A new methodology for the real-time limited-stop bus service design problem

Jintao Hu
Department of Civil Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

W.Y. Szeto
Department of Civil Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong
The University of Hong Kong Shenzhen Institute of Research and Innovation, Shenzhen, China
Guangdong-Hong Kong-Macau Joint Laboratory for Smart Cities, China
Tel: 852-28578552; Email: ceszeto@hku.hk

Abstract

Providing limited-stop bus services can improve the efficiency of bus systems. This paper proposes a new two-stage strategy for providing real-time limited-stop bus services for a corridor and the corresponding model is developed. In the first (tactical planning) stage, given the maximum number of different limited-stop services, an operator determines a set of limited-stop services based on historical bus travel times and passenger arrival rates. In the second (operational) stage, an operator selects one service from the set of limited-stop services obtained in the first stage for each limited-stop vehicle based on (short-term) predictive travel times and passenger arrival rates. Prediction errors are considered in the second stage. An enhanced artificial bee colony algorithm is developed to solve the first-stage model and the Monte Carlo Simulation method is adopted to solve the second-stage model. Numerical results are presented to illustrate the effectiveness and efficiency of the strategy and the effect of prediction errors.

Keywords:

Limited-stop bus service; A two-stage strategy; Real-time optimization; Prediction errors

1. Introduction

Bus operation control strategies are important instruments for bus operators to improve the efficiency of bus systems (Liu et al., 2013). One of the strategies is providing limited-stop bus services. In these services, buses can skip some intermediate stops on a given route. It has been proved that providing limited-stop bus services can benefit passengers (e.g., Ercolano, 1984; El-Geneidy and Surprenant-Legault, 2010) and operators (e.g., Silverman, 1998; Tetreault and El-Geneidy, 2010). Moreover, if limited-stop bus services are applied to electric bus systems, the energy utilization rate of buses can be improved (Tang et al., 2023). Because of these benefits, limited-stop bus services have drawn much attention from researchers in recent years. This paper focuses on providing these services, which is referred to as the limited-stop bus service design problem (LSBSDP) in the literature.

In the literature, the strategy of providing limited-stop bus services on a given route can be divided into two broad categories. One is the tactical planning strategy. In this strategy, an operator determines one limited-stop service on a given route at the tactical planning level and then all limited-stop vehicles (a limited-stop vehicle means the vehicle can provide a limited-stop service) provide the same limited-stop service at the operational level (e.g., Wirasinghe and Vandebona, 2011; Chiraphadhanakul and Barnhart, 2013; Yi et al., 2016; Albarracin and Jaramillo-Ramirez, 2019; Nesheli et al., 2022). This strategy is attractive as passengers can plan for it. The other is the dynamic stop-skipping strategy (e.g., Fu et al., 2003; Wu et al., 2019; Zhang et al., 2017a; Zhang et al., 2017b; Gkiotsalitis, 2021; Zhang et al., 2021). In this strategy, an operator does not consider the tactical planning level. Each limited-stop vehicle can provide one limited-stop service out of all possible different services at the operational level and can offer a different limited-stop service from the other (or the same limited-stop service as the other) on a given route. It is implicitly assumed that passengers get real-time information about the limited-stop service through mobile applications. Compared with the tactical planning strategy, the dynamic stop-skipping strategy has higher effectiveness (i.e., it provides a more system cost saving) due to its higher flexibility to skip stops. However, the strategy creates a more difficult optimization problem and a higher computational burden for the real-time application. Considering the lower effectiveness of the tactical planning strategy and the requirement for higher computational efficiency of the dynamic stop-skipping strategy for real-time applications, we propose a new two-stage strategy in this paper. The two-stage strategy has 1) higher effectiveness than the tactical planning strategy and 2) higher computational efficiency than the dynamic stop-skipping strategy. Furthermore, the new strategy is more general than these two strategies, which are only special cases of the new strategy.

At the operational level of providing limited-stop services, the limited-stop service scheme of a limited-stop vehicle is fixed once it departs from the starting terminal (Fu et al., 2003). Therefore, when we determine its real-time limited-stop service scheme, the real (future) values of bus travel times and passenger arrival rates associated with the vehicle are unknown. An operator needs to predict these values before determining the limited-stop service scheme. It is a common assumption that these predictive values (also named average or expected values) are known and given in the literature of the real-time LSBSDP (e.g., Fu et al., 2003; Gkiotsalitis, 2021). This paper also adopts the same assumption. However, in previous studies, authors determined the limited-stop service scheme by these predictive values directly (e.g., Fu et al., 2003; Wu et al., 2019; Gkiotsalitis, 2021). Different from them, we also take prediction errors into consideration. In other words, we determine the limited-stop service scheme by both the predictive values and prediction errors. We believe that the prediction errors need to be considered for two reasons. The first reason is that there must be prediction errors between the predictive and real values. In other words, the prediction errors do not equal 0 in reality. The second reason is that prediction errors can decrease the effectiveness of bus systems with dynamic limited-stop bus services. For example, when an operator obtains the predictive values by a prediction model with low effectiveness, if he/she directly adopts the predictive values as the real values, he/she is very likely to make a wrong determination of the limited-stop service scheme and the wrong determination can reduce the benefit of providing limited-stop services. In our study, new models (i.e., the first- and second-stage models) for the new two-stage strategy are developed to address the LSBSDP. Prediction errors are considered in the second-stage model. To our best knowledge, no study has dealt with these errors when addressing the LSBSDP. Moreover, the new models can provide more effective solutions than the model derived from the tactical planning strategy and take less computational time than the model derived from the dynamic stop-skipping strategy.

There are roughly two approaches to developing a model for the LSBSDP. The first one is the schedule-based approach (e.g., Fu et al., 2003; Chen et al., 2015; Yu et al., 2015; Gkiotsalitis, 2019; Mou et al., 2020; Zhao et al., 2021; Sadrani et al., 2022). With this approach, models first calculate the arrival and departure times of buses at each bus stop. Then passengers' waiting and in-vehicle times can be obtained. Specifically, the waiting time of one passenger is expressed as the difference between the arrival times of a bus and the passenger at his/her origin bus stop, and his/her in-vehicle travel time equals the gap between the departure time of the bus at his/her origin and the arrival time of the bus at his/her destination. The second one is the frequency-based approach (e.g., Tang et al., 2016, 2018, 2019, 2020, 2022; Wang et al., 2018). Models of this approach do not focus on the arrival and departure times of buses. Passengers' waiting time is calculated directly by the bus frequency (it usually equals the reciprocal of the frequency), and their in-vehicle travel time between two successive bus stops is the mean of historical in-vehicle travel times. Unlike the schedule-based approach, the frequency-based approach is not applicable to modeling the LSBSDP at the operational level. As a result, we adopt the schedule-based approach for our model development in our study.

The solution methods to solve the models of LSBSDP can be broadly classified into exact methods and meta-heuristics. In terms of exact methods, some researchers (e.g., Ulusoy et al., 2010; Huang et al., 2021) solved their models by an enumeration while others (e.g., Leiva et al., 2010; Larrain et al., 2015; Soto et al., 2017; Tang et al., 2017) adopted non-linear programming or mixed-integer non-linear programming solvers. These methods can derive optimal solutions, but they do not apply to a long bus corridor because of their low computational efficiency. To overcome this problem, some researchers (e.g., Ulusoy and Chien, 2015; Yi et al., 2016; Torabi and Salari, 2019; Jiang and Ma, 2021; Liang et al., 2021) attempted to use meta-heuristics, e.g., genetic algorithms and artificial bee colony (ABC) algorithms. The solution method adopted in our study is also a meta-heuristic. Specifically, we develop an enhanced ABC algorithm to solve our model. We demonstrate its higher effectiveness and computational efficiency than genetic and ABC algorithms, which is shown in sub-section 5.3.

In summary, the major contributions of the paper are shown as follows:

(1) A more general two-stage strategy for the LSBSDP than the tactical planning strategy and the dynamic stop-skipping strategy is proposed.

(2) The corresponding model is developed, and prediction errors are considered in the second-stage model.

(3) An enhanced ABC algorithm is developed, with higher effectiveness and computational efficiency than genetic and traditional ABC algorithms.

The remainder of this paper is structured as follows: Section 2 is the problem statement. Section 3 describes the formulation of the model for each stage. Section 4 depicts the solution method. Section 5 shows numerical results, and section 6 concludes the paper.

2. Problem statement

The paper proposes a two-stage strategy to address the real-time limited-stop bus service design problem for a corridor, as shown in Figure 1. In the first stage (i.e., at the tactical planning level), an operator determines a set of limited-stop services (denoted as L) based on historical bus travel times and passenger arrival rates. The maximum number of limited-stop services in L is a parameter, which is predetermined and denoted as NL. However, the stop sequence of each of these limited-stop services is required to determine. In the second stage (i.e., at the operational level), for each limited-stop vehicle, an operator selects one limited-stop service (denoted as $l*$) from L based on (short-term) predictive travel times and passenger arrival rates. Predictive travel times and passenger arrival rates are assumed to be known, which can be obtained by prediction models in practice (e.g., Chien et al., 2002; Sheu, 2005). Prediction errors are considered in the second stage. After Stage 2 (i.e., $l*$ is determined), 1) a bus captain drives a vehicle departing from the bus terminal and provides the corresponding service; 2) passengers are informed of service $l*$ by mobile applications.

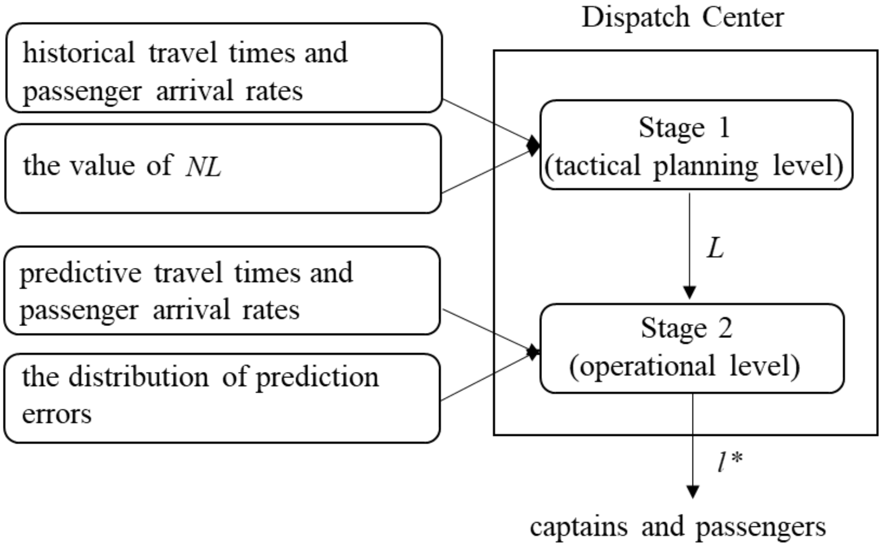

Figure 1. The two-stage strategy

As in the study of Liu et al. (2013), we assume that all-stop and limited-stop vehicles depart from the bus terminal alternately. This assumption is imposed to guarantee a minimum level of service for each origin-destination (OD) pair of passengers: The changed bus headway at each stop due to stop-skipping would not exceed two times the standard headway, to avoid large waiting time for the passengers (Liu et al., 2013). Because of this assumption, when an operator wants to determine the service of one limited-stop vehicle, the operations of all-stop vehicles before and after the limited-stop vehicle need to be considered together with the operation of the limited-stop vehicle. We let i be the vehicle type and use $i = 0,1,2$ to represent the previous all-stop vehicle, the limited-stop vehicle (whose service is undetermined), and the next all-stop vehicle, respectively. These three vehicles form a vehicle group (VG) and then all vehicles in one day can be grouped into a certain number of VGs, as illustrated in Figure 2. We also use N to denote the number of stops in a bus corridor. The total system cost for one VG is comprised of 1) the waiting cost for all passengers between the departure of vehicle 0 and the arrival of vehicle 2, 2) the in-vehicle travel cost associated with passengers in vehicles 1 and 2, and 3) the operating cost associated with vehicles 1 and 2.

