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POSTGLITCH RELAXATION OF THE CRAB PULSAR AFTER ITS FIRST FOUR MAJOR GLITCHES: THE COMBINED EFFECTS OF CRUST CRACKING, FORMATION OF VORTEX DEPLETION REGION AND VORTEX CREEP

M. A. ALPAR,[1,2] H. F. CHAU,[3,4] K. S. CHENG,[5] AND D. PINES[3]

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ABSTRACT

Following the application of vortex creep theory (Alpar et al. 1984; Alpar et al. 1993; Chau et al. 1993) to the postglitch behavior of the Vela pulsar, we extend the model to cover the postglitch behavior of the Crab pulsar (Alpar et al. 1994). We propose that the comparatively modest ($\Delta\Omega/\Omega \sim 10^{-8}$) and somewhat infrequent ($\sim$6 yr interglitch intervals) Crab pulsar glitches are caused by crust cracking during starquakes induced by pulsar spin-down (Ruderman 1976; Baym & Pines 1971). We attribute the anomalous postglitch behavior (an occasional extended spin-up and a long-term response opposite in sign to that seen in the Vela pulsar [see Lyne, Graham-Smith, & Pritchard 1992]) to postglitch response to inward vortex bunching into newly formed vortex traps during a quake. The persistent shift in the angular acceleration $\dot{\Omega}_c$ following a glitch is attributed to the creation of a new vortex depletion region (Alpar & Pines 1993). The different postglitch behavior for the Crab and Vela pulsars can be understood on evolutionary grounds (Alpar et al. 1994).

Subject headings: dense matter — pulsars: individual (Crab Nebula) — stars: neutron

1. INTRODUCTION

Pulsar glitches are probably experienced by pulsars of all ages, but occur more frequently in younger ones (Alpar & Baykal 1994). In particular, during 27 yr of continuous radio timing observations a total of six glitches have been found for the Crab: the first two observed glitches (in 1969 and 1975, respectively) were detected with an uncertainty of about 1 week (Boynton et al. 1972; Demiański & Prószyński 1983); the third (in 1981) took place between timing observations (Lyne, Pritchard, & Graham-Smith 1993); the fourth and the fifth (in 1986 and 1989, respectively) were detected during observation sessions (Lyne & Pritchard 1987; Lyne, Graham-Smith, & Pritchard 1992) in the course of the continuous monitoring program at Jodrell Bank. We do not have enough data so far to analyze the most recent event (in 1992); we therefore concentrate on the 1969, 1975, 1986, and 1989 glitches of the Crab pulsar in the present analysis (Alpar et al. 1994).

The pattern of glitches and postglitch behavior observed for the Crab pulsar is strikingly different from that observed for the Vela pulsar. A key question is whether the differences can be understood on evolutionary grounds. The 10 glitches of the Vela pulsar which have been observed during the period 1969–1995 have offered to both observers and theorists the opportunity of studying postglitch behavior in considerable detail. It is natural to inquire to what extent the theoretical framework developed to explain the Vela pulsar glitches and postglitch relaxation can be extended to the Crab pulsar. That framework is a phenomenological description of the rotational dynamics of a superfluid in the neutron star's inner crust, whose spin-down under the pulsar torque is achieved by the motion of quantized vortices in the presence of pinning forces (Alpar et al. 1993). Our discussion of the Crab pulsar glitches will be built on a comparison with this description of the Vela pulsar.

Before proceeding to the discussion of the Crab pulsar glitches, let us summarize the basic concepts involved in the model applied to the Vela pulsar:

1. There is no observational evidence that the pulsar electromagnetic torque changes in a glitch.
2. If there are no sudden changes in the external torque, the observed sudden changes in the rotation frequency Ω and spin-down rate $\dot{\Omega}$ must be due either to sudden changes in moment of inertia (starquakes), or to sudden changes in the angular momentum transport between the components of the star. In the Vela pulsar starquakes can be ruled out for a number of reasons (Alpar 1995). Hence, the Vela pulsar glitches must reflect sudden changes in the angular momentum transport.
3. When a rotating classical fluid or superfluid spins down as a result of external torques exerted on its container angular momentum is transferred to the container, and, in the presence of differential rotation, from the inner parts of the fluid to the outer parts. The outward angular momentum transfer that spins the fluid down is thereby spinning up the container (the observed outer crust in the case of the neutron star). This spin-up cancels a part of the external spin-down torque, according to the ratio of moments of inertia of fluid and container, so that the net result in dynamical equilibrium is the spin-down of the container at the rate given by the external torque divided by the total moment of inertia of fluid and container.

In a superfluid, the rotation is carried by quantized vortices. The rotational state of the fluid is determined by the distribution of the quantized vortices, and spin-down is achieved by motion of vortices (a vortex current) away from the rotation axis. The discreteness of the vorticity actually makes the rotational dynamics of a superfluid simpler to

[1] Physics Department, Middle East Technical University, Ankara 06531, Turkey.
[2] Adjunct Professor, Physics Department, University of Wisconsin, Madison, WI 53706.
[3] Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, IL 61801-3080.
[4] Present address: School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540.
[5] Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong.

model and to visualize than is the case with a rotating viscous fluid described by the Navier-Stokes equation.

4. Vortex lines in the neutron star crust, where the superfluid coexists with the lattice, can experience pinning forces from the lattice. If the pinning forces could be infinite, vortices would be absolutely pinned down and the superfluid would not be able to spin down at all. In practice, pinning forces are finite. We use the term "pinned superfluid" to indicate that pinning forces are present, not to imply that all vortices are absolutely pinned. There are two qualitatively different modes of pinned superfluid spin-down.

5. One possible mode is continuous (in time) spin-down, corresponding to a continuous vortex current, as vortex lines unpin and repin, either by being activated thermally (in which case their motion is called "vortex creep") or by quantum mechanical tunneling. On the average the vortex lines move outward (away from the rotation axis), as a result of the bias on the system arising from the external spin-down torque. Pinning brings about an excess density of vortex lines and so keeps the superfluid rotation rate higher than the rotation rate of the container (the neutron star crust). In equilibrium, the lag ω between the superfluid and crust lattice rotation rates, $\omega = \Omega_s - \Omega_c$, will be just large enough to drive the vortex current that makes the superfluid spin-down at the same rate as the crust lattice. This continuous vortex current prevails at all times between glitches. Its sensitivity to offsets of ω from its equilibrium value gives rise to the changes in the spin-down rate associated with glitches and postglitch relaxation.

6. If there are large gradients in the pinning forces supplied by the lattice, these will produce inhomogeneities in the vortex density distribution. One then has traps containing a high density of pinned vortices, surrounded by relatively vortex-free regions. Vortex currents between the traps are much reduced, except at those times when the vortex density inside a trap reaches a critical value. Vortices released from a given trap act to induce the discharge of vortices from other traps: the resulting "avalanche" of vortices moving outward decreases the angular momentum of the pinned superfluid and spins the crust up. These are the glitches. This discontinuous vortex current also contributes to the spin-down of the superfluid. Thus, in the long run the glitches are just part of the spin-down of the superfluid required by the external torque. Hence it is natural, as observed in the Vela pulsar glitches, that the associated spin-up of the crust does not relax back. The time-averaged long-term behavior of the crust is always spin-down, as dictated by the external (pulsar) torque.

7. These two modes, vortex creep and glitches, find a natural analogue in electric circuits, in the continuous current through resistors and discharges of capacitors, respectively.

In the case of the Vela pulsar, both the interval between glitches and the postglitch relaxation observed for the nine giant glitches for which detailed observations exist to date can be quantitatively explained within the framework of vortex creep theory (Alpar et al. 1993; Chau et al. 1993). In this approach, glitches are the result of sudden vortex unpinning. This temporarily decouples the continuous mode of angular momentum transfer from the crustal superfluid to the crust. Postglitch relaxation reflects a gradual recoupling of vortex creep. Various estimates on the inertial moment of the pinned superfluid have been carried out (Chau et al. 1993; Datta & Alpar 1993). In the framework of vortex creep theory these suggest that the equation of state (EOS) of the neutron star must by sufficiently stiff that for a 1.4 $M_\odot$ star, the crust is thick enough to contain some 2.6% of the stellar moment of inertia.

