Learning and Individual Differences 119 (2025) 102645

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Distinct roles of cognitive and mathematics skills in different levels of mathematics development☆

Charles Chiu Hung Yip [a], Xiangzi Ouyang [b], Eason Sai-Kit Yip [a], Christine Kong-Yan Tong [a], Terry Tin-Yau Wong [a,*]

[a] Department of Psychology, The University of Hong Kong, Hong Kong
[b] Department of Psychology, Lingnan University, Hong Kong

ARTICLE INFO

Keywords:
Mathematics learning disability
Mathematics achievement
Latent profile analysis
ANOVA
Logistic regression

ABSTRACT

Mathematics development draws on various cognitive and mathematics skills, which may have distinct influence on students at different achievement levels. The current study explored how students' skill profiles contribute to their mathematics achievement levels. Two-hundred-and-seventy-two fourth graders completed assessments on various cognitive and mathematics abilities. Latent profile analysis identified four math achievement classes, namely, mathematics learning disability (MLD), average achievers, high achievers, and mathematical giftedness. Multinomial logistic regression further revealed that, compared to average achievers, students struggling with fraction magnitude understanding and number sentence construction in word problems are more likely to have MLD, students with better spatial skills and fraction magnitude understanding are more likely to be high achievers, and students with better arithmetic principle understanding are more likely to be mathematically gifted. The current findings illustrate the unique cognitive characteristics of students at different achievement levels, which allow practitioners to make level-specific adjustments to their teaching.

Education relevance statement: The current study identified four mathematics achievement classes and examined the skills that contributed to the cognitive profile of these ability groups. Our results revealed the critical skills that differentiated between these achievement groups. Notably, number sentence construction and fraction number line differentiated students with mathematics learning difficulties from average performers. Understanding of abstract arithmetic principles was also found to be the distinctive skill for the highest achievers. The findings informed assessment and subsequent intervention for learners at different mathematics achievement levels. Further research and educational practices (remediation, curriculum differentiation, acceleration) could be developed to tailor their unique learning needs.

1. Introduction

Mathematics is a compulsory subject in primary and secondary schools around the world. Mathematics achievement during school years has a significant impact on various life outcomes, such as academic attainment, psychological well-being, and occupational status (Geary, 2011; Parsons & Bynner, 2005; Ritchie & Bates, 2013). Considering the profound influence of mathematics achievement, it is crucial to identify specific strategies to support the mathematics learning of students with different needs. At the low and high ends of the mathematics performance spectrum, both students with mathematics learning disability (MLD) and mathematically gifted (MG) students deserve attention and support to thrive and fulfill their potential in mathematics. In order to facilitate appropriate educational intervention, it is important to identify potential students with MLD or MG. Prior literature typically examined children's mathematical development and its predictors by treating them as a homogeneous group while MLD- and MG- related research remains limited (Caviola et al., 2022; Lewis & Fisher, 2016). This limits potential efforts in addressing the unique needs of these students through relevant intervention strategies.

☆ This article is part of a special issue entitled: Math difficulties/talent published in learning and individual differences.

* Corresponding author.
E-mail address: terrytyw@hku.hk (T.T.-Y. Wong).

https://doi.org/10.1016/j.lindif.2025.102645
Received 1 December 2023; Received in revised form 8 October 2024; Accepted 4 February 2025
Available online 10 February 2025

1.1. Mathematics learning disability (MLD) and mathematical giftedness (MG)

Children with MLD comprise approximately 5–8 % of the school-aged population (Lewis & Fisher, 2016; Morsanyi et al., 2018). Students with MLD often struggle with mathematics learning and understanding, which subsequently leads to poorer academic motivation, emotional well-being, and even socioeconomic status in adulthood (Geary, 2011; Ritchie & Bates, 2013). Given the consequences of mathematics difficulties, it is important to identify students with MLD and provide suitable assistance in their mathematics learning. Common ways to identify MLD cases involve comparing students' mathematics performance against their peers using standardized achievement tests. These tests utilize certain cutoff thresholds (e.g., 10th percentile, 1.5 *SD* below average) to define MLD status (Lewis & Fisher, 2016). Recent research has highlighted the utility of data-driven approaches (e.g., latent analyses, cognitive diagnostic models) to classify and categorize students' mathematics achievement levels (Ouyang et al., 2023; Zhang et al., 2020). Understanding the specific deficits in cognitive and mathematics skills exhibited by students with MLD is especially important, since such knowledge enables educators to provide specific instructional accommodations that help students with MLD overcome challenges related to particular mathematics skills, which may be especially pertinent to the etiology of MLD (Karagiannakis & Cooreman, 2014).

On the other hand, it is equally important to identify MG students and provide support for their mathematics learning and development. Recognizing MG students is especially important in the STEM age, as these students possess the potential to contribute to and drive meaningful advancements in relevant fields, which could foster scientific progress and societal development (Ficici & Siegle, 2008; Myers et al., 2017). Nevertheless, there is much less research focusing on their needs compared to their MLD peers (Caviola et al., 2022; Myers et al., 2017). In the standardized education landscape, MG students are often presented with comparable educational opportunities as their average achieving peers, without any adjustment to address their exceptional talents and needs (Maggio & Sayler, 2013; Özdemir & Bostan, 2021). This may often be attributed to teachers' lack of knowledge and skills needed to identify and support potential MG students, leaving MG students undetected and their needs unsatisfied (Chamberlin & Chamberlin, 2010; Gadanidis et al., 2011). Under such circumstances, MG students may experience boredom due to a lack of challenge and demotivation (Özdemir & Bostan, 2021), which could further lead to students dropping out and losing their gifted potentials (Landis & Reschly, 2013; Renzulli & Park, 2000). Therefore, it is important to identify the specific mathematics skills that are essential for students with potential to flourish, enabling the development of specific support strategies for the growth of these students.

1.2. Theoretical motivation

To identify skills that may support mathematics development and benefit children in attaining higher mathematics achievement levels, several theories related to mathematics development and relevant cognitive skills were reviewed to guide the selection of the current study variables.

The works of Cragg et al. (2017) and LeFevre et al. (2010) both highlighted the potential roles of domain-general skills in supporting mathematics learning. In Cragg et al.'s (2017) theoretical framework, it was proposed that various executive functions, such as working memory, inhibition, and shifting, support students' mathematics learning through facilitating their arithmetic fact retrieval, procedural skills, and conceptual understanding. Their findings also showed that working memory and inhibition are particularly relevant to students' mathematics achievement. Meanwhile, LeFevre et al.'s (2010) Pathway model emphasized the unique role of spatial attention in early mathematical success, which was found to be distinct from quantitative and linguistic processes. Notably, existing evidence also supported the relevance of spatial processes in numerical magnitude, problem representation, and overall mathematics competence across different developmental stages (Dehaene, 2004; Leung & Wong, 2023; Siegler, 2016; Tam et al., 2019). Based on these theories, verbal working memory, visuospatial working memory, inhibitory control, and spatial skills were selected as measures of domain-general skills that may predict students' mathematics achievement classes.

For domain-specific skills, Wong's (2021) three-component framework of mathematical competence was referred to better understand how school-aged children's mathematics and cognitive skills construe profiles for learners at different achievement levels. In the framework, the author identified three underlying domain-specific skill components that contribute to mathematical competence (i.e., numerical magnitude, problem representation, understanding of mathematical symbols and their relevant principles) by decomposing the mathematical problem-solving process. Other than numerical magnitude, which has been widely investigated in the field of mathematical cognition (Schneider et al., 2018), the framework further highlights the potential significance of problem representation and the understanding of mathematical principles. Deficits in these skill components may also warrant investigation on their relationship with MLD. Guided by this framework, fraction magnitude understanding, number sentence construction, and arithmetic principles were selected as measures of domain-specific skills that may predict students' mathematics achievement classes.

Based on the above theoretical models, the current study aimed to investigate how these identified domain-general and domain-specific skill components predict membership of different ability groups in mathematical competence. The following section offers a detailed explanation of these variables and their relations with mathematics achievement across students in different achievement groups.

1.3. Executive functions and mathematics achievement

Working memory, encompassing verbal and visuospatial components, have consistently been found to associate with overall mathematics achievement (Friso-van den Bos et al., 2013; Peng & Fuchs, 2016). Specifically, verbal working memory enables the retention of mathematics terms, procedures, and problem-solving steps (Raghubar et al., 2010), facilitating tasks such as organizing numerical sequences, operating mathematical rules, and maintaining intermediate results during multi-step arithmetic (Ashkenazi & Danan, 2017; Attout et al., 2014). On the other hand, visuospatial working memory allows the effective organization of visual elements in mathematics such as geometric figures and spatial relations in mathematics problems (Giofrè et al., 2013). These cognitive functions aid students retain and process numbers, operations, and spatial relations, supporting problem-solving and comprehension in mathematical contexts (Lee et al., 2009; Raghubar et al., 2010; Toll et al., 2016).

Additionally, research has found inhibitory control to be essential for mathematics achievement (Cragg & Gilmore, 2014; cf. Lee & Lee, 2019). It aids students in suppressing impulsive response, selecting efficient strategies, and improving speed and accuracy during problem solving (Khng & Lee, 2009; Lemaire & Lecacheur, 2011). Particularly, students with strong inhibitory control may filter distracting information that could impede problem-solving and concentrate on relevant elements of the given tasks (Gilmore et al., 2015; cf. Ng et al., 2017). Moreover, inhibitory control helps students to resist instinctive but incorrect decisions and develop a more thoughtful approach to problems (McNeil et al., 2017; Ng & Lee, 2005).

These skills may play a role in determining students' mathematics class membership (i.e., MLD, typical, MG). Ample evidence has supported the relationships between MLD status and verbal working memory, visuospatial working memory, and inhibitory control. Longitudinal and meta-analytic studies have highlighted the underperformance in both working memory domains among students with MLD compared to their typically-developing peers (Klesczewski et al., 2018; Kroesbergen et al., 2023). Research also indicated that MLD children may have reduced capacity to resist interference that contributes to difficulties in arithmetic problem solving (Barrouillet et al., 1997; De Visscher & Noël, 2013). Relatively little research has examined the cognitive precursors of MG. Gifted students were found to have superior verbal and visuospatial working memory and perform better in tasks requiring inhibitory control compared to typical children (Berg & McDonald, 2018; Johnson et al., 2003). These findings were further corroborated by neuroimaging studies indicating higher activation in brain regions associated with visuospatial processing and quicker inhibitory responses (Desco et al., 2011; Duan et al., 2009).

1.4. Spatial skills and mathematics achievement

Spatial skills, which is the ability to visualize and manipulate spatial information for problem solving, has been consistently found to be linked to mathematics achievement (Atit et al., 2022; Hawes et al., 2022; Newcombe, 2010; Xie et al., 2020). It was found to be important for computation and problem solving (Tam et al., 2019; Thompson et al., 2013). Specifically, spatial skills may promote a mature number-magnitude representation in a spatial format, which could facilitate arithmetic computation (Tam et al., 2019; Yang & Yu, 2021). Moreover, spatial skills may facilitate learners to form visual representations for word problems that organize relevant problem parts in a spatial manner, which was found to be important for mathematics problem solving among students with learning difficulties and those with exceptional abilities (Krawec, 2014; van Garderen & Montague, 2003).

