COOPERATIVE DESIGN OF FEEDER BUS AND BIKE-SHARING SYSTEMS

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ABSTRACT

Feeder services, which provide travelers effective access from local areas to trunk-line systems, are essential, especially under the circumstance that the long-haul trunk routes are designed with large spacing between stations. The most common feeder service for trunk-line systems is the short-haul bus, transporting travelers on a predefined route. In recent years, the development of bike-sharing systems has led to a powerful supplementary feeder service. Most research on feeder services is devoted to the separate design of shared bike station locations or feeder bus routes (including station locations), but their joint design deserves further study. This research proposes a bi-level mixed-integer programming model to simultaneously design the locations of shared bike stations as well as the routes and station locations of feeder buses. A modified genetic algorithm that encapsulates the fixed-point iteration and Frank-Wolfe algorithms is developed to solve the model for large network applications. Numerical examples are given to compare the joint design with the separate design, and the key factors that impact the design outcomes are investigated. The effects of joint design on bus and bike operators as well as travelers are also discussed. The result shows that the joint design of bus and bike systems with one public operator has a better performance than that with one private operator or the separate design with two private operators from the view of the whole society. However, some operators can lose their profits while different travelers gain benefits from the joint design to different extents. Moreover, the bi-modal feeder service system can lead to a non-negligible loss to the operator compared with the unimodal feeder system. This implies that the introduction of the second type of feeder service to the unimodal system by the public sector may bring a strike by the previous operator and a subsidy is recommended to give to the operator to compensate for its loss.

Keywords: station location design, feeder service, shared bikes, bus route design, multi-modal network design

1. INTRODUCTION

The development of urbanization increases city scale and promotes the commute demand between districts with different functions, making public transit play an important role in the daily mobility of residents. To reduce the cost of transit operations, transit operators often provide both long-haul trunk services and short-haul feeder services, and travelers need to make transfers at major transfer stations to finish their trips.

The most common feeder service for long-haul transit is the short-haul bus, also known as the community bus, minibus, or shuttle bus. This kind of bus has a relatively short operation distance and aims at transporting travelers from their origins to the nearest transfer station so that they can transfer to another transit mode like metros.

A well-designed feeder bus service can help solve the “first/last mile problem” efficiently. In Shanghai, there are more than 90 community bus routes connecting the residential areas with metro stations, hospitals, schools, and super food markets. The total route distance of a community bus route is less than 5 km and the distance between two adjacent stops is between 0.4 and 0.6 km. Figure 1.1 shows an example of the community bus route in Shanghai.

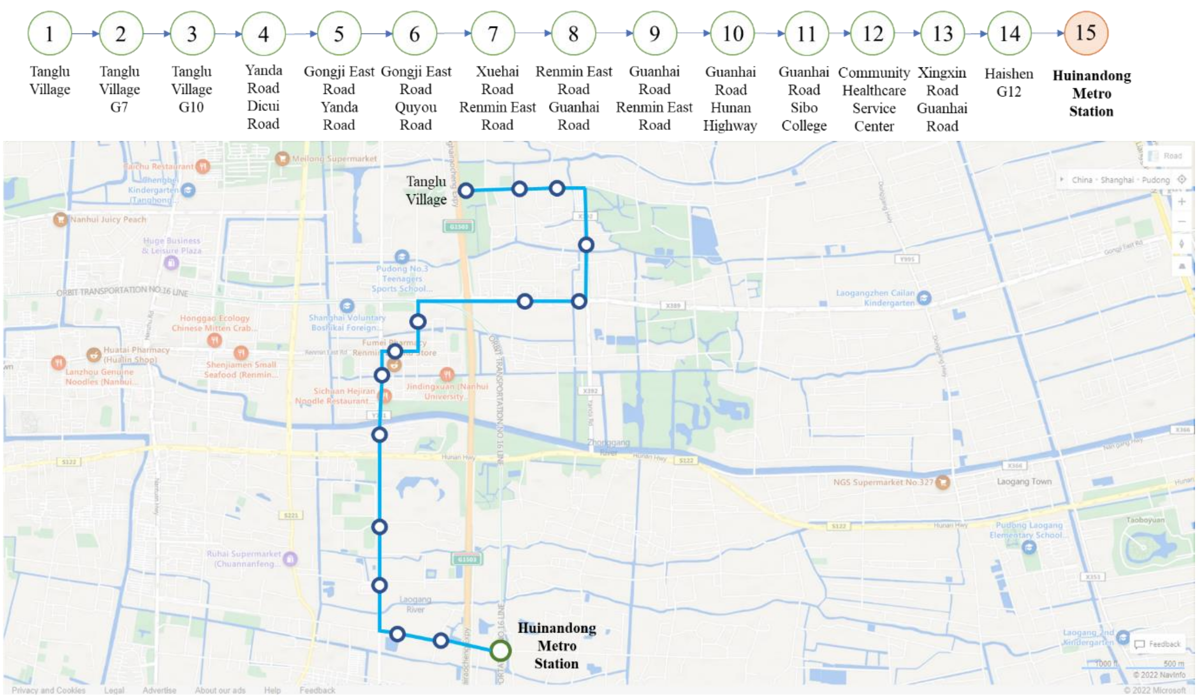


Figure 1.1 Route 1073 from Tanglu Village to Huinandong metro station in Shanghai.

Feeder bus services, as tools for filling the service gap of existing long-haul transit and improving the integration of public transport operations, have received quite a lot of attention in the past three decades. Kuah and Perl (1989) defined the feeder-bus network-design problem (FBNDP) as the problem of designing a feeder-bus network to access an existing rail system. They also presented a mathematical programming model to solve this problem. After that, a lot of research has been done in this discipline.

Traditionally, research on the design of feeder bus services deals with two problems. One is to determine the location of the bus stations (e.g., Ibeas et al., 2010; Cui et al., 2015), and the other is to design the bus route (e.g., Kuah and Perl, 1989; Kuan et al., 2006; Almasi et al., 2015; Zhu et al., 2017). Later, some researchers also tried to solve the two problems simultaneously (e.g., Xiong et al., 2013; Zheng et al., 2020). Only a few studies considered the variation in the bus travel demand (e.g., Ibeas et al., 2010; Zhu et al., 2017), while others treated the travel demand as predetermined and fixed during the bus station location and route design. Most of the previous studies construct the objective function from the perspectives of users and the operator, aiming to optimize the total cost of the two stakeholders.

With the fast development of bike-sharing systems, the role of shared bikes in solving the "first/last mile" problem has also been realized. Compared with short-haul buses, shared bikes have higher flexibility and lower construction costs. Depending on the facility type, there are two types of shared bikes: free-floating bikes and station-based bikes. The former has no physical bike station and every bike has a lock on itself (Shui and Szeto, 2020). The latter needs physical bike stations to store bikes. Users are required to pick up and return bikes at proper stations. Minhang District, the first district to use public shared bikes in Shanghai, started the public shared bike program in 2009. The government paid construction and operation costs and the bike company conducted bike production as well as daily operation, while users only needed to prepay a deposit but no fare. Figure 1.2 shows the public station-based shared bike in Shanghai (Yu, 2020). The locations of bike stations affect the user cost. If bike stations are far away from their origins or destinations, the walking time costs associated with bike stations are high and bike users may give up cycling. To reduce the user cost and meet the cycling demand, it is therefore important to design the location of bike stations carefully.

Figure 1.2 The public station-based shared bike in Shanghai.

According to the design objectives, the location design of shared bike stations can broadly be classified into three different types: maximizing bike user demand, maximizing the accessibility to bike stations, and minimizing the total costs of the users and the investor (Caggiani et al., 2020). For example, Banerjee et al. (2020) determined the optimal locations of bikeshare stations in Baltimore to maximize the number of bike users attracted by the new location, and this belongs to the first type of objective (i.e., maximizing demand). Chen and Sun (2015) maximized the accessibility in terms of total travel time to optimize the location of bike stations. Lin and Yang (2011) determined the number and locations of bike stations considering the total cost of the investor as well as the users. While the aforementioned studies consider a single objective, some studies use multiple objectives (e.g., Chow and Sayarshad, 2014; Nikiforiadis et al., 2021).

Shared bikes provide powerful supplements to the starts and ends of trips. Moreover, shared bikes coexist with feeder buses in many cities with such a bike-sharing system, giving travelers more options to complete their trips. Some studies focus on the effects of shared bikes on the multi-modal network and traveler behavior before and after the introduction of shared bikes (e.g., Li et al., 2014; Fan et al., 2019). Some studies focus on the bike allocation or relocation strategies in the bike-sharing system (e.g., Angeloudis et al., 2014; Zhang et al., 2019; Zhang et al., 2022; Wang et al., 2023; Zhang et al., 2023). Others have provided an outstanding comprehension of bus and bike station design. However, the interaction between the two travel modes needs further investigation. Table 1.1 summarizes and classifies the existing outstanding research on feeder bus and shared bike service design in terms of decision variables, the type of travel demand, and objectives.

Table 1.1 Characteristics of feeder bus and shared bike service design problems

	Reference	**Bus**		**Bike**	**Travel Demand**	**Objective(s)**	
		Station location	**Route**	**Station location**		**Users**	**Operators**
Solely feeder bus service design	Kuah and Perl (1989)	×	√	×	Fixed	√	√
	Kuan et al. (2006)	×	√	×	Fixed	√	√
	Ibeas et al. (2010)	√	×	×	Elastic	√	√
	Xiong et al. (2013)	√	√	×	Fixed	√	√
	Almasi et al. (2015)	×	√	×	Fixed	√	√
	Cui et al. (2015)	√	×	×	Fixed	√	×
	Zhu et al. (2017)	×	√	×	Elastic	×	√
	Zheng et al. (2020)	√	√	×	Fixed	√	×
Solely bike service design	Lin and Yang (2011)	×	×	√	Fixed	√	√
	Chen and Sun (2015)	×	×	√	Fixed	√	×
	Banerjee et al. (2020)	×	×	√	Elastic	√	×

Joint service design	Liu et al. (2015)	√	×	√	Fixed	√	×
	Wu et al. (2020)	√	×	√	Fixed	√	√
	Luo et al. (2021)	√	×	√	Elastic	√	√
	Li et al. (2023)	√	×	√	Fixed	√	√
	This research	√	√	√	Elastic	√	√

To our knowledge, the simultaneous design of feeder bus and shared bike services is a new trend and only a few studies focus on this. For example, Liu et al. (2015) considered both bus station location design and public bike network design at the same time in a public transportation system design problem. However, this work did not design the route of the feeder bus service and only considered minimizing travelers' costs. Wu et al. (2020) worked on the joint optimization of the shared bike feeder system and trunk network. In their research, the trunk network was composed of ordinary buses, bus rapid transit, and metros. However, travel demands were assumed uniformly distributed over the network in this study, and no bus route was designed as well. Luo et al. (2021) developed a continuum model to jointly design the shared bike and transit services in corridors. However, the bus route design was not considered in their model. Li et al. (2023) proposed a novel Demand Responsive Connector (DRC) fed by shared bikes to minimize the total cost of operators and travelers. Again, bus routing was not considered. Bus routes affect both travelers' and operators' costs and hence the total cost and benefit of the system. Therefore, in addition to the locations of bus and shared bike stations, it is important to design bus routes simultaneously from the perspective of both travelers and operators in the joint service design of feeder buses and shared bikes.

One of the most important foundations of joint service design is that the two types of feeder services have the same operator. In most big cities of China, e.g., Shanghai, the bus company is owned and operated by the government. While the recent popularity of private bike-sharing companies attracts more attention in the city, public shared bikes operated by the government are still working unobtrusively. Therefore, the assumption that these two types of feeder services have the same operator exists in reality. Furthermore, the joint design of two feeder service modes with the same public operator may intuitively bring better performance of the two systems and save some public shared-bike schemes from the verge of failure.

This study proposes a bi-level mixed-integer programming model to simultaneously design the locations of shared bike stations as well as the routes and station locations of feeder buses. As the basis of the joint design, this study assumes that both feeder bus and shared bike services are provided by the same public operator, aiming at maximizing the improvement in the total welfare of society. To capture the relationship between different modes, we build a multimodal transportation network consisting of feeder buses, shared bikes, and walking. The locations of both bus and bike stations are selected from candidate station sets, while the routes of feeder buses are designed after bus stations are selected. The upper-level objective is to maximize the increase in social welfare of the whole system, including consumer surplus and the total profit of bus and shared bike operators. The lower-level problem is a combined mode split and traffic assignment problem. Numerical examples at different scales are given to verify the validity of this method and provide insights into this problem.

The main contributions of this study are as follows.

(1) We propose a bi-level optimization model to simultaneously design the feeder bus and bike-sharing systems while considering both users' travel costs and operators' profits. This joint design determines the location of both bike and bus stations as well as bus routes.

(2) We identify the factors that affect the joint design of feeder bus and bike-sharing systems including the demand level, bike fare, and feeder bus fare. We show that the variations in these three factors affect the optimal layout of stations and also the change in social welfare.

