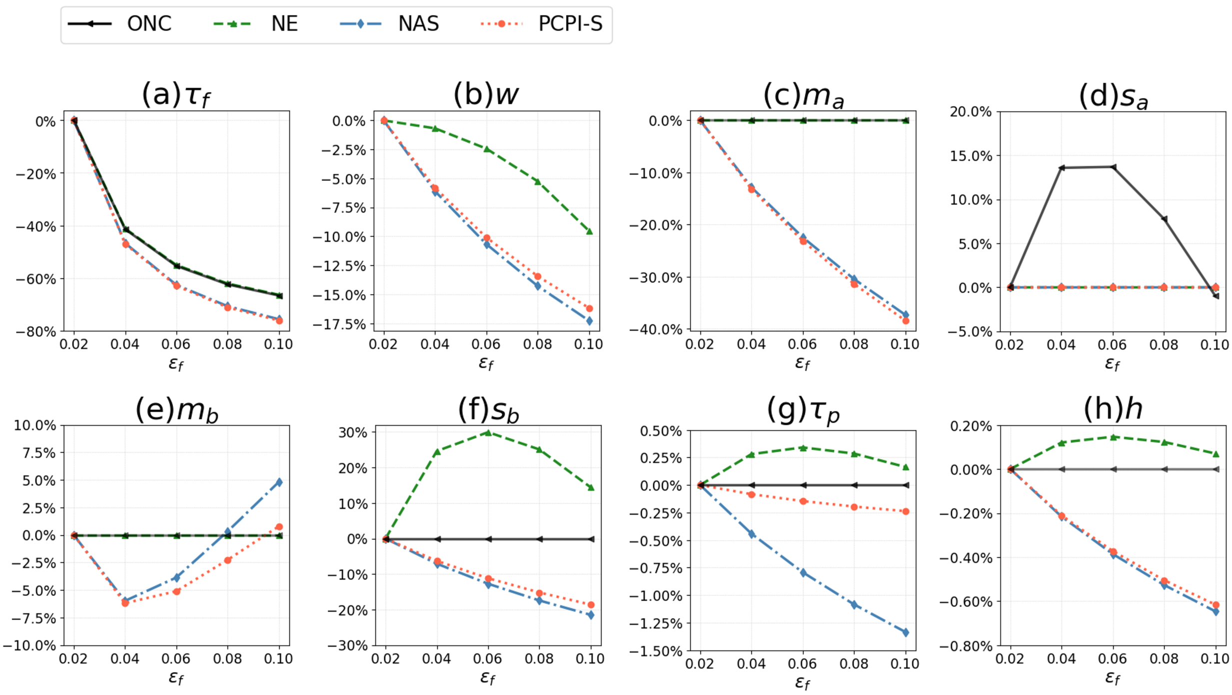

Fig. 4. Result of sensitivity analysis regarding ε_f: operation decisions

Regarding the trucking capacity s_a in Fig. 4d, under ONC, it displays a concave curve, where the trucking capacity s_a peaks at around $\varepsilon_f = 0.06$, then decreases with ε_f. This observation is explained as follows. When the freight market has a more favorable freight demand condition (i.e., ε_f is smaller), the demand is less sensitive to cost. Then, with this information, the carrier only needs to set s_a to a moderate level. This is because the further cost reduction induced by greater s_a will be less likely to attract additional freight demand due to the low cost sensitivity. Secondly, as the demand becomes more sensitive to cost (i.e., ε_f increases), the carrier has to increase s_a to enhance the levels of service and thus attracting more freight demand. Finally, when the freight demand condition becomes less favorable (here $\varepsilon > 0.06$), the freight demand significantly decreases. In response to a small freight demand, the carrier also set a smaller trucking capacity s_a.

We now turn to NE, NAS and PCPI-S. Regarding the FOT capacity s_b in Fig. 4f, one can find a similar concave curve under NE, where the reasoning is comparable to that for s_a as discussed in the above. In comparison with NE, under NAS and PCPI-S, s_b decreases continuously with ε_f which is induced by the less favorable freight demand condition. The reduced FOT capacity of the co-modal channel decreases the levels of service of the co-modal channel, and thus triggers fewer proportion of freight assigned to the co-modal channel (i.e., w in Fig. 4b). Next, regarding