Spatial skills matter to learners at different achievement levels in mathematics (Zhang et al., 2020). Specifically, deficits in spatial skills were found to differentiate students with MLD and low achievers (Ouyang et al., 2023; Zhang et al., 2020). However, a recent meta-analysis did not find significant differences in the strength of the spatial-math effect between typically-developing students and those with learning difficulties (Xie et al., 2020). On the other hand, spatial visualization was found to be one of the important skills that differentiated students with exceptional mathematical performance and typical learners, indicating the importance of spatial skills for high achieving students (Bakker et al., 2022). The current study aimed to further clarify whether spatial skills could explain the differences between students across mathematics ability groups.

1.5. Fraction magnitude understanding and mathematics achievement

In terms of domain-specific mathematics skills, fraction magnitude estimation may differentiate students' mathematics achievement levels. Number magnitude understanding is commonly measured by number line tasks, which assess students' ability to locate specific numbers on an empty number line between a pair of lower and upper limit (Siegler & Opfer, 2003). Students with better numerical magnitude understanding may have a more refined mental number line, which was argued to be fundamental to the acquisition of broader mathematical competencies (Siegler, 2016).

Number magnitude understanding of different types of number (e.g., whole numbers, fractions) were shown to be consistently associated with mathematics performance (Schneider et al., 2018). In particular, fraction magnitude understanding was shown to correlate stronger with mathematics achievement than whole number magnitude understanding (Schneider et al., 2018). Indeed, measuring magnitude understanding with fractions is age-appropriate for Grade 4 students – the current sample age. Evidently, Grade 4 students may perform reasonably well in number line tasks with whole number estimates (Zhu et al., 2017), such that these tasks may not be able to effectively separate students across achievement levels. Fraction number line, on the other hand, may represent a more suitable task to measure number magnitude understanding in these students since there may be greater range of individual differences (Schneider et al., 2018).

A handful of studies have investigated whole number magnitude understanding, but not fraction magnitude understanding, in the MLD population. Geary et al. (2008) and Andersson and Östergren (2012) showed that MLD children had lower levels of number magnitude understanding compared to their typically-achieving peers. Resnick et al. (2016) reported a group of students that were persistently weak in fraction magnitude understanding who also showed minimal growth over time, which is in line with the characteristics of the growth trajectories of students with MLD (Murphy et al., 2007). These findings suggested that students with MLD may have poorer fraction magnitude understanding than their typically-achieving peers.

1.6. Number sentence construction and mathematics achievement

Number sentence construction, referring to the ability to convert word problems into corresponding mathematical expressions (also known as number sentences), may also be critical to students' membership in mathematics achievement classes. Given the prominence of word problems in the modern mathematics curriculum, they have become an indicator of mathematics performance in teaching and assessing students' understanding across all mathematical topics (Verschaffel et al., 2020). When students solve word problems, they would first form an arithmetic or algebraic number sentence and then perform the necessary computations to obtain the solution (Tolar et al., 2012; Wong & Ho, 2017). Although both steps are fundamental to solving word problems, research has uncovered that more word problem mistakes are caused by errors in forming number sentence than performing computation (Lewis & Mayer, 1987; Wong & Ho, 2017). However, less attention has been directed towards the potential role of number sentence construction in the mathematical problem-solving process.

Despite the scant research in this area, Yip et al. (2020) has recently found that students with MLD had poorer understanding of the underlying semantic structures of word problems, which is a crucial factor for forming correct number sentences (Reusser, 1990). Yip et al. (2020) has further demonstrated that the ability to recognize problem types in addition or subtraction problems could predict MLD class membership after controlling for students' arithmetic skills. Intervention studies with students with MLD targeting their ability to form accurate problem representations have also shown positive effects on word problem performance and overall mathematics achievement (Lein et al., 2020). Relatively less attention has been directed towards number sentence construction skills among MG students, though gifted students may indeed have superior number sentence construction skills for having better understanding and representation of word problems when compared to their typical peers (Heinze, 2005).

1.7. Arithmetic principle understanding and mathematics achievement

Arithmetic principle understanding may be another skill that predicts students' level of mathematics achievement. Arithmetic principle understanding refers to students' ability to draw out rules and regularities that apply to arithmetic operations (Prather & Alibali, 2009). Examples of arithmetic principle include commutativity (e.g., $a + b = b + a$), associativity (e.g., $a + (b + c) = (a + b) + c$), inversion (e.g., $a \times b \div b = a$), and complement (e.g., if $a - b = c$, then $a - c = b$). Students with better arithmetic principle understanding may take advantage of the regularities in arithmetic operations, leading to reduced cognitive load and increased accuracy in solving arithmetic problems. For example, students with commutativity knowledge may be able to store and retrieve certain arithmetic facts (e.g., single-digit addition and multiplication) more efficiently, as these facts can be retrieved from two sources and are activated twice as frequently (e.g., 4 + 7 and 7 + 4; Rickard et al., 1994). Previous evidence has shown students with better arithmetic principle understanding demonstrated higher levels of computational skills (Ching & Nunes, 2017; Yip et al., 2023).

Given the benefits of arithmetic principle understanding on mathematical computation, it is surprising that little research has investigated arithmetic principle understanding among students with MLD. Jordan et al. (2003) found that students with MLD in second grade had poorer arithmetic principle understanding than their typically-achieving peers. Andersson (2010) reported a similar effect in third and fourth grade students. No research so far has explored this skill among MG students. The current study attempted to extend prior findings on the group differences between students with MLD and their peers in arithmetic principle understanding by investigating whether such performance difference was also observed between MG students and their typically-achieving peers.

1.8. The current study

The current study aimed to identify the mathematics achievement classes among Grade 4 students, compare skill profiles across the identified achievement classes, and explore the specific skills that may predict achievement class memberships.

Previous studies have found significant associations between nonverbal intelligence and various mathematics skills, such as numerical processing, calculation, and algebra, as well as overall mathematics achievement (Peng et al., 2019). Thus, we will control for nonverbal intelligence in our models to elucidate the unique contributions of both domain-general and domain-specific skills towards classifying students into different mathematics learning profiles.

Existing findings reported significant differences in the cognitive and mathematics skill levels between students at different mathematics achievement levels, which were also expected and hypothesized in the current study. However, these findings did not allow the investigation of whether specific skills differentiated students' achievement class membership. The current study aims to fill this research gap by exploring the specific skills that may predict students' mathematics achievement class membership.

2. Methods

2.1. Participants

A total of 273 Cantonese-speaking fourth graders were recruited from nine local mainstream primary schools in Hong Kong as part of a longitudinal study. One participant was excluded due to anomalous performance in the arithmetic principle understanding task (see last paragraph of Section 2.4.1.). The final sample was 272 students (111 male), with a mean age of 10.29 years ($SD = 0.86$ years).

2.2. Procedures

Ethical approval was obtained from the Human Research Ethics Committee of the corresponding author's affiliated university. Invitations letters were sent to primary schools in Hong Kong. Consent forms were then distributed to the parents. After obtaining parental consents, participants were invited for individual assessments at home or at school. Each data collection session lasted about 90 to 120 min, with breaks given when needed. Parents of the participants received HK $50 supermarket or book coupons as a token of gratitude.

All assessments were carried out by trained test evaluators, who were postgraduate and undergraduate students majoring in psychology. Prior to data collection, test evaluators satisfactorily completed a training module conducted by a doctoral student in psychology who was experienced in educational assessment. The training started with a group didactic component, followed by individual supervised data collection sessions in which the test evaluators administered all study measures in one-to-one data collection sessions with randomly assigned study participants under the supervision of the doctoral student. These procedures help ensure the accuracy and fidelity of data collection procedures.

2.3. Transparency and openness

This study was not preregistered. It was part of a longitudinal research project on arithmetic principle understanding and its relationship with mathematics learning. Data used in this study are available at the Open Science Framework (https://osf.io/3ce5p). Research materials are available upon request.

2.4. Measures

2.4.1. Domain-general and domain-specific skills

Verbal working memory was assessed by the backward syllable span task. In each trial, a string of Cantonese syllables was played at the pace of one syllable per second. Participants were instructed to recall the syllables in reverse order. There were three trials per level, and seven levels in total. One syllable was added to each level as the task proceeded. The task was terminated when participants scored zero in all trials on a level. Two practice trials were provided for familiarization before the task began. Each correct trial was awarded one point. Higher scores indicated greater verbal working memory span.

Visuospatial working memory was assessed by the backward Corsi block tapping task (Corsi, 1973). In each trial, participants were shown a board with nine paper boxes randomly built on it. Then, they were shown a video of box tapping in a specific sequence. Each tap was separated by a one-second interval. Participants were asked to recreate the sequence in the video in reverse order by tapping the boxes on the board. There were three trials per level, and seven levels in total. One tap was added to each level as the task proceeded. The task was terminated when participants scored zero in all trials on a level. Two practice trials were provided for familiarization before the task began. Each correct trial was awarded one point. Higher scores indicated greater visuospatial working memory span.

Inhibitory control was assessed by the colour Stroop task (Stroop, 1935). The task consisted of three parts. The first part involved naming colours (red, yellow, green, blue, purple) in 40 colour boxes. The second part involved naming 40 Chinese colour words printed in black. The third and final part involved naming the colours of 40 coloured Chinese words printed with incongruent ink colours (e.g., the word "red" was printed in green). Participants were instructed to name the colour (for the first and third parts) or words (for the second part) as quickly as they could. The first five items in each part served as practice for task familiarization. Participants' performance was indicated by the inverse difference in the time taken for the final part and the mean reaction time taken for the first and second part (i.e., $\text{Time}_{\text{Part 3}} - (\text{Time}_{\text{Part 1}} + \text{Time}_{\text{Part 2}})/2)^{-1}$. Higher scores indicated better inhibitory control.

Spatial skills were assessed by the Card Rotation Test (Ekstrom et al., 1976). The test consisted of two identically-structured sessions, each with 10 items and a 3-min time limit. Participants were presented with one example and two practice items before the test began. Each item in the test consisted of an irregular polygon (the target shape) along with eight alternative shapes (options). Participants were instructed to select all the options that were planar rotations of the target shape, among distractors of rotated mirror images mixed in the options. One point was awarded to each correct choice, and one point was deducted for each incorrect choice. No penalty was given for omitting correct answers. Higher scores indicated better spatial skills.