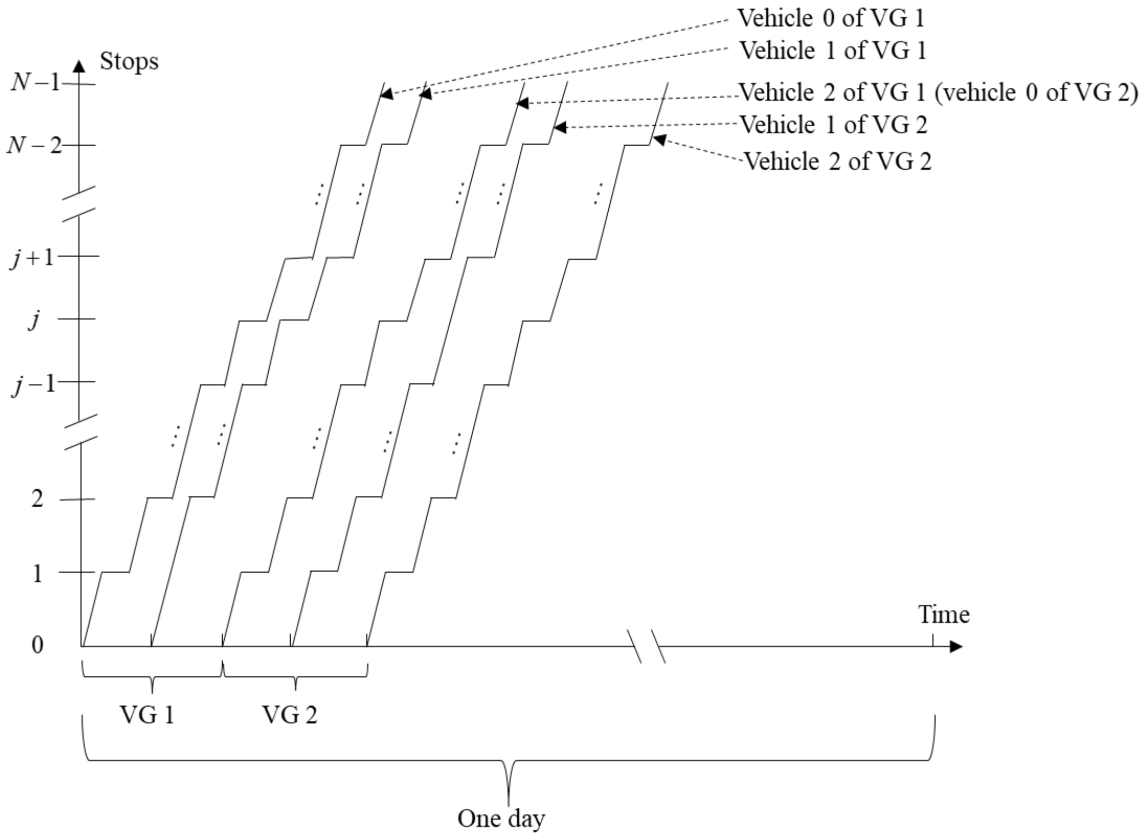

Figure 2. The space-time diagram of vehicles for VG 1 and VG 2

3. Model formulation

3.1. Notations

Since our model involves two stages, we classify notations into common notations, first-stage notations, and second-stage notations, as shown below.

Common notations

Indices and sets

i	Vehicle type, $i = 0,1,2$.
j	Bus stop index, $j = 0,1,...,N-1$, where 0 and $N-1$ mean the starting and ending terminals, respectively. N is the number of stops in a bus corridor.
e	The destination stop index of passengers.
l	The index of a limited-stop service, $l = 1,2,...,NL$. In this paper, an all-stop

service is regarded as a special case of a limited-stop service.

p VG index, representing the order of vehicle groups.

d Day index.

J The set of bus stops, i.e., $J = \{0,1,...,N-1\}$.

E The set of passenger destinations, $E \subset J$.

L The set of limited-stop services.

P The set of VGs in one day.

D The set of days for collecting historical bus travel times and passenger arrival rates in the first stage.

Parameters

NL The maximum number of limited-stop services predetermined in Stage 1.

WV Value of waiting time.

IV Value of in-vehicle travel time.

OV Value of operating time.

Cap The capacity of a vehicle.

b The average boarding time per passenger.

a The average alighting time per passenger.

τ_1 The time of opening and closing doors.

τ_2 Acceleration time.

τ_3 Deceleration time.

First-stage notations

Parameters

$r_{i,j}^{d,p}$ The historical travel time of vehicle i of VG p between stops $j-1$ and j on day d.

$\lambda_{i,j,e}^{d,p}$ The historical arrival rate of passengers heading to stop e from stop j between the arrival times of vehicles $i-1$ and i of VG p at stop j on day d.

Parameter vectors

$\mathbf{r}^{d,p}$ $\left(r_{i,j}^{d,p}\right)_{\forall i\in I, j\in J}$.

$\boldsymbol{\lambda}^{d,p}$ $\left(\lambda_{i,j,e}^{d,p}\right)_{\forall i\in I, j\in J, e\in E}$.

Decision variables

y_{ij}^{l} A binary variable. It equals 1 if vehicle i associated with service l does not skip stop j, and 0 otherwise. Please note that vehicles 0 and 2 cannot skip stops, i.e., $y_{ij}^{l} = 1$, for $i = 0,2;\ j = 0,1,...,N-1;\ l = 1,2,...,NL$.

$\mathbf{y}^{l}$ $\left(y_{ij}^{l}\right)_{\forall i\in I, j\in J}$.

$\mathbf{y}$ $\left(y_{ij}^{l}\right)_{\forall i\in I, j\in J, l\in L}$.

Functions

$z(\mathbf{y})$ The minimum total operator and passenger costs (associated with all $p \in P$ in all $d \in D$).

$f(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y})$ The minimum total operator and passenger costs associated with day d and VG p.

$h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$ The total operator and passenger costs associated with day d and VG p when limited-stop service l is adopted.

Auxiliary variables

$H_{i,j,l}^{d,p}$ The headway between vehicles $i-1$ and i of VG p at stop j on day d if limited-stop service l is adopted.

$Z_{1,l}^{d,p}$ The waiting cost of passengers of VG p on day d if limited-stop service l is adopted.

$Z_{2,l}^{d,p}$ The in-vehicle travel cost of passengers on VG p on day d if limited-stop service l is adopted.

$Z_{3,l}^{d,p}$ The operating cost of VG p on day d if limited-stop service l is adopted.

$A_{i,j,l}^{d,p}$ The arrival time of vehicle i of VG p at stop j on day d if limited-stop service l is adopted.

$A_{2,j}^{d,p-1}$ The arrival time of vehicle 2 of VG $p-1$ at stop j on day d. It has been determined when $f(\mathbf{r}^{d,p-1},\boldsymbol{\lambda}^{d,p-1},\mathbf{y})$ is calculated.

$D_{i,j,l}^{d,p}$ The departure time of vehicle i of VG p at stop j on day d if limited-stop service l is adopted.

$S_{i,j,l}^{d,p}$ The dwell time of vehicle i of VG p at stop j on day d if limited-stop service l is adopted.

$FBP_{i,j,e,l}^{d,p}$ The number of passengers who want to travel from stop j to stop e and fail to board vehicle i of VG p on day d if limited-stop service l is adopted.

$FBP_{2,j,e}^{d,p-1}$ The number of passengers who want to travel from stop j to stop e and fail to board vehicle 2 of VG $p-1$ on day d. It has been determined when $f(\mathbf{r}^{d,p-1},\boldsymbol{\lambda}^{d,p-1},\mathbf{y})$ is calculated.

$IP_{i,j,l}^{d,p}$ The number of passengers on vehicle i of VG p on day d when the vehicle arrives stop j if limited-stop service l is adopted.

$SBP_{i,j,e,l}^{d,p}$ The number of passengers who want to travel from stop j to stop e and succeed in boarding vehicle i of VG p on day d if limited-stop service l is adopted.

$AP_{i,j,l}^{d,p}$ The number of passengers who alight at stop j from vehicle i of VG p on day d if limited-stop service l is adopted.

$W_{i,j,e,l}^{d,p}$ The number of waiting passengers who want to travel from stop j to stop e when vehicle i of VG p on day d arrives stop j.

$TSBP_{i,j,l}^{d,p}$ The number of passengers who succeed in boarding vehicle i of VG p on day d at stop j if limited-stop service l is adopted.

Second-stage notations

Random variables

$\tilde{r}_{i,j}$ The real travel time of vehicle i of the next VG between stops $j-1$ and j. (The value of $\tilde{r}_{i,j}$ is unknown, but the mean and variance satisfy Equations (41) and (42).)

$r_{i,j}$ — The predictive travel time of vehicle i of the next VG between stops $j-1$ and j.

$\varepsilon^{\mathrm{r}}_{i,j}$ — The predictive error of $\tilde{r}_{i,j}$.

$\tilde{\mathbf{r}}$ — $\left(\tilde{r}_{i,j}\right)_{\forall i\in I, j\in J}$.

$\tilde{\lambda}_{i,j,e}$ — The real arrival rate of passengers heading to stop e from stop j between the arrival times of vehicles $i-1$ and i of the next VG at stop j. (The value of $\tilde{\lambda}_{i,j,e}$ is unknown, but the mean and variance satisfy Equations (43) and (44).)

$\lambda_{i,j,e}$ — The predictive arrival rate of passengers heading to stop e from stop j between the arrival times of vehicles $i-1$ and i of the next VG at stop j.

$\varepsilon^{\lambda}_{i,j,e}$ — The predictive error of $\tilde{\lambda}_{i,j,e}$.

$\tilde{\boldsymbol{\lambda}}$ — $\left(\tilde{\lambda}_{i,j,e}\right)_{\forall i\in I, j\in J, e\in E}$.

Parameters

$r_{i,j}{*}$ — A value of $r_{i,j}$.

$\lambda_{i,j,e}{*}$ — A value of $\lambda_{i,j,e}$.

y^{l}_{ij} — The decision variable in the first stage, but it is a parameter in the second stage.

$\bar{\tilde{r}}_{i,j}$ — The expected value of $\tilde{r}_{i,j}$. It can be estimated by calculating the mean of $r^{d,p}_{i,j}, \forall d\in D, p\in P$.

$\sigma^{2,\tilde{\mathrm{r}}}_{i,j}$ — The variance of $\tilde{r}_{i,j}$. It can be estimated by calculating the variance of $r^{d,p}_{i,j}, \forall d\in D, p\in P$.

$\sigma^{2,\mathrm{r}}_{i,j}$ — The variance of the prediction error when we predict $\tilde{r}_{i,j}$ by a prediction model.

$\bar{\tilde{\lambda}}_{i,j,e}$ — The expected value of $\tilde{\lambda}_{i,j,e}$. It can be estimated by calculating the mean of $\lambda^{d,p}_{i,j,e}, \forall d\in D, p\in P$.

$\sigma^{2,\tilde{\lambda}}_{i,j,e}$ — The variance of $\tilde{\lambda}_{i,j,e}$. It can be estimated by calculating the variance of $\lambda^{d,p}_{i,j,e}, \forall d\in D, p\in P$.

$\sigma^{2,\lambda}_{i,j,e}$ — The variance of the prediction error when we predict $\tilde{\lambda}_{i,j,e}$ by a prediction model.

Decision variable

$l{*}$ — The index of the best limited-stop service in L (associated with the next VG).

Function

$g(\tilde{\mathbf{r}}, \tilde{\boldsymbol{\lambda}}, \mathbf{y}^{l})$ — The total operator and passenger costs associated with the next VG when limited-stop service l is adopted.

