Anisotropic charged fluid spheres in *D* space–time dimensions

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The equations describing the hydrostatic equilibrium (mass continuity and Tolman–Oppenheimer–Volkoff) of a static anisotropic general relativistic fluid sphere are obtained in D ($D\geqslant 4$) space–time dimensions in the presence of a cosmological constant. The formalism thus developed is used to study homogeneous anisotropic constant density charged fluid spheres and homogeneous anisotropic charged spheres with a neutral isotropic core in higher dimensions. For these configurations and with a particular choice of the proper charge density a complete solution of the coupled Einstein–Maxwell equations is obtained. © *2000 American Institute of Physics.* [S0022-2488(00)04807-6]

I. INTRODUCTION

The study of the static anisotropic fluid spheres is important for relativistic astrophysics. The theoretical investigations of Ruderman[1] about more realistic stellar models show that the stellar matter may be anisotropic at least in certain very high density ranges ($\rho > 10^{15}$ g/cm^3), where the nuclear interactions must be treated relativistically. According to these views in such massive stellar objects the radial pressure may not be equal to the tangential one. No celestial body is composed of purely perfect fluid. Anisotropy in fluid pressure could be introduced by the existence of a solid core, by the presence of type 3A superfluid or by other physical phenomena. The starting point in the study of fluid spheres is represented by the interior Schwarzschild solution from which all problems involving spherical symmetry can be modelled. Bowers and Liang[2] have investigated the possible importance of locally anisotropic equations of state for relativistic fluid spheres by generalizing the equations of hydrostatic equilibrium to include the effects of local anisotropy. Their study shows that anisotropy may have non-negligible effects on such parameters as maximum equilibrium mass and surface red-shift. Consenza, Herrera, Esculpi, and Witten,[3] Bayin,[4] Krori, Bargohain and Devi,[5] Maharaj and Maartens[6] have obtained different exact solutions of the Einstein field equations describing the interior gravitational field of anisotropic fluid spheres. Bohra and Mehra[7] and Omote and Sato[8] have studied charged spheres in the presence of matter with mass-charge and radius charge relations emerging from the static solution. Several other anisotropic fluid sphere configurations have been analyzed using various ansatz.[9–12] Analytical solutions of the Einstein–Maxwell equations for various charged static spherically symmetric configurations (both isotropic and anisotropic) have been obtained in the papers.[13–19]

Lately there has been an increasing interest in the study of compact astrophysical objects in D space–time dimensions, prior to any compactification. Hence Krori, Borgohain, and Das[20] have extended the interior Schwarzschild solution with vanishing normal pressure of Florides[21] to D space–time dimensions in the presence of a cosmological constant. Wolf has analyzed fluid spheres[22] and charged fluid spheres[23] in D space–time dimensions with the condition of vanishing normal pressure. The Tolman–Oppenheimer–Volkoff (TOV) equation has been generalized to D

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$(D\geqslant 4)$ space–time dimensions with isotropic fluid pressures and the model of the homogeneous star has been solved in the paper[24] while charged isotropic fluid D-dimensional spheres in the presence of a cosmological constant have been considered in the paper.[25]

The purpose of the present paper is to obtain the equations which describe the hydrostatic equilibrium of an anisotropic, spherically symmetric, static fluid configuration in D space–time dimensions, $D\geqslant 4$ and in the presence of a cosmological constant (generalized mass-continuity and TOV equations). The formalism thus developed is used to study the homogeneous charged fluid sphere in D $(D\geqslant 4)$ space–time dimensions with a particular choice of the proper charge density and with anisotropy factor proportional to the electric field. A generalization of the model in the case of a homogeneous anisotropic charged D-dimensional sphere with neutral core is also developed. Exact solutions of the TOV and gravitational field equations are obtained and mass-charge and radius-charge relations are deduced in both cases.

The present paper is organized as follows: In Sec. II, using the Einstein gravitational field equations in D space–time dimensions we deduce the generalized mass-continuity and TOV equations for a static anisotropic fluid sphere. A nondimensional form of these equations is also obtained. The hydrostatic equilibrium equations for an anisotropic charged fluid sphere with constant mass density and with a particular choice of the proper charge are formulated in Sec. III and their exact solutions are found in Sec. IV. The case of a homogeneous D-dimensional homogeneous charged sphere with neutral core is considered in Sec. V. The results are summarized in Sec. VI.

