

# Giant magnetoresistance in magnetic granular systems

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Based on a semiclassical model, the transport properties in systems of cylindrical or spherical magnetic granules are investigated analytically. It is shown that the conductivities as well as the magnetoresistance of these systems depend strongly on the size of the granules. In particular, there is always an optimum granular size for the magnetoresistance. © 1996 American Institute of Physics. [S0021-8979(96)79808-0]

Recently, there has been much interest in the study of giant magnetoresistance (GMR) in magnetic inhomogeneous systems<sup>1-7</sup> both experimentally and theoretically. For multilayer structures, almost all the low-temperature features of GMR could be understood by semiclassical models<sup>8-12</sup> or quantum theories<sup>13-15</sup> by including spin-dependent interface scattering and bulk scattering. Previous theoretical investigations<sup>5,16</sup> on the GMR in granular systems are based on the assumption that transport in these systems is quite close to that in multilayered systems with current perpendicular to the layers. However, differing from those in multilayered structures, both the distributions of the electric field and currents are spatially varied in these three-dimensionally inhomogeneous systems. Therefore, it is valuable and interesting to develop a theory which includes spatial variations in the electric field and in the currents.

In this article, we present a semiclassical description of the GMR in magnetic granular systems, in which the spatial variations of the electric fields and the currents are considered. A new formalism of the position-dependent current is developed. In particular, analytical expressions for the resistivity are obtained. We focus our attention on systems of cylindrical magnetic granules. We find that both the GMR and the resistivities depend strongly on the granular sizes and that there is always an optimum granular size for the GMR.

Let us consider a general inhomogeneous system in which charge carriers are scattered by impurities and rough interfaces. For convenience, we do not include the spin degrees of freedom for a while. In the presence of an external electric field, the steady-state transport properties in this system can be described by the following effective Boltzmann equation:<sup>17</sup>

$$\mathbf{v} \cdot \nabla g + g/\tau = e\mathbf{v} \cdot \mathbf{E}^{\text{eff}}, \quad (1)$$

where  $\tau$  is the position-dependent relaxation time and  $g$  is a function characterizing the deviation of the distribution func-

tion  $f$  from the equilibrium distribution  $f_0$ , which satisfies the relation  $f = f_0 + g(\partial f_0 / \partial \epsilon)$ . Since interface scattering is also considered as impurity scattering in thin mixing films,<sup>14</sup> the entire scattering effect is included in the position-dependent relaxation time  $\tau(\mathbf{r})$ .  $\mathbf{E}^{\text{eff}}(\mathbf{r})$  is the effective internal field to be determined from the continuity condition of the current.

In general, the position-dependent current is related to the electric-field by a two-point conductivity tensor  $\sigma_{\alpha\beta}(\mathbf{r}, \mathbf{r}')$ , such that

$$J_{\alpha}(\mathbf{r}) = \int d^3r' \sigma_{\alpha\beta}(\mathbf{r}, \mathbf{r}') E_{\beta}^{\text{eff}}(\mathbf{r}'), \quad (2)$$

where the summation is indicated by two same coordinate labels. By using the path-integral approach,<sup>12,14</sup> we solve the Boltzmann Eq. (1) and obtain

$$\sigma_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = C_e r_{0\alpha} r_{0\beta} \Phi(\mathbf{r}, \mathbf{r}'), \quad (3)$$

with

$$\Phi(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|^2} \exp\left[-\int_{\mathbf{r}' \rightarrow \mathbf{r}} \frac{du''}{\lambda(\mathbf{r}'')}\right], \quad (4)$$

$C_e = 3n_e e^2 / 2mv_F$ ,  $\hat{\mathbf{r}}_0 = (\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|$ , and  $\lambda(\mathbf{r}) = v_F \tau(\mathbf{r})$ . The integral in Eq. (4) is along the straight line connecting the points  $\mathbf{r}$  and  $\mathbf{r}'$ , and  $du''$  is the element of line at the point  $\mathbf{r}''$ . The effective field  $\mathbf{E}^{\text{eff}}$ , determined by combining Eq. (2) with the continuity condition for the current, is found to be<sup>17</sup>

