Adaptive Lattice Filters for CDMA Overlay

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***Abstract*—This paper presents the behavior of reflection coefficients of a stochastic gradient lattice (SGL) filter applied to a code-division multiple-access overlay system. Analytic expressions for coefficients for a two-stage filter are derived in a Rayleigh fading channel with the presence of narrow-band interference and additive white Gaussian noise. It is shown that the coefficients of the lattice filter exhibit separate tracking and convergent properties, and that compared to an LMS filter, the lattice filter provides fast rate of convergence, while having good capability of narrow-band interference suppression.**

***Index Terms*—CDMA, lattice filters, narrow-band interference suppression.**

I. Introduction

DIRECT-SEQUENCE code-division multiple-access (DS-CDMA) communications is a popular approach in cellular mobile communications, due to its efficient utilization of channel bandwidth, the relative insensitivity to multipath interference and the potential for improved privacy. In addition to providing multiple-access capability and multipath rejection, spread-spectrum communications also offers the possibility of further increasing the overall spectrum efficiency by overlaying a CDMA network on the existing narrow-band users [1]–[5]. Such a procedure must be done very carefully so as not to cause intolerable interference for either the existing narrow-band users or the CDMA users.

A number of studies have been performed using least mean square (LMS) filters to reject narrow-band interference in DS spread-spectrum (or CDMA) systems [1]–[8]. The LMS algorithm performs well except for its slow rate of convergence. A lattice algorithm has been proposed as an alternative and efficient solution since it provides improved rate of convergence when applied to a linear chirp FM signal [9]. This paper studies the performance of a lattice filter applied to rejecting narrow-band interference in a CDMA overlay situation. The reflection coefficients of a stochastic gradient lattice (SGL) filter are updated by a gradient-based algorithm. At each time step, new reflection coefficients are calculated based on the previous values of reflection coefficients and the current values of input signals. The behavior of reflection coefficients of the SGL filter will be described in the presence of narrow-band interference and channel noise in a Rayleigh fading channel.

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This paper is organized as follows. Section II introduces the basic concepts and notations of the CDMA system and derives the reflection coefficients of a two-stage lattice filter. The performance, measured by the signal-to-noise ratio (SNR), of the lattice filter applied to CDMA environment is presented in Section III. In Section IV, numerical results and conclusions are presented.

II. Filter Coefficients

As shown in Fig. 1, the receiver can be categorized into the following parts: a bandpass filter, a lattice filter, a despreader, and a hard decision device. It is assumed that the channel between the CDMA mobile user and its base station is a Rayleigh fading channel. The received signal $r(t)$ at the base station consists of the sum of K independently-fading CDMA signals, a narrow-band interfering signal, and band-limited additive white Gaussian noise (AWGN)

$$r(t)=\sqrt{2P}\sum_{k=1}^{K}\beta_k b_k(t-\tau_k)a_k(t-\tau_k)\cos(2\pi f_o t+\phi_k)+j(t)+n(t) \quad (1)$$

where P represents signal power, f_o denotes the CDMA carrier frequency, $b_k(t)$ is the kth user binary information sequence with bit duration T_b, and $a_k(t)$ is a random spreading sequence of the kth user with chip duration T_c and processing gain N ($N=T_b/T_c$). The random gain β_k and phase ϕ_k of the kth user have a Rayleigh distribution with $E(\beta_k^2)=2\rho$ for all k ($k=1,\cdots,K$), and a uniform distribution in $[0, 2\pi)$, respectively. The path delay τ_k is uniformly distributed in $[0, T_b)$. The CDMA signal bandwidth is $B_c=2/T_c$. As its spectrum is shown in Fig. 2, $j(t)$ is a Gaussian narrow-band signal (or interference), given by

$$j(t)=j_c(t)\cos[2\pi(f_o+\Delta)t]+j_s(t)\sin[2\pi(f_o+\Delta)t] \quad (2)$$

where $j_c(t)$ and $j_s(t)$ stand for the low-pass quadrature terms of the narrow-band interference with bandwidth $B_j/2$, where B_j is the bandwidth of $j(t)$. Δ denotes the frequency offset of the interference from the CDMA carrier frequency. Further, it is assumed that the parameters p ($p=B_j/B_c$) and q ($q=\Delta T_c$) are the ratio of the interference bandwidth to the spread bandwidth and the ratio of the offset of the interference carrier frequency to half of the spread bandwidth, respectively. $n(t)$ is band-limited AWGN with two-sided power spectral density $N_o/2$ and bandwidth B_c.

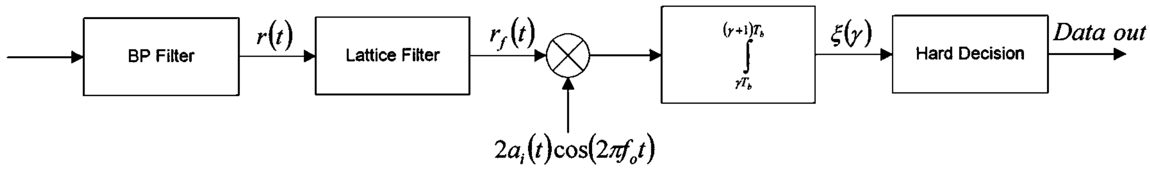


Fig. 1. CDMA receiver model.

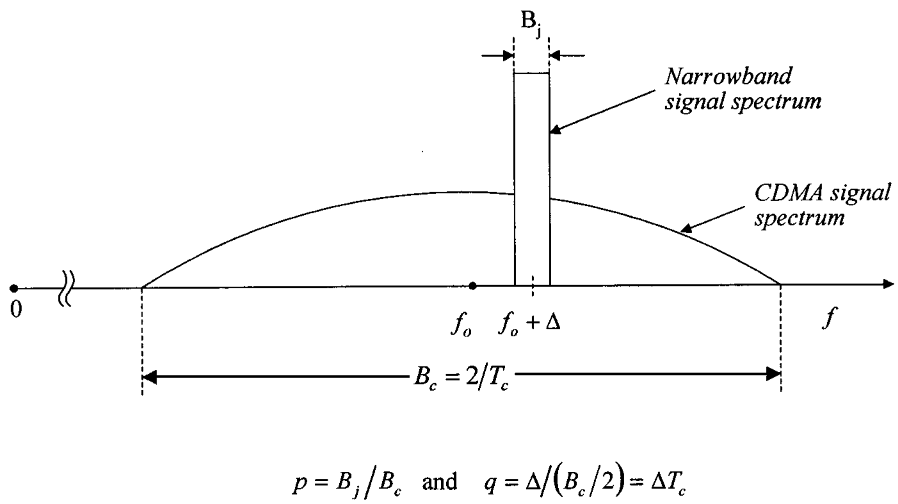


Fig. 2. CDMA and narrow-band signal spectrums.