II. GENERALIZED TOLMAN–OPPENHEIMER–VOLKOFF EQUATION IN *D* SPACE–TIME DIMENSIONS

In D $(D\geqslant 4)$ space–time dimensions the spherically symmetric line element takes the form,[23]

$$\begin{aligned} ds^2 = {} & e^{\nu(r)}(dx^0)^2 - e^{\lambda(r)}dr^2 - r^2 d\theta_1^2 - r^2\sin^2\theta_1 d\theta_2^2 - r^2\sin^2\theta_1\sin^2\theta_2 d\theta_3^2 \\ & - \cdots - r^2\sin^2\theta_1\cdots\sin^2\theta_{D-3}d\varphi^2. \end{aligned} \tag{1}$$

Here,

$$x^0 = ct,\quad x^1 = r,\quad x^2=\theta_1,\quad x^3=\theta_2,\ldots,\quad x^{D-2}=\theta_{D-3},\quad x^{D-1}=\varphi$$

(r is the radial coordinate in D space–time dimensions) with domain $0\leqslant r<\infty$, $0\leqslant\theta_i\leqslant\pi(i=1,\ldots,D-3)$, $0\leqslant\varphi\leqslant 2\pi$. The Einstein gravitational field equations in the presence of a cosmological constant are

$$R_i^k - \frac{1}{2}R\,\delta_i^k = \frac{8\pi G}{c^4}T_i^k + \frac{8\pi G}{c^4}\Lambda\,\delta_i^k. \tag{2}$$

For a spherically symmetric anisotropic matter distribution the components of the energy-momentum tensor are given by

$$T_i^k = (\rho c^2 + p_\perp)u_i u^k - p_\perp\delta_i^k + (p_r - p_\perp)\chi_i\chi^k, \tag{3}$$

where u^i is the D-dimensional velocity, $u^i=\delta_0^i e^{-(\nu/2)}$, χ^i is the unit spacelike vector in the radial direction, $\chi^i=\delta_1^i e^{-(\lambda/2)}$, ρ is the energy density, p_r is the pressure in the direction of χ_i (normal pressure), and $p_\perp$ is the pressure on the $(D-2)$ space orthogonal to χ_i (transversal pressure). In the present paper we suppose $p_r\neq p_\perp$. The case $p_r=p_\perp$ corresponds to the isotropic fluid sphere. $\Delta=p_\perp-p_r$ is a measure of the anisotropy and is called the anisotropy factor.[15]

For the metric (1), the gravitational field equations (2) become

$$\frac{(D-2)\lambda' e^{-\lambda}}{2r} - \frac{(D-2)(D-3)(e^{-\lambda}-1)}{2r^2} = \frac{8\pi G}{c^2}\rho + \frac{8\pi G}{c^4}\Lambda, \tag{4}$$

$$\frac{(D-2)\nu' e^{-\lambda}}{2r}+\frac{(D-2)(D-3)(e^{-\lambda}-1)}{2r^2}=\frac{8\pi G}{c^4}p_r-\frac{8\pi G}{c^4}\Lambda, \tag{5}$$

$$e^{-\lambda}\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\nu'\lambda'}{4}+\frac{(D-2)(\nu'-\lambda')}{4r}\right)+\frac{(D-3)(D-4)(e^{-\lambda}-1)}{2r^2}=\frac{8\pi G}{c^4}p_\perp-\frac{8\pi G}{c^4}\Lambda, \tag{6}$$

where we have denoted $'=d/dr$. From the Bianchi identities $T^k_{i;k}=0$ it follows

$$\nu'=-\frac{2p_r'}{\rho c^2+p_r}+\frac{2(D-2)(p_\perp-p_r)}{(\rho c^2+p_r)r}. \tag{7}$$

From Eq. (4) we immediately obtain

$$\frac{d(r^{D-3}e^{-\lambda})}{dr}=(D-3)r^{D-4}-\frac{8\pi G}{c^2}\frac{2}{D-2}\rho r^{D-2}-\frac{16\pi G\Lambda}{(D-2)c^4}r^{D-2} \tag{8}$$

or

$$e^{-\lambda}=1-\frac{8\pi G}{c^2}\frac{2}{D-2}\frac{1}{r^{D-3}}\int_0^r\rho(r)r^{D-2}dr-\frac{16\pi G\Lambda}{(D-1)(D-2)c^4}r^2 \tag{9}$$

and

$$e^{-\lambda}=1-\frac{2GF(D)M(r)}{r^{D-3}}-\frac{16\pi G\Lambda}{(D-1)(D-2)c^4}r^2, \tag{10}$$

where we have denoted

$$F(D)=\frac{1}{(D-2)2^{D-5}}\quad\text{and}\quad M(r)=\frac{1}{c^2}\int_0^r\pi 2^{D-2}\rho(r)r^{D-2}dr. \tag{11}$$

Using Eqs. (7), (9), and (11) in the gravitational field equation (5) we obtain the generalized TOV equation in arbitrary D space–time dimensions, describing the equilibrium of an anisotropic spherically symmetric configuration in the presence of a cosmological constant,

$$\frac{dp_r}{dr}=-\frac{G(\rho c^2+p_r)\left[\frac{8\pi}{(D-2)c^4}\left(p_r-\frac{2\Lambda}{D-1}\right)r^{D-1}+(D-3)F(D)M(r)\right]}{r^{D-2}\left(1-\frac{2GF(D)M(r)}{r^{D-3}}-\frac{16\pi G\Lambda}{(D-1)(D-2)c^4}r^2\right)}+\frac{(D-2)(p_\perp-p_r)}{r}. \tag{12}$$

A dimensionless form of the generalized TOV equation (12) and of the mass continuity equation,

$$\frac{dM}{dr}=\frac{1}{c^2}2^{D-2}\pi\rho(r)r^{D-2} \tag{13}$$

can be obtained if we introduce a dimensionless independent variable η and the dimensionless functions $\varepsilon(\eta)$, $P_r(\eta)$, $P_\perp(\eta)$ by means of the transformations,

$$r=a\eta,\quad \rho=\rho_c\varepsilon,\quad p_r=\rho_c c^2P_r,\quad p_\perp=\rho_c c^2P_\perp,\quad M=M^*m. \tag{14}$$

Here a is a scale factor (a characteristic length), ρ_c and M^* being a characteristic density and mass, respectively.