(3) We develop a solution method that can be applied to large realistic networks. The upper-level problem is solved by a modified genetic algorithm (GA) and necessary revisions are conducted to make the algorithm adapt to the problem property. The lower-level problem is solved by the fixed-point iteration and Frank-Wolfe algorithms. The experimental results show that the modified GA is effective and has better performance than the Artificial Bee Colony (ABC) algorithm for solving large networks.

The remainder of this paper is organized as follows. Section 2 introduces the multi-modal network and describes the problem. Section 3 depicts the mathematical formulation of this problem. Section 4 proposes a modified GA to solve the large-scale problem and the numerical studies are presented in Section 5. Finally, conclusions are given in Section 6.

2. PROBLEM DESCRIPTION

We consider a multi-modal transportation network consisting of four modes: feeder buses, shared bikes, walking, and others. The first three modes have their own unimodal network as a layer and each layer is composed of its own links and nodes. A bus rapid transit (BRT) station exists in each of the three layers and is the destination of feeder services. Travelers from different origins arrive at the BRT station by feeder buses, shared bikes, walking, or others. Considering that the distance scale of feeder services is limited in reality, we assume that there is no transfer between feeder buses, bikes, and others (i.e., travelers only choose one of the main travel modes). Candidate bus and bike station nodes are found in the network and are the origins of travelers. The location of each candidate node depends on the choice of the designers or transport modelers in reality. In this research, we assume that they are given and known. The last mode between each origin and the BRT station is represented by a virtual path directly connecting the corresponding origin to the BRT station.

Figure 2.1 shows an example of the multi-modal network excluding virtual paths for clarity. In this example, the base network has 4 candidate bus station nodes (nodes 1-4), 4 candidate bike station nodes (nodes 5-8), and 1 BRT station (node 0). Black links in the base network represent roads. Yellow dummy links in the multi-modal network connect the copies of a specific node on different layers. Except for the walking layer which is the same as the base network, the links on the bus and bike layers in the multi-modal network only indicate the connectivity between two points but do not necessarily represent actual paths on the road network. Travel demands are generated from those 8 candidate station nodes and their final destination is node 0.

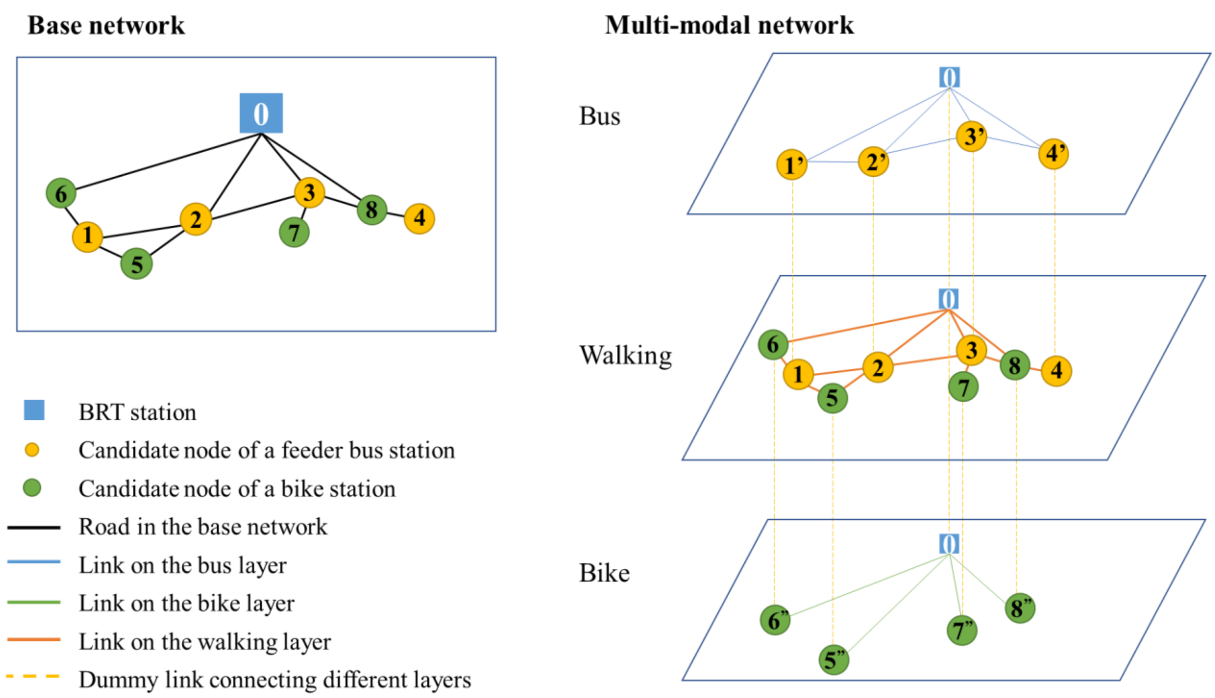


Figure 2.1. Three layers in the multi-modal network.

The bus layer network $G_1 = \{N_1 \cup \{0\}, E_1 \cup E_4\}$ is composed of the node set N_1, the BRT station (denoted as node 0), the link set E_1, and the link set E_4. N_1 is the set of candidate nodes of feeder bus stations. E_1 is the set of links connecting the nodes on the bus layer including the candidate bus stations and the BRT station. E_4 is the set of dummy links to model the dwell time cost of a bus stopping at the open bus station. Figure 2.2 shows an example for explaining the links associated with an open bus station i'. There are three types of links connecting to an open bus station i': link $a_{j'i'}$ from the previous station $j' \in N_1$ to bus station i', link $a_{i'k'}$ from bus station i' to the next station $k' \in N_1$, and dummy link $a_{ii'}$ from the copy of bus station i on the walking layer to itself on the bus layer (the same as the dummy link in Figure 2.1). The traveler flow from the walking layer passes through dummy link $a_{ii'}$ to board the bus, and the cost of dummy link $a_{ii'}$ represents the dwell time cost while the bus waits at the bus station to let the travelers board it. The detailed calculation of dwell time cost is introduced in Section 3.2.2.

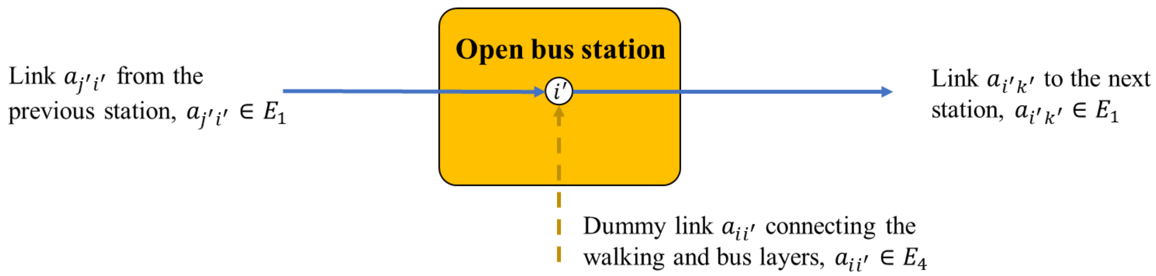


Figure 2.2 Three types of links associated with an open bus station

The bike layer network $G_2 = \{N_2 \cup \{0\}, E_2\}$ is composed of the candidate node set of bike stations N_2, node 0, and the link set E_2, where those links directly connect each of the candidate bike station nodes to the final destination, i.e., the BRT station. The walking layer $G_3 = \{N_3 \cup \{0\}, E_3\}$ is the connection layer between bus and bike layers, and $N_3 = N_1 \cup N_2$, meaning that the candidate nodes on the walking layer include all the copies of the candidate nodes on the bus and bike layers. Any two nodes in the walking layer are connected and the links between them form a set E_3.

Figure 2.3 shows an example of possible travel patterns for a traveler with one origin-destination (OD) pair (i.e., from node 2 to node 0) in this multi-modal network. In this example, the only open bus station is node 1, and the only open bike station is node 5. Travelers departing from node 2 have four choices: taking a bus, taking a bike, walking, or taking other modes (e.g., a taxi) to the destination (i.e., node 0).

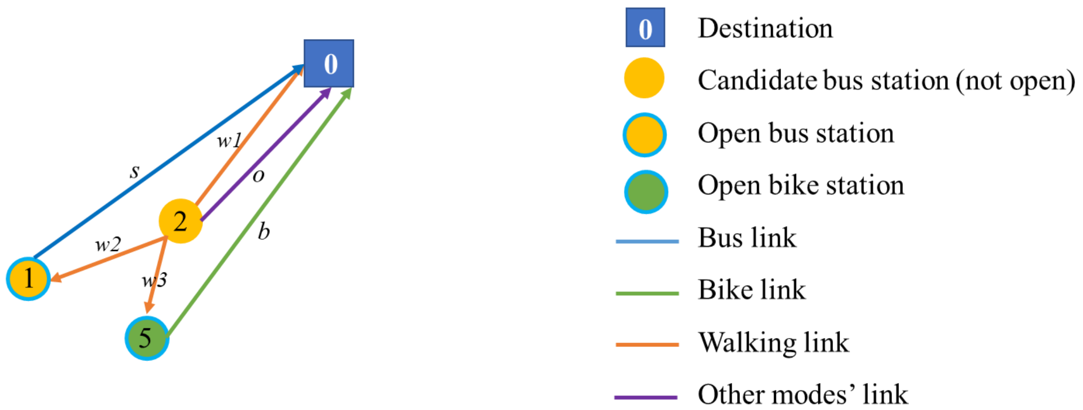


Figure 2.3 Possible travel patterns of a traveler from node 2 to node 0.

In Figure 2.3, as node 2 is not selected as an open bus station, travelers from node 2 who want to take a bus need to walk to node 1 first. Therefore, the path of the bus mode is composed of walking link *w2* and bus link *s*. If there is more than one bus station open, then more than one path exists, and we assume that travelers should choose the path with the lowest cost considering both travel time and fare (The details of cost calculation are introduced in Section 3.2.2). Similar to the bus mode, the path of the bike mode is composed of walking link *w3* and bike link *b*. The walking mode and other modes are not affected by the feeder service design and are simply represented by walking link *w1* and other modes' link *o*, respectively.

For a given multi-modal network as described above, we intend to design (1) the locations of bike stations, (2) the locations of bus stations, and (3) the routes of feeder buses, in order to maximize the improvement in the social welfare of the whole system in the upper-level problem. The mode and route choice of travelers is also considered during the design, and captured by a lower-level problem of mode split and traffic assignment. The mathematical formulation of the problem is presented in Section 3.

3. MATHEMATICAL FORMULATION

We develop a bi-level optimization model to jointly design the feeder bus and bike-sharing systems in this section. The notations adopted in this paper are summarized in Section 3.1. The upper-level problem of determining the station locations and bus routes is described in Section 3.2.1. Section 3.2.2 presents a combined mode split and traffic assignment problem at the lower level to determine the traveler flow of each mode on each link.

3.1 Notations

The notations for sets, parameters, decision variables, and functions of decision variables are defined as follows.

Sets	
N_1	a set of candidate locations of feeder bus stations;
N_2	a set of candidate locations of shared bike stations;
N_3	a set of candidate locations of feeder bus and shared bike stations;
E_1	a set of arcs linking the candidate nodes of bus stations and the BRT station;
E_2	a set of arcs directly linking each of the candidate nodes of bike stations to the BRT station;
E_3	a set of arcs linking the nodes in set N_3 and the BRT station;
E_4	a set of dummy arcs with their costs defined as the dwell time cost of travelers at the corresponding open bus station;
L	a set of bus routes;
J^n	a set of modes that can be chosen by travelers, $n \in \{0,1\}$, where superscripts 0 and 1 refer to before and after the change, respectively; $J^0 = \{\text{walk}, \text{others}\}$; $J^1 = \{\text{bike}, \text{bus}, \text{walk}, \text{others}\}$, where bus, bike, walk, and others, represent the feeder bus, shared bike, walking, and "others" modes, respectively;
P_i	a set of paths of the bus mode connecting node *i* to the destination (node 0), $i \in N_3$.
Parameters	
TV_{walk}	the value of walking time;
TV_{bus}	the value of time of a traveler taking a feeder bus;
TV_{bike}	the value of time of a traveler riding a shared bike;
v_{walk}	walking speed;
v_{bus}	the speed of a bus;
v_{bike}	cycling speed;
C_{conS}	the construction cost of one feeder bus station;
C_{conB}	the construction cost of one shared bike station;
C_{opr}	the operation cost of unit bus route distance;
F_{bus}	the feeder bus fare;
F_{bike}	the rental fare of a shared bike;
d_{ij}	the distance of link (i,j), $i, j \in N_3 \cup \{0\}$;
D_i	the demand from node i, $i \in N_3$;
α	the marginal utility of income;
β_1	the dispersion coefficient of the upper level of the hierarchical logit model;
β_2	the dispersion coefficient of the lower level of the hierarchical logit model;
f_{bus}	the frequency of a feeder bus service;
$DT_{\min}$	the minimum dwell time of a station;
b	the coefficient of boarding flow of the dwell time cost equation.
Decision variables	
Upper-level decisions	