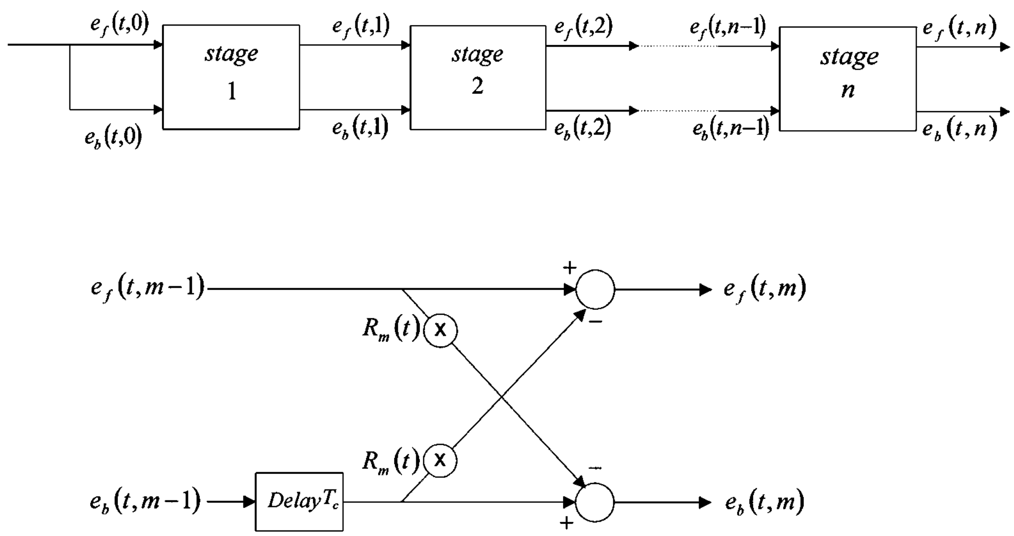


Fig. 3. Lattice structure.

The detail of the lattice filter is shown in Fig. 3, and its input signal is given by

$$e_f(t+sT_c,\,0)=e_b(t+sT_c,\,0)=r(t+sT_c)=r(sT_c) \quad (3)$$

where T_c stands for the delay (or chip duration) of delay element and "s" is an integer. For simple notations, "t" is neglected. The lattice filter can be described by

$$e_f\,(sT_c,n)=e_f\,(sT_c,n{-}1)-R_n\,(sT_c)e_b\,((s{-}1)T_c,n{-}1) \quad (4)$$

and

$$e_b\,(sT_c,\,n)=e_b\,((s{-}1)T_c,n{-}1)-R_n\,(sT_c)e_f\,(sT_c,n{-}1) \quad (5)$$

where $e_f(sT_c,\,n)$ and $e_b(sT_c,\,n)$ are forward and backward prediction errors, n denotes the nth stage of the lattice filter, and $R_n(sT_c)$ is the reflection coefficient of the nth stage of the filter. Note that the lattice filter is assumed symmetric (i.e., the forward and backward coefficients of the nth stage are the same). Finally, assuming that the number of stages is M, the output of the filter is

$$r_f(sT_c)=e_f(sT_c,\,M)=e_b(sT_c,\,M). \quad (6)$$

A. First-Stage Reflection Coefficient

From (1) and (3), a general expression for the input of the filter can be written as

$$\begin{aligned} e_f((s-x)T_c, 0) &= \sqrt{2P}\sum_{k=1}^{K}\beta_k b_k((s-x)T_c-\tau_k)a_k((s-x)T_c-\tau_k) \\ &\quad \cdot\cos[2\pi f_o(s-x)T_c+\phi_k]+j((s-x)T_c) \\ &\quad +n((s-x)T_c). \end{aligned} \tag{7}$$

Similarly, $e_b((s-y)T_c, 0)$ also can be obtained by replacing x with y on the right-hand side of (7). Note that both x and y are integers. The cross correlation of the forward and backward prediction errors can be written as

$$\begin{aligned} E[e_f((s-x)T_c, 0)e_b((s-y)T_c, 0)] \\ = Z_s(x, y)+Z_j(x, y)+Z_n(x, y). \end{aligned} \tag{8}$$

Since $E\{a_{k_1}((s-x)T_c-\tau_{k_1})a_{k_2}((s-y)T_c-\tau_{k_2})\}=0$ for $k_1 \neq k_2$, $Z_s(x, y)$ is given by

$$\begin{aligned} Z_s(x, y) &= P\sum_{k=1}^{K}E[\beta_k^2]E[b_k((s-x)T_c-\tau_k) \\ &\quad \cdot b_k((s-y)T_c-\tau_k)]E[a_k((s-x)T_c-\tau_k) \\ &\quad \cdot a_k((s-y)T_c-\tau_k)]\cdot\cos[2\pi f_o(x-y)T_c] \\ &= 2PK\rho\varepsilon(x, y)\cos[2\pi f_o(x-y)T_c]\delta(x, y) \\ &= 2PK\rho\delta(x, y) \end{aligned} \tag{9}$$

where $E[\beta_k^2]=2\rho$

$$\delta(x, y)=\begin{cases}1, & x=y\\ 0, & \text{otherwise}\end{cases}$$

and $\varepsilon(x, y)$ is the autocorrelation function of the baseband data $b_k(t)$ with $\varepsilon(x, y)=1$ for $x=y$. Assuming that f_oT_c is an integer, the term $Z_j(x, y)$ is given by

$$\begin{aligned} Z_j(x, y) &= E[j((s-x)T_c)j((s-y)T_c)] \\ &= J\frac{\sin(\pi(x-y)T_cB_j)}{\pi(x-y)T_cB_j}\cos[2\pi\Delta(x-y)T_c] \\ &= J\sin c[2\pi(x-y)p]\cos[2\pi(x-y)q] \end{aligned} \tag{10}$$

where J stands for the average power of the narrow-band signal. The function $\sin c[x]$ is defined as $\sin c[x]=\sin(x)/x$. Finally, the term $Z_n(x, y)$ is given by

$$\begin{aligned} Z_n(x, y) &= E[n((s-x)T_c)n((s-y)T_c)] \\ &= \frac{N_o}{2}\cdot 2B_c\delta(x, y) \\ &= N_oB_c\delta(x, y). \end{aligned} \tag{11}$$