With the use of (14) in Eqs. (12) and (13) we obtain the following dimensionless forms of the mass continuity and TOV equations:

$$\frac{dm}{d\eta}=\eta^{D-2}\varepsilon, \tag{15}$$

$$\frac{dP_r}{d\eta}=-\frac{(\varepsilon+P_r)[(P_r-\mu)\eta^{D-1}+(D-3)m]}{\eta^{D-2}\left(1-\frac{2m}{\eta^{D-3}}-\mu\eta^2\right)}+\frac{(D-2)(P_\perp-P_r)}{\eta}, \tag{16}$$

where we have taken

$$M^*=\frac{1}{c^2}\pi 2^{D-2}\rho_c a^{D-1},\quad a^2=\frac{(D-2)c^2}{8\pi G\rho_c},\quad \mu=\frac{2\Lambda}{(D-1)\rho_c c^2}. \tag{17}$$

If the normal and tangential pressures P_r and $P_\perp$ are independent variables then the TOV equation (16) is, from mathematical point of view, a Riccati-type equation of the form[26]

$$\frac{dP_r}{d\eta}=A(\eta)P_r^2+B(\eta)P_r+C(\eta) \tag{18}$$

with

$$A(\eta)=-\frac{\eta}{1-\frac{2m}{\eta^{D-3}}-\mu\eta^2},\quad B(\eta)=-\frac{(D-3)m+(\varepsilon-\mu)\eta^{D-1}}{\eta^{D-2}\left(1-\frac{2m}{\eta^{D-3}}-\mu\eta^2\right)}-\frac{D-2}{\eta}$$

and

$$C(\eta)=-\frac{[(D-3)m-\mu\eta^{D-1}]\varepsilon}{\eta^{D-2}\left(1-\frac{2m}{\eta^{D-3}}-\mu\eta^2\right)}+\frac{(D-2)P_\perp}{\eta}.$$

Equations (15)–(16) form a system of two coupled differential equations in four variables m, ε, P_r, and $P_\perp$. To obtain a general solution of the system we have to specify two physically reasonable functional relations among the four variables. Usually suitable forms of ε and P_r are chosen.

The system (12)–(13) or (15)–(16) must be integrated with some boundary conditions. These conditions depend on the explicit physical meaning of the energy density ρ, normal and tangential pressures p_r and $p_\perp$ and they have to be specified in every given physical situation.

III. HOMOGENEOUS ANISOTROPIC STATIC CHARGED FLUID SPHERES IN *D* SPACE–TIME DIMENSIONS

The Lagrangian of the electromagnetic field in D space–time dimensions is given by[23,27]

$$L=-\frac{1}{K}F_{ik}F^{ik}-\frac{1}{c}j^iA_i, \tag{19}$$

where j^i, $i=0,\dots,D-1$ is the D dimensional current density that for nonconducting fluids becomes $j^i=\rho_e u^i$ (with ρ_e the proper charge density), A_i is the D-dimensional potential, and K is a constant. The electromagnetic field tensor F_{ik} is defined in terms of the potential A_i through $F_{ik}=(\partial A_k/\partial x^i)-(\partial A_i/\partial x^k)$. The field tensor F_{ik} satisfies the Maxwell equations

$$F_{ik;l}+F_{kl;i}+F_{li;k}=0. \tag{20}$$

In the rest frame of reference we adopt the gauge $A_i(\Phi(r),0,\ldots,0)$. Varying (19) with respect to A_i gives the D-dimensional Maxwell equations[23]

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik})=-\frac{K}{4c}j^i. \tag{21}$$

In D ($D\geqslant 4$) space-time dimensions the energy-momentum tensor of the electromagnetic field from (19) can be represented in the form,

$$T_i^k=-\frac{4}{K}F_{il}F^{lk}+\frac{1}{K}F_{lm}F^{lm}\delta_i^k. \tag{22}$$

For a static charged fluid sphere the current density j^i has, for $r<R$ (R is the radius of the sphere), only one component,

$$j^0=\rho_e\frac{dx^0}{ds}=\rho_e e^{-(\nu/2)}. \tag{23}$$

In the following we shall consider static charged spherically symmetric configurations characterized by a particular form of the proper charge density obtained by setting

$$\rho_e=\rho_0 e^{-(\lambda/2)}, \tag{24}$$

and we shall suppose that ρ_0 is a constant.