3.2. Assumptions

3.2.1. The first stage (i.e., tactical planning level)

With the knowledge of $\mathbf{r}^{d,p}$ and $\boldsymbol{\lambda}^{d,p}$, an operator in the first stage aims to determine the stop sequences of a fixed number of limited-stop services to minimize total operator and passenger costs for all $p \in P$ in all $d \in D$.

3.2.2. The second stage (i.e., operational level)

In the second stage, we assume that an operator can get the values of predictive bus travel times and passenger arrival rates for the next VG (i.e., $r_{i,j}*$ and $\lambda_{i,j,e}*$) by prediction models. Then the operator determines one limited-stop service in L to minimize the operator and passenger costs for the next VG, with the consideration of the (historical) prediction errors of prediction models. The mean of these prediction errors is 0, while the variances of these prediction errors are fixed and given.

3.2.3. Passenger behavior, capacity, passenger arrival rate, and waiting time

A passenger is assumed to wait for a vehicle that serves both his/her origin and destination. Since capacity constraints are considered, he/she has to wait for the next vehicle if the arriving vehicle is fully loaded. In a word, a passenger boards the first arriving vehicle that serves both his/her origin and destination and is not fully loaded.

The passenger arrival rate between two successive vehicles is assumed to be uniform. As a result, when a vehicle arrives at a stop, the average waiting time of new passengers at the stop is half of the headway while the additional average waiting of remaining passengers is the headway. New passengers mean that they arrive at the stop after the last vehicle leaves the stop and before the arriving vehicle arrives at the stop. Remaining passengers mean that they arrive at the stop before the last vehicle leaves the stop but fail to board the last vehicle because either 1) the vehicle is fully loaded or 2) the vehicle cannot serve their origin and destination.

3.2.4. Headway, dwell time, and overtaking phenomenon

The headway (and the bus arrival time) at the bus terminal is fixed and given. This implies that the number of buses is known and given and hence the capital cost need not be considered.

Moreover, the headway between all-stop and limited-stop vehicles is assumed to be not greater than 15 min so that the arrival times of passengers are not affected by any stop-skipping strategy of limited-stop vehicles. Furthermore, the dwell time at each stop is determined by the numbers of boarding passengers and alighting passengers at the stop. In addition, overtaking phenomena are not allowed.

3.3. Formulation

3.3.1. The first stage

An operator in the first stage aims to determine the stop sequences of a fixed number of limited-stop services to minimize total operator and passenger costs for all $p \in P$ in all $d \in D$. The first-stage model can be formulated as follows:

$$\min \quad z(\mathbf{y}) = \sum_{d \in D} \sum_{p \in P} f(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y}) \tag{1}$$

Subject to

$$y_{ij}^{l} = 1, \text{for } i = 0, 2;\ j = 0, 1, ..., N-1; l = 1, 2, ..., NL, \tag{2}$$

$$y_{ij}^{l} = 1, \text{for } i = 1;\ j = 0, N-1; l = 1, 2, ..., NL, \tag{3}$$

$$y_{ij}^{l} \in \{0,1\}, \text{for } i = 0, 1, 2;\ j = 0, 1, ..., N-1; l = 1, 2, ..., NL. \tag{4}$$

In Objective function (1), $f(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y})$ is the minimum total operator and passenger costs associated with day d and VG p. Constraint (2) guarantees that vehicles 0 and 2 are all-stop vehicles, and Constraint (3) ensures that vehicle 1 serves the starting and ending terminals. Constraint (4) defines y_{ij}^{l} to be binary variables.

In Objective function (1), $f(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y})$ can be computed by

$$f(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y}) = \min_{l \in \{1,2,...,NL\}} \{h(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y}^{l})\}. \tag{5}$$

$h(\mathbf{r}^{d,p}, \boldsymbol{\lambda}^{d,p}, \mathbf{y}^{l})$ is the total operator and passenger costs associated with day d and VG p when limited-stop service l is adopted.

Let $H_{i,j,l}^{d,p}$ be the headway between vehicles $i-1$ and i of VG p at stop j on day d if limited-stop service l is adopted. When

$$H_{i,j,l}^{d,p} \geq 0, \text{for } i=1,2;\ j=0,1,...,N-2 \quad (6)$$

is satisfied (i.e., overtaking phenomena do not occur), we can obtain $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$ by

$$h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l}) = Z_{1,l}^{d,p} + Z_{2,l}^{d,p} + Z_{3,l}^{d,p}, \quad (7)$$

$$Z_{1,l}^{d,p} = WV\sum_{i=1}^{2}\sum_{j=0}^{N-2}\sum_{e\in E}(\lambda_{i,j,e}^{d,p}H_{i,j,l}^{d,p}\cdot\frac{H_{i,j,l}^{d,p}}{2} + FBP_{i-1,j,e,l}^{d,p}\cdot H_{i,j,l}^{d,p}), \quad (8)$$

$$Z_{2,l}^{d,p} = IV\sum_{i=1}^{2}\sum_{j=0}^{N-2}\sum_{e\in E}SBP_{i,j,e,l}^{d,p}\cdot(\sum_{k=j+1}^{e}r_{i,k}^{d,p} + \sum_{k=j}^{e-1}S_{i,k,l}^{d,p}), \quad (9)$$

$$Z_{3,l}^{d,p} = OV\sum_{i=1}^{2}(\sum_{j=1}^{N-1}r_{i,j}^{d,p} + \sum_{j=0}^{N-2}S_{i,j,l}^{d,p}), \quad (10)$$

$$A_{0,j,l}^{d,p} = A_{2,j}^{d,p-1}, \text{for } j=1,2,...,N-1, \quad (11)$$

$$A_{i,j,l}^{d,p} = D_{i,j-1,l}^{d,p} + r_{i,j}^{d,p}, \text{for } i=1,2;\ j=1,2,...,N-1, \quad (12)$$

$$D_{i,j,l}^{d,p} = A_{i,j,l}^{d,p} + S_{i,j,l}^{d,p}, \text{for } i=1,2;\ j=0,1,...,N-2, \quad (13)$$

$$H_{i,j,l}^{d,p} = A_{i,j,l}^{d,p} - A_{i-1,j,l}^{d,p}, \text{for } i=1,2;\ j=1,2,...,N-2, \quad (14)$$

$$FBP_{0,j,e,l}^{d,p} = FBP_{2,j,e}^{d,p-1}, \text{for } j=1,2,...,N-1; e\in E, \quad (15)$$

$$IP_{i,j,l}^{d,p} = \sum_{k=0}^{j-1}\sum_{e\in E,e\geq j}SBP_{i,k,e,l}^{d,p}, \text{for } i=1,2;\ j=1,2,...,N-1, \quad (16)$$

$$AP_{i,j,l}^{d,p} = \sum_{k=0}^{j-1}\sum_{e\in E,e=j}SBP_{i,k,e,l}^{d,p}, \text{for } i=1,2;\ j=1,2,...,N-1, \quad (17)$$

$$W_{i,j,e,l}^{d,p} = \lambda_{i,j,e}^{d,p}H_{i,j,l}^{d,p} + FBP_{i-1,j,e,l}^{d,p}, \text{for } i=1,2;\ j=0,1,...,N-2; e\in E, \quad (18)$$

$$TSBP_{i,j,l}^{d,p} = y_{i,j}^{l}\cdot\min\left\{\sum_{e\in E,e>j}y_{i,e}^{l}W_{i,j,e,l}^{d,p}, Cap - IP_{i,j,l}^{d,p} + AP_{i,j,l}^{d,p}\right\}, \text{for } i=1,2;\ j=0,.1,..,N-2, \quad (19)$$

$$SBP_{i,j,e,l}^{d,p} = TSBP_{i,j,l}^{d,p}\cdot\frac{y_{i,e}^{l}W_{i,j,e,l}^{d,p}}{\sum_{e'\in E,e'>j}y_{i,e'}^{l}W_{i,j,e',l}^{d,p}}, \text{for } i=1,2;\ j=0,1,...,N-2; e\in E, \quad (20)$$

$$FBP_{i,j,e,l}^{d,p} = W_{i,j,e,l}^{d,p} - SBP_{i,j,e,l}^{d,p}, \text{for } i=1,2;\ j=0,1,...,N-2; e\in E, \text{ and} \quad (21)$$

$$\begin{cases} S_{i,j,l}^{d,p} = y_{i,j}^{l}\cdot(b\cdot TSBP_{i,j,l}^{d,p} + \tau_1 + \tau_2), \text{for } i=1,2;\ j=0, \\ S_{i,j,l}^{d,p} = y_{i,j}^{l}\cdot(\max\{b\cdot TSBP_{i,j,l}^{d,p}, a\cdot AP_{i,j,l}^{d,p}\} + \tau_1 + \tau_2 + \tau_3), \text{for } i=1,2;\ j=1,2,...,N-2. \end{cases} \quad (22)$$

Equation (7) defines that $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$ is comprised of 1) the waiting cost for all passengers between the departure of vehicle 0 and the arrival of vehicle 2, 2) the in-vehicle travel cost associated with passengers in vehicles 1 and 2, and 3) the operating cost associated with vehicles 1 and 2. Equations (8)-(10) are used to calculate these three costs, respectively. In Equation (8), $\lambda_{i,j,e}^{d,p} H_{i,j,l}^{d,p} \cdot \frac{H_{i,j,l}^{d,p}}{2}$ is the waiting time of new passengers, whereas $FBP_{i-1,j,e,l}^{d,p} \cdot H_{i,j,l}^{d,p}$ is the additional waiting time of remaining passengers. In Equation (9), $SBP_{i,j,e,l}^{d,p}$ is the number of in-vehicle passengers for OD pair (j,e) and $(\sum_{k=j+1}^{e} r_{i,k}^{d,p} + \sum_{k=j}^{e-1} S_{i,k,l}^{d,p})$ is the corresponding in-vehicle travel time per passenger. In Equation (10), $(\sum_{j=1}^{N-1} r_{i,j}^{d,p} + \sum_{j=0}^{N-2} S_{i,j,l}^{d,p})$ is the operating time of vehicle i.

Equation (11) determines the arrival time of vehicle 0 of VG p at each stop: vehicle 0 of VG p is just vehicle 2 of VG $p-1$. Equations (12) and (13) are included to calculate bus arrival and departure times at each stop, respectively. Equation (14) computes the headway at each stop.

Equation (15) sets the number of passengers who fail to board vehicle 0 of VG p to be that of vehicle 2 of VG $p-1$. Equations (16), (17), and (18) are used to compute the numbers of in-vehicle passengers, alighting passengers, and waiting passengers, respectively. Equation (19) is used to calculate the number of total passengers who can succeed in boarding.

Equation (20) determines the number of passengers who succeed in boarding for each OD pair, whereas Equation (21) computes the number of passengers who fail to board for each OD pair. Equation (22) defines the dwell time when we assume a 2-door operation.

When Objective function (1) is minimized, the optimal values of y_{ij}^{l} are obtained, and are used to deduce the optimal stop sequence of limited-stop service *l*. (A stop sequence of service *l* offered by vehicle 1 is represented by a string of binary numbers $y_{10}^{l}\ y_{11}^{l} \ldots y_{1,N-1}^{l}$.) The collection of the optimal stop sequence of each limited-stop service is denoted as L'. As there is no constraint to ensure that all elements in L' are different, it is possible that some elements in L' are repeated. As a result, we collect all *different* elements in L' to create a new set, which is denoted as L. The size of L may be different from NL and we use $|L|$ to denote it.

Objective function (1) in Stage 1 represents the total cost of *all* VGs during *all* days. Conducting Stage 1 once can cover all VGs during all days and derive one L for *all* days.