From (9)–(11), $E[e_f((s-x)T_c, 0)e_b((s-y)T_c, 0)]$ is an even function against $x-y$. Therefore, defining

$$c(|x-y|T_c)=E[e_f((s-x)T_c, 0)e_b((s-y)T_c, 0)] \tag{12}$$

one obtains from (8)–(11)

$$c(0)=2PK\rho+J+N_oB_c \tag{13}$$

$$c(T_c)=J\sin c(2\pi p)\cos(2\pi q) \tag{14}$$

$$c(2T_c)=J\sin c(4\pi p)\cos(4\pi q). \tag{15}$$

According to [9] and Fig. 3, the general recursive equation for reflection coefficient of stage n is given by

$$\begin{aligned} R_n((j+1)T_a) &= [1-\alpha e_b^2(jT_a-T_c, n-1)]R_n(jT_a) \\ &\quad +\alpha e_f(jT_a, n-1)e_b(jT_a-T_c, n-1) \end{aligned} \tag{16}$$

where α is a convergence factor (or adaptation step size), j represents the jth adaptation (iteration) of the coefficient, and T_a stands for the adaptation period. The minimum value of T_a is the value which guarantees that both the current (the jth adaptation) and previous (the $(j-1)$th adaptation) input signals of the filter are statistically independent. Normally, $T_a \geq 2/B_j$, where $2/B_j$ is the approximate correlation time of the narrow-band signal. Most often, via a central limit theorem, it is argued that the steady-state coefficients of the filter are jointly Gaussian [6], [10] for small adaptation step size. In (16), the nth-stage derivative of reflection coefficients in terms of forward and backward errors is shown in the following:

$$\begin{aligned} R_n(jT_a) &= \left[1-\alpha e_b^2((j-1)T_a-T_c, n-1)\right] \\ &\quad \cdot R_n((j-1)T_a)+\alpha e_f((j-1)T_a, n-1) \\ &\quad \cdot e_b((j-1)T_a-T_c, n-1) \\ &\quad \cdot \qquad\qquad \cdot \\ &\quad \cdot \qquad\qquad \cdot \\ R_n(2T_a) &= \left[1-\alpha e_b^2(T_a-T_c, n-1)\right]R_n(T_a) \\ &\quad +\alpha e_f(T_a, n-1)e_b(T_a-T_c, n-1) \\ R_n(T_a) &= \left[1-\alpha e_b^2(-T_c, n-1)\right]R_n(0) \\ &\quad +\alpha e_f(0, n-1)e_b(-T_c, n-1). \end{aligned}$$

Assuming $R_n(T_a)=0$ (initialization), one obtains

$$\begin{aligned} R_n((j+1)T_a) &= \alpha e_f(jT_a, n-1)e_b(jT_a-T_c, n-1) \\ &\quad +\alpha\sum_{x=1}^{j}e_f((j-x)T_a, n-1) \\ &\quad \cdot e_b((j-x)T_a-T_c, n-1) \\ &\quad \cdot\prod_{r=1}^{x}\left[1-\alpha e_b^2((j-r)T_a-T_c, n-1)\right]. \end{aligned} \tag{17}$$

Substituting n with 1 in (17), the first-stage reflection coefficient $R_1((j+1)T_a)$ can be obtained

$$\begin{aligned} R_1((j+1)T_a) &= \alpha e_f(jT_a, 0)e_b(jT_a-T_c, 0) \\ &\quad +\alpha\sum_{x=1}^{j}e_f((j-x)T_a, 0)e_b((j-x)T_a-T_c, 0) \\ &\quad \cdot\prod_{r=1}^{x}\left[1-\alpha e_b^2((j-r)T_a-T_c, 0)\right]. \end{aligned} \tag{18}$$

Since the jth and $(j-1)$th adaptation input signals of the filter are independent, the expectation of $R_1((j+1)T_a)$ can be written as

$$\begin{aligned} E[R_1((j+1)T_a)] &= \alpha E[e_f(jT_a, 0)e_b(jT_a - T_c, 0)] \\ &+ \alpha \sum_{x=1}^{j} E\left\{e_f((j-x)T_a, 0)e_b((j-x)T_a - T_c, 0)\right. \\ &\cdot \left.\left[1 - \alpha e_b^2((j-x)T_a - T_c, 0)\right]\right\} \\ &\cdot \prod_{r=1}^{x-1} \left\{1 - \alpha E\left[e_b^2((j-r)T_a - T_c, 0)\right]\right\} \end{aligned} \tag{19}$$

where $E[e_b^2((j-r)T_a - T_c, 0)] = c(0)$, given by (13), represents the average power of the input of the lattice filter. It is assumed that

$$\lambda = \alpha c(0) \tag{20}$$

and

$$g_1 = 1 - \lambda \tag{21}$$

where λ is the scaled adaptation step size, and practically, $0 < \lambda \ll 1$ (or $0 < g_1 < 1$). Thus

$$\begin{aligned} E[R_1((j+1)T_a)] &= \alpha E[e_f(jT_a, 0)e_b(jT_a - T_c, 0)] \\ &+ \alpha \sum_{x=1}^{j} E[e_f((j-x)T_a, 0)e_b((j-x)T_a - T_c, 0)]g_1^{x-1} \\ &- \alpha^2 \sum_{x=1}^{j} E\left[e_f((j-x)T_a, 0)\ e_b^3((j-x)T_a - T_c, 0)\right] g_1^{x-1} \end{aligned} \tag{22}$$

where from (12)

$$\begin{aligned} &E[e_f(jT_a, 0)e_b(jT_a - T_c, 0)] \\ &= E[e_f((j-x)T_a, 0)e_b((j-x)T_a - T_c, 0)] = c(T_c). \end{aligned}$$