The electromagnetic field has only one nonzero component F^{01} and the Maxwell equation (21) gives

$$F^{01}=-\frac{e^{-[(\nu+\lambda)/2]}}{r^{D-2}}Q(r), \tag{25}$$

where we have denoted

$$Q(r)=\frac{K}{4}\int_0^r\rho_0 r^{D-2}dr=\frac{K\rho_0}{4(D-1)}r^{D-1}. \tag{26}$$

The electric field intensity E is defined as usual by $E^2=-F_{01}F^{01}=Q^2(r)/r^{2(D-2)}=K^2\rho_0^2/16(D-1)^2r^2$.

In order that Q represents the charge within the $(D-1)$ dimensional sphere we have[23]

$$K=\frac{4(D-1)\pi^{(D-1)/2}}{\left(\frac{D-1}{2}\right)!} \tag{27}$$

($K=16\pi$ for $D=4$).

For the components of the energy momentum tensor of the D dimensional massive charged anisotropic fluid sphere with proper charge density given by Eq. (24) we find, by using Eqs. (26) and (22),

$$T_0^0=\rho c^2=\rho_m c^2+\frac{2}{K}\frac{Q^2(r)}{r^{2(D-2)}},\quad T_1^1=-p_r=-p_{mr}+\frac{2}{K}\frac{Q^2(r)}{r^{2(D-2)}}, \tag{28}$$

$$T_2^2=\cdots=T_{D-1}^{D-1}=-p_\perp=-p_{m\perp}-\frac{2}{K}\frac{Q^2(r)}{r^{2(D-2)}}, \tag{29}$$

where ρ_m is the mass density and p_{mr} and $p_{m\perp}$ are the normal and transversal hydrostatic pressures of the matter fluid, respectively.

In the following, we shall restrict our analysis only to the case of the homogeneous fluid sphere, that is we shall suppose that the energy density ρ_m of the matter is constant.

We shall introduce now the transformations (14) (with $\rho_c=\rho_m=\text{constant}$) in the form,

$$r=a\eta,\quad \rho=\rho_m\varepsilon,\quad p_{mr}=\rho_m c^2 P_{mr},\quad p_{m\perp}=\rho_m c^2 P_{m\perp},\quad p_r=\rho_m c^2 P_r,\quad p_\perp=\rho_m c^2 P_\perp, \tag{30}$$

which give

$$\varepsilon=1+\frac{\alpha}{2(D-1)}\eta^2,\quad P_r=P_{mr}-\frac{\alpha}{2(D-1)}\eta^2,\quad P_\perp=P_{m\perp}+\frac{\alpha}{2(D-1)}\eta^2, \tag{31}$$

where we have denoted

$$\alpha=\frac{(D-2)}{(D-1)}\frac{K\rho_0^2}{32\pi G\rho_m^2}.$$

By supposing that the dynamical anisotropy in the fluid is due to the presence of the electric field, it is reasonable to prescribe the energy-momentum tensor of the matter and the anisotropy factor such that the tangential and radial matter pressures are linearly related with the electric field. Therefore we assume that

$$\Delta=p_{m\perp}-p_{mr}=\alpha' E^2, \tag{32}$$

with α' a constant. Hence in the dimensionless variables introduced above by Eqs. (30) we have

$$P_{m\perp}-P_{mr}=\frac{\alpha\alpha'}{4(D-1)}\eta^2. \tag{33}$$

By using Eqs. (33) and (31), Eqs. (15)–(16) which describe the hydrostatic equilibrium of an anisotropic homogeneous static charged fluid sphere in D space–time dimensions take the form,

$$\frac{dm}{d\eta}=\eta^{D-2}\left[1+\frac{\alpha}{2(D-1)}\eta^2\right], \tag{34}$$

$$\frac{dP_{mr}}{d\eta}=-\frac{(1+P_{mr})\left[P_{mr}-\mu+\frac{D-3}{D-1}-\frac{2\alpha}{D^2-1}\eta^2\right]\eta}{1-\left(\frac{2}{D-1}+\mu\right)\eta^2-\frac{\alpha}{D^2-1}\eta^4}+\beta\eta, \tag{35}$$

where we have denoted

$$\beta=\frac{\alpha}{D-1}\left[1+(D-2)\left(1+\frac{\alpha'}{4}\right)\right]. \tag{36}$$

For $r>R$ from the Maxwell equations we obtain

$$F^{01} = -\frac{Q}{r^{D-2}}, \tag{37}$$

and, consequently,

$$T_0^0 = T_1^1 = \frac{Q^2}{r^{2(D-2)}}, \tag{38}$$

where $Q=$constant is the charge included within radius R.