Fraction magnitude understanding was assessed by the fraction number line task (Wong, 2018; Wong & Morsanyi, 2023). There were 24 trials in this computerized task. In each trial, participants were shown a number line with 0 on the left and 1 on the right. They were then instructed to move the cursor and indicate the position of a designated fraction on the number line. The absolute difference between participants' marking and the correct location divided by the scale of the number line, known as the percentage absolute error (PAE), was measured for each item. Participants' performance was indicated by one minus the mean PAE across trials. Higher scores indicated better fraction magnitude understanding.

Number sentence construction for word problems was assessed by the number sentence construction task (Wong & Ho, 2017; Wong & Yip, 2023). Participants were instructed to write down the number sentences and answers for 10 word problems, arranged in ascending order of difficulty. Only the number sentence was scored. Each correct number sentence was awarded one point. Number sentences or algebraic equations were both accepted. Higher scores indicated better number sentence construction skills.

Arithmetic principle understanding was assessed by a series of computerized test for commutativity, associativity, inversion, and complement (Torbeyns et al., 2016). In each item, participants were presented with two arithmetic problems: a reference problem at the top of the screen and a target problem at the centre of the screen. Participants were instructed to solve the target problem and were informed that the reference problem may or may not be useful. In half of the items (core items), the reference problem facilitated the problem-solving process through enabling relevant arithmetic principle shortcuts (e.g., for commutativity, reference problem: $375 + 518 = 893$, target problem: $518 + 375 =$?). In the other half (distractor items), the reference problem and the target problem were unrelated (e.g., for commutativity, reference problem: $327 + 453 = 780$, target problem: $443 + 347 =$?). Participants responded by pressing keyboard keys to choose one out of the three answer options provided.

For the arithmetic principle understanding task, time limits were set for each item to ensure that participants solved the target problems through relevant arithmetic principle shortcuts. The time limits for commutativity, associativity, inversion, and complement were 7, 9, 8, and 9 s respectively. These limits were obtained from pilot testing such that participants had sufficient time to use the arithmetic principle shortcuts but not enough for actual computation. One participant was excluded from the sample for displaying significantly above-chance performance in the distractor items. Two examples and four practice items were given before the test began. There were 72 items in this task (36 core items). Each correct response in the core items was awarded one point. Distractor items served as a validity check and were not scored. Higher scores indicated better arithmetic principle understanding.

2.4.2. Mathematics achievement

Numerical operations ability was assessed by the numerical operations subtest of the Weschler Individual Achievement Test – Third Edition (WIAT-III; Wechsler, 2009). There were 61 items in this task, presented in increasing level of difficulty. Participants attempted a subset of items as determined by the grade-specific start point, the basal, and the ceiling of the task (i.e., task terminates upon four consecutive incorrect answers). The items included basic arithmetic operations of whole numbers, fractions, and decimals, and were presented in vertical or horizontal formats. Rough work was allowed, but only the final answers were scored. Each correct answer was awarded one point. Higher scores indicated better numerical operations ability.

Math problem-solving ability was assessed by the math problem solving subtest of the Weschler Individual Achievement Test – Third Edition (WIAT-III; Wechsler, 2009). There were 72 items in this task, presented in increasing level of difficulty. Participants attempted a subset of items as determined by the grade-specific start point, the basal, and the ceiling of the task (i.e., task terminates upon four consecutive incorrect answers). Items were orally presented alongside the corresponding visual stimuli. Participants were instructed to give verbal answers, and rough work was allowed. The subtest was translated by the corresponding author into Chinese and then back-translated by an undergraduate majoring in English. Each correct answer was awarded one point. Higher scores indicated better math problem-solving ability.

2.4.3. Cognitive correlate

Nonverbal intelligence was assessed by the Raven's Standard Progressive Matrices (Raven et al., 2003). The full scale of 60 items was administered. In each item, participants were presented with an incomplete pattern of shapes and were instructed to identify the missing part by selecting the appropriate option. Each correct answer was awarded one point. Higher scores indicated higher nonverbal intelligence.

2.5. Data analyses

The current analysis plan was adapted from prior studies with similar objectives of classifying individuals into subgroups, studying variations in individual characteristics among these subgroups, and identifying predictors of subgroup membership (Sun & Xie, 2020; Vanslambrouck et al., 2019; Zhang et al., 2020).

Latent profile analysis (LPA) was conducted to identify different mathematics achievement profiles, with numerical operations and math problem solving as observed variables. LPA was conducted to multiple models starting from a one-class model and adding one class at a time, until an optimal model was identified. The choice of optimal model was informed based on various criteria: the Bayesian information criterion (BIC), the Lo-Mendell-Rubin likelihood ratio test (LMR-LRT), the bootstrapped likelihood ratio test (BLRT), and entropy of the models. A model solution was considered optimal when it had the lowest BIC value among the models and non-significant *p*-values in the LMR-LRT and BLRT between the model and the next model with one more class. Entropy of the models was also examined, with values over 0.8 indicating minimal classification uncertainty.

Additionally, ANOVA was conducted to examine the differences in domain-general and domain-specific skills between profile groups, as well as to validate the classes obtained from LPA. Post-hoc tests (Tukey) were followed for significant omnibus differences. Finally, a multinomial logistic regression model was fitted to reveal significant predictors of mathematics achievement class membership. A stepwise model was specified for the regression model, where nonverbal intelligence as a cognitive correlate was entered in the first step (Model 1), and domain-general and domain-specific skills (i.e., verbal working memory, visuospatial working memory, spatial skills, fraction magnitude understanding, number sentence construction, arithmetic principle understanding) were entered in the second step (Model 2). The LPA was conducted using R with the mclust package (Scrucca et al., 2023), while the ANOVA, post-hoc tests, and multinomial logistic regression analysis were conducted using SPSS 28.

3. Results

3.1. Descriptive statistics

Table 1 presents the descriptive statistics, internal reliability of each measure, and correlations among the measured variables. All measures displayed satisfactory reliability ($0.75 \leq \alpha s \leq 0.95$). Correlation analyses showed that verbal working memory, visuospatial working memory, nonverbal intelligence, spatial skills, fraction magnitude understanding, number sentence construction, arithmetic principle understanding, numerical operations, and math problem solving had significant positive correlation with one another ($0.19 \leq rs \leq 0.66$, $ps \leq .001$). Inhibitory control had significant positive correlation with most variables ($0.15 \leq rs \leq 0.27$, $ps \leq .01$), except with visuospatial working memory ($r = 0.12$, $p = .052$), fraction magnitude understanding ($r = 0.05$, $p = .412$) and number sentence construction ($r = 0.03$, $p = .622$).

Table 1
Descriptive Statistics, Reliability of Each Measure, and Correlations Among Measured Variables.

Measure	Maximum possible	Mean	*SD*	α	Correlation									
					1	2	3	4	5	6	7	8	9	10
1. VWM	21	7.76	2.81	0.76	–									
2. VSWM	21	9.71	2.87	0.75	0.28***	–								
3. IC	NA	35.48	12.87	NA	0.25***	0.12	–							
4. NVI	60	40.58	7.00	0.86	0.41***	0.35***	0.27***	–						
5. SS	160	43.39	21.85	0.95	0.29***	0.31***	0.15*	0.46***	–					
6. FM	NA	0.14	0.11	0.93	0.23***	0.25***	0.05	0.38***	0.38***	–				
7. NSC	10	5.51	3.03	0.75	0.22***	0.27***	0.03	0.37***	0.26***	0.29***	–			
8. AP	36	19.71	6.90	0.85	0.29***	0.21***	0.16**	0.39***	0.19**	0.21***	0.19**	–		
9. NO	61	34.66	3.81	0.82	0.36***	0.25***	0.17**	0.51***	0.33***	0.43***	0.43***	0.37***	–	
10. MPS	72	50.54	5.20	0.85	0.48***	0.30***	0.17**	0.62***	0.46***	0.54***	0.38***	0.36***	0.66***	–

Note. *** $p < .001$; ** $p < .01$; * $p < .05$. VWM = Visual Working Memory; VSWM = Visuospatial Working Memory; IC = Inhibitory Control; NVI = Nonverbal Intelligence; SS = Spatial Skills; FM = Fraction Magnitude Understanding; NSC = Number Sentence Construction; AP = Arithmetic Principle Understanding; NO = Numerical Operations; MPS = Math Problem Solving. NA = Not Appliable.

3.2. Profile groups of mathematics achievement

Table 2 presents fit indices of LPA models with one to five classes examining mathematics achievement profiles, with numerical operations and math problem solving as observed variables. As the number of classes increased, the BIC values decreased and reached a minimum with the four-class model, where it subsequently increased with the five-class model. Furthermore, increasing model classes stepwise from one class to four classes yielded significant *p*-values for LMR-LRT (*ps* < .001) and BLRT (*ps* = .001) tests, yet the four-class and five-class models showed non-significant differences in both LMR-LRT (*p* = .060) and BLRT (*p* = .212) tests. This indicates that, while the model fits significantly improved as more classes were introduced from one class to four classes, the model fits between the four-class and five-class models did not significantly differ. Hence, a four-class model was selected due to best model fit and parsimony, as consistently supported by the fit indices. The four-class model was further supported by its entropy value 0.844, indicating that the model classification was reliable.

3.3. Differences in measured skills and achievement levels between profile classes

Fig. 1 presents the estimated mean *z*-scores of the measured skills and achievement levels among the four profiled classes. Of note, the mean *z*-scores of the mathematics achievement measures in the lowest achieving class were − 1.25 (i.e., bottom 10.56 %) for numerical operations and − 1.58 (i.e., bottom 5.67 %) for math problem solving. Relatedly, the mean *z*-scores of the mathematics achievement measures in the highest achieving class were 3.09 (i.e., top 0.10 %) for numerical operations and 1.87 (i.e., top 3.09 %) for math problem solving. Hence, the four classes identified were, from lowest to highest achieving: mathematics learning disability (MLD; *n* = 39), average achieving (AA; *n* = 183), high achieving (HA; *n* = 38), and mathematically gifted (MG; *n* = 12). Except for inhibitory control, students with MLD were characterized by poor performance across all measures when compared to other classes, whereas MG students were characterized by superior performance across all measures when compared to AA and MLD students.

Table 2
Fit Indices of Models with Different Numbers of Classes.

Class	BIC	LMR-LRT (*p*-value)	BLRT (*p*-value)	Entropy
One class	1416.696	NA	NA	1
Two classes	1364.247	< .001	.001	0.412
Three classes	1361.533	< .001	.001	0.950
Four classes	**1355.034**	**< .001**	**.001**	**0.844**
Five classes	1367.869	.060	.212	0.810

Note. BIC = Bayesian information criterion; LMR-LRT = Lo-Mendell-Rubin likelihood ratio test; BLRT = bootstrapped likelihood ratio test. Row in boldface indicates final model selected. NA = Not Appliable.

Table 3 presents the descriptive statistics of the measured skills and achievement levels among the four profiled classes, along with ANOVA results examining class differences between the measured skills and achievement levels. Post-hoc power analyses showed that all variables in the ANOVA model demonstrated sufficient power (> 0.80) with the current sample size, except for inhibitory skills (power = 0.29). The main effects were all significant for most measures (*p* < .001), except for inhibitory control (*p* = .133). Since inhibitory control did not significantly differ between students across mathematics achievement levels, it was excluded from subsequent analyses.