s_i^l	1 if location *i* is chosen as a bus station served by route *l*, and 0 otherwise, $\forall\ i \in N_1, l \in L$;
$\boldsymbol{s}$	$\left(s_i^l\right)_{i \in N_1, l \in L}$;
b_i	1 if location *i* is chosen as a bike station, and 0 otherwise, $\forall\ i \in N_2$;
$\boldsymbol{b}$	$(b_i)_{i \in N_2}$;
y_{ij}^l	1 if link $(i,j) \in E_1$ is served by bus route *l*, and 0 otherwise, $\forall\ (i,j) \in E_1, l \in L$;
$\boldsymbol{y}$	$\left(y_{ij}^l\right)_{(i,j) \in E_1, l \in L}$.
Lower-level decisions	
$z_{i,eu}^m$	1 if link (*e*,*u*) is on the lowest cost path from node *i* to node 0 by mode *m*, and 0 otherwise, $\forall\ (e,u) \in E_2 \cup E_3, i \in N_3, m \in \{\text{walk, bike}\}$;
$z_{i,eu,k}^{\text{bus}}$	1 if link (*e*,*u*) is on path k from node i to destination 0 by bus, and 0 otherwise, $\forall\ (e,u) \in E_1 \cup E_3 \cup E_4, k \in P_i, i \in N_3$;
D_i^{S}	the flow of travelers taking buses from node *i*;
D_i^{B}	the flow of travelers riding bikes from node *i*;
D_i^{W}	the flow of travelers walking from node *i*;
D_i^{O}	the flow of travelers taking other modes from node *i*;
x_{eu}	the flow of link (*e*,*u*), $\forall\ (e,u) \in E_1 \cup E_3 \cup E_4$;
$\boldsymbol{x}$	$(x_{eu})_{(e,u) \in E_1 \cup E_3 \cup E_4}$.
Functions of decision variables	
V_i^j	the minimum generalized travel cost of a person departing from node *i* selecting choice j;
CS_i	the consumer surplus for a person departing from node *i*;
TC_i^{bus}	the minimum path travel cost of travelers departing from node *i* by bus;
TC_i^{bike}	the minimum path travel cost of travelers departing from node *i* by bike.

3.2 A bi-level nonlinear mixed-integer programming model

Using the above notations, the proposed problem can be formulated as a bi-level nonlinear mixed-integer programming model. The upper-level model is for the station and route design problem as presented in Section 3.2.1. The lower-level model is for the combined mode split and traffic assignment problem as shown in Section 3.2.2.

3.2.1 Upper-level: station and route design problem

The upper-level problem is defined from the view of bus and bike operators. The problem needs to determine (1) the locations of both bus stations and bike stations and (2) the bus routes with the maximum increase in the total surplus of both operators and travelers (i.e., maximum improvement in social welfare (SW)):

$$\text{P1: } \max_{\boldsymbol{s},\boldsymbol{b},\boldsymbol{y}} f_1(\boldsymbol{s}, \boldsymbol{b}, \boldsymbol{y}) = \Delta TCS + \Delta TOP. \tag{1}$$

ΔTCS is the difference in TCS before and after the design. TCS is the total consumer surplus of all the travelers in the network. The consumer surplus of a person is the utility that she or he receives in the choice situation (de Jong et al., 2007). It describes the difference between the desire that a consumer would like to pay and the price that she or he actually pays.

Logsum is a measure of consumer surplus in the context of logit choice models (de Jong et al., 2007). In our study, we assume all travelers are homogeneous and the difference in marginal utility of people with different levels of income is ignored. Then the expected consumer surplus of a person departing from node $i \in N_3$ can be determined as

$$E(CS_i) = \frac{1}{\alpha}\ln\left(\sum_{j\in J} e^{-\alpha V_i^j}\right) + K, \forall i \in N_3, \tag{2}$$

where J is the set of all choices and $-V_i^j$ is the utility of choice j for a person departing from node i. α is a parameter to reflect the marginal utility of income and is set as 0.03 in this study. K is an unknown constant that represents the fact that the absolute value of utility can never be measured. Therefore, we need to measure the difference between the consumer surplus before and after the provision of feeder services as follows:

$$\Delta E(CS_i) = \frac{1}{\alpha}\left[\ln\left(\sum_{j\in J^1} e^{-\alpha V_i^j}\right) - \ln\left(\sum_{j\in J^0} e^{-\alpha V_i^j}\right)\right], \forall i \in N_3, \tag{3}$$

where superscripts 0 and 1 distinguish the scenario before and after the provision of feeder services, respectively. Here we assume that travelers cannot take any bus or bike to node 0 before the provision of feeder services. V_i^j in this study is the minimum generalized travel cost of a traveler departing from node i.

By summing up the differences in consumer surplus of all travelers in the network, we can calculate ΔTCS as

$$\Delta TCS = \sum_{i\in N_3} \Delta E(CS_i) \times D_i. \tag{4}$$

On the other hand, the second term on the right-hand side of objective (1), i.e., ΔTOP, represents the change in the total operator profit. The total operator profit of both bus and bike operators TOP is determined by three parts: the total fare revenue TF, which includes the bus fare income and the bike fare income from travelers, the total construction cost TCC, and the total operation cost TOC. The relationship between them can be expressed as

$$TOP = TF - TCC - TOC. \tag{5}$$

The total fare revenue can be calculated as

$$TF = \sum_{i\in N_3} D_i^{\mathrm{B}} \times F_{\mathrm{bike}} + \sum_{i\in N_3} D_i^{\mathrm{S}} \times F_{\mathrm{bus}}, \tag{6}$$

where D_i^{B} and D_i^{S} are the number of travelers departing from node i taking a bike or a bus, respectively.

The total construction cost only depends on the number of open stations. Each bus/bike station has its own construction cost, and the sum of these costs can be determined as

$$TCC = C_{\mathrm{conS}} \times \sum_{i\in N_1}\sum_{l\in L} s_i^l + C_{\mathrm{conB}} \times \sum_{i\in N_2} b_i, \tag{7}$$

where C_{conS} and C_{conB} are the construction cost of one bus station and one bike station, respectively.

When it comes to the operation cost, the bus operator and the bike operator have different considerations. The bus operation cost is highly related to the total bus route length and service frequency, while the major bike operation cost comes from inventory level management and bike relocation (Schuijbroek et al., 2017). In this paper, we assume that the capacity of each bike station is infinite, which means that travelers' choices are not limited by the mismatch between supply and demand. In other words, the bike relocation cost is not considered. For the bus mode, the bus frequency is considered to be fixed. Then the operation cost here is only associated with the bus route length and can be determined as

$$TOC = f_{\text{bus}} \cdot C_{\text{opr}} \sum_{l \in L} \sum_{(i,j) \in E_1} d_{ij} \cdot y_{ij}^l \,, \tag{8}$$

where f_{bus} is the fixed bus frequency, C_{opr} is the unit distance cost of bus operation, and $\sum_{(i,j) \in E_1} d_{ij} \cdot y_{ij}^l$ is the length of route *l*.

The change in the total operator profit before and after the provision of feeder services can be calculated as

$$\Delta TOP = TOP^1 - TOP^0 = TOP \,, \tag{9}$$

where superscripts 0 and 1 distinguish the scenario before and after the provision of feeder services, respectively. As we assume that all the travelers cannot take any bus or bike before the construction, the profit of operators under this scenario (i.e., TOP^0) is 0.

The main constraints of the upper-level problem are considered from the view of a vehicle routing problem (VRP). The classical VRP is defined on an undirected graph. Node 0 represents the depot where *n* identical vehicles start and end (Laporte, 2007). Each customer has a non-negative demand and each link has a cost. The objective is to find a path to connect the customers with the minimum cost for the vehicle. In our studied problem, the destination – the long-haul transit station – is treated as a depot (i.e., node 0). All the feeder bus routes start from and end at node 0. Apart from node 0, all the other nodes on the bus route can be visited only once. Constraints (10)-(17) are defined for the upper-level problem, following the typical VRP and location design constraints.

$$\sum_{l \in L} s_i^l \leq 1, \ \forall i \in N_1. \tag{10}$$

$$s_i^l = \sum_{j \in N_1 \cup \{0\},\, j \neq i} y_{ij}^l \,, \ \forall i \in N_1, \ l \in L. \tag{11}$$

$$s_i^l = \sum_{j \in N_1 \cup \{0\},\, j \neq i} y_{ji}^l \,, \ \forall i \in N_1, \ l \in L. \tag{12}$$

$$\sum_{j \in N_1} \sum_{l \in L} y_{j0}^l = |L|. \tag{13}$$

$$\sum_{j \in N_1} \sum_{l \in L} y_{0j}^l = |L|. \tag{14}$$

$$\sum_{i \in H, j \in H, j \neq i} y_{ij}^l \leq \sum_{i \in H \setminus \{k\}} s_i^l \,, \forall l \in L, k \in H, H \subseteq N_1. \tag{15}$$

$$s_i^l \in \{0,1\}, \forall i \in N_1, \ l \in L. \tag{16}$$

$$y_{ij}^l \in \{0,1\}, \forall i \in N_1 \cup \{0\}, \ j \in N_1 \cup \{0\}, l \in L. \tag{17}$$

Constraint (10) means that each selected candidate bus station can only belong to one bus route. Constraints (11) and (12) are the in-degree and out-degree definition constraints for each customer node, respectively, and they must be equal. Constraints (13) and (14) are the in-degree and out-degree definition constraints for node 0, respectively. Constraint (15) is the subtour elimination constraint. Constraints (16) and (17) are domain constraints.

3.2.2 Lower-level: combined mode split and traffic assignment problem

The lower-level problem P2 is a combined mode split and traffic assignment problem from the view of travelers. Let D_i be the total demand from origin $i \in N_3$ to destination 0. D_i is fixed and known. Travelers are assumed to select travel modes based on the hierarchical logit mode choice model, the structure of which is shown in Figure 3.1.

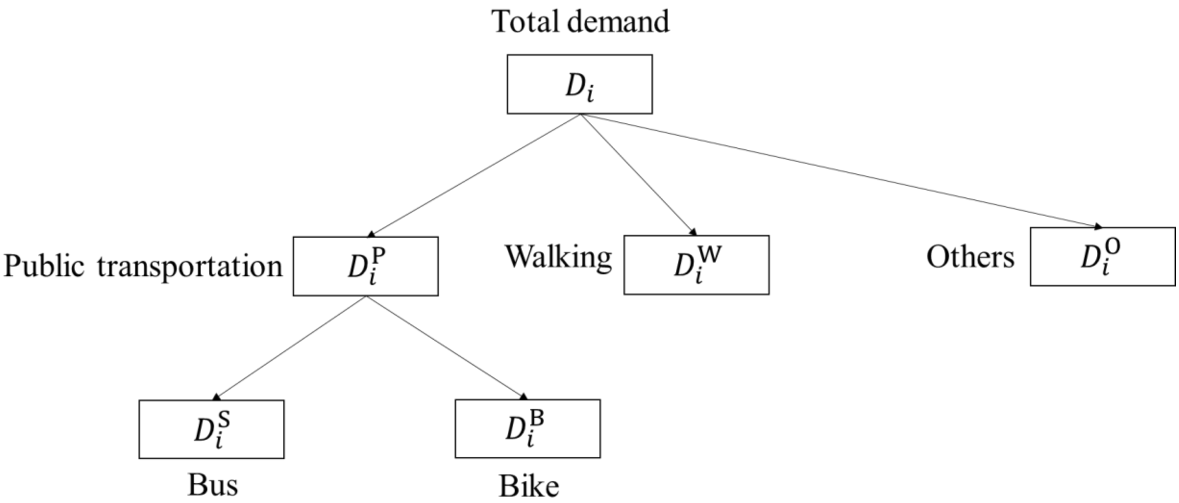


Figure 3.1 A hierarchical mode choice structure

At the first level of mode choice, the whole demand of node *i* is divided into public transportation, walking, and others. Here "others" represents the modes except for bus, bike, and walking, such as taking a taxi. Therefore, D_i can be expressed as

$$D_i = D_i^{\mathrm{P}} + D_i^{\mathrm{W}} + D_i^{\mathrm{O}}, \forall i \in N_3. \tag{18}$$

The probability of travelers walking from node *i* to node 0 is

$$Pr_i^{\mathrm{W}} = \frac{e^{-\beta_1 V_i^{\mathrm{walk}}}}{e^{-\beta_1 L_i^{\mathrm{P}}} + e^{-\beta_1 V_i^{\mathrm{walk}}} + e^{-\beta_1 V_i^{\mathrm{others}}}}, \forall i \in N_3, \tag{19}$$

and the probability of travelers taking the "others" mode from node *i* to node 0 is

$$Pr_i^{\mathrm{O}} = \frac{e^{-\beta_1 V_i^{\mathrm{others}}}}{e^{-\beta_1 L_i^{\mathrm{P}}} + e^{-\beta_1 V_i^{\mathrm{walk}}} + e^{-\beta_1 V_i^{\mathrm{others}}}}, \forall i \in N_3, \tag{20}$$

where

$$L_i^{\mathrm{P}} = -\frac{1}{\beta_2} \ln \left(\sum_{m \in \{\mathrm{bike,bus}\}} e^{-\beta_2 V_i^m}\right), \forall i \in N_3, \tag{21}$$