Based on the central-limit theorem, when the number of active users K is large ($K \gg 1$), the sum of all CDMA signals can be approximated by a Gaussian random variable. Since the narrow-band signal is also Gaussian, the input signals $e_f((j-x)T_a, 0)$ and $e_b((j-x)T_a - T_c, 0)$ are Gaussian. There is a well-known result

$$\begin{aligned} E[X_1X_2X_3X_4] &= E[X_1X_2]\ E[X_3X_4] + E[X_1X_3]\ E[X_2X_4] \\ &+ E[X_1X_4]\ E[X_2X_3] \end{aligned}$$

when all X_1, X_2, X_3, X_4 are Gaussian. Therefore, one obtains

$$\begin{aligned} &E\left[e_f((j-x)T_a, 0)e_b^3((j-x)T_a - T_c, 0)\right] \\ &= E\left[e_b^2((j-x)T_a - T_c, 0)\right] \\ &\quad \cdot E\left[e_f((j-x)T_a, 0)e_b((j-x)T_a - T_c, 0)\right] \\ &= 3c(0)c(T_c). \end{aligned} \tag{23}$$

Therefore, (22) becomes

$$\begin{aligned} E[R_1((j+1)T_a)] &= \alpha c(T_c) + \left[\alpha c(T_c) - 3\alpha^2 c(0)c(T_c)\right] \cdot \sum_{x=1}^{j} g_1^{x-1} \\ &= \alpha c(T_c) + \left[\alpha c(T_c) - 3\alpha^2 c(0)c(T_c)\right] \left[\frac{1 - g_1^j}{1 - g_1}\right]. \end{aligned} \tag{24}$$

It can be seen from (24) that the separate steady state and transient components of the coefficient are shown. Since $\lim_{j\to\infty} g_1^j = 0$, from (13) and (14) and (20) and (21), the steady-state mean of the first-stage coefficient is given by

$$\begin{aligned} E[R_1] &= \lim_{j\to\infty} E[R_1((j+1)T_a)] \\ &= \frac{c(T_c) - 2\alpha c(0)c(T_c)}{c(0)} \\ &= (1 - 2\lambda) \cdot \frac{(J/S) \cdot \sin c(2\pi p) \cos(2\pi q)}{K + J/S + 2N(E_b/N_o)^{-1}} \end{aligned} \tag{25}$$

where $J/S = J/(2P\rho)$ is the ratio of narrow-band interference to signal power, $N = T_b/T_c$ is the processing gain, and $E_b = 2P\rho T_b$.

B. Second-Stage Reflection Coefficient

Since finding closed-form solutions to the coefficients is impossible for the case of the second stage, the second-stage recursion equation is presented. Substituting n with 2 in (16), the second-stage reflection coefficient is given by

$$\begin{aligned} R_2((j+1)T_a) &= \left[1 - \alpha e_b^2(jT_a - T_c, 1)\right] R_2(jT_a) \\ &+ \alpha e_f(jT_a, 1)e_b(jT_a - T_c, 1). \end{aligned} \tag{26}$$

It is seen that $R_2((j+1)T_a)$ is a function of the forward and backward errors of the first stage. Since it is only a function of the past inputs of the second stage, $R_2(jT_a)$ is independent of current inputs $e_f(jT_a, 1)$ or $e_b(jT_a - T_c, 1)$. Therefore

$$\begin{aligned} E[R_2((j+1)T_a)] &= \left\{1 - \alpha E\left[e_b^2(jT_a - T_c, 1)\right]\right\} E[R_2(jT_a)] \\ &+ \alpha E[e_f(jT_a, 1)e_b(jT_a - T_c, 1)]. \end{aligned} \tag{27}$$

In order to obtain the mean of the coefficient, the following equations will be used (see Fig. 3):

$$e_f(jT_a, 1) = e_f(jT_a, 0) - R_1(jT_a)e_b(jT_a - T_c, 0) \tag{28}$$

and

$$e_b(jT_a, 1) = e_b(jT_a - T_c, 0) - R_1(jT_a)e_f(jT_a, 0). \tag{29}$$

Both $E[e_f(jT_a, 1)e_b(jT_a - T_c, 1)]$ and $E[e_b^2(jT_a - T_c, 1)]$ in (27) are given by [11]

$$\alpha E[e_f(jT_a, 1)e_b(jT_a - T_c, 1)] = \lambda \left[\frac{c(2T_c)}{c(0)} + A + B(j)\right] \tag{30}$$

$$\alpha E\left[e_b^2(jT_a - T_c, 1)\right] = \lambda[1 + A + B(j)] \tag{31}$$

where $c(\cdot)$ is given by (12), A and $B(j)$ represent the steady-state and convergent components, respectively, and are given by

$$\begin{aligned} A =& -2\lambda(1-\lambda)[c(T_c)/c(0)]^2 + \lambda^2 - 2(1-\lambda) \\ & \cdot (1-3\lambda)[c(T_c)/c(0)]^2 + \frac{\lambda}{2-3\lambda} \\ & \cdot \left\{1 + 2[c(T_c)/c(0)]^2 - 2\lambda E\left[Z_1^2 Z_2^4\right] + \lambda^2 E\left[Z_1^2 Z_2^6\right]\right\} \\ & + 2[c(T_c)/c(0)] \cdot \frac{(1-3\lambda)}{(2-3\lambda)} \\ & \cdot \left\{[c(T_c)/c(0)](1-6\lambda) + \lambda^2 E\left[Z_1 Z_2^5\right]\right\} \end{aligned} \tag{32}$$