For mathematics achievement measures, the effect sizes (η^2) were 0.693 for numerical operations and 0.769 for math problem solving. Post-hoc comparisons showed that performance in mathematics achievement measures were significantly different between all pairs of classes for numerical operations (*ps* < .006) and math problem solving (*ps* < .001). The current sample also demonstrated sufficient power (> 0.80) for these pairwise tests. The significant differences in the performance between classes for both mathematics achievement measures, in addition to the very large effect sizes, further supported the four-class model from the LPA.

For domain-general and domain-specific skills, the effect sizes (η^2) of significant effects ranged from 0.082 (medium) to 0.307 (large). Post-hoc comparisons showed that students with MLD performed significantly below other classes in all skills. On the contrary, MG students performed significantly better in most skills, except visuospatial working memory, when compared to AA students. The current sample also demonstrated sufficient power (> 0.80) for these pairwise tests, except for visuospatial working memory (power = 0.40).

3.4. Predicting class membership in mathematics achievement

Table 4 presents the results of multinomial logistic regression of students' class membership in mathematics achievement on their performance in the measured skills. The variance inflation factors observed in the regression models were below 1.80, indicating minimal evidence for multi-collinearity (Thompson et al., 2017). Post-hoc power analyses showed that most variables in the regression model demonstrated sufficient power (> 0.80), except for spatial skills (power = 0.66) and visuospatial working memory (power = 0.12). The full logistic model (Model 2) including nonverbal intelligence and study variables demonstrated better fit beyond the partial model (Model 1) with nonverbal intelligence only ($\chi^2(18) = 123.88$, $p < .001$).

Between MLD and AA classes, after controlling for participants' nonverbal intelligence, verbal working memory, fraction magnitude understanding, and number sentence construction skills significantly contributed to the differentiation between the two classes. Specifically,

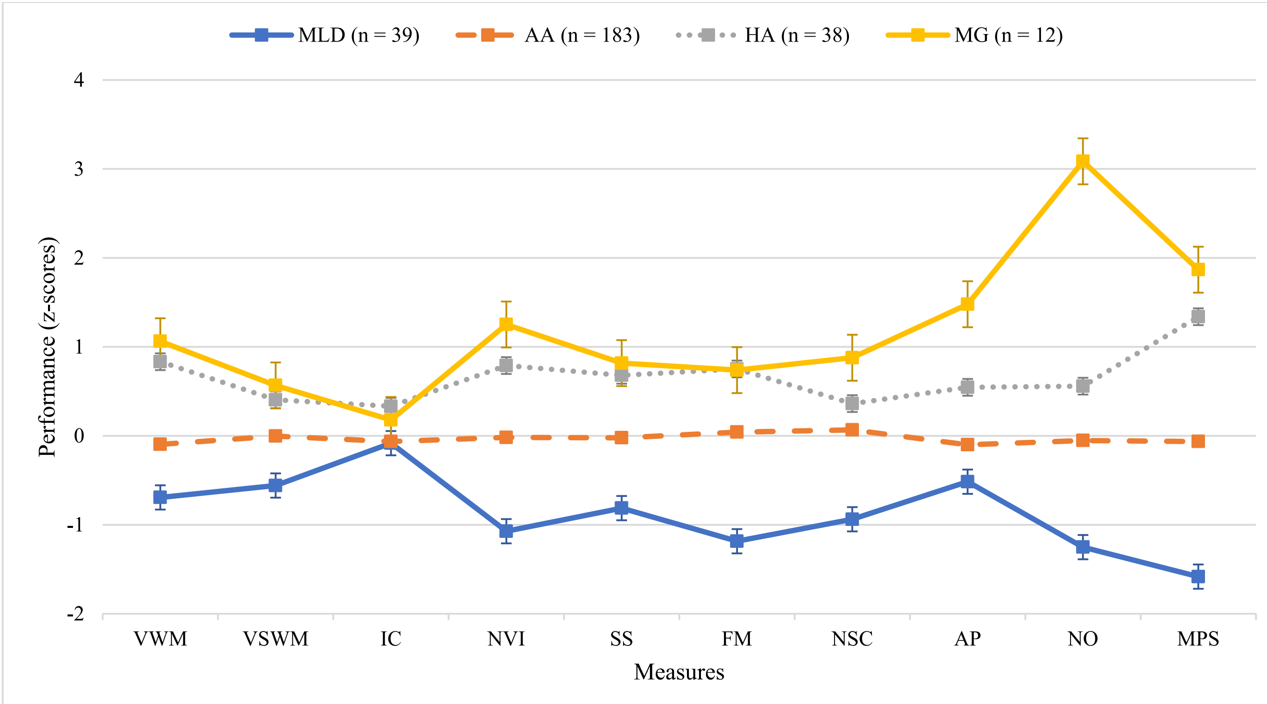

Fig. 1. Estimated Standardized Scores of the Measured Skills and Achievement Levels Among the Four Profiled Classes.
Note. MLD = Students with mathematics learning disability; AA = Average achieving students; HA = High achieving students; MG = Mathematically gifted students. VWM = Visual Working Memory; VSWM = Visuospatial Working Memory; IC = Inhibitory Control; NVI = Nonverbal Intelligence; SS = Spatial Skills; FM = Fraction Magnitude Understanding; NSC = Number Sentence Construction; AP = Arithmetic Principle Understanding; NO = Numerical Operations; MPS = Math Problem Solving. Error bars indicate standard errors.

Table 3
Statistics for MLD, AA, HA, and MG Groups of Each Measure.

Measure	MLD (n = 39)		AA (n = 183)		HA (n = 38)		MG (n = 12)		ANOVAs		
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*F*	η^2	Post-hoc
VWM	5.821	1.848	7.497	2.467	10.105	2.768	10.750	3.571	25.575***	0.223	MG = HA > AA > MLD
VSWM	8.103	2.789	9.699	2.850	10.868	2.171	11.333	3.055	7.988***	0.082	MG = HA = AA > MLD
IC	3.080	0.941	3.101	1.047	3.545	1.503	3.372	1.226	1.880	0.021	NA
NVI	33.077	6.301	40.454	5.908	46.105	4.974	49.333	4.355	42.528***	0.323	MG = HA > AA > MLD
SS	25.641	20.113	42.913	20.184	58.263	17.894	61.250	17.602	20.942***	0.190	MG = HA > AA > MLD
FM	0.734	0.101	0.868	0.100	0.945	0.024	0.944	0.034	39.515***	0.307	MG = HA > AA > MLD
NSC	2.667	2.069	5.710	2.785	6.605	3.184	8.167	2.758	19.902***	0.182	MG > AA > MLD, MG = HA, HA = AA > MLD
AP	16.154	5.756	19.016	6.322	23.474	7.229	29.917	2.392	20.106***	0.184	MG > HA > AA > MLD
NO	29.897	2.827	34.464	1.892	36.789	1.919	46.417	3.260	201.279***	0.693	MG > HA > AA > MLD
MPS	42.308	3.458	50.208	2.258	57.500	2.263	60.250	3.306	297.509***	0.769	MG > HA > AA > MLD

Note. *** $p < .001$. MLD = Students with mathematics learning disability; AA = Average achieving students; HA = High achieving students; MG = Mathematically gifted students. VWM = Visual Working Memory; VSWM = Visuospatial Working Memory; IC = Inhibitory Control; NVI = Nonverbal Intelligence; SS = Spatial Skills; FM = Fraction Magnitude Understanding; NSC = Number Sentence Construction; AP = Arithmetic Principle Understanding; NO = Numerical Operations; MPS = Math Problem Solving. NA = Not Appliable.

when compared to AA students, a one-*SD* increase in verbal working memory, fraction magnitude understanding, and number sentence construction skills were associated, respectively, with 53.4 %, 63.4 %, and 60.7 % reduced odds of MLD class membership.

Between HA and AA classes, after controlling for participants' nonverbal intelligence, verbal working memory, spatial skills, and fraction magnitude understanding significantly contributed to the differentiation between the two classes. Specifically, when compared to AA students, a one-*SD* increase in verbal working memory and spatial skills were associated, respectively, with 122 % and 82.1 % increased odds of HA class membership. A one-*SD* increase in the percentage absolute error in fraction magnitude understanding, moreover, was associated with 98.2 % reduced odds of HA class membership.

Between MG and AA classes, after controlling for participants' nonverbal intelligence, verbal working memory and arithmetic principle understanding significantly contributed to the differentiation between the two classes. Specifically, when compared to AA students, a one-*SD* increase on verbal working memory and arithmetic principle understanding was associated, respectively, with 109 % and 539 % increased odds of MG class membership.

4. Discussion

The current study identified mathematics achievement profile groups among Grade 4 students, compared skill profiles across the identified achievement classes, and explored specific skills that predicted achievement class memberships. Four classes of students with different mathematics achievement levels were identified, namely, the

Table 4
Multinomial Logistic Regression Results.

Measure	MLD/AA		HA/AA		MG/AA	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
Cognitive correlate, OR						
Nonverbal intelligence	0.263***	0.539*	4.030***	1.733	12.540***	3.224
Study variables, OR						
Verbal working memory		0.466*		2.215**		2.085*
Visuospatial working memory		1.025		0.951		0.922
Spatial skills		0.736		1.821*		1.720
Fraction magnitude understanding		0.366***		54.483***		4.769
Number sentence construction		0.393**		1.023		2.034
Arithmetic principle understanding		0.779		1.421		6.390**

Note. *** $p < .001$; ** $p < .01$; * $p < .05$. OR = Odds ratio. MLD = Students with mathematics learning disability; AA = Average achieving students; HA = High achieving students; MG = Mathematically gifted students.

mathematics learning disability (MLD), average achieving (AA), high achieving (HA), and mathematically gifted (MG) classes. Consistent with our hypotheses, students between the bottom (i.e., MLD), average (i.e., AA), and top (i.e., MG) classes significantly differed in their verbal working memory, visuospatial working memory, spatial skills, fraction magnitude understanding, number sentence construction skills, and arithmetic principle understanding. Specifically, MG students performed better in all these skills than AA students, and AA students performed better in all these skills than students with MLD. Furthermore, when compared to the AA class, while verbal working memory, fraction magnitude understanding, and number sentence construction predicted MLD class membership, verbal working memory, spatial skills, and fraction magnitude understanding predicted HA class membership, and verbal working memory and arithmetic principle understanding predicted MG class membership.

4.1. Executive functions and mathematics achievement classes

In line with previous findings (Friso-van den Bos et al., 2013; Peng & Fuchs, 2016), the current research found significant differences in students' visual and visuospatial working memory between mathematics achievements levels. Extending these findings, the current results revealed the significant role of verbal working memory in classifying students across all mathematics achievement classes.