β_1 and β_2 are the dispersion coefficients of the upper and lower levels of the hierarchical logit model, respectively. In this study, $\beta_1 = 0.03$ and $\beta_2 = 0.06$. V_i^m is the generalized travel cost for travelers from node *i* to node 0 by mode *m*. Therefore, the demand for walking and the "others" mode can be, respectively, calculated as

$$D_i^{\mathrm{W}} = D_i \times Pr_i^{\mathrm{W}}, \forall i \in N_3, \tag{22}$$

and

$$D_i^{\mathrm{O}} = D_i \times Pr_i^{\mathrm{O}}, \forall i \in N_3. \tag{23}$$

The probability of travelers taking public transportation is

$$Pr_i^{\mathrm{P}} = 1 - Pr_i^{\mathrm{W}} - Pr_i^{\mathrm{O}}, \forall i \in N_3. \tag{24}$$

If a traveler has decided to take public transportation, then the probability of taking a bus is

$$Pr_i^{\mathrm{P,bus}} = \frac{e^{-\beta_2 V_i^{\mathrm{bus}}}}{e^{-\beta_2 V_i^{\mathrm{bus}}} + e^{-\beta_2 V_i^{\mathrm{bike}}}}, \forall i \in N_3. \tag{25}$$

Similarly, the probability of taking a bike is

$$Pr_i^{\mathrm{P,bike}} = \frac{e^{-\beta_2 V_i^{\mathrm{bike}}}}{e^{-\beta_2 V_i^{\mathrm{bus}}} + e^{-\beta_2 V_i^{\mathrm{bike}}}}, \forall i \in N_3. \tag{26}$$

Therefore, with any given total travel demand departing from node *i*, the traveler demand for buses and bikes can be calculated as

$$D_i^{\mathrm{S}} = D_i \times Pr_i^{\mathrm{P}} \times Pr_i^{\mathrm{P,bus}}, \forall i \in N_3, \tag{27}$$

and

$$D_i^{\mathrm{B}} = D_i \times Pr_i^{\mathrm{P}} \times Pr_i^{\mathrm{P,bike}}, \forall i \in N_3, \tag{28}$$

respectively.

The mode split and traffic assignment problem is tightly related to the minimum generalized travel cost. Equations (29)-(31) present the formulas of the minimum generalized travel costs for travelers by bus, bike, and walking, respectively. The minimum generalized travel cost for travelers by the "others" mode is set as twice as walking time cost.

$$V_i^{\mathrm{bus}} = TC_i^{\mathrm{bus}} + F_{\mathrm{bus}}. \tag{29}$$

$$V_i^{\mathrm{bike}} = TC_i^{\mathrm{bike}} + F_{\mathrm{bike}}. \tag{30}$$

$$V_i^{\mathrm{walk}} = TC_i^{\mathrm{walk}}. \tag{31}$$

TC_i^m represents the minimum path travel cost from node *i* to node 0 by mode *m*; it includes walking, dwell time, and in-vehicle time costs if *m* represents bus, and it includes walking and cycling time costs if *m* represents bike. F_{bus} and F_{bike} are the bus and bike fares. Considering that the scale of the feeder service problem is small, both two fares are set to fixed values and unrelated to the travel distance in this study.

For the walking mode, the minimum generalized travel cost is fixed as the road network remains unchanged during the design of feeder services. Therefore, the problem of finding the minimum generalized travel cost of walking is a shortest path problem. Equation (32) shows the way to calculate TC_i^{walk}.

$$TC_i^{\mathrm{walk}} = \sum_{(e,u)\in E_3} z_{i,eu}^{\mathrm{walk}} \times \frac{d_{eu}}{v_{\mathrm{walk}}} \times TV_{\mathrm{walk}}, \tag{32}$$

where $z_{i,eu}^{\mathrm{walk}}$ is a binary decision variable, which is equal to 1 if link (*e*,*u*) is on the lowest cost path from node *i* to node 0 by walking. d_{eu} is the distance of link (*e*,*u*); v_{walk} is the walking speed; TV_{walk} is the value of walking time of travelers. When the path with the minimum travel cost of walking is obtained, all the flows of walking mode are assigned to this path.

For the bike mode, since the design of bikeways is not considered in this study, once the upper level of the bike station design is determined, the walking time cost from the origin to the bike station and the riding time cost from the bike station to the destination are also determined. Similar to the walking mode, the problem of finding the path with the minimum travel cost for the bike mode becomes a shortest path problem and all the bike flows are assigned to the path with the minimum cost TC_i^{bike}. Equation (33) shows the expression of TC_i^{bike}.

$$TC_i^{\mathrm{bike}} = \sum_{(e,u)\in E_3} z_{i,eu}^{\mathrm{bike}} \times \frac{d_{eu}}{v_{\mathrm{walk}}} \times TV_{\mathrm{walk}} + \sum_{(e,0)\in E_2} z_{i,e0}^{\mathrm{bike}} \times \frac{d_{e0}}{v_{\mathrm{bike}}} \times TV_{\mathrm{bike}}. \tag{33}$$

For the bus mode, the cost of the whole path is composed of three parts: walking time cost from the origin to the nearest open bus station, dwell time cost at the bus station, and in-vehicle travel cost from the bus station to the destination. The dwell time cost is the cost that the bus stopped at a bus station, which is tightly related to the number of passengers boarding the bus at this station. Therefore, the paths used by travelers involving the bus mode cannot be simply obtained by solving the shortest path problem. To find out their paths and the corresponding minimum path travel cost, we need to solve the following user equilibrium (UE) traffic assignment problem:

$$\min g(\boldsymbol{x}) = \sum_{(e,u)\in E_1\cup E_3\cup E_4} \int_0^{x_{eu}} t_{eu}(w)dw \tag{34}$$

$$\text{s.t.} \sum_{k\in P_i} f_k^i = D_i^{\text{bus}},\ \forall i \in N_3, \tag{35}$$

$$f_k^i \geq 0,\ \forall k \in P_i, i \in N_3, \tag{36}$$

$$x_{eu} = \sum_{i\in N_3} \sum_{k\in P_i} f_k^i\, z_{i,eu,k}^{\text{bus}},\ \forall (e,u) \in E_1 \cup E_3 \cup E_4. \tag{37}$$

Expression (34) defines the objective of this traffic assignment problem. x_{eu} is the flow of link (*e,u*). $t_{eu}(w)$ is the travel cost function of link (*e,u*). f_k^i is the flow of path k from origin *i* by bus. $z_{i,eu,k}^{\text{bus}}$ is a binary decision variable, which is equal to 1 if link (*e,u*) is on path *k* from node *i* to destination 0 by bus. All the links in E_1 and E_3 have a certain travel cost independent of the flow of the link.

To model the dwell time at each open bus station, a dummy link set E_4 is introduced. The dummy link in E_4 has a dwell time cost function related to the flow of travelers boarding at this open bus station. Equation (38) shows the way to calculate the dwell time cost of link (*e,u*), denoted as DT_{eu}:

$$DT_{eu}(x_{eu}) = TV_{\text{bus}} \times (DT_{\min} + bx_{eu}), \forall (e,u) \in E_4, \tag{38}$$

where x_{eu} is the flow of link (*e,u*). According to the study of Sun et al. (2013), the parameter value of $DT_{\min}$ is set as 2.17 s and b is 1.84 s in this paper. Therefore, the objective function in Expression (34) can be rewritten as

$$g(\boldsymbol{x}) = \frac{TV_{\text{bus}}}{v_{\text{bus}}} \sum_{(e,u)\in E_1} x_{eu} d_{eu} + \frac{TV_{\text{walk}}}{v_{\text{walk}}} \sum_{(e,u)\in E_3} x_{eu} d_{eu} + TV_{\text{bus}} \sum_{(e,u)\in E_4} \int_0^{x_{eu}} (DT_{\min} + bw)dw. \tag{39}$$

By solving the UE problem defined by Expressions (35)-(37) and (39), the equilibrium flow of each link can be obtained. Then TC_i^{bus} can be calculated using a shortest path algorithm.

4. SOLUTION METHOD

The proposed bi-level optimization problem includes a vehicle routing problem, a mode choice problem captured by a hierarchical logit model, a shortest path problem for the walking and bike modes, and a UE problem for the bus mode. This bi-level optimization problem is nonlinear and nonconvex. It has also been noted that the VRP is NP-hard (Laporte, 2007). Therefore, the computational time by the exact method grows exponentially with network size. In this study, we introduce a modified genetic algorithm (GA) to overcome this problem. The upper-level problem of feeder service design, including station location design and bus route design, are solved by the modified GA. For the lower-level problem, a fixed-point iteration algorithm is used to solve the combined mode split and traffic assignment problem. This algorithm incorporates the Frank-Wolfe algorithm to solve the UE problem for the bus mode. Figure 4.1 shows the overall framework of the solution algorithm for the bi-level optimization problem.

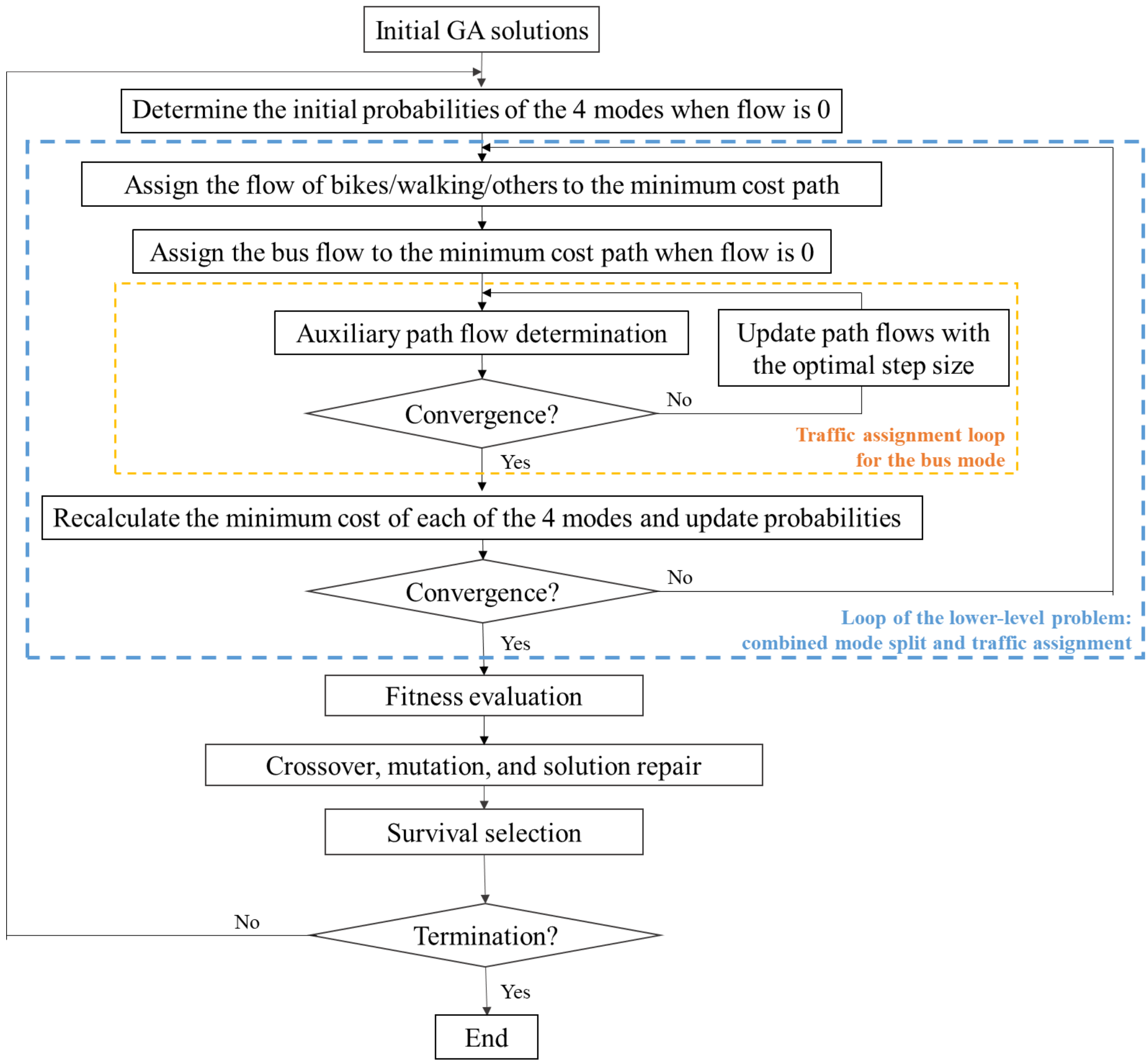


Figure 4.1 The overall framework of the solution algorithm.