$$\begin{aligned} B(j) =& 2(1-\lambda)(1-3\lambda)[c(T_c)/c(0)]^2 g_1^{j-1} - \lambda \frac{g_2^{j-1}}{2-3\lambda} \\ & \cdot \left\{1 + 2[c(T_c)/c(0)]^2 - 2\lambda E\left[Z_1^2 Z_2^4\right] + \lambda^2 E\left[Z_1^2 Z_2^6\right]\right\} \\ & - 2[c(T_c)/c(0)] \left[g_1^{j-1} - \frac{g_2^{j-1}}{(2-3\lambda)}\right] \\ & \cdot \left\{[c(T_c)c(0)](1-6\lambda) + \lambda^2 E\left[Z_1 Z_2^5\right]\right\} \end{aligned} \tag{33}$$

where g_2 is defined as

$$g_2 = 1 - 2\alpha c(0) + 3\alpha^2 c^2(0) = 1 - 2\lambda + 3\lambda^2 \tag{34}$$

and $0 < g_2 < 1$ when $\lambda < 0.8$. Z_1 and Z_2 are zero-mean Gaussian variables with $E[Z_1 Z_2] = c(T_c)/c(0)$ and $E[Z_1^2] = E[Z_2^2] = 1$. Starting with the initial value of $j = 1$ and setting $E[R_2(T_a)] = 0$, substitute the mean of $E[R_2(jT_a)]$ at each iteration in (27) in order to obtain the mean of the second-stage coefficient, $E[R_2((j+1)T_a)]$. Finally, by letting j approach infinity in (27), one obtains

$$\begin{aligned} &\lim_{j\to\infty} E[R_2((j+1)T_a)] \\ &\quad = \lim_{j\to\infty} \left\{\left[1 - \alpha E\left(e_b^2(jT_a - T_c, 1)\right)\right] E[R_2(jT_a)] \right. \\ &\qquad \left. + \alpha E[e_f(jT_a, 1) e_b(jT_a - T_c, 1)]\right\}. \end{aligned} \tag{35}$$

Since

$$E[R_2] = \lim_{j\to\infty} E[R_2((j+1)T_a)] = \lim_{j\to\infty} E[R_2(jT_a)]$$

and $\lim_{j\to\infty} B(j) = 0$, the steady-state mean of the second-stage coefficient is given by

$$\begin{aligned} E[R_2] &= \frac{\lim_{j\to\infty} \alpha E[e_f(jT_a, 1) e_b(jT_a - T_c, 1)]}{\lim_{j\to\infty} \alpha E[e_b^2(jT_a - T_c, 1)]} \\ &= \frac{c(2T_c)/c(0) + A}{1 + A}. \end{aligned} \tag{36}$$

III. SNR

The output of the lattice filter is given by

$$r_f(t) = \sum_{m=0}^{M} w_m r(t - mT_c) = e_f(t, M) \tag{37}$$

where M denotes the number of stages of the lattice filter, w_m are weights which can be obtained from reflection coefficients. The transformation between the weights and the set of reflection coefficients is highly nonlinear. However, both implementations are mathematically equivalent. But there are some practical differences such as convergence and tracking performances. The relationship between coefficients and weights can easily be derived.

For a two-stage filter ($M = 2$), from Fig. 3, the first-stage forward and backward errors are given by

$$e_f(t, 1) = r(t) - R_1(t) \cdot r(t - T_c) \tag{38}$$

and

$$e_b(t, 1) = r(t - T_c) - R_1(t) \cdot r(t). \tag{39}$$

The second-stage forward error is given by

$$e_f(t, 2) = e_f(t, 1) - R_2(t) \cdot e_b(t - T_c, 1). \tag{40}$$

Substitute (38) and (39) into (40), and one obtains

$$\begin{aligned} r_f(t) &= e_f(t, 2) \\ &= r(t) - [R_1(t) - R_1(t)R_2(t)] \\ &\quad \cdot r(t - T_c) - R_2(t) \cdot r(t - 2T_c) \\ &= \sum_{m=0}^{2} w_m r(t - mT_c). \end{aligned} \tag{41}$$

Then, the corresponding weights are

$$\begin{cases} w_0 = 1 \\ w_1 = -R_1(t)\,(1 - R_2(t)) \\ w_2 = -R_2(t). \end{cases} \tag{42}$$

Assuming that the ith user is the reference user and $\tau_i = \phi_i = 0$, the despreader output is given by

$$\xi(\gamma) = \int_{\gamma T_b}^{(\gamma+1)T_b} r_f(t) 2a_i(t) \cos(2\pi f_o t)\, dt \tag{43}$$

where $2\cos(2\pi f_o t)$ is the local recovered carrier and $a_i(t)$ is the spreading sequence of the reference user. Since the high-frequency terms are removed by the integrator, the above expression reduces to

$$\xi(\gamma) \approx \sqrt{2P}\, \beta_i T_b b_i^{(\gamma)} + J(\gamma) + N(\gamma) + \sum_{\substack{k=1 \\ k\neq i}}^{K} I_k \tag{44}$$

where $b_i^{(\gamma)}$ stands for the γth bit of the ith user. The first term on the right-hand side of (37) is the desired signal and the useful signal power is $4P\rho T_b^2$. $J(\gamma)$ is due to the Gaussian narrow-band signal, given by

$$\begin{aligned} J(\gamma) = \sum_{m=0}^{M} w_m \int_{\gamma T_b}^{(\gamma+1)T_b} & \{j_c(t - mT_c) \\ & \cdot \cos[2\pi(f_o + \Delta)(t - mT_c)] + j_s(t - mT_c) \\ & \cdot \sin[2\pi(f_o + \Delta)(t - mT_c)]\}\, 2a_i(t) \cos(2\pi f_o t)\, dt. \end{aligned}$$