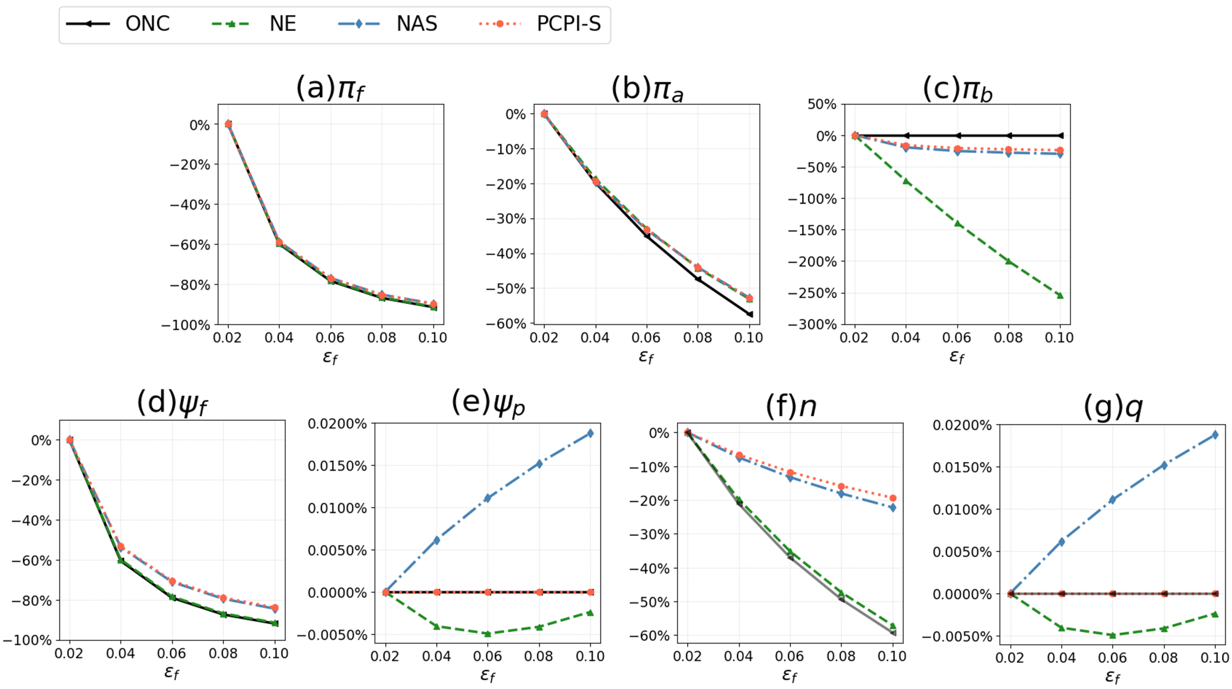

Fig. 5. Result of sensitivity analysis regarding ε_f: profits, consumer surpluses and demands

τ_p and h in Figs. 4g and 4h, since these two variables are governed by the value of s_b, where a smaller s_b leads to a smaller service frequency h and a smaller h further induces a smaller transit fare with a larger q (the reasoning is detailed earlier in Section 5.2). Such an interdependent relationship among s_b, h, τ_p and q justifies why τ_p and h display similar trends (refer to Figs. 4g and 4h) as s_b (with respect to each model), and why q exhibits an exact opposite trend compared with s_b (refer to Fig. 4f).

We now discuss the effect of ε_f on the direct road transportation price (per freight unit) m_a and the co-modal price m_b. Regarding m_a in Fig. 4c, under NAS and PCPI-S, m_a decreases with ε_f. This is because given a less favorable freight demand condition where the freight forwarder might have a decreased revenue from freight customers, to ensure that the freight forwarder would not have a profit loss due to the decreased demand, the direct road price jointly determined by the three operators should be also reduced. Regarding m_b under NAS and PCPI-S in Fig. 4e, one can find that they both exhibit a convex curve. The reasoning is as follows. First, when the ε_f is small, s_b and w are higher (the reasoning is discussed in the above), meaning greater freight volume on the co-modal channel. To ensure the transit operator would have additional benefit gain, the three operators jointly set m_b at a larger value. Secondly, as ε_f increases, m_b is reduced to protect the freight forwarder against the further profit loss due to less favorable freight demand condition, where m_b attains its minimum at around $\varepsilon_f = 0.4$. Finally, when the freight demand condition becomes less and less favorable, m_b goes up again. This is because w decreases with ε_f. It follows a decrease in the freight volume on the co-modal channel and thus a decreased revenue gain from the co-modality for the transit operator. To counter such a condition while guaranteeing the transit operator could also benefit from the co-modality, a higher m_b is jointly set by the three operators.

We now discuss how the three operators' profits are impacted by the value of ε_f. Generally, when the co-modality is introduced, given a less favorable freight demand condition (larger ε_f), the three operators' profits decrease continuously under NE, NAS and PCPI-S (refer to Figs. 5a-5c). This highlights that the freight demand is a critical factor governing the operators' profits.

(Passengers' sensitivity to transit cost, ε_p**)** A greater ε_p (> 0) means that the passenger demand decreases more sharply with respect to the generalized transit cost.