In this case the Einstein equations (4)–(6) give $(\nu+\lambda=0)$,

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2GM_tF(D)}{c^2}\frac{1}{r^{D-3}} + \frac{32\pi GQ^2}{(D-2)(D-3)c^4}\frac{1}{r^{2(D-3)}} - \frac{16\pi G\Lambda}{(D-1)(D-2)c^4}r^2, \tag{39}$$

where M_t is the total mass of the charged fluid sphere. Equation (39) represents the D-dimensional generalization of the Reissner–Nordstrom–de Sitter solution for a central charge Q.

In order to represent a physically acceptable anisotropic fluid sphere the TOV equations (34)–(35) must be integrated with the boundary conditions,

$$m(0) = 0, \tag{40}$$

$$P_{mr}(0) = P_c. \tag{41}$$

Equation (41) assumes that the radial matter pressure P_{mr} remains finite at the center of the sphere. We shall also require vanishing of the radial pressure at the boundary

$$P_{mr}(\eta_S) = 0, \tag{42}$$

where $\eta_s = R/a$ is the value of the variable η at the surface of the sphere. In the absence of surface concentration of charge at $r=R$, we require the continuity of the field tensor F_{ik}. From the continuity of the radial electric field we obtain the condition,

$$E(R) = \frac{Q}{R^2}. \tag{43}$$

Finally, we have to match the interior line element (1) with the D-dimensional Reissner–Nordstrom–de Sitter metric across the boundary, requiring the continuity of the gravitational potentials e^{ν} and e^{λ} at $r=R$.

IV. GENERAL SOLUTION OF THE GRAVITATIONAL FIELD EQUATIONS FOR A HOMOGENEOUS ANISOTROPIC CHARGED FLUID SPHERE

With the use of the boundary condition (40) the mass-continuity Eq. (34) can be immediately integrated to give

$$m(\eta) = \frac{\eta^{D-1}}{D-1}\left[1 + \frac{\alpha}{2(D+1)}\eta^2\right]. \tag{44}$$

In order to solve the TOV equation (35) we shall introduce a new variable,

$$y = \frac{\eta^2}{2} + \frac{(D^2-1)\gamma}{4\alpha} = \frac{\eta^2}{2} + y_c, \quad y \in [y_c, y_S], \tag{45}$$

where we have denoted

$$\gamma = \frac{2}{D-1} + \mu$$

and the values of the new variable y at the center of the D-dimensional sphere and at the surface are

$$y_c = \frac{(D^2-1)\gamma}{4\alpha}$$

and

$$y_S = \frac{\eta_S^2}{2} + \frac{(D^2-1)\gamma}{4\alpha},$$

respectively. We also denote

$$\delta^2 = \frac{\left(\gamma + \frac{1}{y_c}\right) y_c^2}{\gamma}, \quad P_0 = 1 + P_{mr}. \tag{46}$$

Hence Eq. (35) becomes

$$\frac{dP_0}{dy} = -\frac{P_0\left(P_0 - \frac{\gamma}{y_c} y\right)}{\frac{\gamma}{y_c}(\delta^2 - y^2)} + \beta, \tag{47}$$

and must be integrated with the boundary condition,

$$P_0(y_c) = 1 + P_c. \tag{48}$$

Equation (47) is a Riccati-type equation. After trying many forms of P_0, we have obtained two particular solutions of the form,

$$P_{01,2} = \frac{\gamma}{y_c}(y \pm b(\delta^2 - y^2)^{1/2}), \tag{49}$$

where $b = \sqrt{(\beta y_c - \gamma)/\gamma}$. By means of the standard transformation,

$$w = \frac{P_0 - P_{01}}{P_0 - P_{02}}, \tag{50}$$

the Riccati equation (47) is transformed into a first order linear differential equation of the form

$$\frac{dw}{dy} + \frac{2b}{(\delta^2 - y^2)^{1/2}} w = 0 \tag{51}$$

with the general solution given by

$$w(y) = C \exp\left[-2b \sin^{-1}\left(\frac{y}{\delta}\right)\right], \tag{52}$$

with $C>0$ a constant of integration. The constant of integration is determined from the boundary condition $P_{mr}(y_c)=1+P_0(y_c)$, thus leading to the following expression of the normal matter pressure P_{mr}:

$$P_{mr}(y)=\frac{\gamma}{y_c}\left[y-b(\delta^2-y^2)^{1/2}\operatorname{cotanh}\left(\Phi-b\sin^{-1}\left(\frac{y}{\delta}\right)\right)\right]-1, \tag{53}$$

where we have introduced a new constant denoted by

$$\Phi=\frac{1}{2}\ln\left|\frac{\left(-1+\gamma+\sqrt{\frac{\beta y_c-\gamma}{y_c}}-P_c\right)\exp\left[2\sqrt{\frac{\beta y_c-\gamma}{y_c}}\sin^{-1}\left(\sqrt{\frac{\gamma y_c}{\gamma y_c+1}}\right)\right]}{-1+\gamma-\sqrt{\frac{\beta y_c-\gamma}{y_c}}-P_c}\right|.$$

The transversal matter pressure of the charged D-dimensional fluid sphere follows from Eq. (33) and is given by

$$P_{m\perp}(y)=\frac{\gamma}{y_c}\left[y-b(\delta^2-y^2)^{1/2}\operatorname{cotanh}\left(\Phi-b\sin^{-1}\left(\frac{y}{\delta}\right)\right)\right]-1+\frac{\alpha\alpha'}{2(D-1)}(y-y_c). \tag{54}$$

Equations (44), (53), (54) represent the exact general solution of the equations which describe the hydrostatic equilibrium of a charged, homogeneous fluid sphere in D space–time dimensions.

