ON THE DESIGN AND IMPLEMENTATION OF FIR AND IIR DIGITAL FILTERS WITH VARIABLE FREQUENCY CHARACTERISTICS

S. C. Chan, Carson, K. S. Pun, K. S. Yeung, and K. L. Ho
Department of Electrical and Electronic Engineering
The University of Hong Kong, Pokfulam Road, Hong Kong
{scchan, kspun, ksyeung, klho}@eee.hku.hk

ABSTRACT

This paper studies the design and implementation of FIR and IIR VDF, whose frequency characteristics can be controlled continuously by some control or tuning parameters. A least squares (LS) approach is proposed to design FIR VDF with its impulse response expressed as a linear combination of basis functions. By choosing the basis functions as piecewise polynomials, VDF with larger tuning range than ordinary polynomial based approach results. This VDF can be efficiently implemented using the Farrow structure. Making use of the FIR VDF, an eigensystem realization algorithm (ERA)-based model reduction technique is proposed to obtain a stable IIR VDF with lower system order. It does not suffer from the undesirable transient response during parameter tuning. For frequency selective VDF, about 40% of the multiplications can be saved using the IIR VDF. The implementation of the proposed FIR VDF using SOPOT coefficient and the multiplier block technique is also studied. Results show that about two-third of the additions in implementing the SOPOT coefficients can be saved using the multiplier block.

Index Terms — Variable or tunable digital filters, design and implementation, FIR and IIR filters, least squares design, model reduction, multiplier block.

I. INTRODUCTION

Variable digital filters (VDF) are digital filters with controllable spectral characteristics such as variable cutoff frequency response, adjustable passband width, controllable fractional delay, etc. They found applications in different areas of signal processing and communications [1][2]. Methods for designing variable digital filters can be broadly classified into two categories: transformation [3] and spectral parameter approximation [4]-[7] methods. In general, transformation method is applicable to VDF with variable cutoff frequencies, but not general variable characteristics say variable fractional delay. The spectral parameter approximation method is more general in that it assumes that either the impulse responses [5][6] or the poles and zeros [4][7] of the filters are polynomials of certain spectral parameters. Most of the works on VDF reported are focused on the design of IIR VDF and methods for guaranteeing their stability [4]. More recently, the design of 1-D [6] and 2-D [5] FIR VDF has received considerably attention due to their simple design procedure and good filter performance. Also, the close link between the Farrow-based fractional delay digital filter and such FIR VDF becomes more apparent [5].

This paper studies the design and implementation of FIR and IIR VDF. First of all, the least squares (LS) approach in [6] for designing FIR VDF is generalized to a linear combination of basis functions. It is shown that the optimal LS solution can also be obtained by solving a system of linear equations. This differs from the weighted least squares approach in [6] in that i) no discretization of the tuning and frequency variables is used; ii) the approximation function is assumed to be a linear combination of basis functions. In particular, it is shown that tunable filter using a piecewise polynomial yields larger tuning range than ordinary polynomial based approach. The resulting VDF can be implemented with the familiar Farrow structure [10]. The piecewise polynomial-based approach also reduces the number of general multipliers required in the Farrow structure because of the lower order of the piecewise polynomial used. Making use of the FIR VDFs obtained by the proposed approach, an Eigensystem Realization Algorithm (ERA)-based model reduction technique is proposed to yield a stable IIR VDF with lower system order. Model order reduction [8] is applied to this FIR filter to obtain the desire IIR filter with lower system order and hence arithmetic complexity. The reduction process is very simple which involves the computation of the singular value decomposition (SVD) of a Hankel matrix. Therefore, time consuming iterative optimization method is not necessary. In addition, the model reduced IIR system is guaranteed to be stable and it tries to preserve the frequency characteristics and approximately linear-phase of the original system. The proposed IIR VDF does not suffer from undesirable transient response during parameter tuning found in other approaches based on direct tuning of filter parameters [4][7]. This is because the states of the IIR sub-filter in the proposed structure are not abruptly changed during the parameter tuning process. The proposed VDF structure involves a number of sub-filter with fixed coefficients. The implementation of these coefficients using the sum-of-powers-of-two (SOPOT) representation and the multiplier-block technique [9] is studied. Results show that about two-third of the additions in implementing the SOPOT coefficients can be saved using MB, which leads to significant savings in hardware complexity.