Given a less favorable transit demand condition (i.e., a greater ε_p), the transit fare (τ_p) is reduced for the purpose

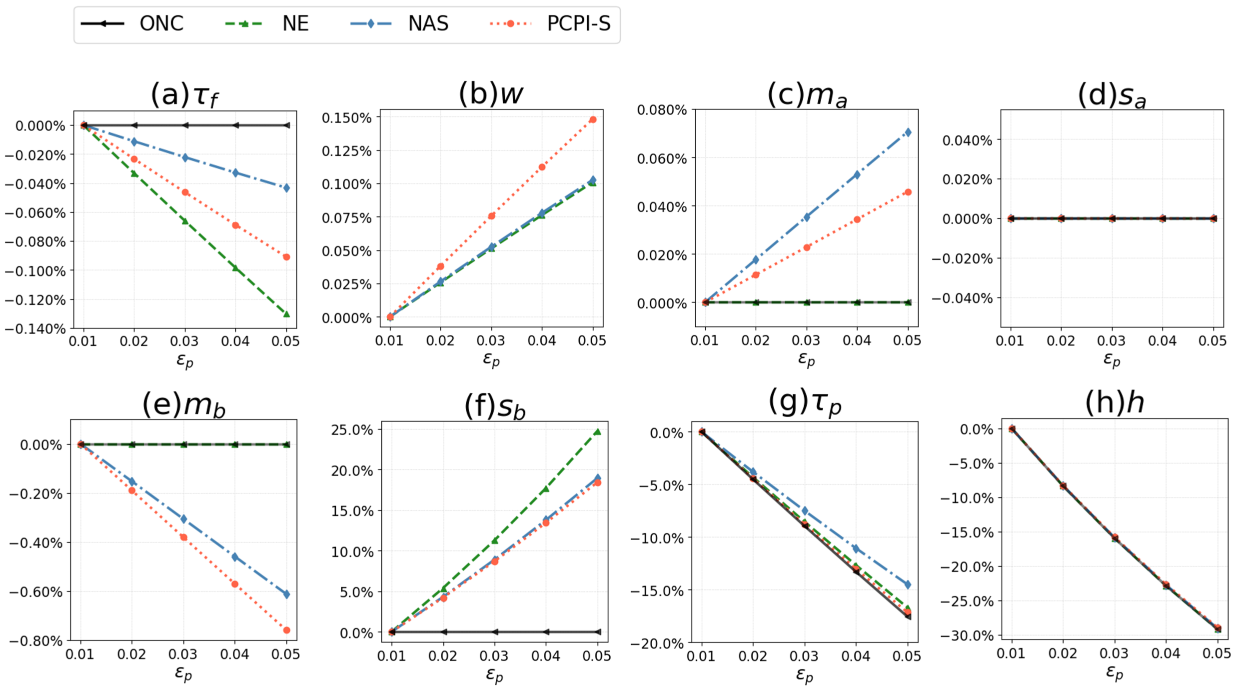

Fig. 6. Result of sensitivity analysis regarding ε_p: operation decisions

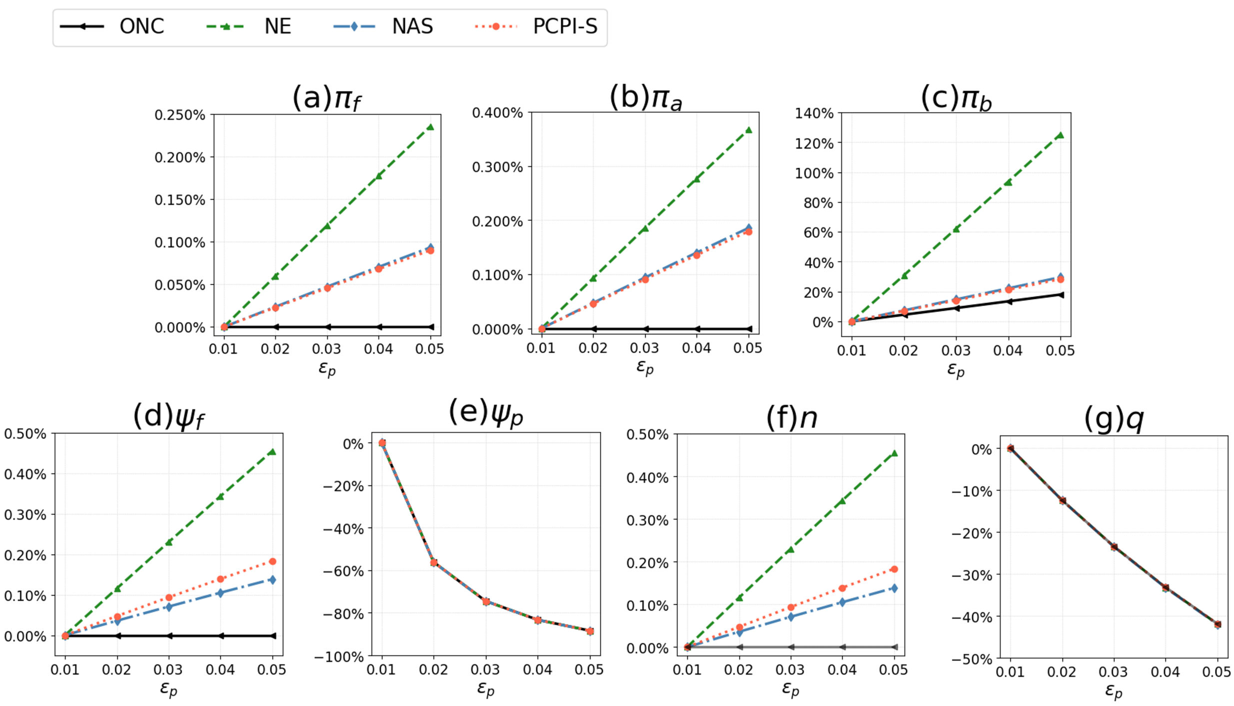

Fig. 7. Result of sensitivity analysis regarding ε_p: profits, consumer surpluses and demands