From Eq. (7) a straightforward integration yields

$$e^{\nu(y)}=\frac{C_0F(y)}{(1+P_{mr}(y))^2}, \tag{55}$$

where we have denoted

$$F(y)=\exp\left[\frac{2\alpha}{D-1}\left[1+(D-2)\left(\frac{\alpha'}{4}+1\right)\right]\int\frac{dy}{1+P_{mr}(y)}\right]$$

and C_0 is a non-negative constant of integration.

In the variable y we can represent the variation of the metric tensor component e^{λ} in the interior of the sphere in the simple form,

$$e^{\lambda(y)}=\frac{y_c}{\gamma}\frac{1}{\delta^2-y^2}. \tag{56}$$

The radius R of the static anisotropic charged D-dimensional fluid configuration is determined from the condition $P_{mr}(R)=0$ and can be represented as

$$R=\sqrt{\frac{(\gamma y_c+1)y_c}{\gamma}}\sin\left(\frac{\gamma\omega_S}{\sqrt{\beta y_c-\gamma}}\right), \tag{57}$$

where ω_s is a solution of the algebraic equation,

$$\frac{\gamma\delta}{y_c}\sin(\omega_S)=\frac{\gamma b\delta}{y_c}\cos(\omega_S)\operatorname{cotanh}(\Phi-\omega_S)+1. \tag{58}$$

In order to match the above metric smoothly on the boundary surface $r=R$ with the Reissner–Nordstrom–de Sitter metric we have to require the continuity of the gravitational potential across that surface. Matching Eq. (56) with the exterior Reissner–Nordstrom–de Sitter gravitational metric tensor component (39) at the boundary $y=y_S=(\eta_S^2/2)+y_c$ gives the value of the integration constant C_0 in the form

$$C_0=\frac{\gamma(\delta^2-y_S^2)}{y_c F(y_S)}. \tag{59}$$

From Eq. (26) we obtain the total charge of the charged fluid configuration as

$$Q=\frac{K\rho_0 R^{D-1}}{4(D-1)}, \tag{60}$$

while the total mass M_t of the sphere is given by

$$M_t=\frac{1}{D-1}\left(\frac{R}{a}\right)^D\left(\frac{a}{R}+\frac{\alpha}{2(D+1)}\frac{R}{a}\right). \tag{61}$$

Equation (61) leads to the following relation relating the total mass of a homogeneous anisotropic fluid sphere with anisotropy factor proportional to the electric field to its total charge:

$$M_t=\frac{Q}{a^D}\left[\frac{4(D-1)a}{K\rho_0}+\frac{\alpha}{2a(D^2-1)}\left(\frac{4(D-1)}{K\rho_0}\right)^{(D+1)/(D-1)}Q^{2/(D-1)}\right]. \tag{62}$$

For the present solution the electric field is also continuous at the boundary of the sphere.

V. HOMOGENEOUS ANISOTROPIC CHARGED FLUID SPHERES WITH NEUTRAL CORE

In the previous section we have analyzed a homogeneous anisotropic D-dimensional charged fluid sphere with the electric charge distributed continuously throughout the sphere. For this configuration and for a particular choice of the proper electric charge density the general solution of the Einstein–Maxwell equations has been obtained. In the present section we shall generalize the previous model by considering a spherical distribution of an anisotropic charged D-dimensional fluid, with proper electric charge density given again by Eq. (24), which surrounds a neutral core of isotropic homogeneous fluid. The energy density of the matter is supposed to be a constant in the whole sphere, $\rho_m=$constant and for simplicity we suppose that it has the same value in both neutral and charged regions. The radius of the core and of the outer surface of the sphere are r_{I} and r_S, respectively.

For the homogeneous neutral core (region I) the mass m_{I}, isotropic matter pressure $P_{m\mathrm{I}}$ $(P_r=P_\perp=P_m)$, metric functions $e_{\mathrm{I}}^{-\lambda}$ and e_{I}^{ν} can be obtained by integrating the mass-continuity, TOV and gravitational field equations for a homogeneous isotropic sphere in the presence of a cosmological constant and are given in the dimensionless variables (14) by

$$m_{\mathrm{I}}(\eta)=\frac{\eta^{D-1}}{D-1}, \tag{63}$$

$$P_{m\mathrm{I}}(\eta)=\frac{(1-\gamma)(1+P_c)-[(1-\gamma)+P_c]\sqrt{1-\gamma\eta^2}}{[(1-\gamma)+P_c]\sqrt{1-\gamma\eta^2}-(1+P_c)}, \tag{64}$$

$$e_{\mathrm{I}}^{-\lambda(\eta)}=1-\frac{2m_{\mathrm{I}}}{\eta^{D-3}}-\mu\eta^2, \tag{65}$$

$$e_{\mathrm{I}}^{\nu(\eta)}=C_{\mathrm{I}}[(1+P_c-\gamma)\sqrt{1-\gamma\eta^2}-(1+P_c)]^2. \tag{66}$$

C_{I} is a constant of integration and the boundary conditions $m_{\mathrm{I}}(0)=0$ and $P_{m\mathrm{I}}(0)=P_c$ have been used.