The following describes the details of the solution algorithm for the whole problem. Specifically, Section 4.1 introduces the modified GA, including the solution representation and initialization (Section 4.1.1), the details of crossover and mutation operators (Section 4.1.2), and the solution repair procedure (Section 4.1.3). Section 4.2 introduces the solution algorithm of the lower-level problem.

4.1 The modified genetic algorithm to solve the upper-level problem

GA is a metaheuristic based on the principles of natural selection and genetics. It has been widely applied to solve the mixed-integer nonlinear problem in transportation research (Tian et al., 2021). In this study, the two decision parts – station location design and bus route design – are all suitable to be coded as chromosomes. Therefore, GA is chosen as a backbone to solve the upper-level problem of a large network and necessary revisions are conducted to make the algorithm adapt to the problem property.

Figure 4.2 illustrates the flowchart of the modified GA. First, initial solutions representing station location and bus route structure are generated. After solving the combined mode split and traffic assignment problem based on the initial GA solutions, the fitness of each solution is evaluated and set to be the objective function value of the bi-level optimization model since we are dealing with a maximization problem. After that, a certain number of offspring are generated through genetic operators, i.e., crossover and mutation operators. Note that we do crossover and mutation for station locations first, and then generate bus routes by inserting the open bus stations randomly into the initial empty routes. The crossover and mutation for bus routes are performed after route generation, distinguishing this algorithm from the canonical GA. After that, repair operators are applied to infeasible solutions to ensure that each station on the bus route must be selected as an open station. Finally, individuals surviving to the next generation are selected from both the parents and the offspring according to their fitness values. This process is repeated until the preset number of iterations has been completed.

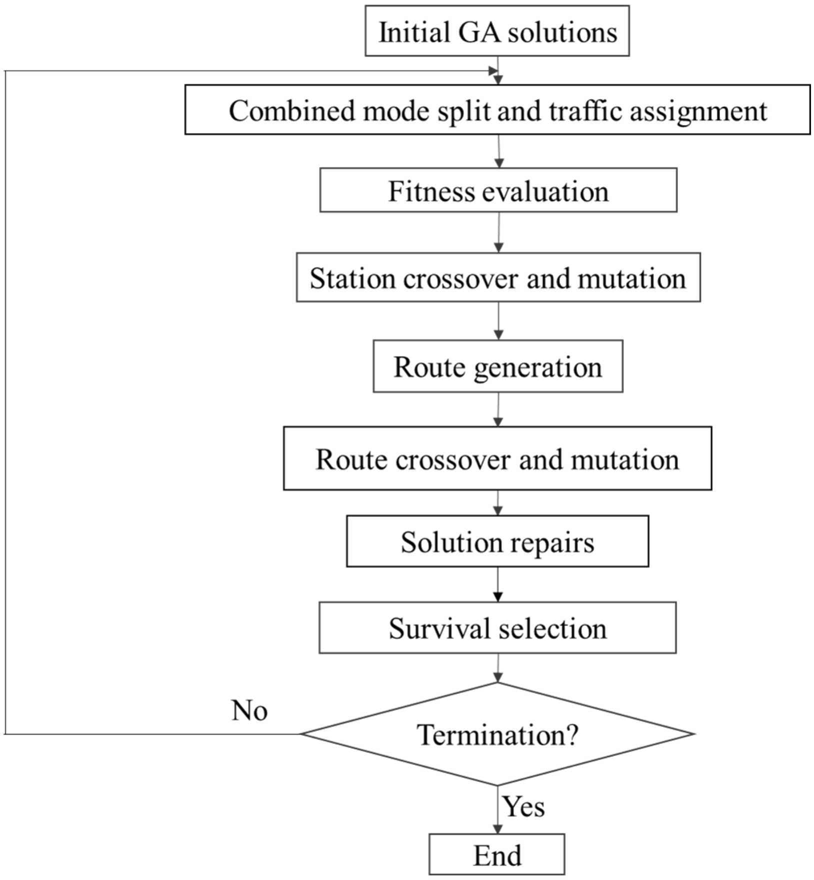
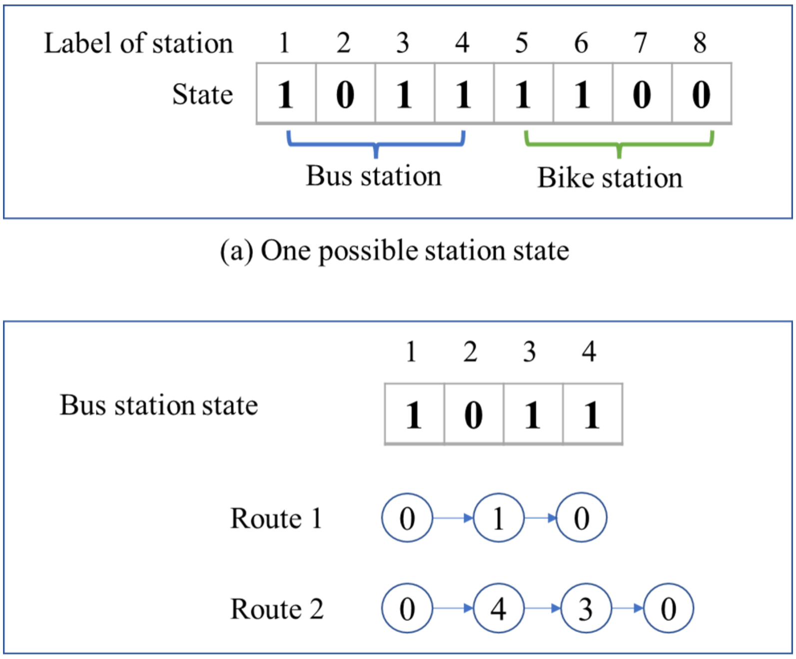


Figure 4.2 The flowchart of the modified genetic algorithm

Compared with the canonical genetic algorithm, the modified GA in this study conducts the crossover and mutation processes twice in one iteration, because the bus routes are generated depending on the station open decision. This modification can adapt to this particular feature in the joint design problem. Besides, the solution repair procedure also helps revise the infeasible generated solutions so that they can join the survival selection procedure.

4.1.1 Solution representation and initialization

In the bus-bike joint design problem, the solution includes two parts: station location design and bus route design. Feeder bus and shared bike networks have their own candidate station node sets. The state of each node can be decided as open or closed. The open state is denoted by 1 while the closed state is denoted by 0. Therefore, a solution for all the states of bus/bike stations (i.e., open or closed) can be represented as a binary number. Taking the network in Figure 2.1 as an example, Figure 4.3(a) gives the representation of a solution with bus stations 1, 3, and 4 and bike stations 5 and 6 open. This solution representation is adopted for the convenience of the following crossover and mutation steps.



(b) One possible outcome of bus routes

Figure 4.3 Solution representation in the modified GA

The station state decision is followed by the feeder bus route design decision. Open bus stations should be connected by bus routes, and each station can only be covered by one route. Figure 4.3(b) gives one possible outcome of bus route initialization using the open bus stations in Figure 4.3(a). All the routes start and end at node 0. To ensure that each route has no less than one stop, the first stop after departing from node 0 is determined by randomly selecting an open bus station and inserting it into each bus route in turn. After all bus routes have their first stop after node 0, the remaining uninserted stations are randomly added to the end of each bus route. This step is repeated until all open bus stations have been added.

4.1.2 Crossover and mutation operators

According to the framework of the modified GA, two groups of crossover and mutation operators are applied to station location design and bus route design, respectively. Section 4.1.2.1 introduces the operators for station location design and Section 4.1.2.2 is for bus route design.

4.1.2.1 Station crossover and mutation

Addressing the location design problem of bus and bike stations by the modified GA is similar to addressing the infrastructure's location in a station-based bike-sharing system. For example, in the bike station location design problem, the typical two-point crossover operator is used to randomly generate new individuals. Therefore, this type of crossover operator is also used in determining the joint design in this study. Figure 4.4 shows the principle of the station crossover operator.

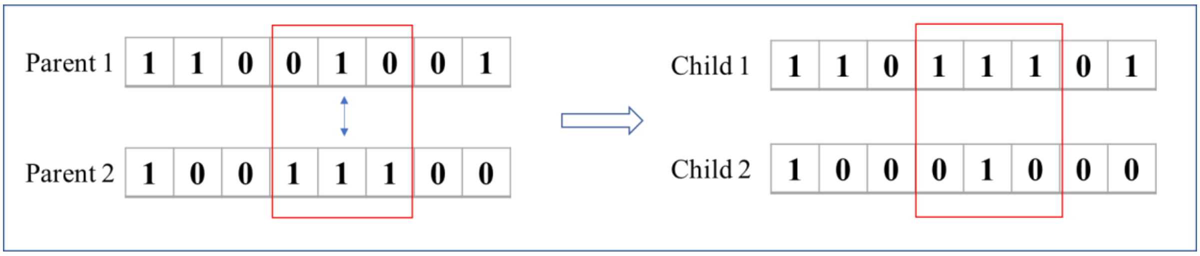


Figure 4.4 The two-point crossover operator for the location design of bus and bike stations.

The mutation operator simply flips the state of a randomly selected candidate station node. For example, if a randomly selected station is open in the chosen solution, it is altered to be closed eventually.

4.1.2.2 Route crossover and mutation

Compared with handling a station location design solution, handling a bus route design solution is more complicated because each solution has multiple bus routes. Although the typical two-point crossover operator is used as well, the length of the genes to be exchanged can be different (In contrast, this length is identical in a station location design solution).

The route crossover operator works as follows. Two routes are randomly selected from different parents and each route provides a sequence of stops with a random length. Then these two sequences of stops exchange their positions, and two new child routes are generated. Figure 4.5 shows the principle of the bus route crossover operator.

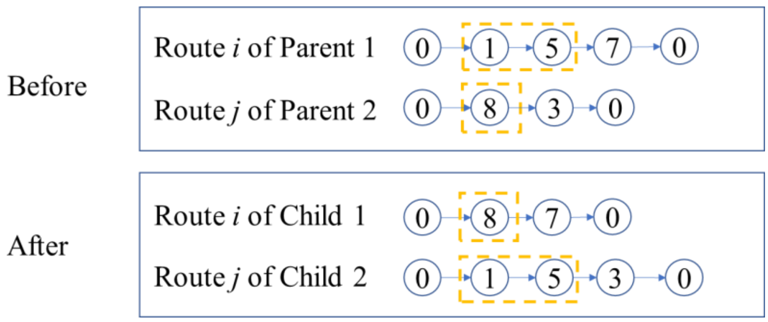


Figure 4.5 The two-point crossover operator for the bus route design.

There are four types of mutation operators applied to bus route design solutions: insert, remove, swap, and transfer. They are commonly used operators in the canonical GA to solve route-related problems. Figure 4.6 shows the illustrations of four mutation operators. In Figure 4.6(a), the insert operator inserts a randomly selected station from candidate bus station set N_1 (node 4) to a randomly selected route (0-1-5-7-0) at a randomly selected position (between node 5 and node 7). The remove operator deletes a random stop (node 5 in Figure 4.6 (b)) from the route, which is also chosen randomly. Figure 4.6 (c) gives an example of the swap operator, which exchanges the two stations at random positions of two randomly selected routes in the same solution (a solution consists of multiple bus routes). The transfer operator moves a random stop (node 5 in Figure 4.6 (d)) in one randomly selected route to another randomly selected route in the same solution.