Since the double frequency components are removed by the integrator, $J(\gamma)$ is approximated by

$$J(\gamma) \approx \sum_{m=0}^{M} w_m \sum_{n=0}^{N-1} a_i^{(n)} \int_{nT_c}^{(n+1)T_c} \{j_c(t+\gamma T_b - mT_c) \cdot \cos[2\pi\Delta(t+\gamma T_b - mT_c)] + j_s(t+\gamma T_b - mT_c) \cdot \sin[2\pi\Delta(t+\gamma T_b - mT_c)]\}\, dt = \sum_{m=0}^{M} w_m \sum_{n=0}^{N-1} a_i^{(n)} \int_{nT_c}^{(n+1)T_c} \hat{j}(t+\gamma T_b - mT_c)\, dt \quad (45)$$

where $a_i^{(n)}$ represents the nth chip of sequence $a_i(t)$ and $\hat{j}(t)$ is the low-frequency version of $j(t)$, given by (2), and is also Gaussian, defined by

$$\hat{j}(t) = j_c(t)\cos(2\pi\Delta t) + j_s(t)\sin(2\pi\Delta t). \quad (46)$$

The autocorrelation function of $\hat{j}(t)$ is the same as (10) and the variance of $J(\gamma)$ is given by (47), shown at the bottom of the page, where $Q(m_1, m_2)$ is defined as

$$Q(m_1, m_2) = \int_{-1}^{1} (1-|\tau|)\sin c[2\pi p(\tau - m_1 + m_2)] \cdot \cos[2\pi q(\tau - m_1 + m_2)]\, d\tau. \quad (48)$$

In (44), $N(\gamma)$ is due to the thermal noise with variance

$$\sigma_N^2 = N_o T_b \sum_{m=0}^{M} E\left[w_m^2\right]. \quad (49)$$

Finally, analogous to [2], the variance of the multiple-access interference $(\sum_{k=1}^{K} I_k)$ is given by

$$\sigma_I^2 = \frac{4P(K-1)\rho T_b^2}{3N} \cdot \left[\sum_{m=0}^{M} E\left[w_m^2\right] + \tfrac{1}{2}\cdot \sum_{m=0}^{M} E[w_m w_{m+1}]\right]. \quad (50)$$

The output SNR is given by

$$\mathrm{SNR} = \frac{4P\rho T_b^2}{\sigma_J^2 + \sigma_N^2 + \sigma_I^2} = \left\{\frac{1}{2}\cdot\left(\frac{E_b}{N_o}\right)^{-1} \sum_{m=0}^{M} E\left[w_m^2\right] + \frac{(J/S)}{2N}\sum_{m_1=0}^{M}\sum_{m_2=0}^{M} \cdot E\left[w_{m_1} w_{m_2}\right] Q(m_1, m_2) + \frac{(K-1)}{3N} \cdot \left[\sum_{m=0}^{M} E\left[w_m^2\right] + \frac{1}{2}\cdot\sum_{m=0}^{M} E[w_m w_{m+1}]\right]\right\}^{-1}. \quad (51)$$

IV. LMS FILTERING

By replacing a lattice filter with an LMS filter, the CDMA receiver with an adaptive LMS filter is constituted [1, Fig. 1]. As in [1], the adaptive LMS filter is modeled as consisting of a Wiener filter and a misadjustment filter operating in parallel. The output of the LMS filter is given by

$$r_f(t) = \sum_{m=-M}^{M} (\alpha_m + v_m)\cdot r(t - mT_c) = \sum_{m=-M}^{M} w_m r(t - mT_c) \quad (52)$$

$$\begin{aligned}\sigma_J^2 &= E[J^2(\gamma)] \\ &= E\left\{\left[\sum_{m_1=0}^{M} w_{m_1}\sum_{n_1=0}^{N-1} a_i^{(n_1)}\int_{n_1T_c}^{(n_1+1)T_c} \hat{j}(t_1+\gamma T_b - m_1T_c)\, dt_1\right] \cdot \left[\sum_{m_2=0}^{M} w_{m_2}\sum_{n_2=0}^{N-1} a_i^{(n_2)}\int_{n_2T_c}^{(n_2+1)T_c} \hat{j}(t_2+\gamma T_b - m_2T_c)\, dt_2\right]\right\} \\ &= \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} E\left[w_{m_1}w_{m_2}\right]\cdot\sum_{n=0}^{N-1}\int_{nT_c}^{(n+1)T_c}\int_{nT_c}^{(n+1)T_c} E[\hat{j}(t_1+\gamma T_b - m_1T_c)\hat{j}(t_2+\gamma T_b - m_2T_c)]\, dt_1\, dt_2 \\ &= \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} E\left[w_{m_1}w_{m_2}\right]\cdot\sum_{n=0}^{N-1}\int_0^{T_c}\int_0^{T_c} E\left[\hat{j}(t_1+\gamma T_b + nT_c - m_1T_c)\ \hat{j}(t_2+\gamma T_b + nT_c - m_2T_c)\right]\, dt_1\, dt_2 \\ &= \sum_{m_1=0}^{M}\sum_{m_2=0}^{M} E\left[w_{m_1}w_{m_2}\right] N\int_{-T_c}^{T_c} J\sin c(2\pi p[\tau/T_c - (m_1-m_2)])\cos(2\pi q[\tau/T_c - (m_1-m_2)])\,(T_c - |\tau|)\, d\tau \\ &= \frac{J\cdot T_b^2}{N}\sum_{m_1=0}^{M}\sum_{m_2=0}^{M} E\left[w_{m_1}w_{m_2}\right] Q(m_1, m_2)\end{aligned} \quad (47)$$

where

$$w_m = \alpha_m + v_m \tag{53}$$

α_m denotes the mth tap coefficient (deterministic) of the Wiener filter, v_m denotes the mth steady-state tap coefficient (random) of the misadjustment filter. Note that v_0 is always zero, because the center tap of the Wiener filter is fixed at 1 (i.e., $\alpha_0 = 1$). It is assumed that $\alpha_m = \alpha_{-m}$ and $v_m = v_{-m}$ for double-sided filters. Since each stage of a lattice filter has two taps (reflection coefficients), the number of taps on each side of the double-sided LMS filter is set the same as the number of stages of the lattice filter for fair comparison (the same number of total taps for both filters). Therefore, the sum in (52) is from $-M$ to M for LMS, whereas the sum in (37) is from 0 to M for lattice.