The important role of verbal working memory in students' achievement classes may be explained by its benefits on students' enhanced processing of various verbal information (e.g., numbers, semantic elements) in mathematics problems. Superior verbal working memory may enable students to retain and manipulate various verbal information simultaneously (Friso-van den Bos et al., 2013; Swanson, 2011). This facilitates better online performance in comprehending problems and selecting appropriate mathematics procedures, leading to more effective and accurate problem-solving approaches (Gilmore et al., 2017). In contrast, students with lower verbal working memory may be overwhelmed by verbal demands associated with more complex problems (Swanson, 2020). They may struggle to hold and manipulate all relevant problem elements concurrently, leading to a reduced capacity to process problem information (Kyttälä et al., 2010). As a result, they may face difficulties in efficiently generating solutions and applying mathematical concepts correctly, contributing to increased susceptibility to MLD (Peng & Fuchs, 2016).

Consistent with some (Cantin et al., 2016; Gerst et al., 2017; Roebers et al., 2012), but not all (Schmerold et al., 2017; St Clair-Thompson & Gathercole, 2006), previous studies, the current results found significant positive correlations between students' inhibitory control and mathematics achievement. Despite being positive, the correlations between inhibitory control and the two indicators for mathematics achievement, i.e., numerical operations (r = 0.17, 95 % CI [0.05, 0.28]) and math problem solving (r = 0.17, 95 % CI [0.05, 0.28]), were weak. These correlations were comparable to recent meta-analytic findings on the association between the Stroop task and mathematics intelligence (r = 0.24, 95 % CI [0.05, 0.43]; Emslander & Scherer, 2022). Further analysis in the current study, however, did not find significant differences in students' inhibitory control skills across mathematics achievement classes. These findings suggest that, while inhibitory control may correlate with mathematics achievement, other cognitive characteristics (e.g., working memory, nonverbal intelligence) may associate more strongly with students' mathematics achievement classes and the latent trait that determined these classes. Consequently, differences in inhibitory control may not be reflected between students across mathematics achievement classes.

4.2. Spatial skills and mathematics achievement classes

Consistent with prior research supporting the link between spatial and mathematics abilities (Atit et al., 2022; Xie et al., 2020), our study found positive associations between students' spatial skills and mathematics achievement levels. The unique contribution of our study, however, lies in the discovery that students who have better spatial skills are more likely to be high achievers than average achievers.

The association between spatial skills with the class status of high achieving students may be explained by the potential positive influence of spatial skills on both their problem-solving ability and arithmetic computation. When challenged with a novel problem, students would need to form a mental representation of the problem from which they could plan strategies to solve it (Sorby et al., 2022). Students with stronger spatial abilities may create a more coherent visual-schematic representation of word problems, which integrates all solution-relevant text elements holistically, rather than only focusing on specific elements in the problem (Boonen et al., 2013; Wang et al., 2022). The production of visual-schematic representations facilitates a clearer understanding of the problem and has been found to mediate the relationships between spatial and mathematics abilities (Boonen et al., 2013). For example, when presented with an arithmetic word problem, students with good spatial ability can depict not only the objects mentioned in the problem but also the relationships (e.g., ratios, relative positions) between them. These visual-schematic representations enable students to form a correct equation that accurately reflects the relationships of the elements in the problem.

With the correct equation, students would proceed to perform arithmetic computations. Spatial skills may also benefit such computation by promoting students' representation and understanding of arithmetic and numerical symbols (Ouyang et al., 2022; Yang & Yu, 2021). Specifically, students with well-developed spatial skills may be able to mentally manipulate the numerical information spatially to connect different problem components, such as aligning digits in multi-digit arithmetic computation. Moreover, spatial skills may also allow students to visualize the processes of arithmetic operations, such as picturing objects being grouped together for addition or objects being removed from a group for subtraction, leading to a deeper understanding of arithmetic operations that can foster a higher level of arithmetic proficiency. Taken together, spatial skills may promote an organized and systematic approach to arithmetic computation, thereby enhancing students' accuracy and efficiency in solving mathematics problems.

4.3. Fraction magnitude understanding and mathematics achievement classes

Echoing previous meta-analytic findings (Schneider et al., 2018), our study found significant differences in students' fraction magnitude understanding across mathematics achievement classes. Building on this work, our study further showed that fraction magnitude understanding could differentiate MLD and high achieving students from average students.

The current finding is in keeping with theories that posit the skills involved in the number line estimation task could support the acquisition of broader mathematical concepts and, thus, facilitate advanced mathematical development (Siegler & Lortie-Forgues, 2014). Specifically, the mental number line may afford a retrieval structure to facilitate the encoding, storage, and retrieval of magnitude-related numerical information (Siegler & Ramani, 2009), which could, in turn, facilitate students' acquisition and retrieval of arithmetic facts around the magnitude of numbers (Tam et al., 2019). Relatedly, number line representation may also facilitate a schema presentation for representing different numerical components and their relations involved in a word problem (Ouyang et al., 2021).

As Grade 4 students start to work with fractions, fraction magnitude understanding may be especially important to their mathematics development (Namkung et al., 2018). When students evaluate the magnitude of a fraction, they have to consider the fact that the numerator and denominator have opposing effects on the magnitude of the fraction. In connection to the aforementioned theories, students may better understand these properties of fractions and the relations between fractions if they have an accurate mental fraction number line. Specifically, the mental number line may render a useful structure to organize simple fraction concepts (e.g., larger denominator with the same nominator means smaller fraction) and thus help students consolidate basic fraction knowledge and generalize to more complex fraction computation (Schneider et al., 2018). This may contribute to reduced computational error and increased efficiency in problem representation during mathematics problem solving. On the contrary, students who do not have sufficient fraction magnitude understanding may suffer pervasive deficits in mathematics learning and performance.

4.4. Number sentence construction and mathematics achievement classes

Previous works have consistently reported difficulties in forming number sentences among students with MLD and their significant underperformance compared to their typically-achieving peers (Yip et al., 2020). Expanding on this work, the current results showed that students' ability to form number sentences also help differentiate between MLD and non-MLD classes.

The complex demands of constructing a number sentence may give insight as to why difficulties in this area are particularly relevant to MLD classification. These demands include comprehension of problem text and mathematics terms, representation of word problem in various schema, and translation of problem schema into mathematical expressions (Reusser, 1990). There is evidence that students with MLD are impaired in all these demands (Lin et al., 2021; Yip et al., 2020). For instance, poor comprehension of mathematical terms may hinder students' understanding of related mathematics concepts in their abstract forms (Lin et al., 2021), leading to erroneous translation between mathematical terms and operations (e.g., directly associating words like "more" or "less" with addition and subtraction; Hegarty et al., 1995). Similarly, not being able to represent word problems in various schema may also hint at students' inadequate conceptual understanding of mathematics operations, resulting in the wrong mental model to approach the problem (Yip et al., 2020). These suboptimal understandings towards mathematics terminologies and concepts prevent students from successfully constructing number sentences and could be detrimental to their overall mathematics development.

4.5. Arithmetic principle understanding and mathematics achievement classes

The current study documented good arithmetic principle understanding among MG students. Extending previous literature on the close association between arithmetic principle understanding and mathematical performance (Wong, 2023; Yip et al., 2023), arithmetic principle understanding was found to significantly discriminate MG students from average achieving students.

There are at least two rationales for good arithmetic principle understanding being a defining feature of high-achievers. First, understanding of arithmetic principles may enhance the use of efficient problem-solving strategies. For example, when calculating $36 + 98 + 2$, students with good associativity understanding may recognize the advantage of adopting a "right-to-left" procedure and begin the computation with $98 + 2$, instead of the typical "left-to-right" procedure. Through facilitating these efficient strategies, arithmetic principle understanding may help reduce students' computational errors in more complex operations (e.g., carrying in $36 + 98$), thereby enhancing their accuracy of arithmetic computation and translating to their high mathematics achievement (Torbeyns et al., 2009).

Second, given their mathematics competence, MG students may experience lower cognitive load during arithmetic computation. Domain-general resources may therefore be freed up for the discovery and application of arithmetic principles. For example, Baroody et al. (1983) found that complement shortcuts were applied more often to addition doubles (e.g., 14–7) than other combinations, which was suggested to be associated with students' higher competence with the double facts. As MG students have gained proficiency in arithmetic computation, they are more likely to reallocate their mental resources towards identifying the underlying regularities, instead of following the typical algorithms (Siegler & Araya, 2005). Although there have been mixed findings on the contribution of calculation skills to arithmetic principle understanding (e.g., Siegler & Araya, 2005; Watchorn et al., 2014; Yip et al., 2023), this provides another possible theoretical account for the finding that high achievers differed from other classes mainly in terms of arithmetic principle understanding.

4.6. Theoretical implications

The current study uncovered the skills that could distinguish between different mathematics achievement classes among late elementary students. In addition to summarizing the significant differences in specific mathematics skill levels between achievement classes, the current results also revealed the contribution of these skills in influencing students' statuses in the mathematics achievement taxonomy. Given that students at different achievement levels require different academic support in terms of intensity and approaches (Karagiannakis & Cooreman, 2014; Özdemir & Bostan, 2021), it is important to pinpoint specific mathematics areas to be targeted that can help advance students' mathematics proficiency. Notably, the current results highlighted the unique contributions of verbal working memory, spatial skills, fraction magnitude understanding, number sentence construction, and arithmetic principle understanding in differentiating performance across the mathematics development spectrum. Improvement in these areas may decrease students' odds of falling into the MLD class and increase their odds of advancing into higher achieving classes (e.g., HA, MG).

4.7. Educational implications

To support the needs of students across different mathematics achievement classes, educators should implement relevant interventions to enhance students' performance in areas that are most pertinent to their needs.

Since number sentence construction was found to discriminate students with MLD from their peers, future practitioners should consider directing more resources in word problem training for students with MLD, particularly in building problem schema and transferring them into mathematical expressions (Lein et al., 2020). As for fraction magnitude understanding, given its encompassing impact on students' achievement categorization across MLD, average achieving, and high achieving classes, future practitioners should consider enhancing students' fraction number sense and magnitude representational system in general to support their advanced mathematical development (e.g., Dyson et al., 2020; Hamdan & Gunderson, 2017).

Additionally, based on the current findings, spatial skills training may help average achieving students progress in their mathematics performance. However, despite the robust spatial-math link found in the literature (Atit et al., 2022; Newcombe, 2010; Xie et al., 2020), not all spatial skills interventions have shown positive results in transferring their effects onto mathematical domains (Hawes et al., 2022; Woolcott et al., 2022). Future research may further investigate the specific spatial processes that could benefit students' mathematics development.

Finally, arithmetic principle understanding was the only mathematics skills investigated that significantly differentiated MG students from other groups of students. This suggests that deepening arithmetic principle understanding could be an avenue for students with considerable potential to further excel in their mathematics achievement. This also points to the utility of training in relevant areas to improve abled students' arithmetic principle understanding, specifically on their ability to recognize the regularities between arithmetic identities and apply them in their mathematics computation (Eaves et al., 2019; Nunes et al., 2012).