The model described below for the Crab pulsar glitches rests mainly in the same framework of these two modes of angular momentum transfer, sudden vortex unpinning, and vortex creep. However, since a pure vortex unpinning model predicts far too short an interval between Crab pulsar glitches (a few months, rather than some years; Alpar, Nandkumar, & Pines 1985), the events must be triggered by some other mechanism. We propose that the comparatively modest ($\Delta\Omega/\Omega \sim 10^{-8}$) and somewhat infrequent ($\sim$6 yr interglitch intervals) Crab pulsar glitches are triggered by starquakes induced by pulsar spin-down (Baym & Pines 1971; Ruderman 1976; Alpar et al. 1994). Starquakes are also a plausible mechanism for the sudden unpinning of neutron vortices in the Crab pulsar, leading to the observed glitch. Further we attribute the anomalous postglitch behavior (an occasional extended spin-up and a long-term response opposite in sign to that seen in the Vela pulsar [Lyne, et al. 1992]) to some of the vortices being transported inward into newly formed vortex traps during a quake, while the persistent change in angular acceleration $\dot{\Omega}_c$ following a glitch represents the creation of a new vortex depletion region, as suggested by Alpar & Pines (1993). We note that pure quake models are also ruled out as they would require the glitch in rotation frequency and the persistent shift in the spin-down rate to be both determined by the change in moment of inertia $\Delta\Omega/\Omega = \Delta\dot{\Omega}/\dot{\Omega} = -\Delta I/I$, while in the Crab pulsar $\Delta\dot{\Omega}/\dot{\Omega} \sim 10^{-4}$ and $\Delta\Omega/\Omega \sim 10^{-8}$. The effect of crust cracking, which is important for hot young pulsars like the Crab, becomes unimportant when they evolve to the age of Vela. This is consistent with the observed time interval between successive glitches in the Crab and Vela pulsars (see § 6 for details).

The data on the postglitch evolution of all four Crab glitches to date provides an opportunity to test our model involving crust cracking in conjunction with vortex creep theory. We follow a "minimalist" phenomenological approach (Alpar et al. 1993) in our analysis of postglitch relaxation. A consistent model is constructed to fit the observed events, with the relaxation timescales constrained to be the same in every glitch, and with the minimum possible number of free parameters. In this way a consistent explanation of the Crab pulsar glitches and postglitch behavior is obtained.

In § 2 we give a brief summary of the observed behavior for the Crab pulsar glitches observed to date. We then present results of our fits to four of these glitches in § 3. A "minimalist" phenomenological model, taking into account vortex creep theory, the formation of vortex depletion regions, and crust cracking, is developed in § 4. Section 5 contains a discussion of the physical origin of the short (0.8 day), intermediate (12 day), and the long (200 day) timescale responses together with the persistent shift in $\dot{\Omega}$. A comparison between the Crab and Vela glitches is carried out in § 6, while our conclusions are presented in § 7.

2. BEHAVIOR OF CRAB GLITCHES

Six glitches have been observed in the Crab pulsar (PSR 0531+21) since its discovery. The fractional increase in angular velocity at the time of glitch ($\Delta\Omega_c/\Omega_c$) ranges from

9×10^{-9} to 6×10^{-8} while the fractional change in the spin-down rate ($\Delta\dot{\Omega}_c/\dot{\Omega}_c$) is of the order of 10^{-3}. Wc concentrate here on the first, second, fourth, and fifth events as these are the glitches for which the most comprehensive observational data presently exist.

The common characteristics of the 1969, 1975, and 1986 glitches of Crab can be summarized as follows (Lyne et al. 1993). First, the rotational frequency of the pulsar decreases rapidly with a timescale of $\lesssim 15$ days which is followed by a further decrease with a timescale of about 100–300 days. Second, unlike the Vela glitches (Cordes, Downs, & Krause-Polstorff 1988), the angular acceleration ($\dot{\Omega}_c$) never returns to its pre-glitch level; there is a permanent offset in $\dot{\Omega}_c$.

The behavior of the 1989 glitch is even more spectacular. Instead of starting to decrease right after the glitch, the rotational frequency of the pulsar further increases by roughly 12% with a timescale of $\lesssim 1$ day (Lyne et al. 1992). Such behavior has never been observed before in any pulsar. It is followed by a simple exponential relaxation with a characteristic time of ≈ 18 days. In addition, Lyne et al. find that there is a further small asymptotic rise which they fitted with an exponential with a characteristic time of ≈ 265 days (Lyne et al. 1992).

3. RESULTS FROM FITTING POSTGLITCH BEHAVIOR

By means of the nonlinear fitting program developed by Chau (Alpar et al. 1993), we find that the simplest good fit to all the above four Crab glitches, with the data taken from Boynton et al. (1972) (1969 glitch), Demiański & Prószyński (1983) (1975 glitch), Lyne & Pritchard (1987) (1986 glitch) and Lyne et al. (1992) (1989 glitch), can be obtained by the following equation (Alpar et al. 1994):

$$\Delta\dot{\Omega}_c(t) = -\sum_{i=1}^{2} a_i e^{-t/\tau_i} - a_3\left[1 - \frac{1}{1 + \alpha e^{-(t+\Delta)/\tau_3}}\right] - b\,, \quad (1)$$

where t is the time since the first postglitch observation, with $\tau_1 = 0.8$ days (± 0.1 days), $\tau_2 = 12$ days (± 1 day), and $\tau_3 = 200$ days (± 20 days). Here Δ is the time lag between the actual time of the glitch and the first postglitch observation. Δ is taken to be 0 for the 1986 and 1989 events. Equation (1) expresses the general postglitch response (eq. [17]) developed in § 4 which takes into account vortex creep, the formation of vortex depletion regions, and crust cracking. The best-fit parameters for the four glitches (that is, those with τ_1, τ_2, and τ_3 treated also as free parameters) are tabulated in Table 1. Table 2 gives the consistent fitting parameters obtained by fixing the response timescales τ_1, τ_2, and τ_3 0.8 days, 12 days, and 200 days, respectively, for all four glitches. The corresponding consistent fits are shown in Figures 1–4. By comparing the χ^2 values, we find that it is possible to describe the postglitch relaxation for all four glitches by a consistent set of relaxation times and a permanent offset in $\dot{\Omega}$, as the χ^2 values for the consistent fits are comparable to those of the best fits in which τ_1, τ_2, and τ_3 are allowed to vary independently for the different glitches. The parameter a_1 is negative for some glitches while a_2 and a_3 are always positive. We have taken the error bars to represent the average amplitudes of the timing residuals. The small χ^2 per degree of freedom for the 1975 and the 1989 glitches suggests a relatively low noise level during these epochs.

The characteristic relaxation times τ_1, τ_2, and τ_3 are determined by the consistent fits of the 1986 and 1989 events. The values of a_1, a_2, and b vary from glitch to glitch. The percentage errors in these parameters are about 3%. The 1989 glitch is the only case where α can be unambiguously determined; it has a percentage error of about 10%. For the other three glitches, the values of α are so small that the third term in equation (1) can be approximated by an exponential decay in the form $-a_3 \alpha e^{-(t+\Delta)/\tau_3}$, and only the amplitude, $a_3 \alpha e^{-\Delta/\tau_3}$, can be deduced from the fit with an error of about 5%. Our fit resembles that proposed by Lyne et al., who used three exponentials, with response times 0.8 days, 18 days, and 265 days, to fit the 1989 glitch (Lyne et al. 1992). The "continuing rise" of the 1989 glitch on the short timescale $\tau_1 = 0.8$ days is an exponentially relaxing, positive contribution to $\Delta\dot{\Omega}(t)$, reflected

TABLE 1

PARAMETERS OF THE BEST FITS OF THE CRAB GLITCHES

	GLITCH YEAR			
PARAMETER	1969	1975	1986	1989
$(a_1)_{-12}$[a]	0.41	...	21.9	−65.1
$(a_2)_{-12}$[a]	0.52	4.00	1.48	13.8
$(a_3)_{-12}$[a]	...	...	...	1.31
α[b]	...	...	...	−0.271
$(a_3 \alpha e^{-\Delta/\tau_3})_{-12}$	0.2	0.02	...	...
τ_1[c]	0.8	...	0.7	0.8
τ_2[c]	8.7	9.7	6.6	14.4
τ_3[c]	200	190	...	210
b_{-13}[d]	0.037	5.6	...	9.2
χ^2[e]	8.6	9.1	22.1	4.5
n[e]	6	29	33	28

[a] $a_i = (I_i/I)(\delta\omega_i(0)/\tau_i)\, e^{-\Delta/\tau_i}$ ($i = 1, 2$) are in unit of 10^{-12} rad s^{-2}, where Δ is the uncertainty in the date of glitch. $a_3 = (I_3/I)\,|\dot{\Omega}|_\infty$ is also in unit of 10^{-12} rad s^{-2}.

[b] $\alpha = e^{t_0/\tau_3} - 1$.

[c] τ_1, τ_2, and τ_3 are in unit of days.

[d] b is the permanent offset of $\Delta\dot{\Omega}_c$ in the unit of 10^{-13} rad s^{-2}.

[e] n and χ^2 are the degree of freedom and χ^2 value of the best fit, respectively.