This paper is organized as follows: In section II, the design method of the FIR VDF using the least squares method is described. The design of the IIR VDF using the ERA model reduction method is then studied in Section III. The implementation of the FIR VDF using the SOPOT representation and the multiplier block techniques is described in Section IV. Several design examples are given in section V. Conclusions of this work are drawn in section VI.

II. LEAST SQUARES DESIGN OF FIR VARIABLE DIGITAL FILTERS

The impulse response of the variable FIR filter under consideration $h(n,\boldsymbol{\Phi})$ is assumed to be a linear combination of some functions $\psi_m(\boldsymbol{\Phi})$ of the spectral parameters $\boldsymbol{\Phi}$, instead of a polynomial. That is

$$h(n,\boldsymbol{\Phi}) = \sum_{m=0}^{M-1} c_{n,m}\psi_m(\boldsymbol{\Phi}), \tag{1}$$

where $c_{m,n}$ is the coefficient of expansion. The z-transform of the VDF in (1) is

$$H(z,\boldsymbol{\Phi}) = \sum_{m=0}^{M-1}\left[\sum_{n=0}^{N-1} c_{n,m} z^{-n}\right]\psi_m(\boldsymbol{\Phi}) = \sum_{m=0}^{M-1} C_m(z)\cdot\psi_m(\boldsymbol{\Phi}). \tag{2}$$

This suggests the general structure for its implementation as shown in figure 1. If $H_I(e^{j\omega},\boldsymbol{\Phi})$ is the desired frequency response, the approximation error is

$$E(\omega,\boldsymbol{\Phi}) = H_I(e^{j\omega},\boldsymbol{\Phi}) - \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} c_{n,m}\psi_m(\boldsymbol{\Phi})e^{-jn\omega}. \tag{3}$$

The L_2 norm of $E(\omega,\boldsymbol{\Phi})$ will therefore be a quadratic function of $c_{m,n}$, which has a unique minimum characterized by a system of linear equation. More precisely, the L_2 norm of $E(\omega,\boldsymbol{\Phi})$ is given by

$$E = \int_{\Phi_S}\int_{\Omega_S} W(e^{j\omega},\boldsymbol{\Phi})\cdot\left|E(\omega,\boldsymbol{\Phi})\right|^2 d\omega d\boldsymbol{\Phi}, \tag{4}$$

where $W(e^{j\omega},\boldsymbol{\Phi})$ is a positive weighting function used to control the amount of approximation error in the frequency and the tuning space. The set Ω_S is the frequency support over which

$H_l(e^{j\omega},\boldsymbol{\Phi})$ is to be approximated. To simplify notation, letting $l = n + Nm$ and $z = e^{j\omega}$ in (2), we have

$$H(e^{j\omega},\boldsymbol{\Phi}) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} c_{n,m}\psi_m(\boldsymbol{\Phi})e^{-jn\omega} = \sum_{l=0}^{NM-1} a_l\phi_l(\omega,\boldsymbol{\Phi}), \quad (5)$$

where $a_l = c_{n,m}$ and $\phi_l(\omega,\boldsymbol{\Phi}) = \psi_m(\boldsymbol{\Phi})e^{-jn\omega}$. Substituting (5) into (4) and simplifying gives the following matrix equation

$$E = \boldsymbol{a}^T\boldsymbol{Q}\boldsymbol{a} - 2\boldsymbol{b}^T\boldsymbol{a} + c, \quad (6)$$

where $\boldsymbol{a} = [a_0 \;\; a_1 \;\; \cdots \;\; a_{NM-1}]^T$, $\boldsymbol{b} = [b_0 \;\; b_1 \;\; \cdots \;\; b_{NM-1}]^T$,