of attracting passengers (refer to Fig. 6g) under all games. Despite the fare reduction, the transit operator will still end up with a smaller demand q^* and thus a smaller consumer surplus for passengers ψ_p (refer to Figs. 7e and 7g). It follows that the transit operator reduces the transit service frequency h to save operating cost (refer to Fig. 6h). Since the passenger demand is small under a less favorable transit demand condition, there exist additional spare spaces in the transit system. Therefore, the transit operator offers more FOT capacity s_b under NE, NAS and PCPI-S so that it could have additional revenue from the co-modality to compensate the revenue loss due to the decreased passenger demand under a larger ε_p (refer to Fig. 6f). Besides, under NAS and PCPI-S, to further attract the freight volume, the co-modal price m_b jointly set by the three operators is reduced (refer to Fig. 6e). The increased FOT capacity on the co-modal mode s_b along with a smaller co-modal price m_b not only attracts a greater proportion of freight units assigned to the co-modal channel w (refer to Fig. 6b) but also increases the total freight transportation capacity $s_a + s_b$ (where the trucking capacity remains unchanged with $s_a = 200$ hitting its lower bound when ε_p varies under NE, NAS and PCPI-S, see Fig. 6d). Furthermore, the cheaper co-modal service reduces the freight forwarder's operating cost, and thus the freight forwarder reduces the freight fare τ_f (refer to Fig. 6a). Overall, with a greater $s_a + s_b$ and a smaller τ_f, the freight customers' generalized cost is further reduced, which leads to an increase in freight demand n and consumer surplus of freight customers ψ_f (refer to Figs. 7d and 7f).

We now discuss the effect of ε_p on the carrier's operation pricing strategy m_a in Fig. 6c. First, ε_p has no effect on m_a under NE, as the price is always set at the upper bound as discussed in Section 5.2. Second, under NAS and PCPI-S, m_a increases with ε_p. This is because a less favorable transit demand condition makes the co-modality become a more favorable freight transportation mode, in which more freight unit will be allocated to the co-modal channel (as discussed earlier) and less freight would be allocated to the direct road channel. To ensure the carrier could also benefit from the co-modal given a decreased freight volume on the direct road channel, m_a that is jointly determined by the three operators is increased.

We now turn to the operators' profits. Regarding the transit operator's profit π_b in Fig. 7c, a greater ε_p yields a larger π_b under all games. The explanation is as follows. As discussed earlier, the transit operator reduces the transit service frequency h to save operating cost given a greater ε_p. Under the numerical setting in Table 2, the operating cost saving due to the reduction in transit service frequency h plus the revenue from the co-modality surpasses the revenue loss due to a less favorable transit demand condition, which results in a greater transit operator's profit. Regarding the freight forwarder's and the carrier's profits (π_f and π_a) in Figs. 7a and 7b, it is evident that π_f and π_a both increase with ε_p under NE, NAS and PCPI-S. This is because the increased FOT capacity in the co-modal channel s_b under a less favorable passenger demand condition (the reasoning is detailed in the above) improves the levels of freight service and thus attracts additional freight demand. It follows that the additional freight demand improves the freight forwarder's and the carrier's profits.

6. Conclusions

This paper analytically analyzes the urban co-modality and examines the system-wide impacts of the co-modal services on the existing urban freight forwarding service and the urban transit service. We model and investigate the characteristics of a co-modal system, where one transit operator serves passengers and provides freight-on-transit (FOT) services (or co-modal services) within the transit system, and one freight forwarder serves freight customers and makes arrangements for freight transportation by using both/either the direct road mode operated by one freight carrier and/or the co-modal mode operated by the transit operator. We derive the analytical conditions for the existence of Pareto-improving co-modal operation decision combinations under which the profits of the freight forwarder, carrier and transit operator, and the consumer surpluses of freight customers and passengers are improved when compared with the status quo without the co-modality. We also explore different games among the freight forwarder, carrier and transit operator, and analyze the optimal operation decisions of the freight forwarder (freight fare and modal-split strategy for freight), carrier (road transportation price and trucking capacity), and transit operator (transit fare, transit service frequency, co-modal transportation price and FOT capacity). Specifically, we both analytically and numerically examine the following three games for comparison: (i) optimal operation under no co-modality, (ii) Nash equilibrium and (iii) cooperative Nash bargaining game. The games that simultaneously enhance the three operators' profits and improve the levels of services for freight customers and passengers are further examined via numerical examples. In addition, we conduct numerical sensitivity analysis regarding passenger and freight demand conditions.