TABLE 2

PARAMETERS OF THE CONSISTENT FITS OF THE CRAB GLITCHES, WITH $\tau_1 = 0.8$ DAYS, $\tau_2 = 12$ DAYS, AND $\tau_3 = 200$ DAYS FOR ALL FOUR GLITCHES

	GLITCH YEAR			
PARAMETER	1969	1975	1986	1989
$(a_1)_{-12}$[a]	0.49	4.01	25.7	−79.6
$(a_2)_{-12}$[a]	0.45	3.15	0.68	17.0
$(a_3)_{-12}$	...	...	...	1.1
α	...	...	...	−0.321
$(a_3 \alpha e^{-\Delta/\tau_3})_{-14}$	8.5	2.7	0.5	...
$(b)_{-13}$[b]	0.08	5.7	0.44	9.2
$[\Delta\Omega_c(0)/\Omega_c]_{-9}$[c]	4.0	44.0	23.7	77.1
Δ_{max}[d]	3.0	6.4	0.0	0.0
χ^2[e]	10.2	7.9	43.6	11.4
n[e]	9	32	36	31

[a] Both a_1 and a_2 are determined up to error of about 3%.

[b] b is determined up to an error of 3%.

[c] $\Delta\Omega_c(0)$ is the initial jump in rotational speed of the pulsar right after the glitch. And $\Delta\Omega_c(0)/\Omega$ is in units of 10^{-9}.

[d] Δ_{max} is the time gap between the first observation after the glitch and the last observation before the same glitch in days. Note that both the 1986 and the 1989 glitches are caught right at the time of glitch.

[e] n and χ^2 are the degree of freedom and χ^2 value of the consistent fit, respectively.

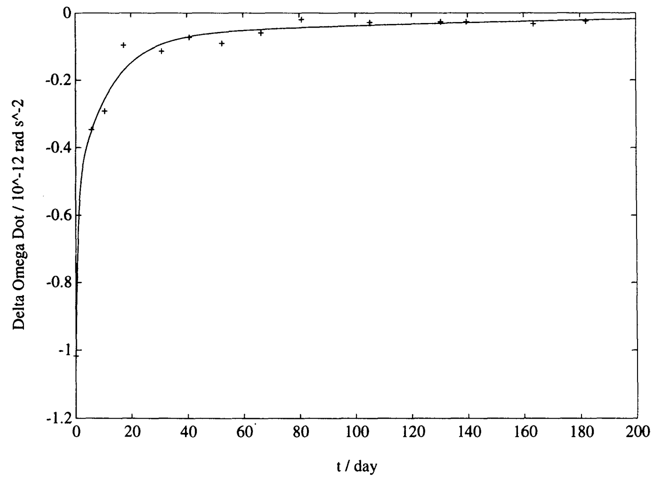

FIG. 1.—Consistent fit of $\Delta\dot{\Omega}_c$ for the 1969 glitch of Crab

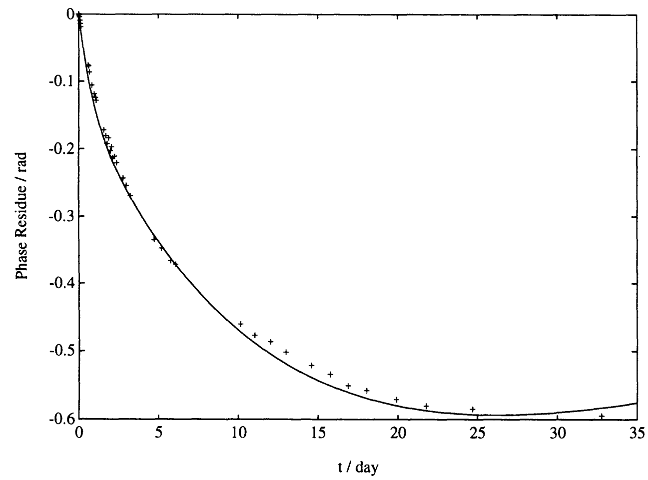

FIG. 3.—Consistent fit of the phase residue for the 1986 glitch of Crab

in the negative value of a_1 for this glitch. The similar but long-term positive $\Delta\dot{\Omega}(t)$ contribution, fitted with an exponential relaxation time of 265 days by Lyne et al. (1992) requires a negative α in our nonlinear response term. The unprecedented occurrence of the short and long timescale "rise" terms after the 1989 glitch (requiring negative a_1 and α_3) is a crucial clue to the physics of the Crab glitches. Since we are using a different fitting function, the relaxation times we obtain differ slightly from those obtained by Lyne et al. (1992).

4. PHYSICAL PROCESSES IN THE CRAB GLITCHES

4.1. *Vortex Pinning and Creep*

According to vortex creep theory (Alpar et al. 1993; Chau et al. 1993), the inner crust of the neutron star is divided into a number of regions of different pinning strengths for the neutron vortex lines. These vortex lines carry the angular momentum of the crustal superfluid, and the crustal superfluid exchanges angular momentum with the crust through the motion of the vortex lines. As the crustal superfluid coexists with the crust lattice, the distribution and motion of the vortex lines is determined by their interactions with the lattice, in particular by the pinning of the vortex lines to the lattice. Glitches involve the motion of the unpinned neutron superfluid vortex lines while postglitch evolution is the response of thermally activated continuous vortex creep to the sudden changes introduced by the glitch. The general form of the postglitch response in vortex creep theory is (Alpar et al. 1993)

$$\begin{aligned}\frac{\Delta\dot{\Omega}_c(t)}{\dot{\Omega}_\infty} = &-\int \frac{dI_l(r)}{I}\frac{\delta\omega_l(0)e^{-t/\tau_l}}{\tau_l\dot{\Omega}_\infty} + \int \frac{dI_n(r)}{I} \\ &\times\left\{1 - \frac{1}{1+[e^{t_0(r)/\tau_n}-1]e^{-t/\tau_n}}\right\} \\ &+\left(\frac{I_A}{I}+\frac{I_B}{I}\right)\theta(t) - \left(\frac{I_A}{I}+\frac{I_B}{I}\right)\theta(t-t_g) \\ &-\frac{I_A}{I}\frac{t}{t_g}\theta(t)\theta(t_g-t)\ . \qquad (2)\end{aligned}$$

Equation (2) is the most general form of postglitch response within the framework of vortex creep model. Some of the terms in equation (2) may be absent in a particular postglitch response of a given pulsar, depending on the nature of the glitch, the pinning energies, and the temperature.

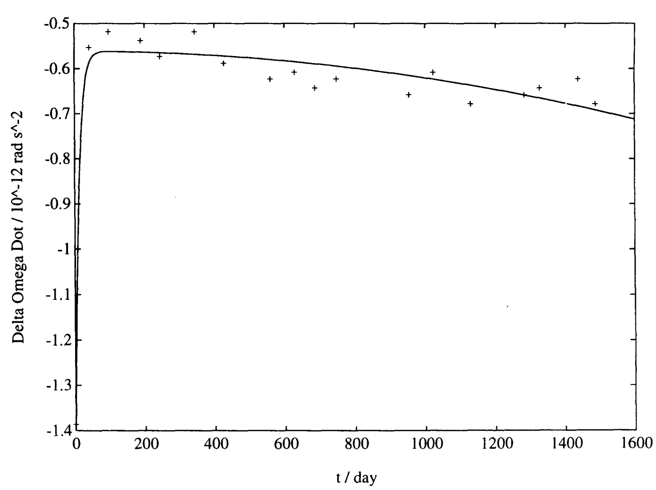

FIG. 2*a*

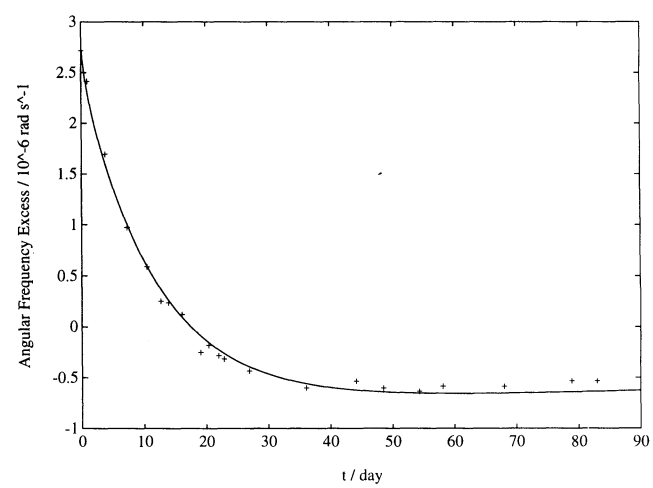

FIG. 2*b*

FIG. 2.—Consistent fits of (*a*) $\Delta\dot{\Omega}_c$ and (*b*) angular frequency excess for the 1975 glitch of Crab. Because of the long time span between the 1975 and 1986 glitch, the effect due to the change of values of Ω and $\dot{\Omega}$ as the pulsar slows down is also important. They show up as a term $\frac{1}{2}\ddot{\Omega}t^2$ in the angular acceleration where $\ddot{\Omega} = \dot{\Omega}[2(\dot{\Omega}/\Omega) - (\dot{\Omega}/\Omega)]$ [Lyne, Pritchard, & Graham-Smith 1993].