In the second region (II), which contains an anisotropic charged D-dimensional homogeneous fluid distribution restricted in the domain $\eta_{\mathrm{I}}\leqslant\eta\leqslant\eta_s$ of the dimensionless space variable the general solution of the mass continuity, TOV and gravitational field equations are

$$m_{\mathrm{II}}(\eta)=\frac{\eta^{D-1}}{D-1}\left[1+\frac{\alpha}{2(D+1)}\eta^2\right]-\frac{\eta_{\mathrm{I}}^{D-1}}{D-1}\left[1+\frac{\alpha}{2(D+1)}\eta_{\mathrm{I}}^2\right]. \tag{67}$$

$$P_{mr\mathrm{II}}(y)=\frac{\gamma}{y_c}\left[\frac{C_{\mathrm{II}}e^{-2b\sin^{-1}(y/\delta)}(y-b\sqrt{\delta^2-y^2})-(y+b\sqrt{\delta^2-y^2})}{C_{\mathrm{II}}e^{-2b\sin^{-1}(y/\delta)}-1}\right]-1, \tag{68}$$

$$e_{\mathrm{II}}^{-\lambda}=1-\eta^2\left[\frac{2}{D-1}\left(1+\frac{\alpha}{2(D+1)}\eta^2\right)+\mu\right]+\frac{2\eta_{\mathrm{I}}^2}{D-1}\left(1+\frac{\alpha}{2(D+1)}\eta_{\mathrm{I}}^2\right), \tag{69}$$

$$e_{\mathrm{II}}^{\nu(y)}=\frac{C_{0\mathrm{II}}F(y)}{(1+P_{mr\mathrm{II}}(y))^2}. \tag{70}$$

The total charge contained in the second region is given by

$$Q=\frac{K\rho_0}{4(D-1)}(R^{D-1}-r_{\mathrm{I}}^{D-1}), \tag{71}$$

where R is the total radius of the sphere.

In the third region (III), $r>R$, the geometry of the D-dimensional spherically symmetric static space–time is described by the Reissner–Nordstrom–de Sitter solution of the gravitational field equations,

$$e_{\mathrm{III}}^{\nu}=e_{\mathrm{III}}^{-\lambda}=1-\frac{2m_s}{\eta^{D-3}}+\frac{\alpha(\eta_S^{D-1}-\eta_{\mathrm{I}}^{D-1})^2}{(D-2)(D-3)\eta^{2(D-3)}}-\mu\eta^2. \tag{72}$$

Matching the radial pressure with the matter pressure of the neutral core at $r=\eta_{\mathrm{I}}$, $P_{m\mathrm{I}}(\eta_{\mathrm{I}})=P_{mr\mathrm{II}}(\eta_{\mathrm{I}})$ leads to the expressions of the constant C_{II} and of the complete form of the radial pressure given by

$$P_{mr\mathrm{II}}(y)=\frac{\gamma}{y_c}\left[y-b(\delta^2-y^2)^{1/2}\operatorname{cotanh}\left(\Phi_{\mathrm{II}}-b\sin^{-1}\left(\frac{y}{\delta}\right)\right)\right]-1, \tag{73}$$

where we denoted the newly introduced constants by $y_{\mathrm{I}}=\eta_{\mathrm{I}}^2/2+y_c$ and

$$\Phi_{\mathrm{II}}=b\sin^{-1}\left(\frac{y_{\mathrm{I}}}{\delta}\right)+\frac{1}{2}\ln\left|\frac{\frac{\gamma}{y_c}(y_{\mathrm{I}}+b\sqrt{\delta^2-y_{\mathrm{I}}^2})-1-\frac{(1-\gamma)(1+P_c)-(1+P_c-\gamma)\sqrt{1-\gamma\eta_{\mathrm{I}}^2}}{(1+P_c-\gamma)\sqrt{1-\gamma\eta_{\mathrm{I}}^2}-(1+P_c)}}{\frac{\gamma}{y_c}(y_{\mathrm{I}}-b\sqrt{\delta^2-y_{\mathrm{I}}^2})-1-\frac{(1-\gamma)(1+P_c)-(1+P_c-\gamma)\sqrt{1-\gamma\eta_{\mathrm{I}}^2}}{(1+P_c-\gamma)\sqrt{1-\gamma\eta_{\mathrm{I}}^2}-(1+P_c)}}\right|.$$