$[\boldsymbol{Q}]_{ij} = \int_{\Phi_S}\int_{\Omega_S} W(e^{j\omega},\boldsymbol{\Phi})\cdot\phi_i(\omega,\boldsymbol{\Phi})\overline{\phi_j(\omega,\boldsymbol{\Phi})}\,d\omega d\boldsymbol{\Phi}$,

$[\boldsymbol{b}]_l = \int_{\Phi_S}\int_{\Omega_S} W(e^{j\omega},\boldsymbol{\Phi})\cdot\mathrm{Re}\{H_I(e^{j\omega},\boldsymbol{\Phi})\overline{\phi_l(\omega,\boldsymbol{\Phi})}\}d\boldsymbol{\Phi}d\omega$,

and $c = \int_{\Phi_S}\int_{\Omega_S} W(e^{j\omega},\boldsymbol{\Phi})\cdot\left|H_I(e^{j\omega},\boldsymbol{\Phi})\right|^2 d\omega d\boldsymbol{\Phi}$.

The optimal *LS* solution, $\boldsymbol{a}_{LS}$, is:

$$\boldsymbol{a}_{LS} = \boldsymbol{Q}^{-1}\boldsymbol{b}. \quad (7)$$

If $h(n,\phi)$ is approximated by a polynomial, then the function $\psi_m(\phi)$ is simply given by ϕ^m. Putting the weighting function $W(e^{j\omega},\boldsymbol{\Phi}) = \begin{cases} K_p & \omega\in S_p \\ K_s & \omega\in S_s \end{cases}$ into (6), the equations can then readily be calculated by the reduction formula or in general numerical integration. The optimal weighted least square solution can be calculated from (7). The design of other VDF such as variable bandpass filters and two-dimensional VDFs can be derived similarly. One problem with approximating $h(n,\phi)$ by a polynomial is that the order of the polynomial and hence the number of sub-filter grows rapidly with the tuning range. To overcome this problem, it is desirable to approximate $h(n,\phi)$ by a piecewise polynomial. The tuning range is divided into disjoint intervals and $h(n,\phi)$ in each interval is approximated by a polynomial in ϕ with lower order. Figure 3 shows a simple example where two piecewise polynomials with order 2 are employed. The operator $\boldsymbol{\Psi}^{-1}$ is only necessary for the IIR VDF to be discussed in section III. For FIR VDF, $\boldsymbol{\Psi}^{-1}$ is not needed and $H_i(z)$ are just the sub-filter $C_i(z)$ for the two 2nd order piecewise polynomials.

III. Design of IIR VDF Using Model Reduction

Model reduction is a useful technique for designing IIR filter, especially for approximately linear-phase IIR filter, from FIR filters. There are several advantages of the model reduction approach: i) it is computational simple which only requires the computation of the SVD of a Hankel matrix, ii) the IIR VDF is guaranteed to be stable, iii) the frequency response such as the phase response of the FIR prototype is well preserved. Direct application of model reduction to the sub-filter $C_m(z)$, however, does not lead to satisfactory results. Its coefficients are in fact the coefficients of the interpolating polynomial. Most of the singular values of the Hankel matrix of the impulse response are rather large. Model reduction, which removes the less significant singular values, is therefore unable to offer great reduction in system order. In what follows, a transformation is used so that another set of sub-filter, which is more amendable to model reduction, is implemented instead of $C_m(z)$. Sampling the transfer function $H(z,\boldsymbol{\Phi})$ in (2) at M values of the tuning parameter $\boldsymbol{\Phi} = \boldsymbol{\Phi}_i, i = 0,\cdots M-1$, in matrix form as

$$H(z,\boldsymbol{\Phi}_i) = \boldsymbol{C}^T\boldsymbol{P}(\boldsymbol{\Phi}_i) = \boldsymbol{C}^T\boldsymbol{P}_i \quad (8)$$