The key findings of this study are summarized below. (i) There exists at least one operation decision combination that increases or at least does not decrease the profits of the freight forwarder, carrier and transit operator and consumer surpluses of freight customers and transit passengers (with one metric strictly improved) after the introduction of the co-modal services if the direct road transportation price (per freight unit) is greater than the sum of the marginal operating cost for the co-modal service and the marginal transit's operating cost associated with a marginal increase in the freight volume on the co-modal channel. (ii) When the three operators are under a non-cooperative relationship, both the freight forwarder and the transit operator will have profit gain from the co-modality; whereas, the carrier will suffer from profit loss due to less freight assigned to the direct road mode. (iii) If the connection trip cost is borne by the transit operator, the freight forwarder will utilize the co-modal transportation more. (iv) The freight forwarder should avoid over-utilizing the co-modal transportation if the aim is to guarantee an improvement in the three operators' profits (against the status quo). (v) To ensure a Pareto-improving co-modal system, the operators have to compromise on their profits by lowering the freight/transit service fare and the co-modal transportation price.

This paper can be further extended in several ways. Firstly, the proposed analytical model is based on a stylized network representation with simplified first- and last-mile consideration. Future studies may extend the proposed model by considering a detailed cost formulation of the first- and last-mile operations with respect to the two channels, including pick-up, routing, sorting, dispatching and drop-off operations. Secondly, considering that co-modal channel might be less flexible in terms of the pick-up and drop-off operations due to the fixed routes and schedules of transit services, carriers might be appointed for performing first-mile collection and last-mile delivery tasks by the transit operator. Thus, a further extension could also consider modeling such an outsourcing arrangement between the transit operator and carriers, and investigate complicated multi-sided strategic interactions among the operators. Thirdly, it is of our interest to investigate the co-modality system with multiple co-modal channels operated on different transportation modes, such as bus, light rail, metro, taxi, and ride-sourcing services. Specifically, with the consideration of the multiple co-modal channels, the competition among the co-modal channels can be investigated, including pricing and capacity allocation strategy; the cooperation among the co-modal channels can also be studied, including synchronization and collaboration strategies among different modes of public transportation modes for an efficient urban freight transportation with the minimal impact on the reliability of public transit systems. Moreover, with the modeling framework proposed in this study, how the existing freight carriers/operators would be influenced under the multiple co-modal channels can be analytically examined.

Future studies may also consider more operational-level optimization problems, including vehicle routing problem for first- and last-mile delivery with transit service schedule as constraints, transit capacity allocation problem with passengers and freight with demand uncertainties, transshipment and synchronization problem among multiple co-modal channels, transit service design problem with loading/unloading transit stops for freight (Wang et al., 2018; Tian et al., 2022). Furthermore, the emergence of urban co-modality will result in the mixed passenger-freight flow. Commuters' (negative) perception of the mixed flow might affect their travel behavior (e.g., mode and route choices), which would impact the demand for public transit services, operators' profits and overall system performance. Future studies may also examine commuters' travel choices with consideration of freight flow in the transit system, and optimize operators' decision-making under a general multi-modal network. A bi-level approach can be adopted for this problem, where the lower-level problem solves the equilibrium passenger and freight flow pattern and the upper-level problem solves the optimal operation decisions, such as freight routing strategy and pricing decisions.

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Appendix A

Proof for Proposition 1. To prove Proposition 1, it suffices to show that, under conditions Eq. (16), there exists at least one combination of operation decisions that gives a PCPI outcome. In the following, we firstly specify one decision combination, and then prove that this decision combination can lead to a system equilibrium that exists and is a PCPI outcome.

Given an operation decision at the status quo without the co-modal service, i.e., $(\tilde{\tau}_f, \tilde{m}_a, \tilde{s}_a, \tilde{\tau}_p, \tilde{h})$, the equilibrium freight demand is denoted by $\tilde{n}$ and is characterized by Eqs. (1)-(2); and the equilibrium transit demand is denoted by $\tilde{q}$ and is characterized by Eqs. (9)-(10). The resultant profits of freight forwarder, carrier and transit operator, consumer surpluses of freight customers and passengers are denoted by $\tilde{\pi}_f$, $\tilde{\pi}_a$, $\tilde{\pi}_b$, $\tilde{\psi}_f$ and $\tilde{\psi}_p$ and calculated based on Eqs. (8), (6), (4), (12), and (14), respectively.

To facilitate the proof, we define the following function with respect to co-modal capacity s_b:

$$g(s_b; w) \equiv l_f(\tilde{n} - \tilde{n}w, \tilde{n}w, \tilde{s}_a, s_b) - l_f(\tilde{n}, 0, \tilde{s}_a, 0) \tag{34}$$

where $g(s_b; w)$ is a function of s_b, treating $\tilde{n}$, $\tilde{s}_a$, and w as parameters. In particular, $\tilde{n}$ and $\tilde{s}_a$ are the freight demand and trucking capacity specified in status quo, respectively, and w is the modal split decision of freight forwarder with co-modality. We also define the following function with respect to transit frequency h:

$$y(h; s_b) \equiv l_p(\tilde{q}, h, s_b) - l_p(\tilde{q}, \tilde{h}, 0) \tag{35}$$

where $y(h; s_b)$ is a function of h, treating $\tilde{q}$, $\tilde{h}$ and s_b as parameters. We now show that the inverse functions of g and y exist. Given the property that $\partial l_f/\partial s_b < 0$ and $\partial l_p/\partial h < 0$, i.e., l_f and l_p are strictly decreasing with respect to s_b and h, respectively, $g(\cdot)$ and $y(\cdot)$ are one-to-one functions. Thus, the inverse functions of g and y exist. We let $g^{-1}(0; w)$ denote the solution to $g(s_b; w) = 0$ given the parameter w, and let $y^{-1}(0)$ denote the solution to $y(h) = 0$.