4.8. Limitations and future directions

The current study was not without limitations. First, the current study did not explore the subtypes of MLD (e.g., procedural, semantic memory, spatial; Geary, 1993) due to the limited sample size. As far as verbal working memory, fraction magnitude understanding, and number sentence construction was found to associate with MLD classification, it remains uncertain whether these skill deficits are associated with different subtypes of MLD. Future studies may examine the contribution of these skills in elucidating MLD subtypes.

Second, our cross-sectional design did not allow us to examine the causal relations between specific skills and overall mathematics achievement. Relatedly, the current analyses were not able to explore whether students' performance in specific skills could contribute to their mathematics learning and growth. Future studies may utilise longitudinal designs and growth models to extend the current findings.

Third, the current participants were tested in varied environments (either at school or home) based on parental or school preferences. This may introduce potential variability in their task performances. Future studies may conduct assessments in a standardized testing environment as long as it is practically feasible.

Fourth, post-hoc power analyses revealed low statistical power for spatial skills and visuospatial working memory in the current multinomial logistic regression model. These low power values suggest potential limitations in detecting true effects associated with these variables. Future studies may recruit a larger sample to verify the current findings.

Fifth, the current study measured mental rotation as a proxy of students' overall spatial skills for its stable and strong associations with mathematics achievement (Ganley & Vasilyeva, 2011; Tam et al., 2019). Nevertheless, other spatial domains may also contribute to students' mathematics development and achievement classes (Pring et al., 2010; Yazdani et al., 2021). Future researchers may utilise a wider range of spatial measures to study the relationships between different spatial skills domains and mathematics achievement, and how spatial skills differentiate students between mathematics achievement classes.

Relatedly, the current study measured arithmetic principle understanding with the application of procedures paradigm, which captures students' competence in directly applying the principles when solving arithmetic equations. However, students may be aware of certain arithmetic principles but do not apply the principles themselves (Siegler & Crowley, 1994). Future researchers may utilise other assessment modalities (e.g., explicit recognition, evaluation of examples; Prather & Alibali, 2009; Wong et al., 2021) to gain a more comprehensive account of the relationship between arithmetic principle understanding and mathematics development.

4.9. Conclusion

In conclusion, our findings uncovered the specific contributions of various skills in classifying students into mathematics learning disability (MLD), average achieving (AA), high achieving (HA), and mathematically gifted (MG) classes. Verbal working memory, fraction magnitude understanding, and number sentence construction was associated with MLD membership, verbal working memory, spatial skills, and fraction magnitude understanding was associated with HA membership, and verbal working memory and arithmetic principle understanding was associated with MG membership. The current findings pointed to the utility of specific skill trainings that could benefit students at different mathematics achievement levels. Educators may tailor mathematics interventions based on students' mathematics achievement levels to better suit their specific learning needs.

CRediT authorship contribution statement

Charles Chiu Hung Yip: Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis. **Xiangzi Ouyang:** Writing – review & editing, Methodology. **Eason Sai-Kit Yip:** Writing – review & editing, Writing – original draft, Project administration, Investigation, Data curation. **Christine Kong-Yan Tong:** Writing – review & editing, Writing – original draft. **Terry Tin-Yau Wong:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Funding acquisition, Data curation, Conceptualization.

Declaration of competing interest

The authors declare no conflict of interest.

Acknowledgements

This work was supported by the General Research Fund of the Research Grants Council of Hong Kong (Project Numbers: 17605721, 17607323, 18600318) awarded to the corresponding author.

References

Andersson, U. (2010). Skill development in different components of arithmetic and basic cognitive functions: Findings from a 3-year longitudinal study of children with different types of learning difficulties. *Journal of Educational Psychology, 102*(1), 115–134. https://doi.org/10.1037/a0016838

Andersson, U., & Östergren, R. (2012). Number magnitude processing and basic cognitive functions in children with mathematical learning disabilities. *Learning and Individual Differences, 22*(6), 701–714. https://doi.org/10.1016/j.lindif.2012.05.004

Ashkenazi, S., & Danan, Y. (2017). The role of mathematical anxiety and working memory on the performance of different types of arithmetic tasks. *Trends in Neuroscience and Education, 7*, 1–10. https://doi.org/10.1016/j.tine.2017.05.001

Atit, K., Power, J. R., Pigott, T., Lee, J., Geer, E. A., Uttal, D. H., … Sorby, S. A. (2022). Examining the relations between spatial skills and mathematical performance: A

meta-analysis. *Psychonomic Bulletin & Review, 29*, 699–720. https://doi.org/10.3758/s13423-021-02012-w
Attout, L., Noël, M.-P., & Majerus, S. (2014). The relationship between working memory for serial order and numerical development: A longitudinal study. *Developmental Psychology, 50*(6), 1667–1679. https://doi.org/10.1037/a0036496
Bakker, M., Torbeyns, J., Verschaffel, L., & De Smedt, B. (2022). The mathematical, motivational, and cognitive characteristics of high mathematics achievers in primary school. *Journal of Educational Psychology, 114*(5), 992–1004. https://doi.org/10.1037/edu0000678
Baroody, A. J., Ginsburg, H. P., & Waxman, B. (1983). Children's use of mathematical structure. *Journal for Research in Mathematics Education, 14*(3), 156–168. https://doi.org/10.2307/748379
Barrouillet, P., Fayol, M., & Lathulière, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioral Development, 21*(2), 253–276. https://doi.org/10.1080/016502597384857
Berg, D. H., & McDonald, P. A. (2018). Differences in mathematical reasoning between typically achieving and gifted children. *Journal of Cognitive Psychology, 30*(3), 281–291. https://doi.org/10.1080/20445911.2018.1457034
Boonen, A. J. H., van der Schoot, M., van Wesel, F., de Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students. *Contemporary Educational Psychology, 38*(3), 271–279. https://doi.org/10.1016/j.cedpsych.2013.05.001
Cantin, R. H., Gnaedinger, E. K., Gallaway, K. C., Hesson-McInnis, M. S., & Hund, A. M. (2016). Executive functioning predicts reading, mathematics, and theory of mind during the elementary years. *Journal of Experimental Child Psychology, 146*, 66–78. https://doi.org/10.1016/j.jecp.2016.01.014
Caviola, S., Mammarella, I. C., & Szűcs, D. (2022). Individual differences in mathematical abilities and competencies. In M. Danesi (Ed.), *Handbook of cognitive mathematics* (pp. 1–32). Springer. https://doi.org/10.1007/978-3-030-44982-7_28-1.
Chamberlin, M. T., & Chamberlin, S. A. (2010). Enhancing preservice teacher development: Field experiences with gifted students. *Journal for the Education of the Gifted, 33*(3), 381–416. https://doi.org/10.1177/016235321003300305
Ching, B. H. H., & Nunes, T. (2017). The importance of additive reasoning in children's mathematical achievement: A longitudinal study. *Journal of Educational Psychology, 109*(4), 477–508. https://doi.org/10.1037/edu0000154
Corsi, P. M. (1973). Human memory and the medial temporal region of the brain. *Dissertation Abstracts International, 34*, 891B.
Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3*(2), 63–68. https://doi.org/10.1016/j.tine.2013.12.001
Cragg, L., Keeble, S., Richardson, S., Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition, 162*, 12–26. https://doi.org/10.1016/j.cognition.2017.01.014
De Visscher, A., & Noël, M.-P. (2013). A case study of arithmetic facts dyscalculia caused by a hypersensitivity-to-interference in memory. *Cortex, 49*(1), 50–70. https://doi.org/10.1016/j.cortex.2012.01.003
Dehaene, S. (2004). Evolution of human cortical circuits for reading and arithmetic: The "neuronal recycling" hypothesis. In S. Dehaene, J. R. Duhamel, M. Hauser, & G. Rizzolatti (Eds.), *From monkey brain to human brain: A Fyssen Foundation symposium* (pp. 133–157). MIT Press. https://doi.org/10.7551/mitpress/3136.003.0012.
Desco, M., Navas-Sanchez, F. J., Sanchez-González, J., Reig, S., Robles, O., Franco, C., … Arango, C. (2011). Mathematically gifted adolescents use more extensive and more bilateral areas of the fronto-parietal network than controls during executive functioning and fluid reasoning tasks. *Neuroimage, 57*(1), 281–292. https://doi.org/10.1016/j.neuroimage.2011.03.063
Duan, X., Shi, J., Wu, J., Mou, Y., Cui, H., & Wang, G. (2009). Electrophysiological correlates for response inhibition in intellectually gifted children: A Go/NoGo study. *Neuroscience Letters, 457*(1), 45–48. https://doi.org/10.1016/j.neulet.2009.04.006
Dyson, N. I., Jordan, N. C., Rodrigues, J., Barbieri, C., & Rinne, L. (2020). A fraction sense intervention for sixth graders with or at risk for mathematics difficulties. *Remedial and Special Education, 41*(4), 244–254. https://doi.org/10.1177/0741932518807139
Eaves, J., Attridge, N., & Gilmore, C. (2019). Increasing the use of conceptually-derived strategies in arithmetic: Using inversion problems to promote the use of associativity shortcuts. *Learning and Instruction, 61*, 84–98. https://doi.org/10.1016/j.learninstruc.2019.01.004
Ekstrom, R. B., French, J. W., & Harmon, H. H. (1976). *Manual for kit of factor-referenced tests*. Educational Testing Service.
Emslander, V., & Scherer, R. (2022). The relation between executive functions and math intelligence in preschool children: A systematic review and meta-analysis. *Psychological Bulletin, 148*(5–6), 337–369. https://doi.org/10.1037/bul0000369
Ficici, A., & Siegle, D. (2008). International teachers' judgment of gifted mathematics student characteristics. *Gifted and Talented International, 23*(1), 23–38. https://doi.org/10.1080/15332276.2008.11673510
Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review, 10*, 29–44. https://doi.org/10.1016/j.edurev.2013.05.003
Gadanidis, G., Hughes, J., & Cordy, M. (2011). Mathematics for gifted students in an arts- and technology-rich setting. *Journal for the Education of the Gifted, 34*(3), 397–433. https://doi.org/10.1177/016235321103400303
Ganley, C. M., & Vasilyeva, M. (2011). Sex differences in the relation between math performance, spatial skills, and attitudes. *Journal of Applied Developmental Psychology, 32*(4), 235–242. https://doi.org/10.1016/j.appdev.2011.04.001
Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*(2), 345–362. https://doi.org/10.1037/0033-2909.114.2.345
Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental & Behavioral Pediatrics, 32*(3), 250–263. https://doi.org/10.1097/DBP.0b013e318209edef
Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33*(3), 277–299. https://doi.org/10.1080/87565640801982361
Gerst, E. H., Cirino, P. T., Fletcher, J. M., & Yoshida, H. (2017). Cognitive and behavioral rating measures of executive function as predictors of academic outcomes in children. *Child Neuropsychology, 23*(4), 381–407. https://doi.org/10.1080/09297049.2015.1120860
Gilmore, C., Keeble, S., Richardson, S., & Cragg, L. (2015). The role of cognitive inhibition in different components of arithmetic. *ZDM, 47*, 771–782. https://doi.org/10.1007/s11858-014-0659-y
Gilmore, C., Keeble, S., Richardson, S., & Cragg, L. (2017). The interaction of procedural skill, conceptual understanding and working memory in early mathematics achievement. *Journal of Numerical Cognition, 3*(2), 400–416. https://doi.org/10.5964/jnc.v3i2.51
Giofrè, D., Mammarella, I. C., Ronconi, L., & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. *Learning and Individual Differences, 23*, 114–122. https://doi.org/10.1016/j.lindif.2012.09.012
Hamdan, N., & Gunderson, E. A. (2017). The number line is a critical spatial-numerical representation: Evidence from a fraction intervention. *Developmental Psychology, 53*(3), 587–596. https://doi.org/10.1037/dev0000252.supp
Hawes, Z. C. K., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology, 58*(1), 112–137. https://doi.org/10.1037/dev0001281
Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology, 87*(1), 18–32. https://doi.org/10.1037/0022-0663.87.1.18
Heinze, A. (2005). Differences in problem solving strategies of mathematically gifted and non-gifted elementary students. *International Education Journal, 6*(2), 175–183.
Johnson, J., Im-Bolter, N., & Pascual-Leone, J. (2003). Development of mental attention in gifted and mainstream children: The role of mental capacity, inhibition, and speed of processing. *Child Development, 74*(6), 1594–1614. https://doi.org/10.1046/j.1467-8624.2003.00626.x
Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834–850. https://doi.org/10.1111/1467-8624.00571
Karagiannakis, G. N., & Cooreman, A. (2014). Focused MLD intervention based on the classification of MLD subtypes. In S. Chinn (Ed.), *The Routledge international handbook of dyscalculia and mathematical learning difficulties* (pp. 265–276). Routledge. https://doi.org/10.4324/9781315740713.ch19.
Khng, K. H., & Lee, K. (2009). Inhibiting interference from prior knowledge: Arithmetic intrusions in algebra word problem solving. *Learning and Individual Differences, 19*(2), 262–268. https://doi.org/10.1016/j.lindif.2009.01.004
Klesczewski, J., Brandenburg, J., Fischbach, A., Schuchardt, K., Grube, D., Hasselhorn, M., & Büttner, G. (2018). Development of working memory from grade 3 to 5: Differences between children with and without mathematical learning difficulties. *International Journal of Disability, Development and Education, 65*(5), 509–525. https://doi.org/10.1080/1034912X.2017.1419555
Krawec, J. L. (2014). Problem representation and mathematical problem solving of students of varying math ability. *Journal of Learning Disabilities, 47*(2), 103–115. https://doi.org/10.1177/0022219412436976
Kroesbergen, E. H., Huijsmans, M. D. E., & Friso-van den Bos, I. (2023). A meta-analysis on the differences in mathematical and cognitive skills between individuals with and without mathematical learning disabilities. *Review of Educational Research, 93*(5), 718–755. https://doi.org/10.3102/00346543221132773
Kyttälä, M., Aunio, P., & Hautamäki, J. (2010). Working memory resources in young children with mathematical difficulties. *Scandinavian Journal of Psychology, 51*(1), 1–15. https://doi.org/10.1111/j.1467-9450.2009.00736.x
Landis, R. N., & Reschly, A. L. (2013). Reexamining gifted underachievement and dropout through the lens of student engagement. *Journal for the Education of the Gifted, 36*(2), 220–249. https://doi.org/10.1177/0162353213480864
Lee, K., & Lee, H. W. (2019). Inhibition and mathematical performance: Poorly correlated, poorly measured, or poorly matched? *Child Development Perspectives, 13*(1), 28–33. https://doi.org/10.1111/cdep.12304
Lee, K., Ng, E. L., & Ng, S. F. (2009). The contributions of working memory and executive functioning to problem representation and solution generation in algebraic word problems. *Journal of Educational Psychology, 101*(2), 373–387. https://doi.org/10.1037/a0013843
LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x
Lein, A. E., Jitendra, A. K., & Harwell, M. R. (2020). Effectiveness of mathematical word problem solving interventions for students with learning disabilities and/or mathematics difficulties: A meta-analysis. *Journal of Educational Psychology, 112*(7), 1388–1408. https://doi.org/10.1037/edu0000453