3.3.2. The second stage

In the second stage, the operator needs to determine the best limited-stop service in L for the next VG by

$$l^* = \arg \min_{l \in \{1,2,\ldots,|L|\}} \{E(g(\tilde{\mathbf{r}}, \tilde{\boldsymbol{\lambda}}, \mathbf{y}^l))\}. \quad (23)$$

When

$$H_{i,j,l} \geq 0, \text{for } i = 1,2;\ j = 0,1,\ldots,N-2 \quad (24)$$

is satisfied (overtaking phenomena do not occur), $g(\tilde{\mathbf{r}}, \tilde{\boldsymbol{\lambda}}, \mathbf{y}^l)$ can be calculated by

$$g(\tilde{\mathbf{r}}, \tilde{\boldsymbol{\lambda}}, \mathbf{y}^l) = Z_{1,l} + Z_{2,l} + Z_{3,l}, \quad (25)$$

$$Z_{1,l} = WV \sum_{i=1}^{2} \sum_{j=0}^{N-2} \sum_{e \in E} (\tilde{\lambda}_{i,j,e} H_{i,j,l} \cdot \frac{H_{i,j,l}}{2} + FBP_{i-1,j,e,l} \cdot H_{i,j,l}), \quad (26)$$

$$Z_{2,l} = IV \sum_{i=1}^{2} \sum_{j=0}^{N-2} \sum_{e \in E} SBP_{i,j,e,l} \cdot (\sum_{k=j+1}^{e} \tilde{r}_{i,k} + \sum_{k=j}^{e-1} S_{i,k,l}), \quad (27)$$

$$Z_{3,l} = OV \sum_{i=1}^{2} (\sum_{j=1}^{N-1} \tilde{r}_{i,j} + \sum_{j=0}^{N-2} S_{i,j,l}), \quad (28)$$

$$A_{0,j,l} = A_{2,j}, \text{for } j = 1,2,\ldots,N-1, \quad (29)$$

$$A_{i,j,l} = D_{i,j-1,l} + \tilde{r}_{i,j}, \text{for } i = 1,2;\ j = 1,2,\ldots,N-1, \quad (30)$$

$$D_{i,j,l} = A_{i,j,l} + S_{i,j,l}, \text{for } i = 1,2;\ j = 0,1,\ldots,N-2, \quad (31)$$

$$H_{i,j,l} = A_{i,j,l} - A_{i-1,j,l}, \text{for } i = 1,2;\ j = 1,2,\ldots,N-2, \quad (32)$$

$$FBP_{0,j,e,l} = FBP_{2,j,e}, \text{for } j = 1,2,\ldots,N-1; e \in E, \quad (33)$$

$$IP_{i,j,l} = \sum_{k=0}^{j-1} \sum_{e \in E, e \geq j} SBP_{i,k,e,l}, \text{for } i = 1,2;\ j = 1,2,\ldots,N-1, \quad (34)$$

$$AP_{i,j,l} = \sum_{k=0}^{j-1} \sum_{e \in E, e = j} SBP_{i,k,e,l}, \text{for } i = 1,2;\ j = 1,2,\ldots,N-1, \quad (35)$$

$$W_{i,j,e,l} = \tilde{\lambda}_{i,j,e} H_{i,j,l} + FBP_{i-1,j,e,l}, \text{for } i = 1,2;\ j = 0,1,\ldots,N-2; e \in E, \quad (36)$$

$$TSBP_{i,j,l} = y_{i,j}^{l} \cdot \min \left\{ \sum_{e \in E, e > j} y_{i,e}^{l} W_{i,j,e,l}, Cap - IP_{i,j,l} + AP_{i,j,l} \right\}, \text{for } i = 1,2;\ j = 0,.1,..,N-2, \quad (37)$$

$$SBP_{i,j,e,l} = TSBP_{i,j,l} \cdot \frac{y_{i,e}^{l} W_{i,j,e,l}}{\sum_{e' \in E, e' > j} y_{i,e'}^{l} W_{i,j,e',l}}, \text{for } i = 1,2;\ j = 0,1,\ldots,N-2; e \in E, \quad (38)$$

$$FBP_{i,j,e,l} = W_{i,j,e,l} - SBP_{i,j,e,l}, \text{for } i=1,2;\ j=0,1,...,N-2; e \in E, \quad \text{and} \tag{39}$$

$$\begin{cases} S_{i,j,l} = y_{i,j}^{l} \cdot (b \cdot TSBP_{i,j,l} + \tau_1 + \tau_2), \text{for } i=1,2;\ j=0 \\ S_{i,j,l} = y_{i,j}^{l} \cdot (\max\{b \cdot TSBP_{i,j,l},\ a \cdot AP_{i,j,l}\} + \tau_1 + \tau_2 + \tau_3), \text{for } i=1,2;\ j=1,2,...,N-2 \end{cases} \tag{40}$$

The meanings of notations used in the second-stage model basically follow those in the first-stage counterpart, except that the latter notations are for VG p on day d whereas the former notations are for the next VG.

As $\tilde{r}_{i,j}$ and $\tilde{\lambda}_{i,j,e}$ are unknown, the operator needs to predict them by prediction models. We denote their predictive values as $r_{i,j}*$ and $\lambda_{i,j,e}*$, respectively. The predictive error of $\tilde{r}_{i,j}$ $\left(\tilde{\lambda}_{i,j,e}\right)$ is denoted as $\varepsilon_{i,j}^{\mathrm{r}}$ $\left(\varepsilon_{i,j,e}^{\lambda}\right)$.

If we assume

1) $\tilde{r}_{i,j}$ follows a normal distribution, denoted as $N(\bar{\tilde{r}}_{i,j}, \sigma_{i,j}^{2,\tilde{r}})$;

2) $\tilde{\lambda}_{i,j,e}$ follows a normal distribution, denoted as $N(\bar{\tilde{\lambda}}_{i,j,e}, \sigma_{i,j,e}^{2,\tilde{\lambda}})$;

3) $\varepsilon_{i,j}^{\mathrm{r}}$ and $\varepsilon_{i,j,e}^{\lambda}$ follow normal distributions with a mean of 0, denoted as $N(0, \sigma_{i,j}^{2,\mathrm{r}})$ and $N(0, \sigma_{i,j,e}^{2,\lambda})$, respectively,

the means and variances of $\tilde{r}_{i,j}$ and $\tilde{\lambda}_{i,j,e}$ obey the following equations (their derivation process is presented in Appendix A):

$$E(\tilde{r}_{i,j} \mid r_{i,j} = r_{i,j}*) = \frac{\bar{\tilde{r}}_{i,j}\sigma_{i,j}^{2,\mathrm{r}} + r_{i,j}*\sigma_{i,j}^{2,\tilde{r}}}{\sigma_{i,j}^{2,\tilde{r}} + \sigma_{i,j}^{2,\mathrm{r}}}, \text{for } i=0,1,2;\ j=1,2,...,N-1, \tag{41}$$

$$Var(\tilde{r}_{i,j} \mid r_{i,j} = r_{i,j}*) = \frac{\sigma_{i,j}^{2,\tilde{r}}\sigma_{i,j}^{2,\mathrm{r}}}{\sigma_{i,j}^{2,\tilde{r}} + \sigma_{i,j}^{2,\mathrm{r}}}, \text{for } i=0,1,2;\ j=1,2,...,N-1, \tag{42}$$

$$E(\tilde{\lambda}_{i,j,e} \mid \lambda_{i,j,e} = \lambda_{i,j,e}*) = \frac{\bar{\tilde{\lambda}}_{i,j,e}\sigma_{i,j,e}^{2,\lambda} + \lambda_{i,j,e}*\sigma_{i,j,e}^{2,\tilde{\lambda}}}{\sigma_{i,j,e}^{2,\tilde{\lambda}} + \sigma_{i,j,e}^{2,\lambda}}, \text{for } i=1,2;\ j=0,1,...,N-1, \text{ and} \tag{43}$$

$$Var(\tilde{\lambda}_{i,j,e} \mid \lambda_{i,j,e} = \lambda_{i,j,e}*) = \frac{\sigma_{i,j,e}^{2,\tilde{\lambda}}\sigma_{i,j,e}^{2,\lambda}}{\sigma_{i,j,e}^{2,\tilde{\lambda}} + \sigma_{i,j,e}^{2,\lambda}}, \text{for } i=1,2;\ j=0,1,...,N-1. \tag{44}$$

3.3.3. Two special cases

As mentioned earlier, the tactical planning strategy and dynamic stop-skipping strategy are only two special cases of the two-stage strategy.

If we set parameter NL to 1, L derived from the first stage only contains one limited-stop service. It means that all limited-stop vehicles in the second stage must provide the limited-stop service. This situation is just the same as that of the tactical planning strategy.

If we set parameter NL to be the maximum number of all possible different limited-stop services, L derived from the first stage is likely to contain all possible different limited-stop services and it depends on whether there is one optimal solution or there are multiple optimal solutions for Stage 1: (1) If there is only one optimal solution for Stage 1, L' is just the set of all different possible limited-stop services and L is the same as L'. In this situation, each limited-stop vehicle in the second stage can provide one limited-stop service out of all possible different services. It is just the same as the situation of the dynamic stop-skipping strategy; (2) If there are multiple optimal solutions for Stage 1, some elements in L' may be repeated and L' is not the set of all possible different limited-stop services. Then L derived from L' is not the set of all possible different limited-stop services. This situation is not the same as that of the dynamic stop-skipping strategy. However, we can prove that the objective function value of Stage 1 associated with L is the same as that associated with the set of all possible different limited-stop services L'' by the following statements:

1) As L' is obtained by solving the model of Stage 1, L' gives the lowest objective function value of Stage 1.
2) As all elements in L' can be found in L and vice versa, the objective function value of Stage 1 associated with L is the same as that of L'.
3) As L'' is a feasible solution, the objective function value of L'' is not better than that of L', which is the lowest objective value according to statement 1). Moreover, as all elements in L' can be found in L'', L'' can be considered to be formed by introducing more different elements to L'. For any solution including L'', adding more different elements in the solution cannot increase the objective function value of Stage 1. Therefore, the objective function value of L'' must be the same as that of L', which is also the same as that of L according to statement 2).

4. Solution method

We develop an enhanced ABC algorithm and adopt the Monte Carlo Simulation method to solve the first- and second-stage models, respectively.

4.1. The enhanced ABC algorithm for the first-stage model

An enhanced ABC algorithm is developed to solve the first model (i.e., determine $\mathbf{y}$). We first introduce the ABC algorithm proposed by Karaboga (2005) (referred to as the traditional ABC algorithm) and then describe the difference between the traditional and enhanced ABC algorithms.

4.1.1. The traditional ABC algorithm

The algorithmic steps of the traditional ABC algorithm are shown in Figure 3, which are explained below:

Step 0: Parameter setting

Set the number of employed bees N_e, the number of onlooker bees N_o, the maximum number of unimproved iterations (the number of trials that fail to improve the current solution) $U_{\max}$, and the maximum number of iterations $I_{\max}$.

Step 1: Initialization

Set iteration = 0. Randomly generate initial solutions $\mathbf{y}_b$, for $b = 0,1,..., N_e - 1$ and assign one employed bee to each solution. Evaluate the fitness $fit(\mathbf{y}_b)$, for $b = 0,1,..., N_\mathrm{e} - 1$. Record the best solution $\hat{\mathbf{y}}$ in $\{\mathbf{y}_b \mid b = 0,1,..., N_\mathrm{e} - 1\}$. Set the counters of unimproved iterations $u_b = 0$, for $b = 0,1,..., N_\mathrm{e} - 1$. Set iteration = 1.

Step 2: Employed bee phase

For each employed bee $b = 0,1,..., N_\mathrm{e} - 1$

Step 2.1: Generate a neighbor solution $\mathbf{y}_b*$ based on $\mathbf{y}_b$.

Step 2.2: Evaluate the fitness $fit(\mathbf{y}_b*)$. If $fit(\mathbf{y}_b*) > fit(\mathbf{y}_b)$, replace $\mathbf{y}_b$ with $\mathbf{y}_b*$ and $u_b = 0$, else increase u_b by 1.