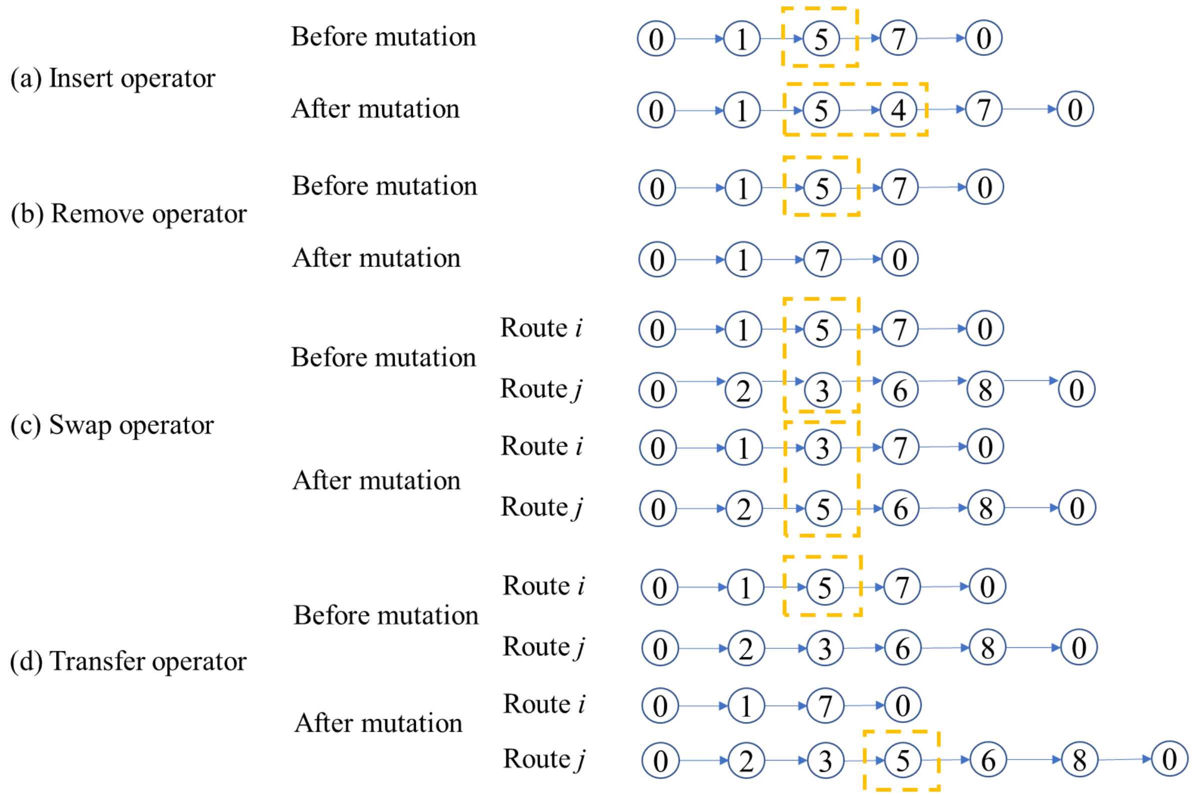


Figure 4.6 Mutation operators for bus routes

4.1.3 Solution repair

After applying the genetic operators to the offspring, some of them may violate the constraints and become infeasible. Therefore, the repair process is implemented to fix infeasible offspring.

The first step is to eliminate the repeated stations in one solution. As the vehicle routing problem has the constraint that each station can only be covered by one route, the repeated stations are removed until all the stations in one solution are different.

The second one is to revise the station state according to the routes after crossover and mutation. The state of each bus station is regenerated according to the routes after crossover and mutation. The stations on the routes are set as open while the others are set as closed. This regenerated state replaces the previous one so that the infeasible solution can become feasible.

The last step is to improve the sequence of stations on one route. Here we use the stop sequence improvement heuristic proposed by Szeto and Wu (2011). All the stations of each route exchange the position on the same route. If the total route time is reduced, the exchange is reserved; otherwise, undo the exchange. The step is repeated until no exchange can be applied. Readers can refer to the study of Szeto and Wu (2011) for more details on the stop sequence improvement heuristic.

4.2 The fixed-point iteration and Frank-Wolfe algorithms to solve the lower-level problem

The lower-level problem is a combined mode split and traffic assignment problem. In this problem, the mode split results affect the demand of each mode, while the demand of each mode determines the flow of each link, and this in turn changes the mode split probability. Therefore, the mode split and traffic assignment affect each other and should be considered as an integrated problem. In this study, we propose a fixed-point iteration algorithm to solve this problem. Figure 4.7 shows the framework of this algorithm. At the beginning, each mode has an initial probability which is calculated according to the upper-level design of the feeder service. Recall that the traffic assignment problem of bike, walking, and “others” modes are all not related to the traveler flow and can be solved as shortest path problems. For the bus mode, as the dwell time cost is a function of traveler flow, there is a traffic assignment loop to solve the UE problem of this mode. After this loop, the cost of each link can be updated and the new probability according to the updated minimum generalized travel cost of each mode can be calculated. This process is repeated until the difference between the probabilities before and after the traffic assignment is acceptable.

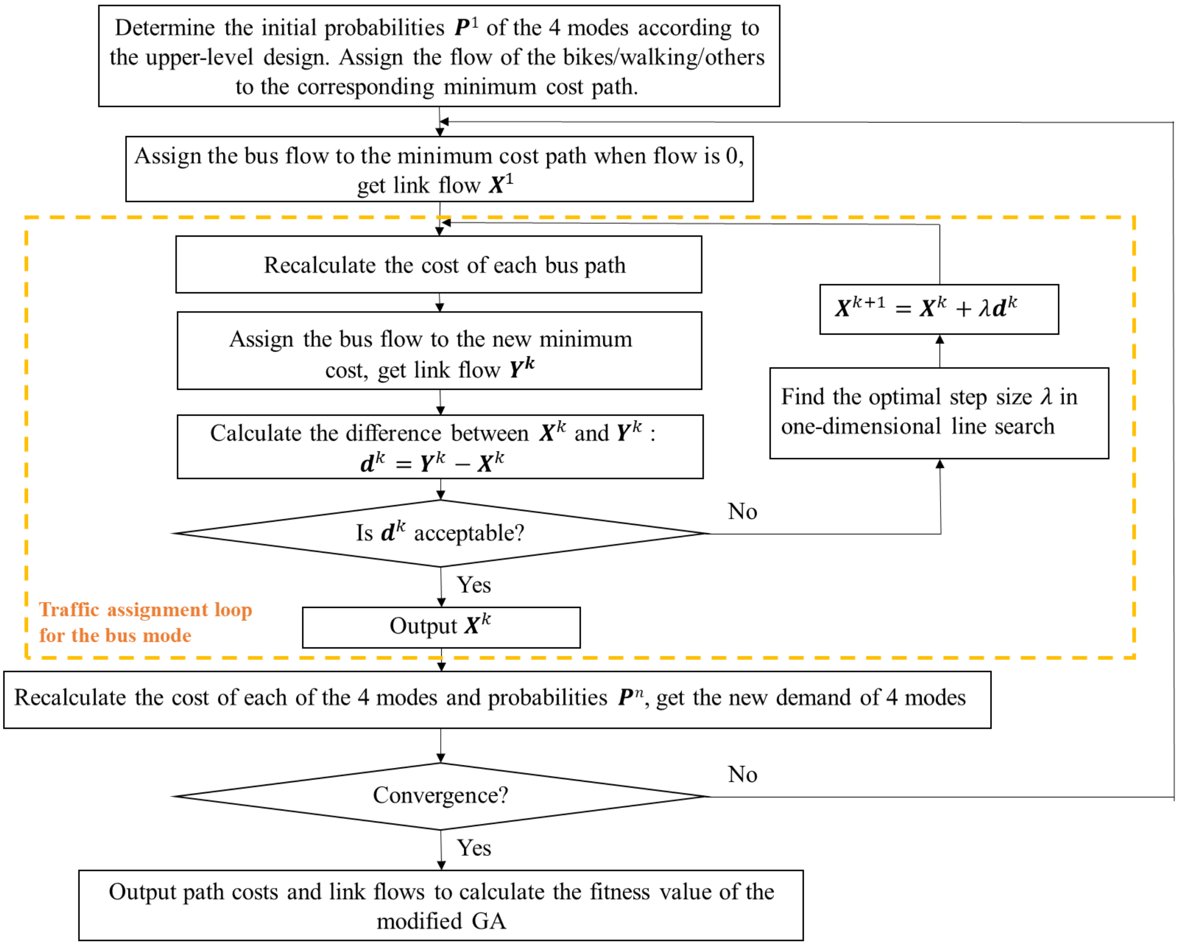


Figure 4.7 The framework of the solution algorithm for the lower-level problem.

When it comes to the traffic assignment loop for the bus mode, considering that the dwell time cost is a linear function of link flow, the problem is solved by the Frank-Wolfe algorithm. The auxiliary flow distribution of the bus network is determined by the all-or-nothing principle.

5. NUMERICAL STUDY

In this section, numerical examples are given to illustrate the key factors that affect the joint design, the importance of joint design, and the performance of the proposed algorithm. Two networks of different scales are selected for these purposes. The small network has 4 candidate bus station nodes and 4 candidate bike station nodes (as shown in Figure 2.1). The large one is a realistic network in Berlin, Germany (Transportation Networks for Research Core Team, 2022). Figure 5.1 shows part of the city scenery of this place named Prenzlauer Berg Center (Delenk, 2021).

Figure 5.1 City scenery of Prenzlauer Berg Center

For the small network problem, the enumeration method was used to get exact solutions. The insights into the joint design problem are provided from the view of bus operators, bike operators, and travelers. For the large network, the modified genetic algorithm proposed in Section 4 was applied. The modified GA was coded in C++ and ran on a laptop with an Intel Core i7-1260P CPU@2.10 GHz and 16 GB RAM.

The values of time for walking, taking a bus, and riding a bike are set as 30 $/h, 20 $/h, and 17 $/h, respectively. Here the value of time for taking a bus was obtained from the report of He et al. (2016) in Shenzhen. As there is no value of time for walking in this report, we calculated it based on the ratio between the value of time for taking a bus and that for walking. This ratio was obtained from the government transport report of the UK (2015). To the best of our knowledge, little research paid attention to the value of time for riding bikes. Therefore, this value is set artificially in the following experiments. The construction cost of a station is set according to the real case of Shenzhen. The income of the operators is the total fare revenue, which is highly related to the travel demand (see Equation (6) in Section 3.2.1). Moreover, the OD demand in the numerical cases is given as the number of persons per hour. To make the income and the cost of operators comparable, the construction cost of stations is converted into unit construction cost per hour by dividing the predicted life of the station. In this study, the unit construction cost of a bus station is 100 $/h and the unit construction cost of a bike station is 50 $/h. Walking, bus, and cycling speeds are set as 5 km/h, 50 km/h, and 20 km/h, respectively. The operation cost of the unit bus route distance is 2 $/km based on the situation in Shenzhen. The frequency of feeder buses is set as 5 buses per hour according to the demand setting in this study to make sure that the supply of feeder bus service can satisfy the demand of bus passengers.

5.1 Convergence and tuning of parameters of the modified GA

Before analyzing the effects of joint design, it is essential to determine the parameters of the modified GA. Several experiments were conducted to find the suitable population size and mutation rate of the algorithm. The large network in Germany (Transportation Networks for Research Core Team, 2022) was used to determine the parameters. This network has 352 nodes. 120 of them are candidate bus station nodes and the remaining are candidate bike station nodes.

The demand of each node was randomly generated as an integer ranging from 10 to 50 (persons per hour). The modified GA with a population size of 10, 20, 30, 40, and 50 were run. Each modified GA was run 10 times. The corresponding average objective values and the average running time of 10 runs are shown in Figure 5.2. It can be seen that the objective values of different population sizes become relatively stable when the population size reaches 20, but the computation time keeps increasing from 569.75 s to 1572.20 s. Therefore, in the following experiments, the population size is set as 20. Figure 5.3 shows the convergence process of the modified GA over 500 iterations with a population size of 20 and a mutation rate of 0.8. It can be seen that the algorithm requires about 400 iterations for convergence.

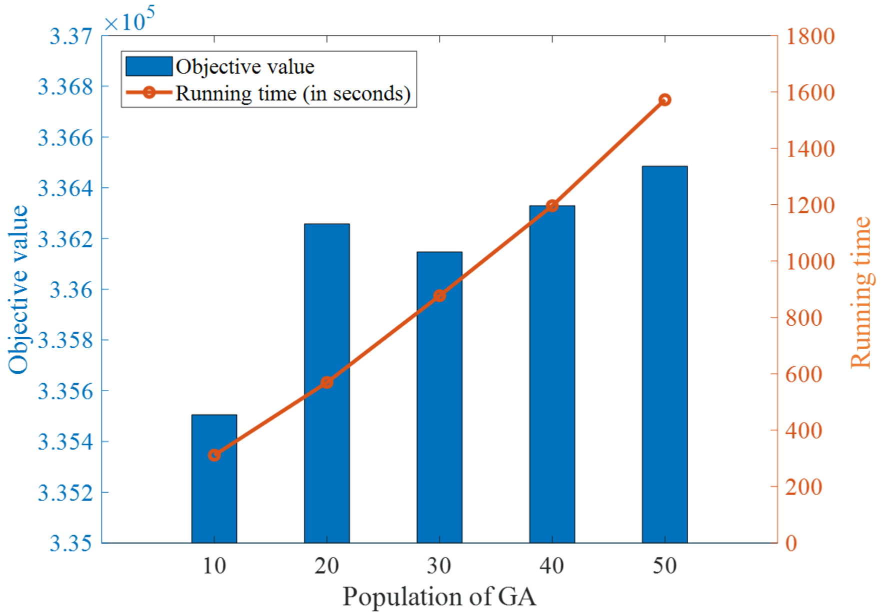


Figure 5.2 Objective values and running times of different populations

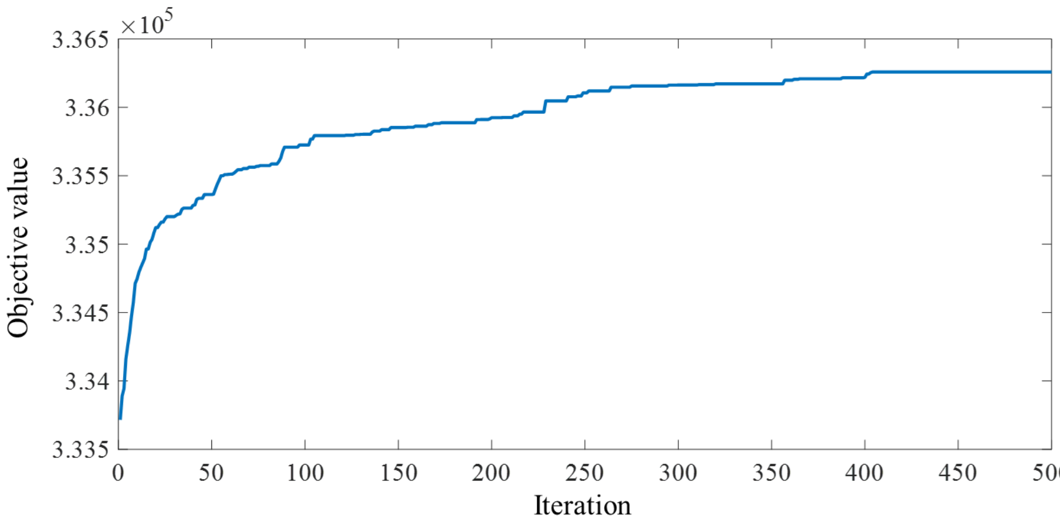


Figure 5.3 Convergence process of the modified GA

When it comes to the mutation rate, four settings were considered including 0.2, 0.4, 0.6, and 0.8. Figure 5.4 shows the convergence process of the algorithm under different mutation rates. The modified GA with a mutation rate of 0.8 has the best performance while the running times under the four settings are similar. Therefore, the mutation rate is set as 0.8 in the following experiments.