Lemaire, P., & Lecacheur, M. (2011). Age-related changes in children's executive functions and strategy selection: A study in computational estimation. *Cognitive Development, 26*(3), 282–294. https://doi.org/10.1016/j.cogdev.2011.01.002

Leung, C. O.-Y., & Wong, T. T.-Y. (2023). How are spatial ability and math related? The mediating roles of numerical magnitude knowledge, understanding of arithmetic operations, and word-problem representation. *Journal of Educational Psychology, 115* (7), 969–984. https://doi.org/10.1037/edu0000817

Lewis, A. B., & Mayer, R. E. (1987). Students' miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology, 79*, 363–371. https://doi.org/10.1037/0022-0663.79.4.363

Lewis, K. E., & Fisher, M. B. (2016). Taking stock of 40 years of research on mathematical learning disability: Methodological issues and future directions. *Journal for Research in Mathematics Education, 47*(4), 338–371. https://doi.org/10.5951/jresematheduc.47.4.0338

Lin, X., Peng, P., & Zeng, J. (2021). Understanding the relation between mathematics vocabulary and mathematics performance: A meta-analysis. *The Elementary School Journal, 121*(3), 504–540. https://doi.org/10.1086/712504

Maggio, M. R., & Sayler, M. (2013). Trying out acceleration for mathematically talented fifth graders. *Gifted Child Today, 36*(1), 20–26. https://doi.org/10.1177/1076217512465284

McNeil, N. M., Hornburg, C. B., Fuhs, M. W., & O'Rear, C. D. (2017). Understanding children's difficulties with mathematical equivalence. In D. C. Geary, D. B. Berch, R. Ochsendorf, & K. M. Koepke (Eds.), *Acquisition of complex arithmetic skills and higher-order mathematics concepts* (pp. 167–195). Elsevier Academic Press. https://doi.org/10.1016/B978-0-12-805086-6.00008-4.

Morsanyi, K., van Bers, B. M. C. W., McCormack, T., & McGourty, J. (2018). The prevalence of specific learning disorder in mathematics and comorbidity with other developmental disorders in primary school-age children. *British Journal of Psychology, 109*(4), 917–940. https://doi.org/10.1111/bjop.12322

Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities, 40* (5), 458–478. https://doi.org/10.1177/00222194070400050901

Myers, T., Carey, E., & Szűcs, D. (2017). Cognitive and neural correlates of mathematical giftedness in adults and children: A review. *Frontiers in Psychology, 8*, 1646. https://doi.org/10.3389/fpsyg.2017.01646

Namkung, J. M., Fuchs, L. S., & Koziol, N. (2018). Does initial learning about the meaning of fractions present similar challenges for students with and without adequate whole-number skill? *Learning and Individual Differences, 61*, 151–157. https://doi.org/10.1016/j.lindif.2017.11.018

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator, 34*(2), 29–36. https://doi.org/10.1037/a0016127

Ng, J., Lee, K., & Khng, K. H. (2017). Irrelevant information in math problems need not be inhibited: Students might just need to spot them. *Learning and Individual Differences, 60*, 46–55. https://doi.org/10.1016/j.lindif.2017.09.008

Ng, S. F., & Lee, K. (2005). How primary five pupils use the model method to solve word problems. *The Mathematics Educator, 9*(1), 60–83.

Nunes, T., Bryant, P., Evans, D., Bell, D., & Barros, R. (2012). Teaching children how to include the inversion principle in their reasoning about quantitative relations. *Educational Studies in Mathematics, 79*(3), 371–388. https://doi.org/10.1007/s10649-011-9314-5

Ouyang, X., Yang, Y., Zhang, X., & Zhang, Q. (2021). Longitudinal relations between the approximate number system and symbolic number skills in preschool children. *Journal of Experimental Child Psychology, 212*, Article 105254. https://doi.org/10.1016/j.jecp.2021.105254

Ouyang, X., Zhang, X., Räsänen, P., Koponen, T., & Lerkkanen, M. K. (2023). Subtypes of mathematical learning disability and their antecedents: A cognitive diagnostic approach. *Child Development, 94*(3), 633–647. https://doi.org/10.1111/cdev.13884

Ouyang, X., Zhang, X., & Zhang, Q. (2022). Spatial skills and number skills in preschool children: The moderating role of spatial anxiety. *Cognition, 225*, Article 105165. https://doi.org/10.1016/j.cognition.2022.105165

Özdemir, D. A., & Bostan, M. I. (2021). Mathematically gifted students' differentiated needs: What kind of support do they need? *International Journal of Mathematical Education in Science and Technology, 52*(1), 65–83. https://doi.org/10.1080/0020739X.2019.1658817

Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?* National Research and Development Centre for Adult Literacy and Numeracy.

Peng, P., & Fuchs, D. (2016). A meta-analysis of working memory deficits in children with learning difficulties: Is there a difference between verbal domain and numerical domain? *Journal of Learning Disabilities, 49*(1), 3–20. https://doi.org/10.1177/0022219414521667

Peng, P., Wang, T., Wang, C., & Lin, X. (2019). A meta-analysis on the relation between fluid intelligence and reading/mathematics: Effects of tasks, age, and social economics status. *Psychological Bulletin, 145*(2), 189–236. https://doi.org/10.1037/bul0000182

Prather, R. W., & Alibali, M. W. (2009). The development of arithmetic principle knowledge: How do we know what learners know? *Developmental Review, 29*(4), 221–248. https://doi.org/10.1016/j.dr.2009.09.001

Pring, L., Ryder, N., Crane, L., & Hermelin, B. (2010). Local and global processing in savant artists with autism. *Perception, 39*(8), 1094–1103. https://doi.org/10.1068/p6674

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences, 20*(2), 110–122. https://doi.org/10.1016/j.lindif.2009.10.005

Raven, J., Raven, J. C., & Court, J. H. (2003). *Manual for Raven's progressive matrices and vocabulary scales*. Harcourt Assessment.

Renzulli, J. S., & Park, S. (2000). Gifted dropouts: The who and the why. *Gifted Child Quarterly, 44*(4), 261–271. https://doi.org/10.1177/001698620004400407

Resnick, I., Jordan, N. C., Hansen, N., Rajan, V., Rodrigues, J., Siegler, R. S., & Fuchs, L. S. (2016). Developmental growth trajectories in understanding of fraction magnitude from fourth through sixth grade. *Developmental Psychology, 52*(5), 746–757. https://doi.org/10.1037/dev0000102

Reusser, K. (1990). From text to situation to equation: Cognitive simulation of understanding and solving mathematical word problems. In H. Mandl, E. de Corte, N. Bennett, & H. F. Friedrich (Eds.), *Learning and instruction: European research in an international context* (pp. 477–498). Pergamon Press.