From the LMS adaptation algorithm, the tap coefficient vector of the misadjustment filter can be presented as

$$\begin{aligned} V(j+1) &= \left[I - \alpha X(j)X^T(j)\right] V(j) + \alpha e^*(j)X(j) \\ &= \left[I - \frac{\lambda}{c(0)} X(j)X^T(j)\right] V(j) + \frac{\lambda}{c(0)} e^*(j)X(j) \end{aligned} \tag{54}$$

where $V(j)$ is the column tap weight vector on the jth adaptation, I stands for an identity matrix, $X(j)$ is the column sample vector of the input signal on the jth adaptation, $\lambda = \alpha c(0)$ is the scaled adaptation step size, which is assumed the same for both lattice and LMS filters, and $e^*(j)$ is the prediction error at the Wiener filter output on the jth adaptation, and is given by

$$e^*(j) = \sum_{m=-M}^{M} \alpha_m r(t - mT_c). \tag{55}$$

Analogous to (8)–(15), the steady-state variance of $e^*(j)$ can easily be derived as

$$\begin{aligned} E\left[(e^*)^2\right] =& (2KP\rho + N_oB_c) \sum_{m=-M}^{M} \alpha_m^2 + J \\ &\cdot \sum_{m_1=-M}^{M} \sum_{m_2=-M}^{M} \alpha_{m_1}\alpha_{m_2} \sin c(2\pi(m_1 - m_2)p) \\ &\cdot \cos(2\pi(m_1 - m_2)q). \end{aligned} \tag{56}$$

When the input signal is Gaussian, the steady-state tap coefficient covariance matrix has been shown in [1, eq. (A13)] as

$$\begin{aligned} \left[VV^T\right] &= \lim_{j\to\infty} E\left[V(j+1)V^T(j+1)\right] \\ &\approx \frac{\lambda}{2c(0)} E\left[(e^*)^2\right] I. \end{aligned} \tag{57}$$

With the Gaussian assumption of the input signal, the tap coefficient covariance matrix is a diagonal matrix, which completely defines the statistics of the misadjustment filter for the Gaussian input signal. That is, in the steady state, the variance of different tap coefficients are equal and different tap coefficients are uncorrelated and independent. The performance (SNR) of LMS filtering can be obtained from (51) by substituting w_m with $\alpha_m + v_m$ and 0 in the sum with $-M$, respectively, i.e.,

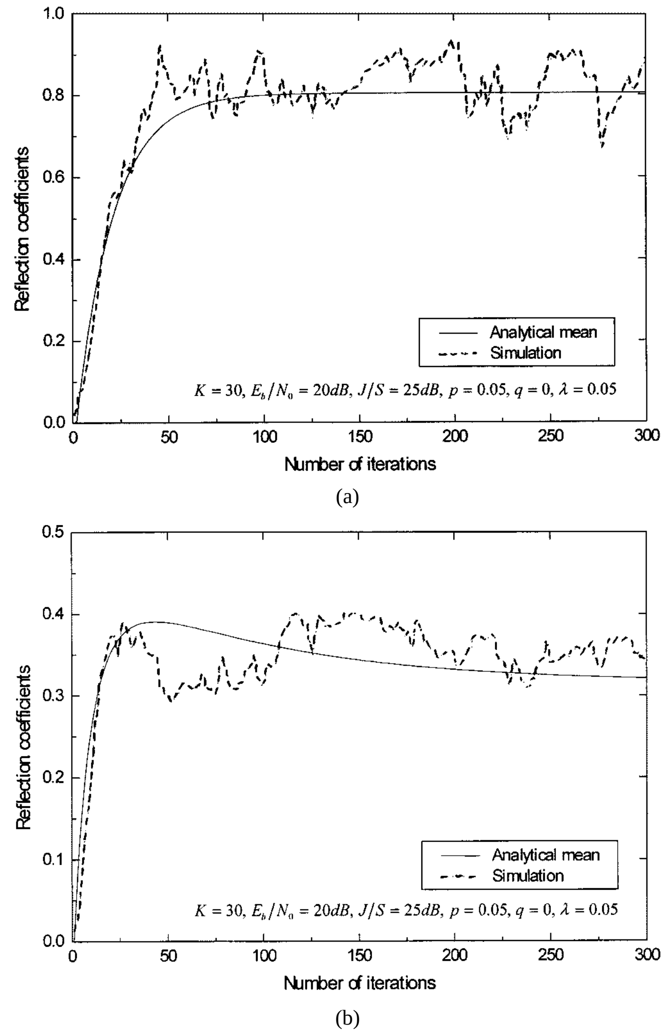
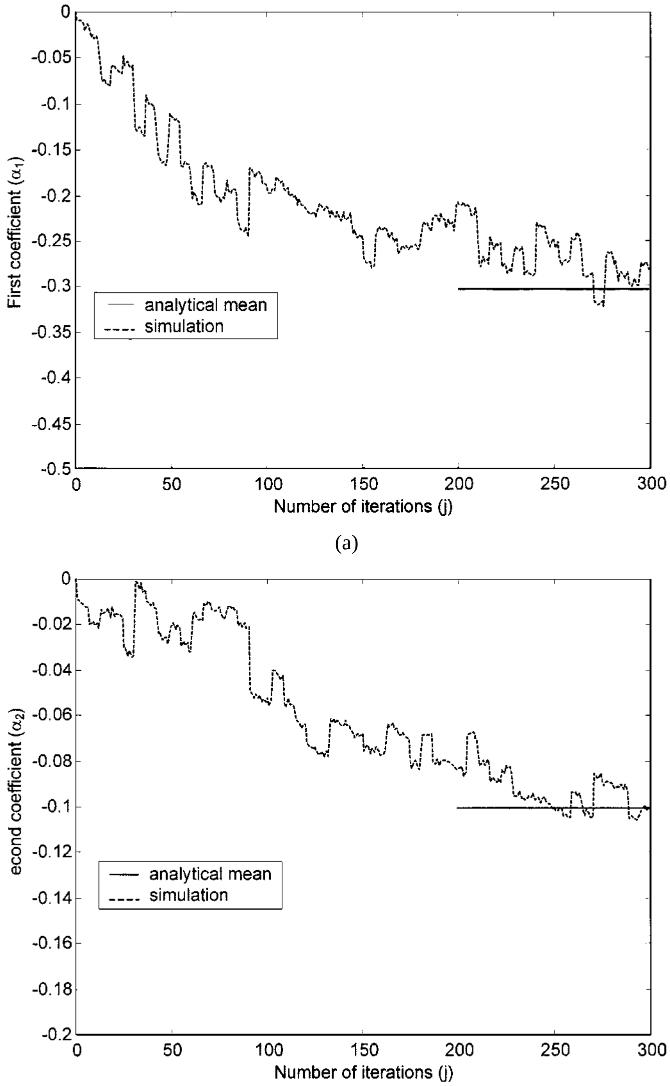