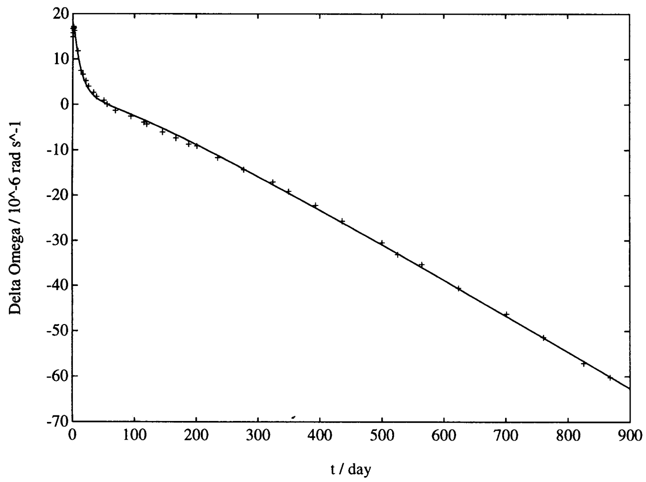

FIG. 4a

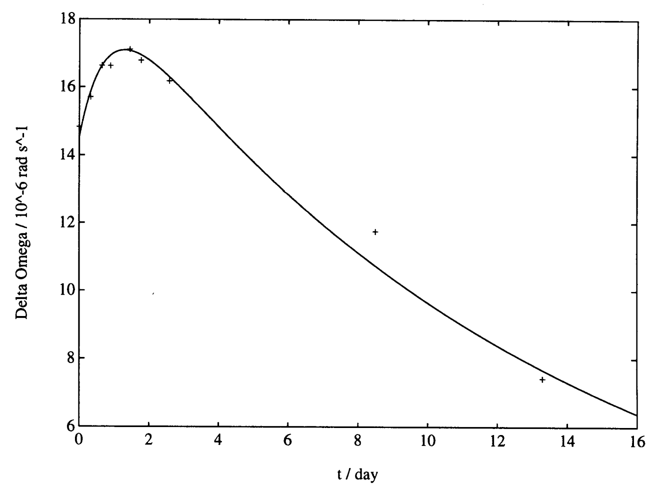

FIG. 4b

FIG. 4.—Consistent fit of $\Delta\Omega_c$ for the 1989 glitch of Crab. (*a*) shows the postglitch behavior for the first 900 days, and (*b*) is the blow-up of the first 16 day response of this glitch.

The first term in equation (2) is the *linear response*, which is the vortex creep response of those parts of the crustal superfluid at which the pinning energy E_p is sufficiently small compared to the internal temperature kT. Here I is the total inertial moment of the star, and I_l is the inertial moment associated with the component of the pinned superfluid having linear response. The characteristic response time τ_l for a linear region is given by (Alpar, Cheng, & Pines 1989):

$$\tau_l = \frac{kT}{E_p} \frac{\omega_{\rm cr} r}{4\Omega v_0} \exp\left(\frac{E_p}{kT}\right), \qquad (3)$$

where $\omega_{\rm cr}$ is the critical angular velocity lag between the superfluid and the crust above which pinning is no longer possible, and $v_0 \approx 10^7$ cm s^{-1} is the typical velocity for microscopic vortex motion. The glitch causes an initial perturbation $\delta\omega(0)$ in the angular velocity lag between the superfluid and the crust, and is given by

$$\delta\omega(0) = \delta\Omega(0) + \Delta\Omega_c , \qquad (4)$$

where $\delta\Omega(0)$ is the *reduction* of local superfluid angular velocity, and $\Delta\Omega_c$ is the *increase* in the crustal angular velocity at the glitch.

The second term in equation (2) is the *nonlinear response* from regions where the pinning energy is sufficiently high compared to the temperature. Here $t_0 \equiv \delta\omega(0)/|\dot{\Omega}|$ is the offset time, and the nonlinear relaxation time τ_n is given by (Alpar et al. 1984)

$$\tau_n = \frac{kT}{E_p} \frac{\omega_{\rm cr}}{|\dot{\Omega}|} . \qquad (5)$$

If the density of vortices unpinned in the glitch is uniform in some part of the superfluid, $\delta\omega(0, r)$ (and hence $t_0[r]$) varies linearly with the distance from the rotational axis of the star r from 0 to $\delta\omega_{\max}(0)$. By explicitly integrating over the second term in equation (2), we obtain (Alpar et al. 1984)

$$\frac{\Delta\dot{\Omega}_c(t)}{\dot{\Omega}_c} = \frac{I_A}{I}\left\{1 - \frac{1 - (\tau_n/t_g)\ln[1 + (e^{t_g/\tau_n} - 1)e^{-t/\tau_n}]}{1 - e^{-t/\tau_n}}\right\}, \qquad (6)$$

where t_g denotes the maximum $t_0(r)$ in the region. In the event that $t_g \gg \tau_n$, equation (6) can be approximated by the last three terms in equation (2). This description of the *long-term response* (Alpar et al. 1984) fits the postglitch data of the Vela pulsar rather well. We believe that the Vela glitches are induced by vortex instability. As the star slows down, vortex density may build up in some regions of the star (Chau & Cheng 1993; Cheng et al. 1988; Mochizuki & Izuyama 1995). The repulsive force between vortices in this region will eventually be so large that vortex unpinning will occur. The time between successive vortex instability induced glitches is therefore characterized by the time required to store up enough vortex density in the unpinning regions. This time is given by $t_g \equiv \delta\omega_{\max}(0)/|\dot{\Omega}|$ from our model fits. Comparison of the estimated t_g with the observed time intervals between the Vela glitches show that about 30% or better agreement in individual interglitch intervals, and 20% agreement in the average. In the Crab pulsar, t_g of the order of a few months are estimated with vortex creep theory on the assumption that the glitches are pure unpinning events, in disagreement with the observed interglitch intervals of years. This has been interpreted as evidence that the Crab glitches are not caused by vortex unpinning alone (Alpar et al. 1985).

The pinning force per unit length along a vortex line is different in different regions of the star. The "strong pinning region" refers to those parts of the crustal superfluid where E_p exceeds the binding energy of a nucleus to its equilibrium position in the lattice so that the typical distance between successive pinning centers equals the typical lattice spacing. Observational upper limits (Alpar et al. 1987) indicate that strong pinning is not present in any substantial part of the crust superfluid. Where the pinning forces are not strong enough to dislodge nuclei, a vortex line can only pin to those nuclei that it encounters geometrically along its orientation in the lattice, giving "weak pinning." At higher densities in the inner crust, vortex cores become large enough to include several lattice sites simultaneously (Alpar et al. 1984; Alpar et al. 1989). Then the nature of pinning changes. The pinning energy will change slightly if a vortex line is displaced in the lattice and the effective strength of pinning is expected to diminish by a large factor. Unfortunately, we do not have any way to estimate the pinning parameters in this "superweak pinning" regime.

The transition between linear and nonlinear response takes place when $\tau_l = \tau_n$. From equations (3) and (5), the pinning energy at this transition is given by (Alpar et al. 1989)

$$\left(\frac{E_p}{kT}\right)_{\rm tr} = \ln\left(\frac{4\Omega_c v_0}{r|\dot{\Omega}_\infty|}\right) \equiv \ln\left(\frac{8\tau_{\rm age} v_0}{r}\right) \approx 28.8 , \quad (7)$$

where $\tau_{\rm age}$ is the spin-down age of the pulsar, and the last equality gives the value for Crab. Provided that the transition region is in the "weak pinning" regime, the corresponding transition relaxation time ($\tau_{\rm tr}$) is given by

$$\tau_{\rm tr} \approx 160 \text{ days} , \quad (8)$$

where we have assumed $\omega_{\rm cr} \approx 1$ rad s^{-1}. If the transition takes place in the "superweak pinning" regime, then $\tau_{\rm tr}$ should be smaller than 160 days. Since we do not have a reliable estimate of $\omega_{\rm cr}$ in the "superweak pinning" regime, the exact value for the "superweak" transitional relaxation time remains uncertain.