$$\mathbf{E}^{\text{eff}} = \mathbf{E}^{\text{ex}} - \nabla \mu, \quad (5)$$

where  $\mu = GD_{\beta} E_{\beta}^{\text{ex}}$  with

$$G = (1 - \Phi\Lambda)^{-1} = 1 + \Phi\Lambda + \dots, \quad (6)$$

$D_{\alpha}(\mathbf{r}, \mathbf{r}') = r_{0\alpha} \Phi(\mathbf{r}, \mathbf{r}')$ , and  $\Lambda(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')/\lambda(\mathbf{r})$ . Here,  $GD_{\beta}$  represents the integral  $\int d\mathbf{r}'' G(\mathbf{r}, \mathbf{r}'') D_{\beta}(\mathbf{r}'', \mathbf{r}')$ . The constant  $\mathbf{E}^{\text{ex}}$  in the above expressions is actually the external field, since

$$\mathbf{E}^{\text{ex}} = \langle \mathbf{E}^{\text{eff}} \rangle, \quad (7)$$

where  $\langle \cdot \rangle$  means taking the average over the whole system.

Now let us turn to investigate transport in granular systems within the framework of a mean-field treatment. We consider the  $N_p$ -particle system to be realized by adding one particle to the system of  $N_p - 1$  particles. The effective field in the system of  $N_p - 1$  particles is written as  $\mathbf{E}^{\text{bc}}$ , which is essentially the sum of the applied field and the field produced by the accumulated charge on the  $N_p - 1$  particles. The  $N_p$ -particle problem is then treated approximately as that of an isolated particle in the background field  $\mathbf{E}^{\text{bc}}$ . The above considerations can be explicitly represented as

$$\mathbf{E}^{\text{eff}} = \mathbf{E}^{\text{bc}} - \nabla(G_1 D_{1,\beta} E_\beta^{\text{bc}}). \quad (8)$$

A Subscript 1 has been used to indicate that the parameters are associated with the single-particle case. The background field  $\mathbf{E}^{\text{bc}}$  will be determined from the boundary condition (7). Equation (8) is highly useful for us to determine the effective electric field in the  $N_p$ -particle system, because we need only to solve the problem of an isolated particle in the background electric field. We wish to point out that Eq. (8) is appropriate only in the small region near the central particle. Nevertheless, using this equation in the region of every particle, we are able to obtain the field in the whole system.

To obtain the average conductivity analytically, we now take the local limit, i.e.,

$$\sigma_{\alpha\beta}^c(\mathbf{r}, \mathbf{r}') = \sigma(\mathbf{r}) \delta_{\alpha\beta} \delta(\mathbf{r} - \mathbf{r}'), \quad (9)$$

where  $\sigma(\mathbf{r}) = ne^2 \lambda(\mathbf{r}) / 2m v_F$ .<sup>18</sup> Since  $\nabla \times \mathbf{E}^{\text{eff}} = 0$ , it is convenient to define an effective scalar potential by

$$\mathbf{E}^{\text{eff}}(\mathbf{r}) = -\nabla U^{\text{eff}}(\mathbf{r}). \quad (10)$$

We consider first the system composed of parallel ferromagnetic cylinders of radius  $a$  in a nonmagnetic medium. The electric field is assumed to be applied along the  $z$  axis and perpendicular to the cylinders. For simplicity,  $\lambda(\mathbf{r})$  is taken to be the constant  $\lambda_0$  in the medium,  $\lambda_F$  in the particles, and  $\lambda_l$  in the mixing films, respectively. If we consider all granules to be identical cylinders, the equation of continuity for the current becomes Poisson's equation in all regions. In this case, the effective potential in the  $x$ - $y$  plane has the form

$$U^{\text{eff}}(\rho, \varphi) = -\sin(\varphi) \begin{cases} E^{\text{bc}} \rho + D / \rho, & \rho > a + d \\ C_1 \rho + C_2 / \rho, & a + d > \rho > a. \\ E^F \rho, & a > \rho \end{cases} \quad (11)$$

Here  $D$ ,  $C_1$ ,  $C_2$ , and  $E^F$  are constants to be determined from the continuity of the scalar potential and the current at  $r = a$  and  $r = a + d$ . For example, the electric field in the inside region of the particles  $E^F$  is found to be  $E^F = \gamma_F E^{\text{bc}}$  with  $\gamma_F = 2\lambda_0 / (\lambda_0 + \lambda_F + \lambda_0 \lambda_F R_l / a)$ . We have assumed that  $d$  is much less than  $a$ , so that only the ratio  $R_l = d / \lambda_l$  is of significance.