where $\boldsymbol{C} = [C_0(z) \;\cdots\; C_{M-1}(z)]^T$, $\boldsymbol{P}_i = \boldsymbol{P}(\boldsymbol{\Phi})\big|_{\boldsymbol{\Phi}=\boldsymbol{\Phi}_i}$, and $\boldsymbol{P}(\boldsymbol{\Phi}) = [\psi_0(\boldsymbol{\Phi}) \;\cdots\; \psi_{M-1}(\boldsymbol{\Phi})]^T$. (8) can also be rewritten as

$$\boldsymbol{H} = \boldsymbol{\Psi}\cdot\boldsymbol{C} \quad (9)$$

where $\boldsymbol{H} = [H(z,\boldsymbol{\Phi}_0) \;\cdots\; H(z,\boldsymbol{\Phi}_{M-1})]^T$, $\boldsymbol{\Psi} = [\boldsymbol{P}_0 \;\cdots\; \boldsymbol{P}_{M-1}]^T$ is a $M\times M$ matrix. If $\boldsymbol{\Psi}$ in (9) is nonsingular, then we can express $\boldsymbol{C}$ in terms of $\boldsymbol{H}$ as

$$\boldsymbol{C} = \boldsymbol{\Psi}^{-1}\cdot\boldsymbol{H}. \quad (10)$$

In other words, the sub-filter $C_m(z)$ can be replaced by another set of sub-filter $H(z,\boldsymbol{\Phi}_i)$ followed by a linear transformation $\boldsymbol{\Psi}^{-1}$. For polynomial basis function, we have $\psi_m(\boldsymbol{\Phi}) = \phi^m$. If the M values of $\boldsymbol{\Phi}$ are evenly spaced, then $\boldsymbol{\Psi}$ is the Vandermonde matrix and it is nonsingular. The model order reduction of $H(z,\boldsymbol{\Phi}_i)$ will produce lower system order than that of $C_m(z)$, if $H(z,\boldsymbol{\Phi}_i)$ is frequency selective. The model order reduction employed is the ERA [8]. Note that the sub-filter can be viewed as a single input and M outputs (SIMO) system. To carry out model reduction of these sub-filter using the ERA algorithm, let's rewrite them in state space model (SSM):

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\cdot\boldsymbol{x}(k) + \boldsymbol{B}u(k), \quad (11)$$

$$\boldsymbol{y}(k) = \boldsymbol{C}\cdot\boldsymbol{x}(k) + \boldsymbol{D}\cdot u(k), \quad (12)$$

where the size of $\boldsymbol{A}$ is $(n\times n)$, $\boldsymbol{B}$ is $(n\times 1)$, $\boldsymbol{C}$ is $(M\times n)$, and $\boldsymbol{D}$ is $(M\times 1)$. Let $\boldsymbol{Y}_k$, $k = 1, 2, \ldots$, be the $(n\times 1)$ pulse-response matrix or Markov parameters, one gets:

$$\boldsymbol{Y}_0 = \boldsymbol{D},\;\; \boldsymbol{Y}_1 = \boldsymbol{CB},\;\; \boldsymbol{Y}_2 = \boldsymbol{CAB},\;\; \ldots,\;\; \boldsymbol{Y}_k = \boldsymbol{CA}^{k-1}\boldsymbol{B},\ldots \quad (13)$$

The ERA system begins by forming the generalized $\alpha M\times\beta$ Hankel matrix $H(k-1)$ composed of (13) as

$$\boldsymbol{H}(k-1) = \begin{bmatrix} \boldsymbol{Y}_k & \boldsymbol{Y}_{k+1} & \cdots & \boldsymbol{Y}_{k+\beta-1} \\ \boldsymbol{Y}_{k+1} & \boldsymbol{Y}_{k+2} & \cdots & \boldsymbol{Y}_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{Y}_{k+\alpha-1} & \boldsymbol{Y}_{k+\alpha} & \cdots & \boldsymbol{Y}_{k+\alpha+\beta-2} \end{bmatrix}. \quad (14)$$