With respect to the status quo, we construct a combination of operation decisions for the co-modal system denoted by $(\tau_f^*, w^*, m_a^*, s_a^*, \tau_p^*, h^*, m_b^*, s_b^*)$, which includes eight decision variables. Treating w^* as a parameter, we construct the other seven operation decisions as follows:

$$\tau_f^* = \tilde{\tau}_f;\ \tau_p^* = \tilde{\tau}_p;\ s_a^* = \tilde{s}_a \tag{36a}$$

$$s_b^* = g^{-1}(0; w^*) \tag{36b}$$

$$h^* = y^{-1}(0; s_b^*) \tag{36c}$$

$$m_a^* = \frac{k_a(\tilde{n} - \tilde{n}w^*, \tilde{s}_a) - k_a(\tilde{n}, \tilde{s}_a) + \tilde{m}_a\tilde{n}}{\tilde{n} - \tilde{n}w^*} \tag{36d}$$

$$m_b^* = \frac{k_p(\tilde{q}, h^*) + k_b(\tilde{n}w^*) + (1-\alpha)k_c(\tilde{n}w^*) - k_p(\tilde{q}, \tilde{h})}{\tilde{n}w^*} \tag{36e}$$

It suffices to show that there exists at least a positive w^* such that the decision variables in Eqs. (36d)-(36e) are positive and the resulting equilibrium is a PCPI outcome. We will analyze the decision variables $(s_b^*, h^*, m_a^*, m_b^*)$ one by one.

Define a function $f(\varphi) \equiv l_f(\tilde{n} - \varphi, \varphi, \tilde{s}_a, 0) - l_f(\tilde{n}, 0, \tilde{s}_a, 0)$. Given that $l_f(\cdot)$ is a smooth function of n_a, n_b, s_a and s_b, $f(\cdot)$ is also a smooth function. It can be readily shown that $f(0) = 0$. Under the condition Eq. (16b) in Proposition 1 (i.e., $l'_{f,n_b}(\tilde{n}, 0, \tilde{s}_a, 0) > l'_{f,n_a}(\tilde{n}, 0, \tilde{s}_a, 0)$), one can show that $f'(0) = -l'_{f,n_a}(\tilde{n}, 0, \tilde{s}_a, 0) + l'_{f,n_b}(\tilde{n}, 0, \tilde{s}_a, 0) > 0$. It follows that one can find a $\bar{\varphi} \in (0, \tilde{n})$ such at $\forall \varphi \in (0, \bar{\varphi}]$, $f(\varphi) > 0$.

Denote $w_1 = \frac{\bar{\varphi}}{\tilde{n}} \in (0, 1)$. Thus, $\forall w \in (0, w_1]$, $\tilde{n}w \in (0, \tilde{n})$, and $f(\tilde{n}w) > 0$. Then, we have $g(0; w) = l_f(\tilde{n} - \tilde{n}w, \tilde{n}w, \tilde{s}_a, 0) - l_f(\tilde{n}, 0, \tilde{s}_a, 0) = f(\tilde{n}w) > 0$, $\forall w \in (0, w_1]$. Moreover, given the property of delay function, when $s_b \to +\infty$, $l_f \to 0$, i.e., when $s_b \to +\infty$, $g(s_b; w) < 0$. Thus, $\forall w \in (0, w_1]$, there exists at least one positive $s_b^* = g^{-1}(0; w)$ that gives the solution to function $g(s_b^*; w) = 0$.

Applying the implicit function theorem on $g(s_b^*; w) = 0$, one can derive that:

$$\frac{\partial s_b^*}{\partial w} = -\frac{l'_{f,n_b} - l'_{f,n_a}}{l'_{f,s_b}} \tag{37}$$

We now show that when $s_b^* > 0$, $h^* = y^{-1}(0; s_b^*) > 0$. Since l_p is strictly increasing with s_b, when $s_b > 0$, $y(\tilde{h}; s_b) = l_p(\tilde{q}, \tilde{h}, s_b) - l_p(\tilde{q}, \tilde{h}, 0) > 0$. Moreover, given the property $h \to +\infty$, $l_p \to 0$, i.e., when $h \to +\infty$, $y(h; s_b) < 0$. Thus, when $s_b^* > 0$, there exists at least one positive $h^* = y^{-1}(0; s_b^*)(> \tilde{h})$ that gives the solution to function $y(h^*; s_b^*) = 0$.