The radius of the stellar configuration is obtained from the condition of the vanishing radial pressure at the outer surface of the sphere, that is, by solving the algebraic equation $P_{mr\mathrm{II}}(y_S)=0$, and can again be represented in the following parametrical form:

$$R=\sqrt{\frac{(\gamma y_c+1)y_c}{\gamma}}\sin\left(\frac{\gamma\omega_S}{\sqrt{\beta y_c-\gamma}}\right), \tag{74}$$

where ω_S is a solution of the algebraic equation,

$$\frac{\gamma\delta}{y_c}\sin(\omega_S)=\frac{\gamma b\,\delta}{y_c}\cos(\omega_S)\mathrm{cotanh}(\Phi-\omega_S)+1. \tag{75}$$

By applying the condition of continuity of the metric function e^{ν} at the boundaries $r=\eta$ and $r=R$, $e_{\mathrm{I}}^{\nu}(\eta_{\mathrm{I}})=e_{\mathrm{II}}^{\nu}(\eta_{\mathrm{I}})$ and $e_{\mathrm{II}}^{\nu}(\eta_S)=e_{\mathrm{III}}^{\nu}(\eta_S)$ we obtain the expressions of the constants C_{I} and $C_{0\mathrm{II}}$,

$$C_{\mathrm{I}}=\frac{\gamma}{y_c}\,\frac{(\delta^2-y_S^2)F(y_S)}{[(1+P_c-\gamma)\sqrt{1-\gamma\eta_{\mathrm{I}}^2}-(1+P_c)]^2(1+P_{mr\mathrm{II}}(y_{\mathrm{I}}))}, \tag{76}$$

$$C_{0\mathrm{II}}=\frac{\gamma(\delta^2-y_S^2)F(y_S)}{y_c}. \tag{77}$$

The continuity of the metric tensor component e^{λ} and of the radial electric field E across the two boundaries separating the neutral core and the charged region and the outer surface of the sphere from the vacuum is also satisfied by the present solutions.

VI. DISCUSSIONS AND FINAL REMARKS

In the present paper we have obtained in D space–time dimensions a complete solution for a homogeneous anisotropic charged fluid sphere, whose proper charge density is represented by $\rho_e=\rho_0 e^{-(\lambda/2)}$, where ρ_0 is a constant and for an anisotropic charged fluid sphere with a neutral core. The obtained solutions, corresponding to this particular functional form of the charge density are nonsingular throughout the sphere. We have not discussed the stability of such spheres but it would most likely be unstable since the electrostatic repulsion would tend to destabilize it.

From Eqs. (54) and (68) it follows that the variable y must satisfy the condition $y<\delta$, for all y. Particularly, for $y=y_S$, with y_S the value of y at the surface of the sphere, we obtain

$$\eta_S<(2y_c)^{1/2}\left[\left(1+\frac{1}{\gamma y_c}\right)^{1/2}-1\right]^{1/2}=\eta_{\max}. \tag{78}$$

Hence, $\eta_{\max}$ gives an upper limit of the radius of the charged fluid sphere with proper charge density given by Eq. (24) as a function of the values of the cosmological constant and of the electrical charge. Similarly, from Eq. (44) we obtain for the total mass of the charged sphere,

$$m_s\leqslant\frac{\eta_{\max}^{D-1}}{(D-1)}\left[1+\frac{\alpha}{2(D+1)}\,\eta_{\max}^2\right]=m_{\max}. \tag{79}$$

Therefore $m_{\max}$ is the upper limit of the total mass of the charged fluid sphere corresponding to the particular charge density (24).

The results obtained in the present paper essentially depend on the functional form of the proper charge density given by Eq. (24). This form has been chosen mainly for mathematical convenience, in order to provide an exact closed form solution of the gravitational field equations. Other, physically better motivated charge density profiles, could lead to different mass and charge distributions inside the higher dimensional charged fluid sphere and, consequently, to different results on the maximum allowable mass and radius of this type of general relativistic object.

An interesting question is the possibility of observing such higher dimensional charged relativistic objects in an astrophysical setting. The observation of γ-ray bursts prompted investigators to suggest that there might be a relation between the strong-coupling phase of QED and the detected γ-ray bursts. The presence of certain anomalies in the spectrum of γ-ray bursts led some scientists to speculate that these very violent cosmic events are emissions from charged objects in more than four space–time dimensions.[28] On the other hand compact stellar objects formed from a mixture of quarks and gluons are also supposed to form at the final stages of stellar evolution. The quark-gluon plasma could exist at sufficiently high densities as a result of the gravitational collapse. In the case of neutron stars a phase transition of neutron matter to quark matter at zero temperature or temperatures small compared to degeneracy temperature allows the existence of hybrid stars, i.e., stars having a quark core and a crust of neutron matter with appropriate pressure balancing at the interface. In fact, quark matter with nonzero electrically charged constituents rather than neutron matter could hold the large magnetic field of the pulsars[29] and hence it is possible that for strange-matter made stars the effects of the nonzero electrical charge be important.

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