For simplicity, we choose $\alpha = \beta = n$, and $k = 1$. The SVD of the Hankel matrix with $k = 1$ is computed

$$\boldsymbol{H}(0) = \boldsymbol{R}\cdot\Sigma\cdot\boldsymbol{S}^T, \quad (15)$$

where the columns of matrices $\boldsymbol{R}$ and $\boldsymbol{S}$ are orthonormal and $\boldsymbol{\Sigma}$ is a diagonal matrix. Let r be the order of the model-reduced system and $\boldsymbol{R}_r$ and $\boldsymbol{S}_r$ be the matrices formed by the first r columns of $\boldsymbol{R}$ and $\boldsymbol{S}$, respectively. Similarly, let $\boldsymbol{\Sigma}_r$ be the matrix formed by the first r columns and first r rows of $\boldsymbol{\Sigma}$. To reduce $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$ to the reduced system $(\boldsymbol{A}_r, \boldsymbol{B}_r, \boldsymbol{C}_r, \boldsymbol{D}_r)$, let's form the following reduced Hankel matrix

$$\boldsymbol{H}_r(0) = \boldsymbol{R}_r\Sigma_r\boldsymbol{S}_r^T. \quad (16)$$

It can be shown that $\boldsymbol{H}_r(0)$ is composed of the controllability matrix $\boldsymbol{M}_c$ and the observability matrix $\boldsymbol{M}_o$ as

$$\boldsymbol{H}_r(0) = \boldsymbol{M}_c\cdot\boldsymbol{M}_o, \quad (17)$$

where $\boldsymbol{M}_c = [\boldsymbol{C}_r \;\; \boldsymbol{C}_r\boldsymbol{A}_r \;\; \cdots \;\; \boldsymbol{C}_r\boldsymbol{A}_r^{\alpha-1}]^T$, $\boldsymbol{M}_o = [\boldsymbol{B}_r \;\; \boldsymbol{A}_r\boldsymbol{B}_r \;\; \cdots \;\; \boldsymbol{A}_r^{\beta-1}\boldsymbol{B}_r]$.

More generally, we have

$$\boldsymbol{H}_r(k) = \boldsymbol{M}_c\boldsymbol{A}_r^{\,k}\boldsymbol{M}_o, \quad (18)$$

Comparing Eqs. (16) and (17) with $k = 0$ gives

$$\boldsymbol{M}_c = \boldsymbol{R}_r\Sigma_r^{1/2}, \text{ and } \boldsymbol{M}_o = \Sigma_r^{1/2}\boldsymbol{S}_r^T. \quad (19)$$

From Eq. (17), it is clear that the first column of $\boldsymbol{M}_o$ forms the reduced input matrix $\boldsymbol{B}_r$ whereas the first M rows of $\boldsymbol{M}_c$ form the reduced output matrix $\boldsymbol{C}_r$. The reduced $\boldsymbol{D}_r$ matrix is exactly equal to the $\boldsymbol{D}$ matrix. To determine $\boldsymbol{A}_r$, consider Eq. (18) and (19) with $k = 1$,

$$\boldsymbol{A}_r = \Sigma_r^{-1/2}\boldsymbol{R}_r^{\,T}\boldsymbol{H}(1)\boldsymbol{S}_r\Sigma_r^{-1/2}. \quad (20)$$

The reduced system $(\boldsymbol{A}_r, \boldsymbol{B}_r, \boldsymbol{C}_r, \boldsymbol{D}_r)$ is now determined. Let the model-reduced vector of $\boldsymbol{H}$ be denoted as $\widetilde{\boldsymbol{H}}$. The new reduced transfer function will have the same denominator $D(z)$ and the M numerators, denoted by $N_i(z)$. Therefore, we have

$$\widetilde{\boldsymbol{H}} = \boldsymbol{N}/D(z), \quad (21)$$

where $N = [N_0(z) \cdots N_{M-1}(z)]^T$. Since there is only one denominator, the implementation complexity associated with the denominator of the transfer functions is greatly reduced. The structure of the final IIR VDF is shown in Fig. 2.