Step 3: Onlooker bee phase

For each onlooker bee

Step 3.1: Select a solution $\mathbf{y}_b$ by the fitness-based roulette wheel selection method and then generate a neighbor solution $\mathbf{y}_b*$ based on $\mathbf{y}_b$.

Step 3.2: Evaluate the fitness $fit(\mathbf{y}_b*)$. If $fit(\mathbf{y}_b*) > fit(\mathbf{y}_b)$, replace $\mathbf{y}_b$ with $\mathbf{y}_b*$ and $u_b = 0$, else increase u_b by 1.

Step 4: Updating the best solution

For $b = 0,1,..., N_e - 1$, if $fit(\mathbf{y}_b) > fit(\hat{\mathbf{y}})$, set $\hat{\mathbf{y}}$ to be $\mathbf{y}_b$.

Step 5: Scout bee phase

For $b = 0,1,..., N_e - 1$, if $u_b \geq U_{\max}$, replace $\mathbf{y}_b$ with a new randomly generated solution and evaluate the fitness of the new solution.

Step 6: Termination criterion checking

If iteration < $I_{\max}$, iteration = iteration + 1 and return to Step 2. Otherwise, stop and output the best solution $\hat{\mathbf{y}}$.

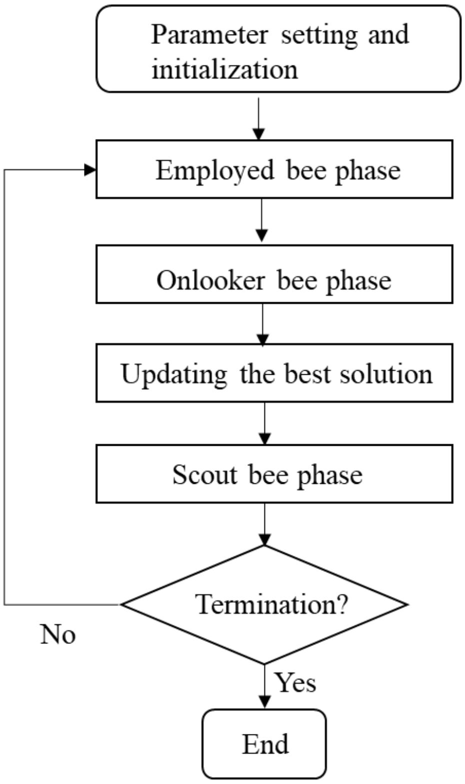

Figure 3. The flowchart of the traditional ABC algorithm

The representation of a solution in the traditional ABC algorithm

Based on Constraints (2)-(4), y_{ij}^l, for $i = 1; j = 1, 2, ..., N-2; l = 1, 2, ..., NL$ can be 0 or 1, and y_{ij}^l must be 1 for other values of i, (i.e., $i = 0, 2$). Because of that, a solution in the traditional ABC algorithm can be represented by Figure 4, which only considers y_{ij}^l, for $i = 1; j = 1, 2, ..., N-2; l = 1, 2, ..., NL$ as binary values.

l_1	$y_{1,1}^{1}$	$y_{1,2}^{1}$	$y_{1,3}^{1}$	$\cdots$	$y_{1,N-2}^{1}$
l_2	$y_{1,1}^{2}$	$y_{1,2}^{2}$	$y_{1,3}^{2}$	$\cdots$	$y_{1,N-2}^{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Figure 4. The representation of a solution in the traditional ABC algorithm

Fitness function

The fitness function $fit(\mathbf{y})$ in the traditional ABC algorithm equals the reciprocal of the objective function value (i.e., $1/z(\mathbf{y})$). $z(\mathbf{y})$ is computed by Equations (1) and (5) and $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$. $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$ can be obtained by Equations (7)-(22) if $y_{i,j}^{l}$ is given, which is presented in detail in Appendix B.

Random generation of a solution and non-neighborhood/neighborhood operators

In Steps 1 and 5, a solution in the traditional ABC algorithm is generated randomly. It means that each element y_{ij}^{l}, for $i=1; j=1,2,...,N-2; l=1,2,...,NL$ is randomly determined to be 0 or 1.

In Steps 2.1 and 3.1, each employed or onlooker bee generates a neighbor solution based on $\mathbf{y}_b$ by one of the neighborhood operators:

Neighborhood operator 1: Single change

This operator randomly selects one element y_{ij}^{l} in $\mathbf{y}_b$ and then changes the value from 0 to 1 or from 1 to 0.

Neighborhood operator 2: Swap within the limited-stop service

This operator randomly selects one limited-stop service in $\mathbf{y}_b$ and two elements y_{ij}^{l} of the service. Then we swap the values of these two elements.

Neighborhood operator 3: Swap between two limited-stop services

If $NL>1$, this operator randomly selects two limited-stop services and one stop ($j=1,2,...,N-2$) in $\mathbf{y}_b$. Then we swap the values of these two elements y_{ij}^{l} associated with these limited-stop services and the stop.

4.1.2. The difference between the traditional and enhanced ABC algorithms

It is easy for the traditional ABC algorithm to fall into a local optimum as employed and onlooker bees can only search neighbor solutions. We enhance the traditional ABC algorithm by allowing employed bees to search non-neighbor solutions. For this purpose, an enhanced step is introduced between Steps 1 and 2 of the traditional ABC algorithm, which is shown as follows:

Enhanced step: Enhanced employed bee phase

For each employed bee $b = 0,1,...,N_\mathrm{e} - 1$

Step ①: Generate a non-neighbor solution $\mathbf{y}_b*$ based on $\mathbf{y}_b$.

Step ②: Evaluate the fitness $fit(\mathbf{y}_b*)$. If $fit(\mathbf{y}_b*) > fit(\mathbf{y}_b)$, replace $\mathbf{y}_b$ with $\mathbf{y}_b*$ and $u_b = 0$, else increase u_b by 1.

In Step ①, each employed bee generates a non-neighbor solution based on $\mathbf{y}_b$ by a non-neighborhood operator. The non-neighborhood operator is to 1) select solution $\mathbf{y}_{b'}$ by the fitness-based roulette wheel selection method, 2) select a limited-stop service index and two stop indices randomly, and 3) change all elements y_{ij}^l with the stop index between the two selected stop indices inclusively and with the selected limited-stop service index in $\mathbf{y}_b$ to be the same as those in $\mathbf{y}_{b'}$.

4.2. The Monte Carlo Simulation method for the second-stage model

We adopt the Monte Carlo Simulation method to solve the second-stage model, i.e., to calculate $E(g(\tilde{\mathbf{r}}, \tilde{\boldsymbol{\lambda}}, \mathbf{y}^l)), \text{for } l = 1, 2, ..., |L|$ and then determine the best limited-stop service $l*$ in L for the next VG. The steps of the method are as follows:

Step 0: Parameter setting

Set the maximum number of simulations $m_{\max}$.

Step 1: Initialization

Set the simulation counter $m = 1$ and $E(g(\tilde{\mathbf{r}}, \tilde{\boldsymbol{\lambda}}, \mathbf{y}^l))$=0.

Step 2: Sampling

Randomly generate a value of $\tilde{r}_{i,j}$ and a value of $\tilde{\lambda}_{i,j,e}$ from normal distributions with their means and variances defined by Equations (41), (42), (43), and (44).

Step 3: Calculation

Based on the values of $\tilde{r}_{i,j}$ and $\tilde{\lambda}_{i,j,e}$ generated in Step 2, calculate $g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l)$ by Equations (25)-(40). Its calculation process is similar to that of $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^l)$, which is detailed in Appendix B.

Step 4: Update

Update $E(g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l))$ by $E(g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l))=\dfrac{(m-1)E(g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l))+g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l)}{m}$.

Step 5: Stop test

If $m<m_{\max}$, set $m=m+1$ and return Step 2; Otherwise, stop and output $E(g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l))$.

After the above Monte Carlo Simulation method, we get the values of $E(g(\tilde{\mathbf{r}},\tilde{\boldsymbol{\lambda}},\mathbf{y}^l)),\text{for } l=1,2,...,|L|$. l^* can be determined by comparing these values as stated by Equation (23).

5. Numerical study

In this section, 1) the effectiveness and efficiency of our strategy, 2) the effect of variances of prediction errors, and 3) the effectiveness and efficiency of the enhanced ABC algorithm were examined. All solution methods were coded with C++ in Visual Studio 2019 and run on a computer with a 2.30 GHz CPU and 16.0 GB RAM.

In the following numerical studies, a real-world bus route (Route 63 in Harbin City, China) is adopted. It is a 34-stop bus corridor and around 17 km. The average running times (i.e., $\bar{\tilde{r}}_{i,j}$) between neighbor stops are presented in Table 1. On each day, the operation time of the bus system is from 6:00 to 24:00. Headway at the starting terminal is 5 minutes. Cap, b, a, τ_1, τ_2, and τ_3 are 150 passengers, 1 second, 2 seconds, 6 seconds, 7 seconds, and 7 seconds, respectively. As in the study of Chen et al. (2015), WV, IV, and OV are \$15/h, \$10/h, and \$150/h, respectively.

Table 1. The average running times (seconds) between neighbor stops

Stop	0	1	2	3	4	5	6	7	8	9
$\bar{\tilde{r}}_{i,j}$	0	54	43	45	68	111	97	70	60	53
Stop	10	11	12	13	14	15	16	17	18	19
$\bar{\tilde{r}}_{i,j}$	68	76	92	65	78	36	87	61	59	39
Stop	20	21	22	23	24	25	26	27	28	29

$\bar{\bar{r}}_{i,j}$	51	111	55	45	97	83	85	37	42	32
Stop	30	31	32	33						
$\bar{\bar{r}}_{i,j}$	69	52	23	47						

In the first stage, we adopted 1 day for data collection. The historical bus travel time (i.e., $r_{i,j}^{d,p}$) between any two neighbor stops was randomly generated by a normal distribution with a mean of $\bar{\bar{r}}_{i,j}$ and a variance of $(0.3\times\bar{\bar{r}}_{i,j})^2$ (i.e., $\sigma_{i,j}^{2,\tilde{\mathrm{r}}}=(0.3\times\bar{\bar{r}}_{i,j})^2$). Similarly, the historical passenger arrival rate (i.e., $\lambda_{i,j,e}^{d,p}$) from any upstream stop to any downstream destination was randomly generated by a normal distribution with a mean of 0.5 person/minute (i.e., $\bar{\bar{\lambda}}_{i,j,e}$= 0.5 person/minute) and a variance of $(0.3\times\bar{\bar{\lambda}}_{i,j,e})^2$ (i.e., $\sigma_{i,j,e}^{2,\tilde{\lambda}}=(0.3\times\bar{\bar{\lambda}}_{i,j,e})^2$). Destinations are comprised of stops 11, 22, and 33.

In the enhanced ABC algorithm, the values of N_{e}, N_{o}, $U_{\max}$, and $I_{\max}$ are $10\times NL$, $5\times NL$, $10\times NL$, and 400, respectively. The usage probabilities of neighborhood operators 1, 2, and 3 are 0.3, 0.3, and 0.4, respectively. NL is set to 4 unless otherwise specified.

In the second stage, we generate $r_{i,j}*$ by introducing an auxiliary parameter $\tilde{r}_{i,j}*$ (a value of $\tilde{r}_{i,j}$). The generation method is to 1) generate $\tilde{r}_{i,j}*$ by the distribution of $\tilde{r}_{i,j}$, 2) get the distribution of $r_{i,j}$ by equation $r_{i,j}=\tilde{r}_{i,j}*+\varepsilon_{i,j}^{\mathrm{r}}$, and 3) generate $r_{i,j}*$ by the distribution of $r_{i,j}$. This generation method was also adopted by Schinckel et al. (2007), Guo and Yang (2020), and Khalilisamani et al. (2021). The generation of $\lambda_{i,j,e}*$ is similar to that of $r_{i,j}*$. The variances of prediction errors $\sigma_{i,j}^{2,\mathrm{r}}$ and $\sigma_{i,j,e}^{2,\lambda}$ are set to 0 unless otherwise specified.