4.2. *Effects of Crust Cracking and Formation of Capacitor Regions*

As the pulsar spins down, the solid crust will deviate farther and farther from its reference configuration, and stresses will build up in the crustal lattice. Whenever the local stress is too high for the lattice to sustain, that part of the crust has to crack, causing a starquake. The processes of crust cracking and starquakes have been analyzed by various authors (Baym & Pines 1971; Pandharipande, Pines, & Smith 1976; Ruderman 1969, 1976, 1991a, b, c).

The oblateness ϵ of a star is defined as $(I_c - I_{c0})/I_{c0}$, where I_c is the moment of inertia of the crust, and I_{c0} is the inertial moment of the crust when the star is nonrotating. The equilibrium value of ϵ is given by (Baym & Pines 1971)

$$\epsilon = \frac{\Omega_c^2}{4(A+B)}\frac{\partial I}{\partial \epsilon} + \frac{B}{A+B}\epsilon_0 \approx \frac{I_0\Omega_c^2}{4A} + \frac{B}{A}\epsilon_0 , \quad (9)$$

where I_0 is the moment of inertia of the nonrotating star, ϵ_0 is the reference value for ϵ which can be changed by a quake. Provided that the effect of plastic flow is not important, ϵ_0 is a constant between two successive glitches. Here $A = 3M^2G/25R$ and $B \approx (57/50)\,(4\pi/3\;R^3)\;C_{44}$, where C_{44} is the shear modulus which has a characteristic value of $\approx 10^{30}$ ergs cc^{-1} over most parts of the crust. In general, $A \gg B$ (Baym & Pines 1971).

When the crust cracks and the oblateness is reduced suddenly, the initial oblateness ϵ_0 is changed by a negative shift $\Delta\epsilon_0$ and from equation (9), the equilibrium oblateness ϵ also shifts by

$$\Delta\epsilon = \Delta\left(\frac{I_0\Omega^2}{4A}\right) + \frac{B}{A}\Delta\epsilon_0 \approx \frac{B}{A}\Delta\epsilon_0 . \quad (10)$$

We now discuss a mixed model for the glitch, involving both crust breaking and vortex unpinning. As discussed below, when taken singly both models have difficulties in explaining the Crab pulsar glitches and interglitch behavior, while there are attractive arguments that the two mechanisms may be operating cooperatively. The angular momentum balance at the time of a glitch reads

$$\Delta(I_c\Omega_c) = I_c\Delta\Omega_c + \Omega_c\Delta I_c = \sum_i I_i\delta\omega_i , \quad (11)$$

where the sum is over all the superfluid components transferring angular momentum to the crust as a result of sudden vortex motion. Spin-down of the superfluid, as dictated by the external (pulsar) torque, is realized by radially outward motion of the vortices away from the rotation axis. Pinned vortices are held back from this radially outward motion by pinning forces exerted by the lattice. When unpinned suddenly, these vortices will tend to move radially outward. Since most of the unpinned vortices move outward, making $\delta\omega_i > 0$ in most regions, the right-hand side of equation (11) must be positive. Thus the upper bound for $|\Delta\epsilon|$ is given by

$$|\Delta\epsilon| = \left|\frac{\Delta I_c}{I_c}\right| \leq \left(\frac{\Delta\Omega_c}{\Omega_c}\right)_{\rm res} , \quad (12)$$

where $(\Delta\Omega_c)_{\rm res} = \Delta\Omega_c(0) - \sum_i I_i/I_c\,\delta\omega_i$ is the initial jump in Ω_c at the time of glitch, after subtracting all the known contributions from the angular momentum exchange with the superfluid, and is of the order of 10^{-9} to 10^{-8} for a typical Crab glitch.

The permanent offset in $\dot{\Omega}_c$ observed in the Crab glitches may result from a change in oblateness of the star or the creation of new vortex depletion regions (Alpar & Pines 1993; Alpar et al. 1994). Suppose that the permanent offset observed is due entirely to the reduction in oblateness; then a remnant change in frequency

$$\left(\frac{\Delta\Omega_c}{\Omega_c}\right)_{\rm res} = \left(\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c}\right)_{\rm res} = \left|\frac{\Delta I}{I}\right| \quad (13)$$

is also expected. As the persistent shift in $\Delta\dot{\Omega}_c/\dot{\Omega}_c \sim 10^{-4}$ is much larger than the glitch $\Delta\Omega_c/\Omega_c \sim 10^{-8}$, this is out of the question. We therefore pursue the idea that the formation of a vortex depletion region, which we have called a "capacitor region" (Alpar & Pines 1993; Alpar & Pines 1995), is the major cause of the permanent offset in $\dot{\Omega}_c$. In this case the permanent offset is not due to a reduction in the moment of inertia of the star, but to a permanent change that decouples some part of the star from angular momentum transfer. Physically, a capacitor region is a place in the inner crust of the star where a vortex current cannot be sustained. There are two components of a vortex capacitor. First, a vortex trap where a local excess in pinning force associated with some inhomogeneity in the crust lattice leads to excess vortex density. This is analogous to capacitor plates in an electronic circuit. Within the vortex trap itself vortex motion (creep) continues. The vortex current can be affected by, and can respond to, a glitch. We have called such vortex traps collectively regions A in our earlier work (Alpar et al. 1994). The second component of the capacitor is a vortex depletion region around the vortex trap (between adjacent vortex traps). Such vortex depletion regions accompany the traps because the extra number of vortices in a trap creates an excess local superfluid velocity around the trap, which will be too large to allow pinned vortices: the local superfluid velocity is larger than the critical velocity that pinning forces can sustain. Pinning forces are extra strong within the trap but not around it. Thus a vortex depletion region will surround a local vortex trap, and if a vortex is introduced into this region, where pinning forces are weak, it will move through the region rapidly, following the high superfluid velocity. The rapid motion conserves the vortex current as the vortex density is relatively low here. The vortex depletion regions will pass a vortex current when the vortex traps have piled up a large enough number of vortices to go unstable and release the excess vorticity. With the rapid motion of these vortices through the vortex depletion regions, these regions will contribute to the angular momentum transfer at glitches, though they do not contribute to vortex current (and hence to the spin-down) between glitches. The contribution of a particular vortex depletion region to $\Delta\dot{\Omega}$ occurs only at the *first* glitch that forms that particular trap and depletion region. We have called the vortex depletion regions collectively as the regions B in earlier work (Alpar et al. 1994). In the Vela pulsar the network of vortex capacitors has long been established; the regions B therefore contribute only to the angular momentum transfer and not to the $\Delta\dot{\Omega}$. The sudden release of vortices at a glitch is analogous to the discharge of a capacitor, so we call the vortex trap plus vortex depletion region a vortex capacitor. Vortex capacitors could be created during a starquake in the following way: the energy released in a quake can easily create lattice imperfections in the crustal lattice, thereby creating pinning centers with large pinning energies, and consequently a local vortex density excess, and a surrounding vortex depletion region (Chau & Cheng 1993). We suggest that a young pulsar like the Crab is still in the process of forming capacitor regions at glitches.

Neglecting the $\Omega_c \Delta I_c$ term, we may approximate the angular momentum balance (eq. [11]) by

$$I_c \Delta\Omega_c \approx \sum_i I_i \delta\omega_i + I_B \delta\omega_B \, , \tag{14}$$

where I_B is the inertial moment of the vortex depletion region. With the creation of new vortex depletion regions at the time of glitch, we may write I_B as

$$I_B = I_{B'} + I_b \, , \tag{15}$$

where $I_{B'}$ and I_b are the inertial moments of the already existing, and newly formed vortex depletion regions, respectively. The inertial moment of the newly formed capacitor regions, I_b, will cause a permanent change in $\dot{\Omega}$, as this region was supporting a vortex current and contributing to $\dot{\Omega}$ before the glitch and is no longer contributing after the glitch. This "persistent shift" is given by

$$\frac{\Delta\dot{\Omega}_{\rm res}}{\dot{\Omega}} = \frac{I_b}{I} \, , \tag{16}$$

where $\Delta\dot{\Omega}_{\rm res}$ is the observed permanent change in $\dot{\Omega}$. An alternative way to explain the permanent offset in $\dot{\Omega}_c$ invokes a glitch-induced change in the external torque (Ruderman 1991a; Link, Epstein, & Baym 1992). The effect is likely to be too small to have any observable glitch-related change in the electromagnetic signature of the pulsar.

At the time of a starquake-induced glitch, the crustal lattice breaks in a particular place; the vortices pinned to that particular crustal region may either unpin and move outward (away from the rotational axis of the star) or may be carried by the breaking lump and move along with it toward the rotational axis. The initial perturbation in the angular velocity lag $\delta\omega(0)$ becomes negative in this region whenever the total angular momentum associated with the inward moving vortices outweighs those moving outward. This seems to be a possible explanation for the "extended spin-up" in the postglitch evolution of the 1989 Crab glitch. However, with this simple interpretation the ratio of the moments of inertia associated with the inward moving vortices to that of the entire star is about 10^{-4}. If this were the true moment of inertia change in the inward breaking of the crust, $\Delta\Omega/\Omega$ should be 10^{-4} also! This is much too large for crust breaking.