Rickard, T. C., Healy, A. F., & Bourne, L. E. (1994). On the cognitive structure of basic arithmetic skills: Operation, order, and symbol transfer effects. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 20*(5), 1139–1153. https://doi.org/10.1037/0278-7393.20.5.1139

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Roebers, C. M., Cimeli, P., Röthlisberger, M., & Neuenschwander, R. (2012). Executive functioning, metacognition, and self-perceived competence in elementary school children: An explorative study on their interrelations and their role for school achievement. *Metacognition and Learning, 7*(3), 151–173. https://doi.org/10.1007/s11409-012-9089-9

Schmerold, K., Bock, A., Peterson, M., Leaf, B., Vennergrund, K., & Pasnak, R. (2017). The relations between patterning, executive function, and mathematics. *The Journal of Psychology, 151*(2), 207–228. https://doi.org/10.1080/00223980.2016.1252708

Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development, 89*(5), 1467–1484. https://doi.org/10.1111/cdev.13068

Scrucca, L., Fraley, C., Murphy, T. B., & Raftery, A. E. (2023). *Model-based clustering, classification, and density estimation using mclust in R. Chapman and Hall/CRC*. https://doi.org/10.1201/9781003277965

Siegler, R. S. (2016). Magnitude knowledge: The common core of numerical development. *Developmental Science, 19*, 341–361. https://doi.org/10.1111/desc.12395

Siegler, R. S., & Araya, R. (2005). A computational model of conscious and unconscious strategy discovery. *Advances in Child Development and Behavior, 33*, 1–42. https://doi.org/10.1016/S0065-2407(05)80003-5

Siegler, R. S., & Crowley, K. (1994). Constraints on learning in nonprivileged domains. *Cognitive Psychology, 27*(2), 194–226. https://doi.org/10.1006/cogp.1994.1016

Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives, 8*(3), 144–150. https://doi.org/10.1111/cdep.12077

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14*(3), 237–243. https://doi.org/10.1111/1467-9280.02438

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games-but not circular ones-improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*(3), 545–560. https://doi.org/10.1037/a0014239

Sorby, S. A., Duffy, G., & Yoon, S. Y. (2022). Math instrument development for examining the relationship between spatial and mathematical problem-solving skills. *Education Sciences, 12*(11), 828. https://doi.org/10.3390/educsci12110828

St Clair-Thompson, H. L., & Gathercole, S. E. (2006). Executive functions and achievements in school: Shifting, updating, inhibition, and working memory. *Quarterly Journal of Experimental Psychology, 59*(4), 745–759. https://doi.org/10.1080/17470210500162854

Stroop, J. R. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology, 18*(6), 643–662. https://doi.org/10.1037/h0054651

Sun, Z., & Xie, K. (2020). How do students prepare in the pre-class setting of a flipped undergraduate math course? A latent profile analysis of learning behavior and the impact of achievement goals. *The Internet and Higher Education, 46*, Article 100731. https://doi.org/10.1016/j.iheduc.2020.100731

Swanson, H. L. (2011). Working memory, attention, and mathematical problem solving: A longitudinal study of elementary school children. *Journal of Educational Psychology, 103*(4), 821–837. https://doi.org/10.1037/a0025114

Swanson, H. L. (2020). Specific learning disabilities as a working memory deficit: A model revisited. In A. J. Martin, R. A. Sperling, & K. J. Newton (Eds.), *Handbook of educational psychology and students with special needs* (pp. 19–51). Routledge/Taylor & Francis Group. https://doi.org/10.4324/9781315100654-3.

Tam, Y. P., Wong, T. T.-Y., & Chan, W. W. L. (2019). The relation between spatial skills and mathematical abilities: The mediating role of mental number line representation. *Contemporary Educational Psychology, 56*, 14–24. https://doi.org/10.1016/j.cedpsych.2018.10.007

Thompson, C. G., Kim, R. S., Aloe, A. M., & Becker, B. J. (2017). Extracting the variance inflation factor and other multicollinearity diagnostics from typical regression results. *Basic and Applied Social Psychology, 39*(2), 81–90. https://doi.org/10.1080/01973533.2016.1277529

Thompson, J. M., Nuerk, H. C., Moeller, K., & Kadosh, R. C. (2013). The link between mental rotation ability and basic numerical representations. *Acta Psychologica, 144* (2), 324–331. https://doi.org/10.1016/j.actpsy.2013.05.009

Tolar, T. D., Fuchs, L., Cirino, P. T., Fuchs, D., Hamlett, C. L., & Fletcher, J. M. (2012). Predicting development of mathematical word problem solving across the intermediate grades. *Journal of Educational Psychology, 104*(4), 1083–1093. https://doi.org/10.1037/a0029020

Toll, S. W. M., Kroesbergen, E. H., & Van Luit, J. E. H. (2016). Visual working memory and number sense: Testing the double deficit hypothesis in mathematics. *British Journal of Educational Psychology, 86*(3), 429–445. https://doi.org/10.1111/bjep.12116

Torbeyns, J., De Smedt, B., Stassens, N., Ghesquiere, P., & Verschaffel, L. (2009). Solving subtraction problems by means of indirect addition. *Mathematical Thinking and Learning, 11*(1–2), 79–91. https://doi.org/10.1080/10986060802583998

Torbeyns, J., Peters, G., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2016). Children's understanding of the addition/subtraction complement principle. *The British Journal of Educational Psychology, 86*(3), 382–396. https://doi.org/10.1111/bjep.12113

van Garderen, D., & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research & Practice, 18*(4), 246–254. https://doi.org/10.1111/1540-5826.00079

Vanslambrouck, S., Zhu, C., Pynoo, B., Lombaerts, K., Tondeur, J., & Scherer, R. (2019). A latent profile analysis of adult students' online self-regulation in blended learning environments. *Computers in Human Behavior, 99*, 126–136. https://doi.org/10.1016/j.chb.2019.05.021

Verschaffel, L., Depaepe, F., & Van Dooren, W. (2020). Word problems in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 908–911). Springer. https://doi.org/10.1007/978-3-030-15789-0_163.

Wang, L., Cao, C., Zhou, X., & Qi, C. (2022). Spatial abilities associated with open math problem solving. *Applied Cognitive Psychology, 36*(2), 306–317. https://doi.org/10.1002/acp.3919

Watchorn, R. P. D., Bisanz, J., Fast, L., LeFevre, J. A., Skwarchuk, S. L., & Smith-Chant, B. L. (2014). Development of mathematical knowledge in young children: Attentional skill and the use of inversion. *Journal of Cognition and Development, 15*(1), 161–180. https://doi.org/10.1080/15248372.2012.742899

Wechsler, D. (2009). *Wechsler Individual Achievement Test-III (WIAT-III)*. Pearson.

Wong, T. T.-Y. (2018). Is conditional reasoning related to mathematical problem solving? *Developmental Science, 21*(5), Article e12644. https://doi.org/10.1111/desc.12644

Wong, T. T.-Y. (2021). Components of mathematical competence in middle childhood. *Child Development Perspectives, 15*(1), 18–23. https://doi.org/10.1111/cdep.12394

Wong, T. T.-Y. (2023). Cognitive predictors of children's arithmetic principle understanding. *Journal of Experimental Child Psychology, 227*, Article 105579. https://doi.org/10.1016/j.jecp.2022.105579

Wong, T. T.-Y., & Ho, C. S.-H. (2017). Component processes in arithmetic word-problem solving and their correlates. *Journal of Educational Psychology, 109*(4), 520–531. https://doi.org/10.1037/edu0000149

Wong, T. T.-Y., Leung, C. O. Y., & Kwan, K.-T. (2021). Multifaceted assessment of children's inversion understanding. *Journal of Experimental Child Psychology, 207*, Article 105121. https://doi.org/10.1016/j.jecp.2021.105121

Wong, T. T.-Y., & Morsanyi, K. (2023). The link between transitive reasoning and mathematics achievement in preadolescence: The role of relational processing and deductive reasoning. *Thinking and Reasoning, 29*(4), 531–558. https://doi.org/10.1080/13546783.2022.2095031

Wong, T. T.-Y., & Yip, E. S.-K. (2023). What is the unknown? The ability to identify the semantic role of the unknown from word problems longitudinally predicts mathematical problem solving performance. *Contemporary Educational Psychology, 73*, Article 102183. https://doi.org/10.1016/j.cedpsych.2023.102183

Woolcott, G., Le Tran, T., Mulligan, J., Davis, B., & Mitchelmore, M. (2022). Towards a framework for spatial reasoning and primary mathematics learning: An analytical synthesis of intervention studies. *Mathematics Education Research Journal, 34*(1), 37–67. https://doi.org/10.1007/s13394-020-00318-x

Xie, F., Zhang, L., Chen, X., & Xin, Z. (2020). Is spatial ability related to mathematical ability: A meta-analysis. *Educational Psychology Review, 32*(1), 113–155. https://doi.org/10.1007/s10648-019-09496-y

Yang, X., & Yu, X. (2021). The relationship between mental rotation and arithmetic: Do number line estimation, working memory, or place-value concept matter? *British Journal of Educational Psychology, 91*(3), 793–810. https://doi.org/10.1111/bjep.12403

Yazdani, S., Soluki, S., Arjmandnia, A. A., Fathabadi, J., Hassanzadeh, S., & Nejati, V. (2021). Spatial ability in children with mathematics learning disorder (MLD) and its impact on executive functions. *Developmental Neuropsychology, 46*(3), 232–248. https://doi.org/10.1080/87565641.2021.1913165

Yip, E. S.-K., Wong, T. T.-Y., Cheung, S. H., & Chan, K. K. W. (2020). Do children with mathematics learning disability in Hong Kong perceive word problems differently? *Learning and Instruction, 68*, Article 101352. https://doi.org/10.1016/j.learninstruc.2020.101352

Yip, E. S.-K., Wong, T. T.-Y., & Kwan, K. T. (2023). The relation between complement understanding and computational skills: A random intercept cross-lagged panel model. *Developmental Psychology, 59*(3), 431–441. https://doi.org/10.1037/dev0001481

Zhang, X., Räsänen, P., Koponen, T., Aunola, K., Lerkkanen, M. K., & Nurmi, J. E. (2020). Early cognitive precursors of children's mathematics learning disability and persistent low achievement: A 5-year longitudinal study. *Child Development, 91*(1), 7–27. https://doi.org/10.1111/cdev.13123

Zhu, M., Cai, D., & Leung, A. W. S. (2017). Number line estimation predicts mathematical skills: Difference in grades 2 and 4. *Frontiers in Psychology, 8*, 1576. https://doi.org/10.3389/fpsyg.2017.01576