Similarly, by applying the implicit function theorem on $y(h^*; s_b) = 0$, one can verify that:

$$\frac{\partial h^*}{\partial s_b} = -\frac{l'_{p,s_b}}{l'_{p,h}} > 0 \tag{38}$$

Since $\partial h^*/\partial s_b > 0$ (i.e., h^* is strictly increasing with s_b), given any $s_b > 0$, the solution h^* to $y(h^*; s_b) = 0$ is unique. Thus, one can also establish a one-to-one mapping between h^* and the parameter s_b, denoted as $h^* = h^*(s_b)$.

To summarize, when $w^* \in (0, w_1]$, there exist at least one positive s_b^* defined by Eq. (36b) and a unique positive h^* defined by Eq. (36c). Furthermore, one can readily verify that, under the condition Eq. (16c) and $w^* \in (0, w_1]$, we have $m_a^* > 0$ and $m_b^* > 0$. This completes the first part of the proof, i.e., when $w^* \in (0, w_1]$, there exist a combination of positive decision variables $(\tau_f^*, m_a^*, s_a^*, \tau_f^*, h^*, m_b^*, s_b^*)$ defined by Eq. (36).

We now proceed to pin down the range of w^* such that the decisions in Eq. (36) yield a PCPI outcome. Define the following function:

$$u(w) \equiv k_a(\tilde{n}, \tilde{s}_a) - k_a(\tilde{n} - \tilde{n}w, \tilde{s}_a) - k_p(\tilde{q}, h^*(s_b^*)) - k_b(\tilde{n}w) - k_c(\tilde{n}w) + k_p(\tilde{q}, \tilde{h}) \tag{39}$$

where $s_b^*(> 0)$ is the solution to $g(s_b^*; w) = 0$ with $w \in (0, w_1]$. One can see that when $w \to 0$, $u(w) \to 0$. Given the condition Eq. (16a) in Proposition 1, we have

$$\lim_{w\to 0^+} u'(w) = k'_{a,n_a} - (\tilde{n}, \tilde{s}_a) > k'_b(0) - k'_c(0) - k'_{p,h}(\tilde{q}, \tilde{h}) \frac{l'_{p,s_b}(\tilde{q}, \tilde{h}, 0)}{l'_{p,h}(\tilde{q}, \tilde{h}, 0)} \frac{l'_{f,n_b}(\tilde{n}, 0, \tilde{s}_a, 0) - l'_{f,n_a}(\tilde{n}, 0, \tilde{s}_a, 0)}{l'_{f,s_b}(\tilde{n}, 0, \tilde{s}_a, 0)} > 0$$

It follows that one can find a $w_2 \in (0, 1)$ such that $\forall w \in (0, \min\{w_1, w_2\}]$, $u(w) > 0$.

We now show that, given a $w^* \in (0, \min\{w_1, w_2\}]$, decisions $(\tau_f^*, w^*, m_a^*, s_a^*, \tau_p^*, h^*, m_b^*, s_b^*)$ governed by Eq. (36) yield a PCPI outcome. Denote $\pi_f^*, \pi_a^*, \pi_b^*, \psi_f^*$ and ψ_p^* the profits of freight forwarder, carrier and transit operator, and the consumer surplus of freight customers and passengers, respectively. We examine them one by one.

We start with the consumer surplus. Substituting Eq. (36b) into Eq. (34) we have $g(s_b^*; w^*) = l_f(\tilde{n} - \tilde{n}w^*, \tilde{n}w^*, \tilde{s}_a, s_b^*) - l_f(\tilde{n}, 0, \tilde{s}_a, 0) = 0$. Adding $\tilde{\tau}_f + t_f$ on both sides of this equation and rearranging it, we have $\tilde{\tau}_f + t_f + l_f(\tilde{n}(1 - w^*), \tilde{n}w^*, \tilde{s}_a, s_b^*) = \tilde{\tau}_f + t_f + l_f(\tilde{n}, 0, \tilde{s}_a, 0)$. With $\tau_f^* = \tilde{\tau}_f$ and $s_a^* = \tilde{s}_a$, this equation can be further rewritten as $\tau_f^* + t_f + l_f(\tilde{n}(1 - w^*), \tilde{n}w^*, s_a^*, s_b^*) = \tilde{\tau}_f + t_f + l_f(\tilde{n}, 0, \tilde{s}_a, 0)$, which is equivalent to $c_f^* = \tilde{c}_f$. Given the fact that supply-demand equilibrium of freight market is unique (as shown in Section 2.2), we have $D_f(c_f^*) = D_f(\tilde{c}_f)$, i.e., $n^* = \tilde{n}$. With $c_f^* = \tilde{c}_f$ and $n^* = \tilde{n}$, based on formula for consumer surplus of freight customers in Eq. (4), one can have $\psi_f^* = \tilde{\psi}_f$. One can use the similar approach to verify that τ_p^*, h^* and s_b^* in Eq. (36) could lead to $c_p^* = \tilde{c}_p$ and $q^* = \tilde{q}$, and thus $\psi_p^* = \tilde{\psi}_p$, in which the details are omitted.