IV. EFFICIENT IMPLEMENTATION OF VDF

To reduce the implementation complexity, the sub-filter are implemented as multiplier-less FIR filters using the SOPOT coefficients in the form

$$\hat{c}_{k,n} = \sum_{j=1}^{L_s} b_{k,n,j} \cdot 2^{a_j}, \tag{22}$$

where $b_{n,k,j}(i) \in \{-1,1\}$ and $a_j \in \{-l,\ldots,-1,0,1,\ldots,l\}$. l is a positive integer and its value determines the range of the coefficients, and L_s is the number of terms used in the coefficient approximation and is usually limited to a small number. The coefficient multiplication can then be implemented as limited number of shifts and additions. To design the SOPOT sub-filter, we minimize the L_∞ norm of its difference in frequency response with the ideal one as

$$\delta_a = \max_{\omega \in S, d \in [0,1]} \left\langle \left| H_I(e^{j\omega}) - \hat{H}(e^{j\omega}) \right| \right\rangle. \tag{23}$$

$H_I(e^{j\omega})$ is the ideal frequency response and $\hat{H}(e^{j\omega})$ is the frequency response calculated for a given SOPOT filter coefficients. The real-valued coefficients $c_{k,n}$ are first determined by the LS method described in section II. Let $\boldsymbol{b}$ be the vector containing these coefficients. The random search algorithm will repetitively calculate a candidate SOPOT vector $\boldsymbol{b}_c$ by adding to $\boldsymbol{b}$ a random perturbation vector $\lambda \boldsymbol{b}_p$ and then rounding it to the nearest SOPOT representation. That is

$$\boldsymbol{b}_c = \lfloor \boldsymbol{b} + \lambda \boldsymbol{b}_p \rfloor_{SOPOT}. \tag{24}$$

The vector $\boldsymbol{b}_p$ is a random vector with elements chosen in the range ± 1, and λ is a user-defined variable used to control the size of the neighborhood to be searched. $\lfloor \cdot \rfloor_{SOPOT}$ is the rounding operator that converts every element inside the input vector to its closest SOPOT value with a given value of l. The performance measure δ_a of the new coefficients is then calculated. The set that yields the minimum peak error δ_a under the given constraints of total number of terms and l is recorded as the final solution. The random search algorithm is similar in concept to the stimulated annealing algorithm. However, we have used the real-valued optimal solution as a starting point to reduce the searching time required. To implement the sub-filter $C_k(z)$ using the MB [9], the structure in Fig. 1 is rewritten in its transposed form. The input is now multiplied with a large number of constant coefficients in SOPOT form. These products can efficiently be implemented using MB, which reduces the redundancies in multiplying a given input with a set of integer coefficients by removing any possible common sub-expressions in their representations. Using MB, it is possible to reduce significantly the additions in implementing the multiplier-less sub-filter leading to great hardware savings.

V. DESIGN EXAMPLES

A tunable linear-phase FIR VDF and an approximately linear-phase IIR VDF are designed. The tuning range of the passband is from 0.2π to 0.4π and the transition band is fixed at 0.2π. We divide the tuning range into 2 intervals with 3 sub-filter per interval. Each sub-filter has 40 taps. The frequency responses of the FIR VDF designed using the LS method are shown in Fig. 4 (a), (b). The frequency responses, group delay, and transient response of the IIR VDF are plotted in figure 4 (c) and (d). Details comparison of the FIR and IIR VDF are summarized in Table 1. Since the order of the IIR VDF is reduced to half and there is only one denominator for all the sub-filter, the total number of multiplications is reduced approximately by 40% as compared with the FIR VDF. The frequency response of the IIR VDF is seen to be comparable to the original FIR VDF.

	FIR VDF	IIR VDF
Filter length	40	21 (numerators) 21 (denominator)
Interpolation order *	2	2
Total No. of Multiplications (due to filter coefficients)	240	147
Required adders before using MB	317	548
Required adders after using MB	103	146
Total No. of Additions (due to the tapped delay line)	234	140
Stopband Attenuation	45.34dB	44.31dB

Table 1. Parameters for the tunable lowpass filters (*2 blocks each with 3 branches, each block use Lagrange interpolator of order-two.)