Instead, we propose that the "extended spin-up" is due to the formation of vortex traps and the subsequent bunching of vortices into these traps. During a starquake, new vortex trapping regions are created and vortices in the neighborhood are bunched into these regions. In bunching into a local trap some vortices will have moved toward the rotation axis while some move away from it. While those vortices moving radially outward in this process have a positive contribution to $\delta\omega(0)$ (a decrease in the superfluid velocity), those moving radially inward into these traps will give a negative contribution (an increase in the superfluid velocity). As the latter regions with a local spin-up in the superfluid relax after the glitch, the subsequent spin-down of the superfluid during the relaxation process will be reflected in an "extended spin-up" of the observed pulsar crust in postglitch response.

On combining the effects of pinned superfluid response, and creation of capacitor regions, we write the general postglitch evolution equation for the Crab as

$$\begin{aligned} \frac{\Delta\dot{\Omega}_c(t)}{\dot{\Omega}_\infty} = &-\int \frac{dI_l(r)}{I} \frac{\delta\omega_l(0)e^{-t/\tau_l}}{\tau_l \dot{\Omega}_\infty} + \int \frac{dI_n(r)}{I} \\ &\times \left\{1 - \frac{1}{1 + [e^{t_0(r)/\tau_n} - 1]e^{-t/\tau_n}}\right\} \\ &+ \frac{I_A}{I}\,\theta(t) - \frac{I_A}{I}\,\theta(t - t_g) \\ &- \frac{I_A}{I}\frac{t}{t_g}\,\theta(t)\theta(t_g - t) + \frac{I_b}{I}\,\theta(t) \, , \end{aligned} \tag{17}$$

which is consistent with equation (1). The second term is the "Fermi function" response characteristic of nonlinear creep. Regions associated with the inward bunching of vortices into newly created vortex traps may have a negative $\delta\omega(0)$. In general, not every term in equation (17) shows up after a Crab glitch.

The above arguments together with the result from the fits to postglitch response of the Crab pulsar lead to the glitch scenario illustrated in Figure 5. Crust cracking occurs in region 1. Vortices which unpin in this region move through regions 2, 3, and B, and possibly repin in A_2. New vortex traps may be created anywhere in the inner crust during the starquake. The regions immediately radially outward from newly formed vortex traps, where vortices

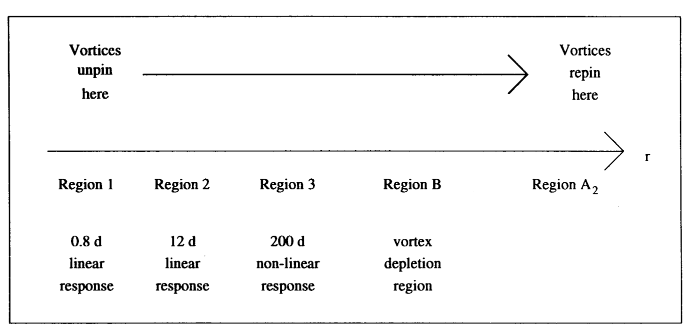

FIG. 5.—Schematic representation of the "minimalist" model of a glitch in the Crab pulsar. Vortices unpin around region 1 and move through regions 2, 3, and B. They may finally repin at regions A_2.

move inward in being bunched into these newly formed vortex traps, give rise to the positive $\Delta\dot{\Omega}_c$ components of the postglitch response (extended spin-ups).

5. DISCUSSION

5.1. *The* 0.8 Day *Short Timescale Response*

Is the 0.8 day response linear or nonlinear vortex creep? If the short timescale response were nonlinear, then the second term in equation (2) would be well approximated by

$$\left[\frac{\Delta\dot{\Omega}_c(t)}{\dot{\Omega}}\right]_{0.8\ \mathrm{days}} \approx \frac{I_1}{I}\left(1-\frac{1}{1-e^{-t/\tau_1}}\right) = -\frac{I_1}{I}\sum_{k=1}^{+\infty} e^{-kt/\tau_1} \tag{18}$$

whenever $1-e^{-t/\tau_1} \gg e^{(t_0-t)/\tau_1}$; or $t \gtrsim \tau_1 \ln(1+e^{t_0/\tau_1}) \approx \tau_1 e^{t_0/\tau_1} \simeq \tau_1$. In other words, the very short timescale response should be seen as a series of equal amplitude "harmonics" ($\tau_k = \tau/k$) of exponential decays of positive $\Delta\dot{\Omega}_c$, with a fundamental response timescale of $\tau_l = 0.8$ days. The absence of even the $k = 2$ term in the postglitch response tells us that the 0.8 day response comes from a linear creep or other linear response region.

Table 3 summarizes the deduced parameters for the four glitches of the Crab pulsar. The "extended" 0.8 day spin-up for the 1989 glitch is associated in our model with the formation of new vortex traps. If vortex traps were not created in this region, then most of the unpinned vortices would simply move radially outward during the glitch. This makes $\delta\omega(0) > 0$, and one would observe only the "conventional" relaxation, $\Delta\dot{\Omega}_c < 0$, right after the glitch. An "extended" 0.8 day spin-up arising from a starquake-induced glitch with formation of new vortex traps may be expected in future Crab glitches.

TABLE 3

Deduced Parameters of the Four Crab Glitches from the Consistent Fits

Parameter	Glitch Year 1969	1975	1986	1989
$\left(\frac{I_1}{I}\,\delta\omega_1\right)_{-7}$ [a]	≥ 0.34	≥ 2.77	17.8	-55
$\left(\frac{I_2}{I}\,\delta\omega_2\right)_{-7}$ [a]	≥ 4.7	≥ 32.7	7.1	177
$\left(\frac{I_3}{I}\right)_{-4}$	...	...	...	4.4
$[\delta\omega_3(0)]_{-2}$ [b]	...	...	...	-1.62
$\left(\frac{I_b}{I}\right)_{-4}$	0.03	2.2	0.18	3.8
$\left[\frac{I_B}{I}\,\delta\omega_B(0)\right]_{-7}$ [a]	≤ 2.46	≤ 47.4	2.0	94.7
$[\delta\omega_B(0)]_{-2}$ [b,c]	≤ 8.2	≤ 2.2	≤ 1.1	≤ 2.5
$\left(\frac{I_2}{I}\right)_{-4}$ [c]	≥ 0.06	≥ 1.49	≥ 0.65	≥ 0.71
$\left(\frac{I_p}{I}\right)_{-3}$ [c]	≥ 0.01	≥ 1.74	≥ 0.25	≥ 1.87

[a] $(I_1/I)\delta\omega_1$, $(I_2/I)\delta\omega_2$, and $(I_B/I)\delta\omega_B$ are in unit of 10^{-7} rad s^{-1}.

[b] $\delta\omega_3(0)$, and $\delta\omega_B(0)$ are in unit of 10^{-2} rad s^{-1}.

[c] $\delta\omega_B(0)$, I_2/I and hence the total pinned crustal moment observed (I_p/I) are deduced using the assumptions that $\delta\omega_B(0) = \delta\omega_2(0)$, and $I_b = I_B$.

The exact location where the vortex traps are formed varies between glitches. But it is likely that vortex traps are formed at places close to the "epicenter" of the corresponding starquake where the crust breaks. Dislocation rich regions with small critical strain angles are likely to break first. The critical strain angle (φ_{cr}) can be estimated by (Ruderman 1969),

$$\varphi_{\mathrm{cr}} = \frac{7R^3}{8GM}(\Omega_{\mathrm{init}}^2 - \Omega_{\mathrm{final}}^2)\sin^2 2\theta\ , \tag{19}$$

where θ is the latitude measured from the equator, Ω_{init} and Ω_{final} are the angular velocities of the star just after the last glitch and just before the next glitch, respectively. From equation (19), the time to the next glitch, t_g, is given by

$$\varphi_{\mathrm{cr}} \approx \frac{7R^3}{4GM}\Omega_c\,|\dot{\Omega}_\infty|\,t_g\ . \tag{20}$$

Taking a standard 1.4 $M_\odot$ star, and $\langle t_g \rangle = 4.5$ yr (i.e., six glitches in some 27 yr), equation (20) gives us $\varphi_{\mathrm{cr}} \approx 6\times 10^{-7} R_6^3$, where R_6 is the radius of the star in units of 10^6 cm. For the Friedman-Pandharipande EOS (Friedman & Pandharipande 1981), the radius of the star is some 11 km and equation (20) gives us $\varphi_{\mathrm{cr}} \approx 1.1\times 10^{-6}(M/M_\odot)^{-1}$. For $M \lesssim 1.4\ M_\odot$, this value is consistent with that of imperfect terrestrial metals. If φ_{cr} in the neutron crystal is much larger than the terrestrial value, the expected interglitch interval predicted by pure crust breaking models would far exceed the observed t_g.