In the Monte Carlo Simulation method, $m_{\max}$ is set as 1000.

5.1. The effectiveness and computational efficiency of our strategy

In this sub-section, the effectiveness and computational efficiency of our strategy were tested by comparing it with the tactical planning strategy and the dynamic stop-skipping strategy. The tactical planning strategy means that an operator determines one limited-stop service at the tactical planning level and then all limited-stop vehicles provide the same limited-stop service at the operational level. The dynamic stop-skipping strategy means that an operator does not consider the tactical planning level. Each limited-stop vehicle can provide one limited-stop service out of all possible different services at the operational level and each limited-stop vehicle can provide a different limited-stop service from the other. This strategy still considers VGs. There are also 3 vehicles $(i = 0,1,2)$ in one VG and only vehicle 1 is a limited-stop vehicle. The difference between our strategy and the dynamic stop-skipping strategy is that vehicle 1 of our strategy can provide one limited-stop service in L (the subset of all possible different limited stop services) but vehicle 1 of the dynamic stop-skipping strategy can provide one limited-stop service out of all possible different limited-stop services.

The value of NL has an influence on the effectiveness and computational efficiency of our strategy. We adopted $NL = 1,2,3,4$ and got the corresponding number of limited-stop services and the sequence of stops from the first stage, as shown in Table 2.

Table 2. The stop sequences of the limited-stop services obtained from the first stage

NL	The stop sequences of the limited-stop services
1	0-1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-22-23-24-25-26-28-33
2	0-1-2-3-4-5-6-7-8-9-10-11-13-15-17-18-19-22-23-25-28-29-33
	0-1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-33
3	0-1-2-3-4-5-6-7-8-9-10-11-13-15-19-20-22-23-24-25-33
	0-1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-19-20-22-23-24-26-28-33
	0-1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31-32-33
4	0-1-2-3-4-5-6-7-8-9-10-11-14-15-16-19-22-23-26-33 (service 1)
	0-1-2-3-4-5-6-7-8-9-10-11-12-16-17-18-19-20-22-23-24-26-27-28-33 (service 2)
	0-1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-28-33 (service 3)
	0-1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31-32-33 (service 4)

Table 3 shows the relationship between NL and the system cost saving (i.e., reduction) and running times at the operational level for one day. The system cost saving is the difference between the total operator and passenger costs in the situations with and without limited-stop services. In this table,

(1) $NL = 1$ implies that each limited-stop vehicle at the operational level provides the same limited-stop service, which is the same as the situation of the tactical planning strategy.

(2) $NL = 2,\ 3,\ \text{or}\ 4$ is the situation of our strategy.

(3) $NL = \max$ represents the situation of the dynamic stop-skipping strategy. The first stage is not considered and any limited-stop vehicle can provide any limited-stop service at the operational level. In our numerical study, the number of all possible different limited-stop services equals $2^{32} = 4294967296$ (there are 32 intermediate stops in the corridor).

At the operational level, we exhausted all possible different limited-stop services and then selected the best one.

(4) The reduction is x when NL is y means that if the number of limited-stop services obtained from the first stage is y, we can save \$ x per day at the operational level. The benchmark is the situation without limited-stop services (i.e., $NL=0$).

Table 3. The system cost savings (\$) and running times (seconds) under different values of NL

NL	1 (tactical planning strategy)	2	3	4	Max (dynamic stop-skipping strategy)
Reduction	1177	2987	3151	3195	3586 (optimal)
Reduction percentage*	32.8%	83.3%	87.9%	89.15%	100.0%
Running time	0	8.3×10^{-3}	1.1×10^{-2}	1.3×10^{-2}	2.8×10^{6}

Reduction percentage*: reduction/optimal reduction $\times$ 100%.

The effectiveness of our strategy

From Table 3, we can see that our strategy can increase reduction significantly, compared with the tactical planning strategy (i.e., $NL=1$). The tactical planning strategy can only give 32.8% of the optimal reduction, which is relatively small. On the contrary, our strategy can lead to more than 80% of the optimal reduction. Moreover, we observe that a large value of NL leads to higher effectiveness. We also find our strategy has lower effectiveness, in terms of a reduction percentage, compared with the dynamic stop-skipping strategy. However, it is acceptable because the reduction percentages are already more than 85% when we provide three/four limited-stop services.

The computational efficiency of our strategy

The computational efficiency of the tactical planning strategy, our strategy, and the dynamic stop-skipping strategy were also tested. We compared their running times at the operational level under different numbers of stops in a corridor, and the result is presented in Table 3. In Table 3, all running times of the tactical planning strategy are 0 s. This is because an operator does not need to determine which limited-stop service is the best at the operational level since there is only one limited-stop service. With our strategy, it can be observed that running time increases with the value of NL. However, all running times are very small. The maximum running time is only 1.3×10^{-2} s, which implies the high computational efficiency of our strategy. However, with the dynamic stop-skipping strategy, the minimum running time is 2.8×10^{6} s, which is obviously unacceptable in real-time operations.

In conclusion, 1) the tactical planning strategy has the highest computational efficiency but its effectiveness can be low; 2) the dynamic stop-skipping strategy has the highest effectiveness but its computational efficiency is unacceptable when the corridor is long; 3) our strategy has both high effectiveness and high efficiency, which provides a better trade-off between effectiveness and computational efficiency than the above two strategies; 4) a larger value of NL leads to higher effectiveness and lower efficiency. If an operator prefers effectiveness, a larger value of NL, e.g., 4, is recommended; if an operator prefers efficiency, a smaller value of NL, e.g., 2, is recommended.

5.2. The effect of variances of prediction errors

In this sub-section, the performance of our strategy under different variances of prediction errors is studied. At the operational level, the number of limited-stop vehicles used to provide each of the four limited-stop services in one day under different variances of prediction errors are presented in Table 4. (For services 1 to 4 in Table 4, please refer to Table 2.) The result illustrates that the number of limited-stop vehicles for each service varies with the variances of prediction errors. To be more specific, for the same VG, an operator may offer very different limited-stop services in L under different variances of prediction errors.

Since the variances of prediction errors change the number of limited-stop vehicles in each service, it is obvious that the system cost saving at the operational level for one day is also affected. The saving under different variances of prediction errors is shown in Table 5. From the table, we can find that the saving (i.e., reduction) decreases as the variances of prediction errors increase. When $\sigma_{i,j}^{2,\mathrm{r}} = 0.100\cdot\bar{\bar{r}}_{i,j}$ minute^2 and $\sigma_{i,j,e}^{2,\lambda} = 0.100\cdot\bar{\bar{\lambda}}_{i,j,e}$ (person/minute)^2, the saving is only \$2112, which is much less than the one with $\sigma_{i,j}^{2,\mathrm{r}} = 0.000\cdot\bar{\bar{r}}_{i,j}$ minute^2 and $\sigma_{i,j,e}^{2,\lambda} = 0.000\cdot\bar{\bar{\lambda}}_{i,j,e}$ (person/minute)^2 (i.e., \$3195). From this result, we can conclude that the variances of prediction errors cannot be ignored, especially in situations with large variances of prediction errors.

Table 4. The numbers of limited-stop vehicles under different variances of prediction errors

Variances of prediction errors	$\sigma_{i,j}^{2,\mathrm{r}} = 0.000\cdot\bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.000\cdot\bar{\bar{\lambda}}_{i,j,e}$	$\sigma_{i,j}^{2,\mathrm{r}} = 0.001\cdot\bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.001\cdot\bar{\bar{\lambda}}_{i,j,e}$	$\sigma_{i,j}^{2,\mathrm{r}} = 0.010\cdot\bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.010\cdot\bar{\bar{\lambda}}_{i,j,e}$	$\sigma_{i,j}^{2,\mathrm{r}} = 0.100\cdot\bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.100\cdot\bar{\bar{\lambda}}_{i,j,e}$

Service 1	1	1	1	0
Service 2	16	18	15	12
Service 3	36	35	38	32
Service 4	55	54	54	64

Table 5. The system cost savings ($) under different variances of prediction errors

Variances of prediction errors	$\sigma_{i,j}^{2,r} = 0.000 \cdot \bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.000 \cdot \bar{\bar{\lambda}}_{i,j,e}$	$\sigma_{i,j}^{2,r} = 0.001 \cdot \bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.001 \cdot \bar{\bar{\lambda}}_{i,j,e}$	$\sigma_{i,j}^{2,r} = 0.010 \cdot \bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.010 \cdot \bar{\bar{\lambda}}_{i,j,e}$	$\sigma_{i,j}^{2,r} = 0.100 \cdot \bar{\bar{r}}_{i,j}$ $\sigma_{i,j,e}^{2,\lambda} = 0.100 \cdot \bar{\bar{\lambda}}_{i,j,e}$
Reduction	3195	3118	2746	2112

5.3. The accuracy and computational efficiency of the enhanced ABC algorithm

To test the accuracy and computational efficiency of the enhanced ABC algorithm, we carried out an enumeration to search for the global optimum when $NL = 1$. As limited-stop services must serve the first and last stop, there are 32 intermediate stops. Therefore, the number of possible limited-stop services is $2^{32} = 4294967296$. All possible different limited-stop services were evaluated and the optimal limited-stop service is 1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-23-24-25-26-27-29-34, which is the same as the result obtained from the enhanced ABC algorithm in Table 2. However, the enumeration took 2.3×10^6 s, whereas the enhanced ABC algorithm only took 11.0 s (in 400 iterations). The results illustrate that the enhanced ABC algorithm is accurate and efficient.

We also studied the accuracy and computational efficiency of the enhanced ABC algorithm when $NL = 4$, compared with a genetic algorithm (GA) and the traditional ABC algorithm. Their reductions over running time are shown in Figure 5. It is easy to see that 1) the enhanced ABC algorithm provides a higher-quality solution after convergence than GA and the traditional ABC algorithm, which shows higher effectiveness; 2) the enhanced ABC algorithm takes less running time to get the same solution quality (with more than a $2000 reduction) than GA and the traditional ABC algorithm, which shows higher computational efficiency.

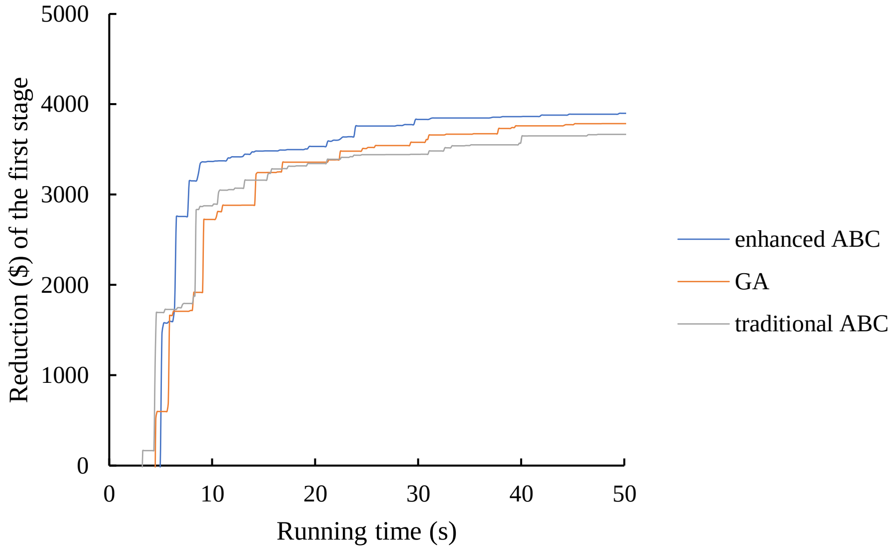

Figure 5. The convergence of the enhanced ABC algorithm, GA, and the traditional ABC algorithm

6. Conclusion

The paper proposes a new two-stage strategy to address the LSBSDP. The two-stage strategy is more general than the tactical planning strategy and the dynamic stop-skipping strategy. Numerical studies show that our strategy has both high effectiveness and high efficiency, which provides a better trade-off between effectiveness and computational efficiency than the tactical planning strategy and the dynamic stop-skipping strategy.