Fig. 4. Analytical mean and simulation of the reflection coefficients of lattice filters. (a) First stage. (b) Second stage.

$$\begin{aligned} \text{SNR} = \Bigg\{ & \frac{1}{2}\cdot\left(\frac{E_b}{N_o}\right)^{-1} \left(\sum_{m=-M}^{M} E\left[\alpha_m^2\right] + M\frac{\lambda}{c(0)} E\left[(e^*)^2\right] \right) \\ & + \frac{(J/S)}{2N} \Bigg(\sum_{m_1=-M}^{M} \sum_{m_2=-M}^{M} E\left[\alpha_{m_1}\alpha_{m_2}\right] Q(m_1, m_2) \\ & + M \frac{\lambda}{c(0)} E\left[(e^*)^2\right] Q(0, 0) \Bigg) \\ & \cdot \frac{(K-1)}{3N} \left[\sum_{m=-M}^{M} E\left[\alpha_m^2\right] + M \frac{\lambda}{c(0)} E\left[(e^*)^2\right] \right. \\ & \left. + \frac{1}{2} \cdot \sum_{m=-M}^{M} E[\alpha_m\alpha_{m+1}] \right] \Bigg\}^{-1}. \end{aligned} \tag{58}$$

V. Numerical and Simulation Results

It is assumed that the ratio of interference bandwidth to that of spread spectrum is 5% ($p = 0.05$) and the ratio of the offset



(b)

Fig. 5. Analytical mean and simulation of the coefficients of LMS filters. (a) First coefficient (α_1). (b) Second coefficient (α_2).

of the interference carrier frequency to half that of spread spectrum is 0 $(q = 0)$. That is, the narrow-band interference is centered at the CDMA spectrum. The processing gain is 750 and the number of stages of the filter is 2 $(M = 2)$. The number of simultaneous users is 30 $(K = 30)$. The normalized adaptation step size is 0.05 ($\lambda = 5\%$). The ratio of narrow-band interference to signal is 25 dB ($J/S = 25$ dB).

Fig. 4(a) and (b) illustrates the mean of the first-stage and the second-stage reflection coefficients of the lattice filter, respectively, as a function of the number of iterations. Also, simulation results with single runs are shown. It is seen from the solid curves of the two figures that it takes about 100 iterations for the coefficients to complete the convergent state. Since the ratio of interference bandwidth to that of spread spectrum is 5% $(p = 0.05)$, the bandwidth of the narrow-band interference is $0.05 \times 2/T_c$. Therefore, the correlation time of the narrow-band interference is $20T_c$. Assuming the minimum adaptation period $T_a = 20T_c$, 100 iterations means $100T_a = 2000T_c \approx 8T_b$. That is, it needs around 8 bits for completion of convergence. It is also seen from the dotted curves of both figures that the simulated coefficients fluctuate around the analytic means. Note that recursive (17) is used for the simulation with $n = 1$ and $n = 2$, respectively, $\alpha = \lambda/c(0)$ and $R_1(0) = R_2(0) = 0$.

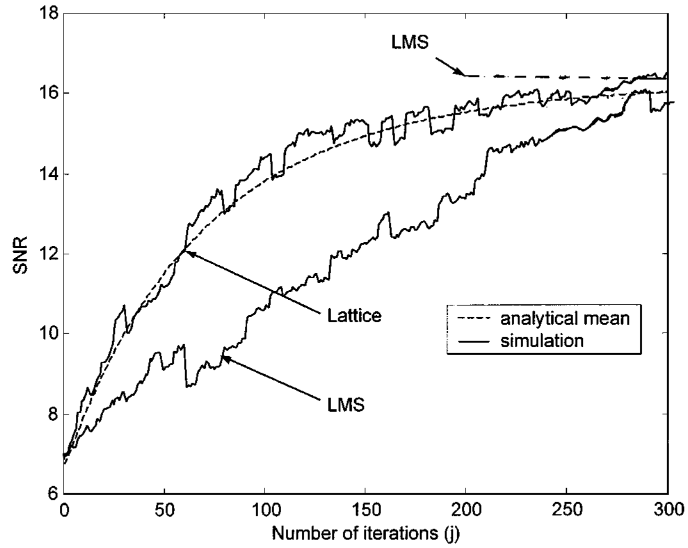


Fig. 6. SNR of the CDMA overlay system against the number of iterations.

For comparison, Fig. 5(a) and (b) shows the coefficients of the LMS filter with the same parameters as in Fig. 4(a) and (b). It is seen from Fig. 5(a) and (b) that it takes more than 200 iterations for the coefficients to converge. That is, the convergence of LMS filters takes longer (about twice) than that of lattice filters. The reason for the lattice to converge fast is that each stage of the lattice converges individually [see (16)], independent of the remaining stages (i.e., the various stages of the lattice are decoupled from one another). However, the adaptation of the LMS is related with all taps (or coefficients) together [see (54)].

Fig. 6 shows the output SNR's of CDMA overlay systems with both lattice and LMS filters. It can be seen that the lattice filter converges faster (about twice) than LMS filter, which is consistent with Fig. 5(a) and (b). However, the SNR performance for both filters is very close in the stable state (i.e., the iteration number $j > 200$). By use of the filters, the SNR performance in the stable state can be improved by as much as 10 dB, compared to that without filters (i.e., $j = 0$).

In conclusion, because each stage of the lattice converges individually, independent of the remaining stages, the lattice filter provides fast rate of convergence, while having good capability of narrow-band interference suppression.

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