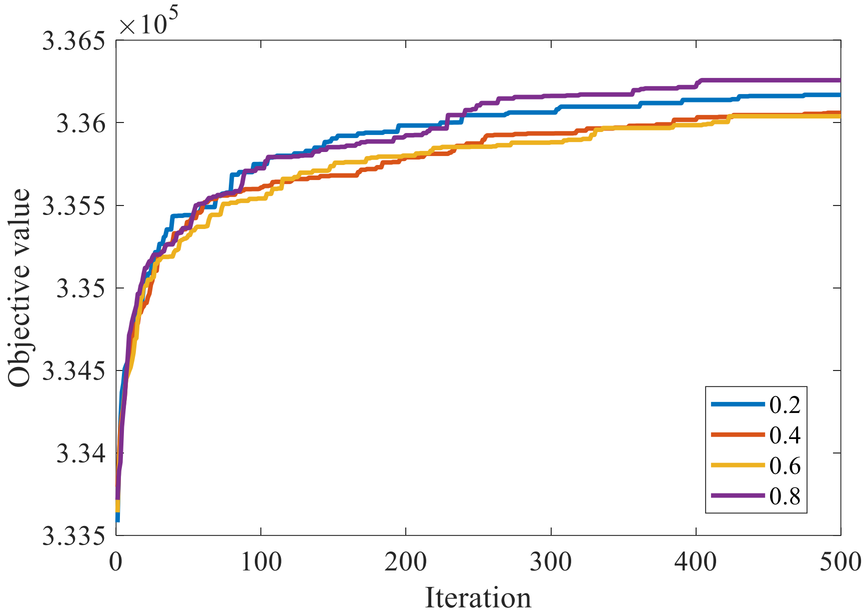


Figure 5.4 Convergence process of different mutation rates

5.2 Comparison of the modified GA and benchmark methods

To verify the effectiveness of the modified GA in Section 4, the results of the enumeration method of the small network are set as the benchmark. Three instances with the demand levels of 10, 50, and 100 (persons per hour at each node) of the small network were used. The program of the modified GA was run 10 times. All of the experiments gave the same solution as the enumeration method. According to this fact, the modified GA method is verified to be able to give exact solutions.

To further evaluate the performance of the modified GA, another metaheuristic named Artificial Bee Colony (ABC) algorithm was also developed in this study. The neighborhood operators to get new solutions of ABC algorithm mainly refer to the study of Szeto et al. (2011), including random swaps, random insertions, and reversing a subsequence. The sum of the number of employed bees and onlookers of ABC algorithm is set the same as the population size in the modified GA, 20, to make the comparison unbiased. 10 instances were used to compare the performance of ABC algorithm and the modified GA. Two algorithms were provided with the same set of initial solutions, the same random seeds, and the same running time (1000 s). Each algorithm was run 10 times. Table 5.1 shows the experimental results of 10 instances. n represents the total number of candidate stations, while n_{bus} is the number of candidate bus stations and n_{bike} is the number of candidate bike stations. Both the average value and the best value were obtained from the 10 runs of ABC algorithm and the modified GA. According to these results, when the network has a scale of 50 candidate stations in total, the performance of ABC algorithm and the modified GA are similar. When the network scale expands to 100 candidate stations, the modified GA has better performance than the ABC algorithm. With the continuous increase of network scale, the difference between the two algorithms keeps increasing. Therefore, the following analysis of the large network in Germany (with 352 candidate stations) was solved by the modified GA.

Table 5.1 Comparison of experimental results between ABC algorithm and the modified GA.

Instance	n	n_{bus}	n_{bike}	**ABC algorithm**		**The modified GA**		**Improvement**	
				Average ($)	Best ($)	Average ($)	Best ($)	Average ($)	Best ($)
1	50	10	40	46020	46036	46021	46049	1	13
2	50	20	30	46236	46274	46319	46333	83	59
3	100	30	70	90590	90625	90861	90923	271	298
4	100	40	60	90573	90739	90950	91067	377	328
5	150	50	100	140374	140546	141204	141419	830	873
6	150	60	90	140129	140426	141180	141275	1051	849
7	200	80	120	186098	186574	187787	187981	1689	1407
8	250	100	150	235590	236123	237940	238064	2350	1941
9	300	110	180	278304	278605	280133	280286	1829	1681
10	352	120	232	333288	333695	336258	336572	2970	2877

5.3 Small network

This section provides insights into the bus-bike joint design problem using the exact solutions for a small network with 4 candidate bus station nodes, 4 candidate bike station nodes, and a BRT station. The link distances of this network are shown in Table 5.2. Analysis of the key factors that affect the joint design was conducted based on this instance. Moreover, the comparison between separate design and joint design was also conducted.

Table 5.2 Link travel distance table (in meters)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	0	5	3	2	4	3.9	5	2.7	2.9
1	5	0	2	5	7	1.3	0.8	5.7	5.9
2	3	2	0	3	5	0.9	2.8	3.7	3.9
3	2	5	3	0	2	3.9	5.8	0.7	0.9
4	4	7	5	2	0	5.9	7.8	2.7	1.4
5	3.9	1.3	0.9	3.9	5.9	0	2.1	4.6	4.8
6	5	0.8	2.8	5.8	7.8	2.1	0	6.5	6.7
7	2.7	5.7	3.7	0.7	2.7	4.6	6.5	0	1.6
8	2.9	5.9	3.9	0.9	1.4	4.8	6.7	1.6	0

5.3.1 Key factors that affect the joint design

In this research, different design layouts can affect the travel cost and then change the traveler flow of each mode, but the total travel demand for all the modes at each node D_i is invariant to the feeder service design. To identify how the demand level affects the final optimal feeder service design, a range of demand levels from 10 to 100 persons per hour at each node are set in the experiments. Here the bus fare is set as $3 and the bike fare is set as $1.5.

Figure 5.5 shows the number of open bus/bike stations and the average change in social welfare under different demand levels. Since the total change in social welfare is closely related to the total number of users, the average change in social welfare per traveler is used here instead of the total value. When the demand level is quite low, only 2 bus stations and 3 bike stations are open, because opening one more station cannot bring enough improvement in consumer surplus to cover the construction and operation costs of operators. With the increase in total demand, one more bike station is first added to the network when the demand level increases to 20 persons per hour. After that, when the demand keeps growing, the remaining two bus stations become open at the demand levels of 30 and 40 persons per hour, respectively. At the same time, the average change in social welfare keeps growing when the total travel demand increases, but the growing speed decreases.

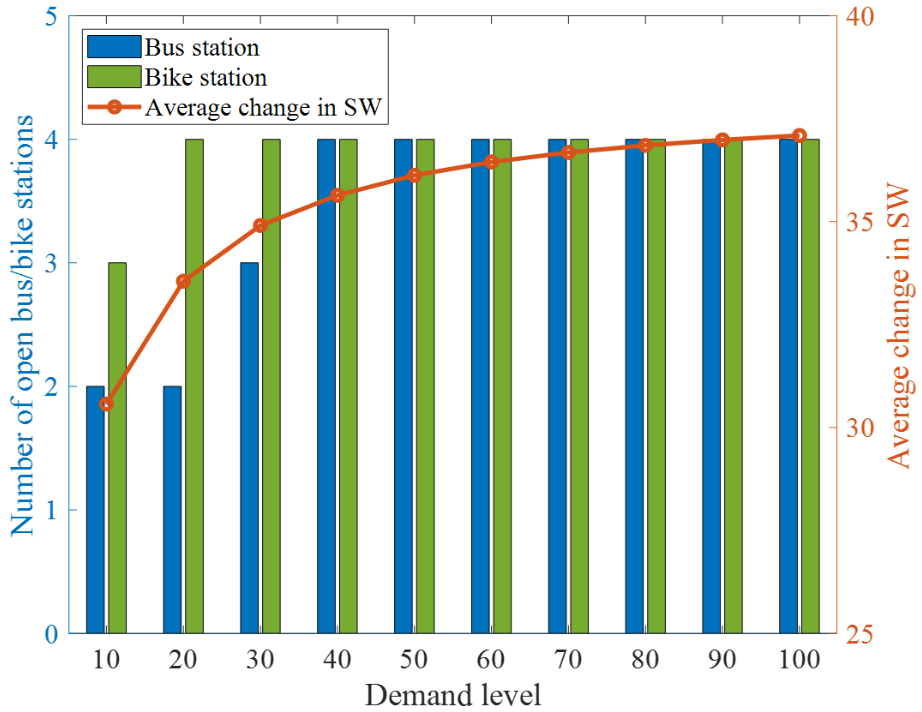


Figure 5.5 Number of open stations and average change in social welfare under different demand levels.

The other factor investigated here is the public transportation fare. To avoid the effect of demand level, the number of travelers at each node is fixed at 100 persons per hour. The optimal number of open bus stations and bike stations under this scenario is 4 and 4, respectively, as can be seen in Figure 5.5. Then we varied the fare of either bus or bike from $1 to $10 at a step of $1 while keeping the fare of the other fixed to $1. The results show that the station state is consistent with the results in Figure 5.5, indicating that the variation in the fare of public transportation barely has an effect on the design layout. However, the expensive public transportation fares indeed affect the decision of travelers in this bi-modal feeder service system. Figure 5.6 shows the changes in the profit of operators (i.e., ΔTOP), changes in the consumer surplus (i.e., ΔTCS), and changes in the social welfare under different bus and bike fares. From the green lines, it can be seen that with the increase in bike fare, the change in the TOP increases a lot (as shown in Figure 5.6(a)). The bike operator gains more profit from a higher bike fare. On the other hand, a higher bike fare makes some of the travelers convert to taking the bus, increasing the fare income of the bus operator. Therefore, the change in the TOP increases. Meanwhile, the change in the TCS decreases (as shown in Figure 5.6(b)), meaning that the interests of travelers are sacrificed. The change in the SW, as the sum of the changes in TOP and TCS, also decreases in Figure 5.6(c), indicating that the reduction in consumer surplus is greater than the improvement in operator profits. Similar trends appear when the bus fare changes (see the blue lines in Figure 5.6).

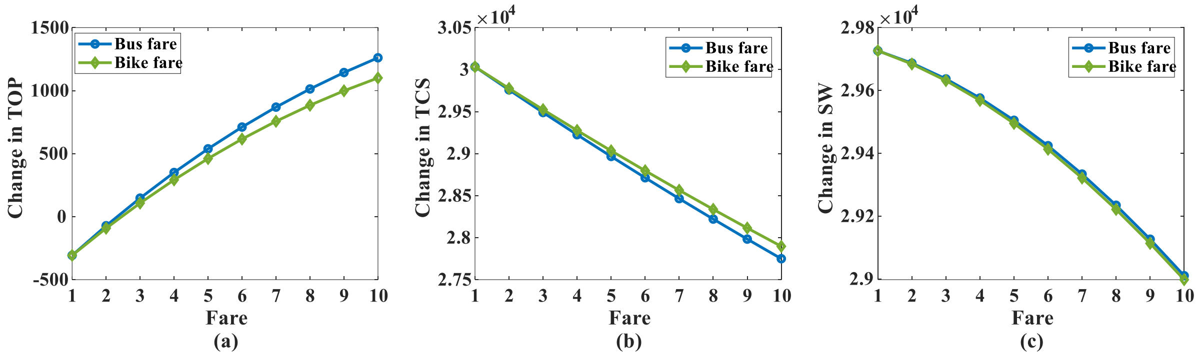


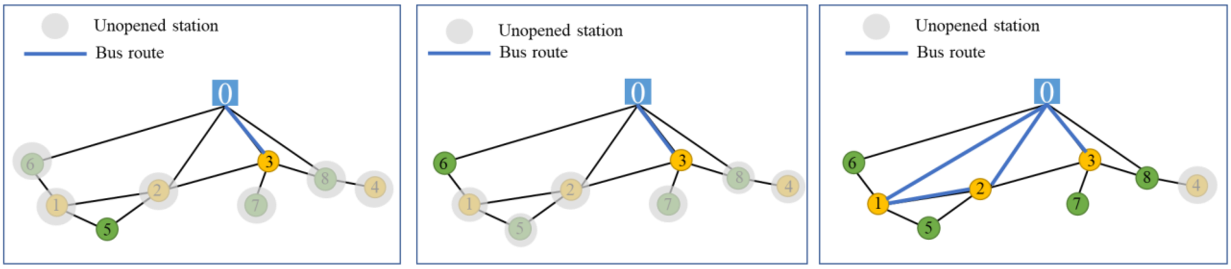
Figure 5.6 The changes in TOP, TCS, and SW under different bus and bike fares.