Prediction errors are considered in this study. In the numerical examples, different variances of prediction errors can lead to very different limited-stop service schemes at the operational level. The system cost saving of providing limited-stop services is, therefore, affected. In particular, the saving decreases as the variances of prediction errors increase. More importantly, the effect of prediction errors cannot be neglected, especially in situations with large variances of prediction errors.

An enhanced ABC algorithm is developed to solve the first-stage model. Its high effectiveness and computational efficiency are verified by comparing it with an enumeration, GA, and the traditional ABC algorithm.

This study opens at least two future research directions. First, the distribution of prediction errors may not follow normal distributions in practice. How to extend the current methodology to tackle other distributions is one important direction. Second, in some situations, the headway between all-stop and limited-stop vehicles can be larger than 15 min. This means the arrival times of passengers may be affected by the stop-skipping strategy. How to model this situation is another interesting direction.

Appendix A: The derivation process of Equations (41), (42), (43), and (44)

To derive Equations (41) and (42), we need to calculate $\tilde{r}_{i,j}$ when $r_{i,j}=r_{i,j}*$. In this appendix, we determine the distribution of $\tilde{r}_{i,j}$ when $r_{i,j}=r_{i,j}*$ by obtaining the probability density function of $\tilde{r}_{i,j}$ when $r_{i,j}=r_{i,j}*$, i.e., $f_{\tilde{r}_{i,j}|r_{i,j}*}(\cdot)$.

Let $\tilde{r}_{i,j}*$ be a variable representing a possible value of $\tilde{r}_{i,j}$. Then by definition,

$$\begin{aligned} f_{\tilde{r}_{i,j}|r_{i,j}*}(\tilde{r}_{i,j}*) &= \frac{f_{\tilde{r}_{i,j}r_{i,j}}(\tilde{r}_{i,j}*,r_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)} \\ &= \frac{f_{\tilde{r}_{i,j}}(\tilde{r}_{i,j}*)\times f_{r_{i,j}|\tilde{r}_{i,j}*}(r_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)}, \end{aligned} \tag{45}$$

where

$f_{\tilde{r}_{i,j}r_{i,j}}(\cdot,\cdot)$	The joint probability density function of $\tilde{r}_{i,j}$ and $r_{i,j}$.
$f_{r_{i,j}}(\cdot)$	The probability density function of $r_{i,j}$.
$f_{\tilde{r}_{i,j}}(\cdot)$	The probability density function of $\tilde{r}_{i,j}$.
$f_{r_{i,j}|\tilde{r}_{i,j}*}(\cdot)$	The probability density function of $r_{i,j}$ when $\tilde{r}_{i,j}=\tilde{r}_{i,j}*$.

Since $\tilde{r}_{i,j}$ follows $N(\bar{\tilde{r}}_{i,j},\sigma^{2,\tilde{r}}_{i,j})$, the corresponding distribution of $f_{\tilde{r}_{i,j}}(\cdot)$ is $N(\bar{\tilde{r}}_{i,j},\sigma^{2,\tilde{r}}_{i,j})$. Since the prediction error of $\tilde{r}_{i,j}$ follows $N(0,\sigma^{2,\mathrm{r}}_{i,j})$, the corresponding distribution of $f_{r_{i,j}|\tilde{r}_{i,j}*}(\cdot)$ is $N(\tilde{r}_{i,j}*,\sigma^{2,\mathrm{r}}_{i,j})$. For simplicity, we denote $f_{\tilde{r}_{i,j}}(\cdot)$ and $f_{r_{i,j}|\tilde{r}_{i,j}*}(\cdot)$ as $n_1(\cdot)$ and $n_2(\cdot)$, respectively. Then Equation (45) becomes

$$\begin{aligned} f_{\tilde{r}_{i,j}|r_{i,j}*}(\tilde{r}_{i,j}*) &= \frac{f_{\tilde{r}_{i,j}}(\tilde{r}_{i,j}*)\times f_{r_{i,j}|\tilde{r}_{i,j}*}(r_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)} \\ &= \frac{n_1(\tilde{r}_{i,j}*)\times n_2(r_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)}. \end{aligned} \tag{46}$$

Since the corresponding distribution of $n_2(\cdot)$ is $N(\tilde{r}_{i,j}*,\sigma^{2,\mathrm{r}}_{i,j})$, we have

$$n_2(r_{i,j}*)=\frac{1}{\sqrt{2\pi\sigma^{2,\mathrm{r}}_{i,j}}}e^{-\frac{(r_{i,j}*-\tilde{r}_{i,j}*)^2}{2\sigma^{2,\mathrm{r}}_{i,j}}}. \tag{47}$$

Let us construct an auxiliary distribution, $N(r_{i,j}*, \sigma_{i,j}^{2,\mathrm{r}})$, and the corresponding probability density function is denoted as $n_3(\cdot)$. Then we have

$$n_3(\tilde{r}_{i,j}*) = \frac{1}{\sqrt{2\pi\sigma_{i,j}^{2,\mathrm{r}}}} e^{-\frac{(\tilde{r}_{i,j}*-r_{i,j}*)^2}{2\sigma_{i,j}^{2,\mathrm{r}}}}. \quad (48)$$

From Equations (47) and (48), it is easy to see that $n_2(r_{i,j}*) = n_3(\tilde{r}_{i,j}*)$. We substitute this equation into Equation (46) and then obtain

$$f_{\tilde{r}_{i,j}|r_{i,j}*}(\tilde{r}_{i,j}*) = \frac{n_1(\tilde{r}_{i,j}*) \times n_3(\tilde{r}_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)}. \quad (49)$$

It is easy to see $\int_{-\infty}^{+\infty} f_{\tilde{r}_{i,j}|r_{i,j}*}(\tilde{r}_{i,j}*) d\tilde{r}_{i,j}* = 1$. Then we have

$$\int_{-\infty}^{+\infty} \frac{n_1(\tilde{r}_{i,j}*) \times n_3(\tilde{r}_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)} d\tilde{r}_{i,j}* = 1. \quad (50)$$

$n_1(\tilde{r}_{i,j}*) \times n_3(\tilde{r}_{i,j}*)$ in the left side of the equation can be rewritten as (to be clear, we use x, u_1, σ_1^2, u_2, and σ_2^2 to denote $\tilde{r}_{i,j}*$, $\bar{\tilde{r}}_{i,j}$, $\sigma_{i,j}^{2,\tilde{\mathrm{r}}}$, $r_{i,j}*$, and $\sigma_{i,j}^{2,\mathrm{r}}$)

$$
\begin{aligned}
n_1(\tilde{r}_{i,j}*)\times n_3(\tilde{r}_{i,j}*) &= \frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-u_1)^2}{2\sigma_1^2}}\times\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x-u_2)^2}{2\sigma_2^2}} \\
&= \frac{1}{2\pi\sigma_1\sigma_2}e^{-(\frac{(x-u_1)^2}{2\sigma_1^2}+\frac{(x-u_2)^2}{2\sigma_2^2})} \\
&= \frac{1}{2\pi\sigma_1\sigma_2}e^{-\frac{(x-u_1)^2\sigma_2^2+(x-u_2)^2\sigma_1^2}{2\sigma_1^2\sigma_2^2}} \\
&= \frac{1}{2\pi\sigma_1\sigma_2}e^{-\frac{x^2-2\frac{u_1\sigma_2^2+u_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}x+\frac{u_1{}^2\sigma_2^2+u_2{}^2\sigma_1^2}{\sigma_1^2+\sigma_2^2}}{2\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}}} \\
&= \frac{1}{2\pi\sigma_1\sigma_2}e^{-\frac{(x-\frac{u_1\sigma_2^2+u_2\sigma_1^2}{\sigma_1^2+\sigma_2^2})^2}{2\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}}-\frac{(u_1-u_2)^2}{2(\sigma_1^2+\sigma_2^2)}} \\
&= \frac{e^{-\frac{(u_1-u_2)^2}{2(\sigma_1^2+\sigma_2^2)}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\frac{1}{\sqrt{2\pi}\frac{\sigma_1\sigma_2}{\sqrt{\sigma_1^2+\sigma_2^2}}}e^{-\frac{(x-\frac{u_1\sigma_2^2+u_2\sigma_1^2}{\sigma_1^2+\sigma_2^2})^2}{2\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}}} \\
&= A\frac{1}{\sqrt{2\pi}\sigma_0}e^{-\frac{(x-u_0)^2}{2\sigma_0^2}},
\end{aligned}
\tag{51}
$$

where $A=\frac{e^{-\frac{(u_1-u_2)^2}{2(\sigma_1^2+\sigma_2^2)}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}$, $u_0=\frac{u_1\sigma_2^2+u_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}$, and $\sigma_0^2=\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}$.

From Equations (50) and (51), we can get $\frac{A}{f_{r_{i,j}}(r_{i,j}*)}=1$ by

$$
\begin{aligned}
&\int_{-\infty}^{+\infty}\frac{n_1(\tilde{r}_{i,j}*)\times n_3(\tilde{r}_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)}d\tilde{r}_{i,j}*=1 \\
&\Rightarrow \frac{A}{f_{r_{i,j}}(r_{i,j}*)}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_0}e^{-\frac{(x-u_0)^2}{2\sigma_0^2}}dx=1 \\
&\Rightarrow \frac{A}{f_{r_{i,j}}(r_{i,j}*)}\times 1=1 \\
&\Rightarrow \frac{A}{f_{r_{i,j}}(r_{i,j}*)}=1.
\end{aligned}
\tag{52}
$$

Based on Equations (49) and (51) and $\frac{A}{f_{r_{i,j}}(r_{i,j}*)}=1$, we have

$$f_{\tilde{r}_{i,j}|r_{i,j}*}(\tilde{r}_{i,j}*)=\frac{n_1(\tilde{r}_{i,j}*)\times n_3(\tilde{r}_{i,j}*)}{f_{r_{i,j}}(r_{i,j}*)}=\frac{A\frac{1}{\sqrt{2\pi}\sigma_0}e^{-\frac{(x-u_0)^2}{2\sigma_0^2}}}{f_{r_{i,j}}(r_{i,j}*)}=\frac{1}{\sqrt{2\pi}\sigma_0}e^{-\frac{(x-u_0)^2}{2\sigma_0^2}}. \qquad (53)$$

Finally, according to Equation (53), we can conclude that the corresponding distribution of $f_{\tilde{r}_{i,j}|r_{i,j}*}(\cdot)$ is $N(u_0,\sigma_0^2)$. As a result, the following equations (i.e., Equations (41) and (42)) hold:

$$E(\tilde{r}_{i,j}\mid r_{i,j}=r_{i,j}*)=u_0=\frac{u_1\sigma_2^2+u_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}=\frac{\bar{\tilde{r}}_{i,j}\sigma_{i,j}^{2,\mathrm{r}}+r_{i,j}*\sigma_{i,j}^{2,\tilde{\mathrm{r}}}}{\sigma_{i,j}^{2,\tilde{\mathrm{r}}}+\sigma_{i,j}^{2,\mathrm{r}}} \quad \text{and}$$

$$Var(\tilde{r}_{i,j}\mid r_{i,j}=r_{i,j}*)=\sigma_0^2=\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}=\frac{\sigma_{i,j}^{2,\tilde{\mathrm{r}}}\sigma_{i,j}^{2,\mathrm{r}}}{\sigma_{i,j}^{2,\tilde{\mathrm{r}}}+\sigma_{i,j}^{2,\mathrm{r}}}.$$

Similarly, the following equations (i.e., Equations (43) and (44)) also hold:

$$E(\tilde{\lambda}_{i,j,e}\mid \lambda_{i,j,e}=\lambda_{i,j,e}*)=\frac{\bar{\tilde{\lambda}}_{i,j,e}\sigma_{i,j,e}^{2,\lambda}+\lambda_{i,j,e}*\sigma_{i,j,e}^{2,\tilde{\lambda}}}{\sigma_{i,j,e}^{2,\tilde{\lambda}}+\sigma_{i,j,e}^{2,\lambda}} \quad \text{and}$$

$$Var(\tilde{\lambda}_{i,j,e}\mid \lambda_{i,j,e}=\lambda_{i,j,e}*)=\frac{\sigma_{i,j,e}^{2,\tilde{\lambda}}\sigma_{i,j,e}^{2,\lambda}}{\sigma_{i,j,e}^{2,\tilde{\lambda}}+\sigma_{i,j,e}^{2,\lambda}}.$$

Appendix B: The calculation process of $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^l)$

In this appendix, we present how to calculate $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^l)$ by Equations (7)-(22) if $y_{i,j}^l$ is given. The detailed calculation process is shown as follows:

1. Set $i=1$. Obtain $A_{i-1,j,l}^{d,p}, \text{for } j=0,1,...N-1$ and $FBP_{i-1,j,e,l}^{d,p}, \text{for } j=0,1,...N-1; e\in E$ by Equations (11) and (15).