Now, we need to calculate the average of the effective electric field. We emphasize that even when the thickness goes to zero, the contribution from the field in the interlayer cannot be neglected. In terms of Eq. (7), we obtain

$$E^{\text{ex}} = [(1-f) + (\gamma_F + \gamma_l)f] E^{\text{bc}}, \quad (12)$$

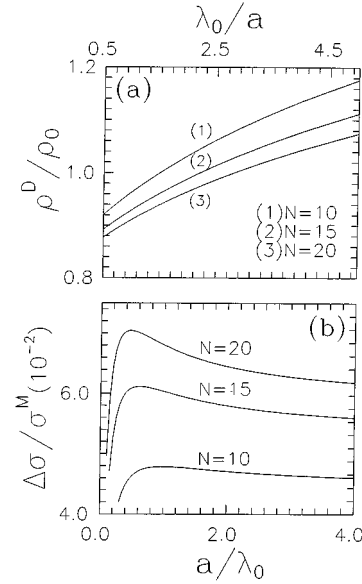


FIG. 1. (a) Resistivity  $\rho^D$  as a function of  $\lambda_0/a$  and (b) percent of GMR  $\Delta\sigma/\sigma^M$  as a function of  $a/\lambda_0$  in systems of cylindrical granules for several spin-asymmetry factors  $N$ . The other parameters are  $f=0.3$ ,  $\lambda_F^\uparrow=0.6\lambda_0$  and  $R_l^\uparrow=1.0$ .

where  $f$  is the volumetric filling factor, and  $\gamma_l = \lambda_F R_l \gamma_F / a$ . The average conductivity is evaluated from  $\langle J \rangle / E^{\text{ex}}$ .

In order to elucidate the GMR effect, the spin degrees of freedom need to be included. We focus on the cases where the spin diffusion length is much larger than the mean free path, so that the total conductivity is the sum from the two spin channels, and for each channel the previous formulas can be extended straightforwardly. Let  $\lambda_l^s$  and  $\lambda_F^s$ , with  $s = \uparrow$  for the majority spin and  $s = \downarrow$  for the minority spin, denote the mean free path of electrons in the mixing films and in the particles,<sup>8</sup> respectively then we arrive at

$$\frac{\sigma^D}{\sigma_0} = \frac{2(1-f) + f(\gamma_F^\uparrow \lambda_F^\uparrow + \gamma_F^\downarrow \lambda_F^\downarrow) / \lambda_0}{2(1-f) + (\gamma_F^\uparrow + \gamma_F^\downarrow + \gamma_l^\uparrow + \gamma_l^\downarrow) f}, \quad (13)$$

for the demagnetized state, and

$$\frac{\sigma^M}{\sigma_0} = \sum_{s=\uparrow, \downarrow} \frac{(1-f) + f \gamma_F^s \lambda_F^s / \lambda_0}{2(1-f) + 2(\gamma_F^s + \gamma_l^s) f}, \quad (14)$$

for the magnetized state, with  $\sigma_0 = ne^2 \lambda_0 / m v_F$ .

The GMR effect is measured by  $\Delta\sigma = \sigma^M - \sigma^D$ , which can be found to be always positive. Its amplitude is defined as  $\Delta\rho/\rho^D = \Delta\sigma/\sigma^M$  with  $\rho^D = 1/\sigma^D$ ,  $\rho^M = 1/\sigma^M$ , and  $\Delta\rho = \rho^D - \rho^M$ . For simplicity, we here assume the spin-asymmetry factors in the particles and in the mixing film are of the same value  $N = \lambda_F^\uparrow / \lambda_F^\downarrow = R_l^\uparrow / R_l^\downarrow$  although they are quite different. In Fig. 1(a), we plot the resistivity as a function of the inverse of radius, which is approximately in linear proportion. The GMR is shown in Fig. 1(b). There is always an optimum radius for the GMR. To understand this feature, we notice that there two factors that determine the GMR. On one hand, since the number of atoms at interfaces is inversely proportional to the radius, the proportion of the spin-dependent interface scattering to the total scattering increases with decreasing size of the cylinders. On the other hand, the

smaller the radius, the more easily the currents pass by the cylinders, an effect which results in the decrease in the spin dependence of the scattering of the electrons. The competition between these two factors leads to a maximum of the GMR.