For reasons to be discusssed in § 5.3, we believe that the 0.8 day response corresponds to a "superweak" pinning region. In the superweak regime we do not have a calculation of the effective pinning energies. E_p of about 1 MeV, a typical value in the weak pinning regime, is a reasonable upper limit for the superweak regime. Using this in equation (3) for the linear response time $\tau_l = 0.8$ days, and taking $kT = 35$ keV for Crab's interior temperature, $r = 10^6$ cm, and the microscopic velocity $v_0 = 10^7$ cm s^{-1}, we find the lower limit $\omega_{\mathrm{cr}} = 5\times 10^{-3}$ rad s^{-1}. This is consistent with superweak pinning.

5.2. *The* 12 Day *Intermediate Timescale Response*

The 12 day intermediate timescale relaxation is likely to be a linear response from superweak pinning regions also. As was the case with the 0.8 day response, equation (3) provides only a lower bound for the critical angular velocity lag in the intermediate-timescale response region, 7×10^{-2} rad s^{-1}, consistent with the interpretation that it reflects a "superweak" pinning regime. The amplitude of the 12 day response indicates that this is a region through which vortices pass at the time of the glitch. The value of $\delta\omega_2(0)$, $\lesssim 2.5\times 10^{-2}$ rad s^{-1} in the 1989 glitch, gives $t_0 \lesssim 119$ days $\gg \tau_2 = 12$ days, so that a Fermi function behavior would have been observed if this region were a nonlinear creep region. As such nonlinear response is not observed, it must be a linear creep region with the 12 day relaxation time. The difference between the pinning energies of the 0.8 day and 12 day linear relaxation regions, from equation (3), is given by

$$E_p(12\ \mathrm{days}) - E_p(0.8\ \mathrm{days}) \approx kT\ln\left(\frac{12}{0.8}\right) = 0.094\ \mathrm{MeV}\ . \tag{21}$$

5.3. *The* 200 Day *Long Timescale Response*

After the intermediate-timescale relaxation, a 200 day nonlinear response provides the simplest good fit to the data (Alpar et al. 1994). The question is whether the 200 day nonlinear response corresponds to a weak or a superweak pinning region. Equation (8) tells us that in the weak pinning regime this nonlinear response corresponds to a region which is rather close to the boundary between linear and nonlinear response. This gives a natural reason to understand the break in relaxation times between the 200 day, the 12 day, and 0.8 day timescales of linear response: 200 days represents the nonlinear response of weak pinning regions. At the transition to linear response, the dependence of the relaxation time τ_l on pinning parameters becomes very sensitive, and it is within this sensitive linear response regime that the transition in pinning parameters to superweak pinning takes place. The two different timescales, 0.8 days and 12 days, represent different superweak pinning regions, with different but consistent values of ω_{cr} we inferred above. We thus associate the 200 day nonlinear response with weak pinning, while the 0.8 day and 12 day linear response are associated with superweak pinning. For $\tau_w = 200$ days, from equation (5), the minimum critical angular velocity lag between the superfluid and the crust in this region is found to be 1.2 rad s^{-1}, which is consistent with the weak pinning region interpretation. Equation (7) implies the minimum pinning energy (E_p) in this nonlinear region of Crab is about 1 MeV, where we have assumed the interior temperature of Crab to be 35 keV from standard cooling calculations (Tsuruta 1986). The superfluid gap energy in this region is given by (Alpar et al. 1989)

$$\Delta_{\rm nl}({\rm MeV}) \gtrsim 1.1[(kT)_{\rm Crab}/35\ {\rm keV}]^{1/2}[\gamma k_{\rm F}({\rm fm}^{-1})]^{-1/2} \quad (22)$$

where $\gamma \lesssim 1$ is a constant, $k_{\rm F}$ is the Fermi momentum of the neutron superfluid, and $\Delta_{\rm nl}$ is the superfluid gap energy in the nonlinear region. For reasonable values of $k_{\rm F}$ and γ, this inequality yields $\Delta_{\rm nl} \gtrsim 1.3$ MeV. However, recent gap energy calculations (Ainsworth, Wambach, & Pines 1989; Wambach, Ainsworth, & Pines 1993) show that $\Delta \leq 1$ MeV over the entire inner crust of a neutron star. The discrepancy suggests that either (1) the internal temperature of Crab is lower than 35 keV predicted from the standard cooling mechanisms; or (2) the actual superfluid gap energy for matter at nuclear matter density is higher than the theoretically calculated one; or (3) the 200 day nonlinear response is in fact from a superweak pinning region.

The previous three observed glitches do not exhibit this 200 day component of postglitch relaxation. For the 1989 event, the value of $\delta\omega_3(0)$ is negative in sign indicating that inward vortex motion associated with the formation of new vortex traps has occurred in this region as well as in region 1.

5.4. *The Persistent Shift*

In addition to the 0.8 day, 12 day, and 200 day responses, there is also a persistent (nonrelaxing) component of $\Delta\dot{\Omega}_c$ after each glitch. This $\Delta\dot{\Omega}_{\rm res}$ varies from -3.7×10^{-14} rad s^{-2} to -7.7×10^{-13} rad s^{-2} in the four observed glitches. As we have discussed in § 4, the persistent shift reflects the formation of new capacitor regions whose inertial moments are tabulated in Table 3. If the present rate of creation of capacitors persists, some 200 more glitches, taking place over an interval of $\approx$2000 yr, are required for the Crab pulsar to build up the total capacitor moment of inertia $I_A/I + I_B/I$ of the order of 10^{-2} as in the Vela pulsar (Alpar et al. 1993). The large glitches for the Vela pulsar and their different signature, in particular the lack of persistent shifts, indicate that capacitor formation has ended and saturated the crust superfluid regions in Vela. (Though occasional small glitches with $\Delta\Omega/\Omega$ of the order of 10^{-8} as in the Crab pulsar may reflect the ongoing formation of some capacitors. No $\Delta\dot{\Omega}_{\rm res}$ associated with these minor glitches in Vela has yet been resolved.) The angular momentum balance (eq. [11]) for the Crab can be written as

$$I_c\,\Delta\Omega_c(0) \approx I_1\,\delta\omega_1(0) + I_2\,\delta\omega_2(0) + I_B\,\delta\omega_2(0)\,, \quad (23)$$

where I_B is the inertial moment of the vortex-depleted region. The number of unpinned vortices that move outward determine $\delta\omega_2(0)$. Assuming that the same number of vortices also move through the vortex depletion regions, the angular momentum transfer associated with these regions is $I_B\,\delta\omega_2(0)$. The values of $I_B\,\delta\omega_2(0)$ are listed in Table 3. Clearly, $I_B \geq I_b$. In particular, if we assume that the unpinned vortices only pass through the newly formed capacitor regions, then $I_B = I_b$ and the inertial moments of the newly formed capacitor region b, and that of 2, together with the angular velocity lag $\delta\omega_2$, can be deduced. These are listed in Table 3. It is easy to see that the total inertial moments deduced in this way represent lower limits for the actual inertial moments of pinned portions of the crust superfluid involved in the glitches. The moment of inertia in the pinned crust superfluid in the Crab pulsar must be at least 0.19% of the total inertial moment of the star. This is consistent with both the present neutron star EOS calculations, and the capacitor model (Alpar & Pines 1993; Alpar & Pines 1995) and provides support for the argument that the Crab pulsar is in the stage of capacitor formation.

The observed linear response thus comes from both regions 1 and 2 through which vortices move at the time of the glitch. The vortex depletion region B which does not appear in the postglitch relaxation in $\dot{\Omega}$ enters only by the angular momentum balance. If vortices in some regions are pulled inward into newly formed vortex traps at the time of glitch, this will cause the "extended rise." Other vortices which simply unpin and move outward contribute the rest of the postglitch relaxation.