We now consider the profit of carrier π_a^*. Substituting $n^* = \tilde{n}$ and Eqs. (36a) and (36d) into Eq. (7), and according to $\tilde{\pi}_a = \tilde{m}_a\tilde{n} - k_a(\tilde{n}, \tilde{s}_a)$, we have:

$$\pi_a^* - \tilde{\pi}_a = k_a(\tilde{n} - \tilde{n}w^*, \tilde{s}_a) - k_a(\tilde{n}, \tilde{s}_a) + \tilde{m}_a\tilde{n} - k_a(\tilde{n} - \tilde{n}w^*, \tilde{s}_a) - \tilde{m}_a\tilde{n} + k_a(\tilde{n}, \tilde{s}_a) = 0 \Leftrightarrow \pi_a^* = \tilde{\pi}_a$$

We now consider the profit of transit operator π_b^*. Substituting $n^* = \tilde{n}$, $q^* = \tilde{q}$, Eqs. (36a), (36b), (36c) and (36e) into Eq. (13), and according to $\tilde{\pi}_b = \tilde{\tau}_p\tilde{q} - k_p(\tilde{q}, \tilde{h})$, we have:

$$\begin{aligned}\pi_b^* - \tilde{\pi}_b = {} & \tilde{\tau}_p\tilde{n} + k_p(\tilde{q}, h^*) + k_b(\tilde{n}w^*) + (1-\alpha)k_c(\tilde{n}w^*) - k_p(\tilde{q}, \tilde{h}) - k_b(\tilde{n}w^*) - (1-\alpha)k_c(\tilde{n}w^*) - k_p(\tilde{q}, h^*) \\ & - \tilde{\tau}_p\tilde{n} + k_p(\tilde{q}, \tilde{h}) = 0 \Leftrightarrow \pi_b^* = \tilde{\pi}_b\end{aligned}$$

Finally, we consider the profit of freight forwarder π_f^*. Substituting $n^* = \tilde{n}$, Eqs. (36a), (36d), and (36e) into Eq. (5), and according to $\tilde{\pi}_f = \tilde{\tau}_p\tilde{n} - \tilde{m}_a\tilde{n} - k_f(\tilde{n})$, we have:

$$\begin{aligned}\pi_f^* - \tilde{\pi}_f = {} & \tilde{\tau}_f\tilde{n} - k_f(\tilde{n}) - \alpha k_c(\tilde{n}w^*) - k_a(\tilde{n} - \tilde{n}w^*, \tilde{s}_a) + k_a(\tilde{n}, \tilde{s}_a) - \tilde{m}_a\tilde{n} - k_p(\tilde{q}, h^*) - k_b(\tilde{n}w^*) - (1-\alpha)k_c(\tilde{n}w^*) \\ & + k_p(\tilde{q}, \tilde{h}) - \tilde{\tau}_f\tilde{n} + k_f(\tilde{n}) + \tilde{m}_a\tilde{n}\end{aligned}$$

It is clear that given $w^* \in (0, \min\{w_1, w_2\}]$, $\pi_f^* - \tilde{\pi}_f$ is equivalent to $u(w^*)$. As shown above, $u(w^*) > 0$ and thus $\pi_f^* > \tilde{\pi}_f$.

To conclude, given the condition in Eq. (16), there exists at least one combination of operation decisions leading to a PCPI outcome that $\pi_f^* > \tilde{\pi}_f$, $\pi_a^* = \tilde{\pi}_a$, $\pi_b^* = \tilde{\pi}_b$, $\psi_f^* = \tilde{\psi}_f$ and $\psi_p^* = \tilde{\psi}_p$. This completes the proof for Proposition 1. □

Appendix B

The main notations in this paper are summarized in Table B1. Those not included in Table B1 are specified in the text.

Table B1 Glossary of notations

Symbol	Description
c_f	Total freight cost of a customer
c_p	Total transit cost of a passenger
h	Transit service frequency
k_a	Operating cost of carrier
k_b	Operating cost of co-modal operation
k_c	Connection trip cost within transit system
k_f	General operating cost of freight forwarder
k_p	Operating cost of transit service
l_f	Non-monetary freight delay cost
l_p	Non-monetary transit cost in relation to waiting, service delay and crowding
m_a	Road transportation price (per freight unit charged to freight forwarder)
m_b	Co-modal transportation price (per freight unit charged to freight forwarder)
n	Freight demand
n_a	Freight volume on direct road channel
n_b	Freight volume on co-modal channel
q	Transit demand
s_a	Trucking capacity (or direct road channel capacity)
s_b	Freight-on-transit capacity (or co-modal channel capacity)
t_f	Expected delivery time cost if there is no delay
t_p	In-vehicle travel time cost
w	Mode-split strategy for freight
z_b	Benefit of transit operator
α	The proportion of connection trip cost covered by freight forwarder
τ_f	Freight fare (charged to freight customers)
τ_p	Transit fare (charged to transit passengers)
π_a	Profit of carrier
π_b	Profit of transit operator
π_f	Profit of freight forwarder
ψ_f	Consumer surplus of freight customers
ψ_p	Consumer surplus of transit passengers

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