5.3.2 Comparison between separate and joint designs

The traditional designs of feeder bus and shared bike systems are separately handled by two different operators. In the separate design, two operators make decisions but do not exchange the related information with each other. However, the two systems coexist in the same urban transportation system and travelers make decisions according to the realistic situation, leading to a deviation from the expectation. Therefore, we compare the separate design and joint design in this section to illustrate the necessity of the latter.

In this section, the travel demand at each node is set as 30 persons per hour. We assume two types of operators: the private operator who aims at maximizing its own profit, and the public operator who aims at maximizing the change in social welfare. Under this assumption, we consider three different scenarios: (1) two systems are separately designed by two different private operators, (2) two systems are jointly designed by one private operator, and (3) two systems are jointly designed by one public operator. Here the separate design by two private operators was obtained by solving a Nash equilibrium problem. One of the operators first determines a temporary design for its system. Then the second operator optimizes the design for its system based on the temporary design of the first operator. With this design of the second operator, the first operator optimizes the design for its system. Afterwards, the second operator optimizes the design for its system based on the latest design of the first operator. The process repeats until the Nash equilibrium is obtained.

Figure 5.7 shows the design results of stations and bus routes under these scenarios in turn. In Figure 5.7(a), one bus station (node 3) and one bike station (node 5) are open. The bus route is 0-3-0. In Figure 5.7(b), one bus station (node 3) and one bike station (node 6) are open. The bus route is 0-3-0. In Figure 5.7(c), the open bus stations are 1, 2, and 3. All the candidate bike stations are open. The two bus routes are 0-1-2-0 and 0-3-0.



(a) Separate design with two private operators (b) Joint design with one private operator (c) Joint design with one public operator

Figure 5.7 Design results of station and bus routes under three scenarios.

Table 5.3 shows the performance of the three designs in Figure 5.7. It can be seen that compared with the first and second designs, which are handled by private operators, the total profit of the operator decreases in the third one, but the change in social welfare improves and outweighs the additional loss in the total profit. Considering this situation, an appropriate amount of subsidy is recommended to make up for the loss of the operator under the joint design system with one public operator. Comparing the separate design with two private operators and the joint design with one private operator, although the two operators are all aimed at maximizing their own profit, because of the absence of communication, the total profit of the two operators in the joint design is higher than that in the separate design.

Table 5.3 Comparison between separate and joint designs

	Separate design		**Joint design**		**Joint design**	
Operator	**Two private operators**		**One private operator**		**One public operator**	
	Bus	Bike	Bus	Bike	Bus	Bike
Objective	Maximize own profit		Maximize the total profit		Maximize the change in social welfare	
Construction cost ($)	100	50	100	50	300	200
Profit ($)	41	41	58	28	-100	-216
Total profit ($)	82		86		-316	
Change in consumer surplus ($)	6,971		6,668		8,694	
Change in social welfare ($)	7,053		6,754		8,378	

From Table 5.3, it can be seen that the change in consumer surplus has a significant improvement after the joint design with one public operator. However, travelers departing from different nodes experience different benefits. Figure 5.8 shows the change in consumer surplus of travelers departing from different origins. For example, the travelers at all nodes can benefit from the joint design compared with the situation with no feeder service, as represented by yellow bars in Figure 5.8. To be more specific, travelers at node 1 gain the most improvement while those at node 3 gain the least. When we compare the joint design by the public operator with the separate design by two private operators (see the difference between blue and yellow bars), we can observe that travelers at node 6 gain the most improvement in consumer surplus from the joint design, while travelers at node 3 and node 5 only have slight improvements. This difference raises the issue of inequity. The joint design should also ensure that there are no significant differences in benefits for travelers from different origins. Therefore, the policymakers may consider giving a discount fare to travelers who obtain slight improvements from the design. Another effective measure is to incorporate a constraint of equity between travelers into the design model to avoid the final feeder service design bringing significant differences in benefits between travelers.

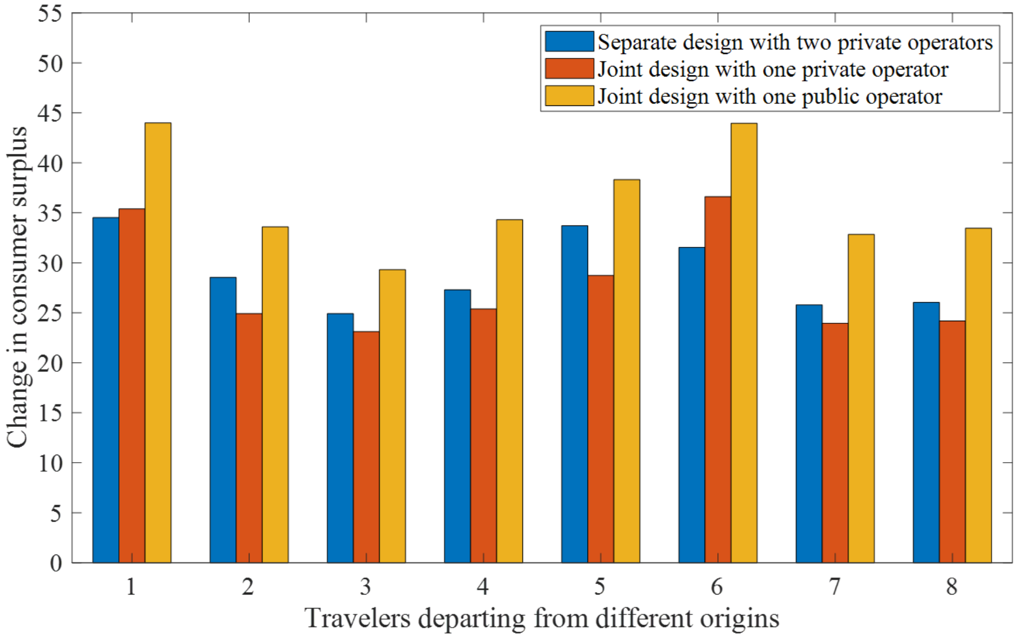


Figure 5.8 Change in consumer surplus for travelers departing from different origins.

5.4 Large network

To illustrate that the developed methodology can be applied to a large realistic instance, the large network and demand setting in Section 5.1 were used in the experiments in this section. To be more specific, we compared the performance of the modified GA with different mutation operators. Computational experiments were also conducted to investigate the difference between unimodal and bi-modal feeder service systems. Here the unimodal system refers to the scenario that only one type of feeder service (i.e., either the bus or bike service) exists in the network.

5.4.1 Effect of mutation operators

To evaluate the effectiveness of the mutation operators, seven scenarios (GA-1 to GA-7) were generated with different combinations of station and route mutation operators. For each scenario, the modified GA was run 10 times and the average fitness value of the results was calculated. Table 5.4 shows the computational results of these experiments.

Table 5.4 Results of computational experiments with different mutation operators.

Scenario	GA-1	GA-2	GA-3	GA-4	GA-5	GA-6	GA-7

Station mutation operator used	No	Yes	Yes	Yes	Yes	Yes	Yes
Route mutation operator used	No	No	Insert	Remove	Swap	Transfer	All four
avg.	336,069	336,074	336,022	336,185	336,154	336,033	336,258
min.	335,945	335,937	335,782	335,425	335,736	335,800	336,069
max.	336,279	336,366	336,320	336,541	336,553	336,256	336,572
std.	98.55	131.68	149.07	330.77	263.14	151.00	172.68
average run time (s)	519.00	549.62	546.39	542.54	553.38	545.97	569.75

From Table 5.4, it can be seen that scenario GA-2 achieves a better fitness value compared with GA-1, which has no mutation on station and route design. This result indicates that the station mutation operator can help improve the average objective value of the problem. The best performance appears in scenario GA-7 which has the combination of all four route mutation operators with the same application probability. This combination provides the highest average fitness value but the computation time does not increase significantly compared with computation times in the scenarios with only one route mutation operator (i.e., GA-3 to GA-6). Therefore, the combination of 1 station mutation operator and 4 route mutation operators is adopted in this algorithm.

5.4.2 Unimodal and bi-modal feeder service systems

After the successful application of this model to a large realistic network, the comparison between unimodal and bi-modal feeder service systems was conducted. The optimization objective for the unimodal system is the same as that of the bi-modal system, i.e., maximizing the change in social welfare. The formulation of a system with only bus/bike feeder service can be obtained by slightly modifying the formulation of the bi-modal system. For the bus-only system, all the candidate bike stations are closed and the travel demand is split into three modes: feeder bus, walking, and “others”. A similar modification was conducted for the bike-only system. Then the results can be calculated using the proposed method.

In this section, the travel demand at each node is set to be the same as in Section 5.1. The population size of the modified GA is 20 and the mutation rate is 0.8. The combination of 1 station mutation operator and 4 route mutation operators was adopted to obtain the results. Table 5.5 shows the comparison of the average results of 10 runs between the two systems. It can be found that the unimodal system brings more profits to the operator, no matter the bus or bike operator, than the bi-modal system. In each of the two unimodal systems, travelers only have one type of public feeder service: bike or bus. This can significantly boost the traveler flow of the corresponding feeder service. In contrast, the bi-modal system can improve social welfare to a great degree but brings a non-negligible loss to the operator at the same time. This result implies that the introduction of the second type of feeder service to the unimodal system by the public sector may bring a strike by the previous operator and a subsidy is recommended to give to the operator to compensate for its loss.

Table 5.5 Comparison between unimodal and bi-modal feeder service systems.

	Change in SW ($)	Profit of bus operator ($)	Profit of bike operator ($)

Bi-modal system	336,258	6,607	2,093
Unimodal system with buses only	208,486	11,601	/
Unimodal system with bikes only	209,133	/	5,142

6. CONCLUSION

In this paper, we propose a bi-level nonlinear mixed-integer model to jointly design the feeder bus and bike-sharing systems. Both the feeder bus and shared bike systems are assumed to be operated by the same public operator. The proposed model is formulated over a multi-modal network to capture the relationship between walking, feeder buses, and shared bikes. This model aims to determine stations' states and bus routes to maximize the improvement in social welfare. Social welfare, which is defined by the total benefits of travelers and operators, is respectively quantified by consumer surplus and operator profit. A solution method based on the modified GA, the fixed-point iteration algorithm, and the Frank-Wolfe algorithm is proposed to solve the model for a large realistic network application.

Computational experiments were conducted on both small and large networks. The parameter setting refers to the investigation of Shenzhen, China. The effects that different factors have on the joint design are discussed in this paper. Increasing the demand level can lead to more open stations, and first for bikes, then for buses. Nevertheless, the additional benefit that a new open station can bring keeps decreasing. As for the fare, a higher fare can actually engage a higher operator income, but the resulting reduction in social welfare can also be obvious at the same time.

The results show that compared with the separate design by two private operators, the joint design of bus and bike systems has a better performance, no matter whether the performance is determined by a single private operator (aiming at maximizing the total profit) or a single public operator (aiming at maximizing the improvement in social welfare). However, in the joint design with one public operator, the operator may lose its profit. Considering this situation, an appropriate amount of subsidy is recommended to make up for the loss of the operator under the joint design system. Moreover, travelers departing from different origins can gain benefits from the joint design to different extents. The implication of the issue of inequity raised from this situation is that the joint design should also ensure there are no significant differences in benefits for travelers from different origins.

The results of a large-scale case study show that the proposed bi-modal feeder service system can improve social welfare to a great extent compared to the unimodal feeder service system, although at the expense of a non-negligible loss to the feeder service operator. It implies that the introduction of the second type of feeder service to the unimodal system by the public sector may bring a strike by the previous operator and a subsidy is recommended to compensate the operator for its loss.

Besides the new view brought by the joint design of feeder bus and shared bike services discussed in this paper, an important extension can be made in future studies on the cooperative design of long-haul transit and feeder services, as the long-haul section has been ignored by this study. Another interesting direction is to incorporate the variations in individuals' value of time.

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