VI. CONCLUSIONS

A systematic method for design and implementation of FIR and IIR VDF is presented. A LS approach for designing FIR VDF with its impulse response expressed as a linear combination of basis functions is first presented. By choosing the basis functions as piecewise polynomial, VDF with larger tuning range than ordinary polynomial based approach can be obtained. The resulting VDF can be efficiently implemented using the Farrow structure. Making use of the FIR VDF, an ERA-based model reduction technique is proposed to yield a stable IIR VDF with lower system order. It does not suffer from undesirable transient response during parameter tuning. For frequency selective VDF, about 40% of the multiplications can be saved using the IIR VDF. The coefficients implementation of the proposed VDF using SOPOT and the MB technique is also presented. Results show that about two-third of the additions in implementing the SOPOT coefficients can be saved using MB, which leads to significant savings in hardware complexity.

ACKNOWLEDGEMENT

The authors would like to thank Dr. T. B. Deng for reprints of his papers on variable digital filters and useful discussion.

REFERENCES

[1] F. M. Gardner, "Interpolation in digital modems I. Fundamentals," *IEEE Trans. Commun.*, vol. 41, pp.501-507, Mar 1993.

[2] L. Erup, F. M. Gardner and R. A. Harris, "Interpolation in digital modems II. Implementation and performance," *IEEE Trans. Commun.*, vol. 41, pp. 998-1008, Jun 1993.

[3] P. Jarske, Y. Neuvo, and S. K. Mitra, "A simple approach to the design of linear phase FIR digital filters with variable characteristics," *IEEE Trans. Signal Processing*, vol. 14, no. 4, pp. 313-326, Jun 1988.

[4] T. B. Deng, "Design of recursive 1-D variable filters with guaranteed stability," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 1035-1040, Aug 1999.

[5] C. K. S. Pun, S. C. Chan and K. L. Ho, "Efficient 1D and circular symmetric 2D FIR filters with variable cutoff frequencies using the Farrow structure and multiplier-block," in *Proc. IEEE ISCAS 2001*, vol. 2, pp. 561-564.

[6] A. Tarczynski, G. D. Gain, E. Hermanowicz and M. Rojewski, "WLS design of variable frequency response FIR filters," in *Proc. IEEE ISCAS 1997*, vol. 4, pp. 2244-2247.

[7] R. Zarour, and M. M. Fahmy, "A design technique for variable digital filters," *IEEE Trans. Circuits Syst. II*, vol. 36, pp. 1473-1478, Nov 1989.

[8] J. N. Juang, and R. S. Pappa, "An eigensystem realization algorithm for modal parameter identification and model reduction," *Journal of Guidance, Control, and Dynamics*, vol. 8, no. 5, pp. 620-627, Sept – Oct, 1985.

[9] A. G. Dempster and M. D. Macleod, "Use of minimum-adder multiplier blocks in FIR digital filters," *IEEE Trans. Circuits Syst. II*, vol. 42, pp. 569-577, Sept 1995.
[10] C. W. Farrow, "A continuously variable digital delay element," in *Proc. ISCAS 1998*, pp. 2641-2645.

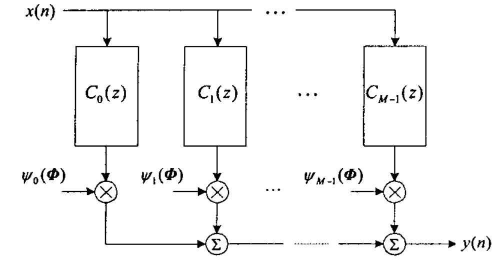


Figure 1. A general FIR VDF structure.