   While $i\le 2$, do

   Set $j=0$.

   While $j\le N-2$, do

   1) As $A_{i-1,j,l}^{d,p}$ and $A_{i,j,l}^{d,p}$ is known, get $H_{i,j,l}^{d,p}$ by Equation (14).

   2) Since $SBP_{i,k,e,l}^{d,p}, \forall k<j$ is known, calculate $IP_{i,j,l}^{d,p}$ and $AP_{i,j,l}^{d,p}$ by Equations (16) and (17), respectively.

3) Obtain $W_{i,j,e,l}^{d,p}$ by Equation (18) as $FBP_{i-1,j,e,l}^{d,p}$ and $H_{i,j,l}^{d,p}$ are known.

4) With the knowledge of $W_{i,j,e,l}^{d,p}$, $IP_{i,j,l}^{d,p}$, and $AP_{i,j,l}^{d,p}$, get $TSBP_{i,j,l}^{d,p}$ by Equation (19).

5) With the knowledge of $W_{i,j,e,l}^{d,p}$ and $TSBP_{i,j,l}^{d,p}$, obtain $SBP_{i,j,e,l}^{d,p}$ by Equation (20).

6) Since $W_{i,j,e,l}^{d,p}$ and $SBP_{i,j,e,l}^{d,p}$ are known, calculate $FBP_{i,j,e,l}^{d,p}$ by Equation (21).

7) Calculate $S_{i,j,l}^{d,p}$ by Equation (22) as $AP_{i,j,l}^{d,p}$ and $TSBP_{i,j,l}^{d,p}$ are known.

8) As $S_{i,j,l}^{d,p}$ is known, compute $A_{i,j+1,l}^{d,p}$ by Equations (12) and (13).

9) $j = j+1$.

endwhile

$i = i+1$.

endwhile

2. As the values of all variables on the right-hand side of Equations (8)-(10) are known, calculate $Z_{1,l}^{d,p}$, $Z_{2,l}^{d,p}$, and $Z_{3,l}^{d,p}$. Obtain $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$ by adding up $Z_{1,l}^{d,p}$, $Z_{2,l}^{d,p}$, and $Z_{3,l}^{d,p}$ using Equation (7).
3. Check whether Inequality (6) is satisfied. If it is satisfied, stop and output $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$; Otherwise, set $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$ equals a sufficiently large positive value, then stop and output $h(\mathbf{r}^{d,p},\boldsymbol{\lambda}^{d,p},\mathbf{y}^{l})$.

Acknowledgments

This research was jointly supported by a grant from the National Natural Science Foundation of China (No. 71771194) and the Research Grants Council of the Hong Kong Special Administrative Region of China (17203223). The authors are grateful to the two reviewers for their constructive comments.

References

Albarracin, A.F.G., Jaramillo-Ramirez, D., 2019. Limited-stop high-frequency service design reducing in-vehicle congestion. Journal of Advanced Transportation 2019, 5745870.

Chen, J., Liu, Z., Zhu, S., Wang, W., 2015. Design of limited-stop bus service with capacity constraint and stochastic travel time. Transportation Research Part E-Logistics and Transportation Review 83, 1-15.

Chien, S.I.J., Ding, Y., Wei, C., 2002. Dynamic bus arrival time prediction with artificial neural networks. Journal of Transportation Engineering 128(5), 429-438.

Chiraphadhanakul, V., Barnhart, C., 2013. Incremental bus service design: combining limited-stop and local bus services. Public Transport 5(1), 53-78.

El-Geneidy, A.M., Surprenant-Legault, J., 2010. Limited-stop bus service: an evaluation of an implementation strategy. Public Transport 2(4), 291-306.

Ercolano, J.M., 1984. Limited-stop bus operations: an evaluation. Transportation Research Record 994, 24-29.

Fu, L., Liu, Q., Calamai, P., 2003. Real-time optimization model for dynamic scheduling of transit operations. Transportation Research Record 1857(1), 48-55.

Gkiotsalitis, K., 2019. Robust stop-skipping at the tactical planning stage with evolutionary optimization. Transportation Research Record 2673(3), 611-623.

Gkiotsalitis, K., 2021. Stop-skipping in rolling horizons. Transportmetrica A-Transport Science 17(4), 492-520.

Guo, J., Yang, C., 2020. Impact of prediction errors on high throughput predictive resource allocation. IEEE Transactions on Vehicular Technology 69(9), 9984-9999.

Huang, D., Xing, J., Liu, Z., An, Q., 2021. A multi-stage stochastic optimization approach to the stop-skipping and bus lane reservation schemes. Transportmetrica A-Transport Science 17(4), 1272-1304.

Jiang, X., Ma, J., 2021. Mixed scheduling model for limited-stop and normal bus service with fleet size constraint. Information 12(10), 400.

Karaboga, D., 2005. An idea based on honey bee swarm for numerical optimization. Technical Report Tr06. Erciyes University.

Khalilisamani, N., Thomson, P., Raadsma, H., Khatkar, M., 2021. Impact of genotypic errors with equal and unequal family contribution on accuracy of genomic prediction in aquaculture using simulation. Scientific Reports 11(1), 18318.

Larrain, H., Munoz, J.C., Giesen, R., 2015. Generation and design heuristics for zonal express services. Transportation Research Part E-Logistics and Transportation Review 79, 201-212.

Leiva, C., Munoz, J.C., Giesen, R., Larrain, H., 2010. Design of limited-stop services for an urban bus corridor with capacity constraints. Transportation Research Part B-Methodological 44(10), 1186-1201.

Liang, M., Zhang, H., Ma, R., Wang, W., Dong, C., 2021. Cooperatively coevolutionary optimization design of limited-stop services and operating frequencies for transit networks. Transportation Research Part C-Emerging Technologies 125, 103038.

Liu, Z., Yan, Y., Qu, X., Zhang, Y., 2013. Bus stop-skipping scheme with random travel time. Transportation Research Part C-Emerging Technologies 35, 46-56.

Mou, Z., Zhang, H., Liang, S., 2020. Reliability optimization model of stop-skipping bus operation with capacity constraints. Journal of Advanced Transportation 2020, 4317402.

Nesheli, M.M., Srikukenthiran, S., Shalaby, A., 2022. An optimization model for planning limited-stop transit operations. Public Transport, https://doi.org/10.1007/s12469-022-00293-5.

Sadrani, M., Jafarian-Moghaddam, A.R., Esfahani, M.A., Rahimi, A.M., 2022. Designing limited-stop bus services for minimizing operator and user costs under crowding conditions. Public Transport, https://doi.org/10.1007/s12469-022-00307-2.

Schinckel, A., Einstein, M., Foster, K., Craig, B., 2007. Evaluation of the impact of errors in the measurement of backfat depth on the prediction of fat-free lean mass. Journal of Animal Science 85(8), 2031-2042.

Sheu, J.B., 2005. A fuzzy clustering approach to real-time demand-responsive bus dispatching control. Fuzzy Sets and Systems 150(3), 437-455.

Silverman, N., 1998. Limited-stop bus service at New York city transit. Journal of Transportation Engineering 124(6), 503-509.

Soto, G., Larrain, H., Munoz, J.C., 2017. A new solution framework for the limited-stop bus service design problem. Transportation Research Part B-Methodological 105, 67-85.

Tang, C., Ceder, A., Zhao, S., Ge, Y., 2016. Determining optimal strategies for single-line bus operation by means of smartphone demand data. Transportation Research Record 2539(1), 130-139.

Tang, C., Ceder, A.A., Ge, Y., 2017. Integrated optimization of bus line fare and operational strategies using elastic demand. Journal of Advanced Transportation 2017, 7058789.

Tang, C., Shi, H., Liu, T., 2023. Optimization of single-line electric bus scheduling with skip-stop operation. Transportation Research Part D-Transport and Environment 117, 103652.

Tetreault, P.R., El-Geneidy, A.M., 2010. Estimating bus run times for new limited-stop service using archived AVL and APC data. Transportation Research Part A-Policy and Practice 44(6), 390-402.

Torabi, M., Salari, M., 2019. Limited-stop bus service: a strategy to reduce the unused capacity of a transit network. Swarm and Evolutionary Computation 44, 972-986.

Ulusoy, Y.Y., Chien, S.I.J., 2015. Optimal bus service patterns and frequencies considering transfer demand elasticity with genetic algorithm. Transportation Planning and Technology 38(4), 409-424.

Ulusoy, Y.Y., Chien, S.I.J., Wei, C.H., 2010. Optimal all-stop, short-turn, and express transit services under heterogeneous demand. Transportation Research Record 2197(1), 8-18.

Wang, D.Z.W., Nayan, A., Szeto, W.Y., 2018. Optimal bus service design with limited stop services in a travel corridor. Transportation Research Part E-Logistics and Transportation Review 111, 70-86.

Wirasinghe, S.C., Vandebona, U., 2011. Route layout analysis for express buses. Transportation Research Part C-Emerging Technologies 19(2), 374-385.

Wu, W., Liu, R., Jin, W., Ma, C., 2019. Simulation-based robust optimization of limited-stop bus service with vehicle overtaking and dynamics: A response surface methodology. Transportation Research Part E-Logistics and Transportation Review 130, 61-81.

Yi, Y., Choi, K., Lee, Y.J., 2016. Optimal limited-stop bus routes selection using a genetic algorithm and smart card data. Journal of Public Transportation 19(4), 178-198.

Yu, B., Wang, K., Peng, Z., Wang, C., Gao, Z., Yao, B., 2015. Dynamic extra buses scheduling strategy in public transport. Promet-Traffic and Transportation 27(3), 205-216.

Zhang, H., Liang, S., Zhao, S., Shang, Q., 2021. Advantages of bus stop skipping and holding control in reducing schedule deviation. Proceedings of the Institution of Civil Engineers-Municipal Engineer 174(1), 14-23.

Zhang, H., Zhao, S., Cao, Y., Liu, H., Liang, S., 2017a. Real-time integrated limited-stop and short-turning bus control with stochastic travel time. Journal of Advanced Transportation 2017, 2960728.

Zhang, H., Zhao, S., Liu, H., Li, J., 2017b. A dynamic short-turning bus control for uncertain demand. Journal of Advanced Transportation 2017, 7392962.

Zhao, H., Feng, S., Ci, Y., 2021. Scheduling a bus fleet for evacuation planning using stop-skipping method. Transportation Research Record 2675(11), 865-876.