The above calculations can be extended straightforwardly to the case of spherical granules. We can find that eqs. (13) and (14) are still valid if the parameters  $\gamma_F^s$  and  $\gamma_I^s$  are represented as

$$\gamma_F^s = \frac{3\lambda_0}{2\lambda_0 + \lambda_F^s + 2\lambda_0\lambda_F^s R_I^s / a}, \quad (15)$$

$$\gamma_I^s = 2\lambda_F^s R_I^s \gamma_F^s / a. \quad (16)$$

In summary, we have presented a new and efficient approach to calculate the conductivity in inhomogeneous systems based on the effective Boltzmann equation. Within a mean-field framework, we employ our formal theory to investigate in detail the GMR effect in cylindrical magnetic granular systems.

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- <sup>1</sup>M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen van dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, *Phys. Rev. Lett.* **61**, 2472 (1988).
- <sup>2</sup>G. Binasch, P. Grunberg, F. Saurenbach, and W. Zinn, *Phys. Rev. B* **39**, 4828 (1989).
- <sup>3</sup>S. S. P. Parkin, N. More, and K. P. Roche, *Phys. Rev. Lett.* **64**, 2304 (1990).
- <sup>4</sup>W. P. Pratt, Jr., S.-F. Lee, J. M. Slaughter, R. Loloee, P. A. Shroeder, and J. Bass, *Phys. Rev. Lett.* **66**, 3060 (1992).
- <sup>5</sup>A. E. Berkowitz, J. R. Mitchell, M. J. Carey, A. P. Young, S. Zhang, F. E. Spada, E. T. Parker, A. Hutten, and C. L. Chien, *Phys. Rev. Lett.* **68**, 3745 (1992).
- <sup>6</sup>J. Q. Xiao, J. S. Jiang, and C. L. Chien, *Phys. Rev. Lett.* **68**, 3749 (1992).
- <sup>7</sup>J. Q. Wang and G. Xiao, *Phys. Rev. B* **49**, 3982 (1994).
- <sup>8</sup>R. E. Camley and J. Barnas, *Phys. Rev. Lett.* **63**, 664 (1989).
- <sup>9</sup>R. Q. Hood and L. M. Falicov, *Phys. Rev. B* **46**, 8287 (1992).
- <sup>10</sup>T. Valet and A. Fert, *Phys. Rev. B* **48**, 7099 (1993).
- <sup>11</sup>M. Liu and D. Y. Xing, *Phys. Rev. B* **47**, 12 272 (1993).
- <sup>12</sup>L. Sheng and D. Y. Xing, *Phys. Rev. B* **50**, 1001 (1994).
- <sup>13</sup>P. M. Levy, S. Zhang, and A. Fert, *Phys. Rev. Lett.* **65**, 1643 (1990).
- <sup>14</sup>H. E. Camblong and P. M. Levy, *Phys. Rev. Lett.* **69**, 2835 (1992); *J. Magn. Magn. Matter.* **121**, 446 (1993); *J. Appl. Phys.* **73**, 5533 (1993).
- <sup>15</sup>H. E. Camblong, S. Zhang, and P. M. Levy, *J. Appl. Phys.* **75**, 6905 (1994).
- <sup>16</sup>S. Zhang and P. M. Levy, *J. Appl. Phys.* **73**, 5315 (1993).
- <sup>17</sup>L. Sheng, Z. D. Wang, D. Y. Xing, and J. X. Zhu, *Phys. Rev. B* (in press).
- <sup>18</sup>Beyond the local limit, only a numerical solution of  $\sigma_{\alpha\beta}(\mathbf{r}, \mathbf{r}')$  can be obtained in terms of Eqs. (5) and (6). As far as the distance of particles is significantly larger than the mean free path, the conclusion obtained in the following is still expected qualitatively.