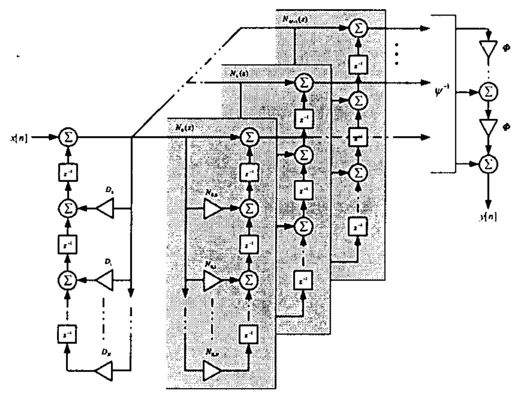


Figure 2. Proposed IIR VDF structure.

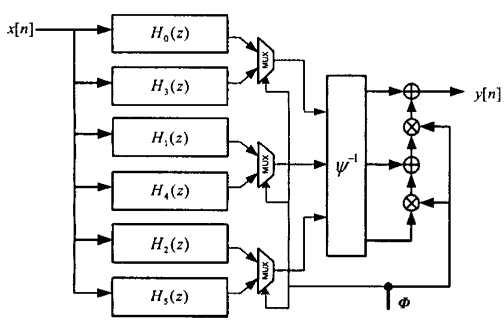
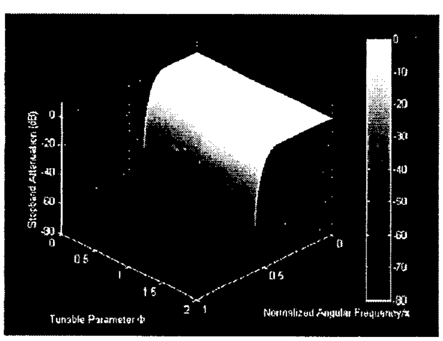
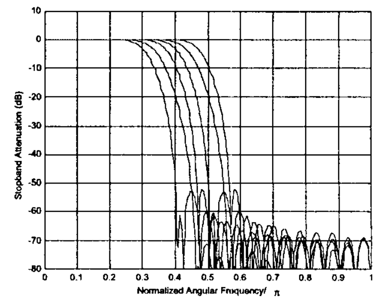


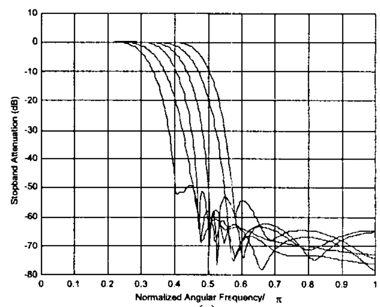
Figure 3. Proposed piecewise polynomial-based VDF structure. For FIR VDF $\Psi^{-1} = I$ and $H_i(z)$ are the sub-filters.



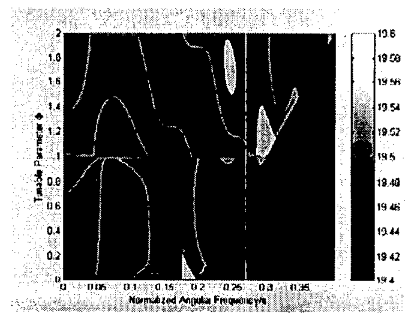
(a)



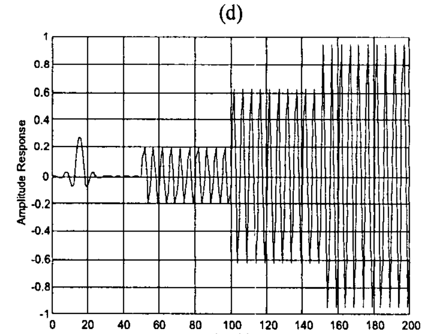
(b)



(c)



(d)



(e)

Figure 4. Design results: (a) 3-D plot of frequency response of FIR VDF (b) Frequency responses of FIR VDF even sampled in the range $\Phi = [0,2]$. (c) Frequency responses of IIR VDF even sampled in the range $\Phi = [0,2]$. (d) Group delay of the IIR VDF. (e) Transient response of the IIR VDF. Tuning parameter Φ is increased by 0.25 every 50 samples, i.e. the passband is increased by 0.05π successively. It can be seen that the amplitude of the VDF output increases at each block of 50 samples. Note there is no transient during parameter switching.