6. COMPARISON WITH VELA GLITCHES

The postglitch evolution of the Vela pulsar contains four components: three of these display exponential relaxation with 10 hr, 3.2 day, and 33 day relaxation times while the fourth represents a long-term relaxation process in which $\Delta\dot{\Omega}_c$ evolves linearly in t (Alpar et al. 1993). The 10 hr and 3.2 day relaxation reflect linear response while the 33 day relaxation comes from the response of the transition region between linear and nonlinear response. Assuming that the structure of Crab and Vela is similar, it is natural to ask how the corresponding regions in Crab behave after a glitch. In the linear regime the relaxation time depends exponentially on the value of E_p/kT. Small changes in E_p/kT may result in a huge change in the linear relaxation time. It is therefore plausible that the value of E_p/kT for the 33 day response of the Vela and the 12 day response of the Crab come from regions of the two stars in which the values of E_p/kT are approximately the same. By means of equation

(3), we have

$$\frac{(\tau_l)_{\rm Crab}}{(\tau_l)_{\rm Vela}} \approx \frac{(\Omega_c)_{\rm Vela}}{(\Omega_c)_{\rm Crab}} = 0.37 \, . \tag{24}$$

Thus, the 33 day relaxation of Vela scales to that of 12 days for the Crab with the assumption that both relaxation times are associated with linear regions of the same E_p/kT.

In similar fashion, $\tau_l = 3.2$ days for the Vela pulsar scales to 1.1 days for the Crab pulsar, close to the 0.8 days observed. The 10 hr response for the Vela pulsar scales down to a corresponding time of 3.6 hr for the Crab pulsar. The absense of a response with a timescale ≈ 3.6 hr in the Crab would then suggest that the value of E_p/kT in the 10 hr response regime of the Vela pulsar is slightly higher than that of the corresponding region in the Crab. The characteristic relaxation time of this region in Crab will then be much smaller than 3.6 hr and is likely to be so short that it cannot be resolved in the 1989 event. Such a fast relaxation time implies that the region is effectively coupled to the crust and hence, we expect the total moment of inertia in the pinned crust superfluid for Crab is less than that of Vela. The situation can be represented schematically as in Figure 6.

The nonlinear relaxation time is less sensitive to the value of E_p/kT. From equation (5) the 200 day nonlinear response timescale of the Crab is scaled to a 2400 day response in the Vela by assuming that the response regions have the same value of $\omega_{\rm cr}$ and E_p, together with the fact that $T_{\rm Crab} \approx 2T_{\rm Vela}$ for various different equations of state and cooling mechanisms (Tsuruta 1986; Shibazaki & Lamb 1989; Cheng et al. 1992; Chong & Cheng 1993). We are not sure if such a response timescale is present in the Vela glitches because it is much longer than the typical time between successive Vela glitches. Its contribution cannot be distinguished from the long-term ($\Delta\dot{\Omega}_c \propto t$) relaxation of the Vela.

The effects of evolution on the postglitch response of pulsars as they age from that of the Crab pulsar to that of Vela can be summarized as follows: for the pulsar at the age of Crab, where kT and $|\dot{\Omega}|$ are both high, a glitch is likely to be triggered by crust breaking. Capacitor regions and vortex traps are in the formation stage and hence it may be possible to observe a positive $\Delta\dot{\Omega}$ component of the response. The formation of a network of vortex depletion regions also accounts for the persistent shift $\Delta\dot{\Omega}_{\rm res}$ after every glitch. In addition, the 0.8 day, 12 day, and 200 day responses correspond to regions through which superfluid vortices actually move at the time of the glitch.

As the star cools and slows down, the characteristic relaxation timescales (especially for the linear response) increase (eqs. [3] and [5]). Furthermore, nonlinear response takes over as a pulsar ages. From equations (3) and (8), $\tau_{\rm tr}$ can be written as

$$\tau_{\rm tr} = \frac{\omega_{\rm cr}}{|\dot{\Omega}_\infty|}\left\{\ln\left[\frac{8v_0\,\tau_{\rm age}}{r}\right]\right\}^{-1} \approx \frac{\omega_{\rm cr}}{K\Omega^n} \tag{25}$$

for some constant K, where $n(>0)$ is the braking index of the pulsar. Provided that $\omega_{\rm cr}$ is not a rapidly varying function, as the star spins down, $\tau_{\rm tr}$ increases. Equation (7) tells us that $(E_p/kT)_{\rm tr}$ increases, but only logarithmically with pulsar age. The cooling decreases the temperature more rapidly, and $(E_p)_{\rm tr}$ decreases, essentially in proportion to the temperature. So the transition region moves to weaker pinning regions, and eventually into superweak pinning regions. Physically, the transition region moves inward in the crust toward the core, and hence the moment of inertia in nonlinear response regions increases as a pulsar gets older (Alpar et al. 1989). The developing importance of nonlinear response with superweak pinning coincides with the development of the vortex capacitor network to saturate the crust superfluid, and to make for a different phenomenology of glitches in the Vela and older pulsars. Crust cracking-induced glitches give way to the vortex instability-induced glitches. The development of the capacitor network gradually ceases as the crustal superfluid becomes filled with vortex capacitors. The persistent shift in $\dot{\Omega}_c$ therefore does not occur in the Vela pulsar. Since the observation of the second Vela glitch, it has been clear that the large $\Delta\Omega/\Omega \sim 10^{-6}$) and frequent ($t_g \sim 2$ yr) Vela glitches cannot be due to crust breaking with reasonable crust lattice parameters. By assuming the critical strain angles for the inner crust of the two stars are the same, one finds that the time between crust cracking-induced glitches in the Vela pulsar is about 100 yr. As a Crab-like young pulsar ages, the number of vortex traps increases to saturate the pinned superfluid region with a network amenable to unpinning in an avalanche where vortices unpinned in one trap induce unpinning in other traps (Alpar & Pines 1995). In this scenario the dynamical evolution of pulsars culminates in Vela pulsar-type glitches, which are pure large-scale unpinning events, and their intervals t_g can be understood on this premise (Alpar et al. 1993).

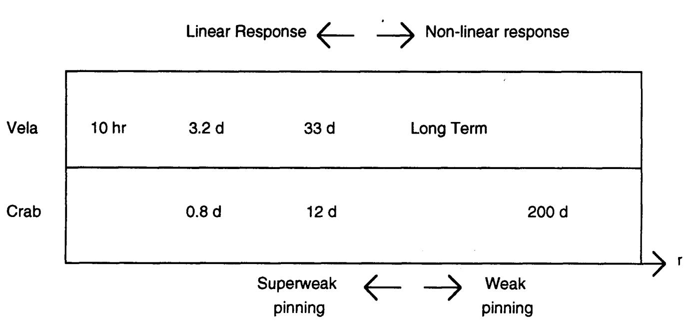

FIG. 6.—Schematic representation of the different response regions in the Crab and the Vela. Regions with similar values of E_p/kT are grouped together in the same column.

7. CONCLUSIONS

We have evaluated the postglitch response for four out of the six Crab pulsar glitches using the vortex creep theory and taking into account as well as the effects of crust breaking and the formation of vortex depletion regions. The occasional "extended" 0.8 day rise provides strong evidence for the formation of vortex traps during a crust-breaking–induced glitch in which some of the pinned neutron vortices are bunched into the newly formed vortex traps. The permanent offset in $\Delta\dot{\Omega}_c$ results from the creation of new capacitor regions which will not allow vortices to pass through except at a glitch, as discussed in earlier work (Cheng et al. 1988; Chau & Cheng 1993). We also showed that within vortex creep theory, the postglitch relaxation of the Crab can be described by two regions of linear response with characteristic timescales of 0.8 days and 12 days, respectively, together with a nonlinear response with characteristic timescale of 200 days. All represent regions through which vortices move at the time of the glitch. Moreover, the 0.8 day and 12 day responses in the Crab pulsar are related to the 3.2 day and the 33 day responses of the Vela pulsar, respectively, if they reflect approximately the same E_p/kT.

The total extent of vortex capacitors in the Crab pulsar will increase with time (Alpar et al. 1994). The Crab pulsar is consistent with the capacitor model (Alpar & Pines 1993, 1995). This suggests that the Crab is in the process of forming capacitor regions; when it evolves to the age of the Vela pulsar, a percolating capacitor network will be fully developed. From the present rate of formation of capacitor regions, some 200 more glitches will be required for the Crab to reach the fully connected capacitor state as seen in the Vela pulsar. We expect that future Crab glitches can be described by equation (17) using the same set of response timescales we have found here, with occasional "extended" spin-ups. As long as the Crab pulsar is in the stage of capacitor formation, there will be a persistent shift in $\dot{\Omega}$ after every Crab glitch. The postglitch response of the 1992 Crab glitch thus offers a good chance to test the validity of our theory.

We also conclude, in accord with earlier work, that the glitches in young pulsar like the Crab pulsar are the result of starquakes induced by spin-down. This proposal is consistent with the recent report by Kaspi et al. (1994) on the spin-down of the young pulsar PSR B1509−58, who find it has not glitched during an 11 yr span. The frequency of starquakes induced by pulsar spin-down is directly proportional to $\Omega\dot{\Omega}$. Since the Crab pulsar has glitched six times in 25 yr, we estimate that the likelihood of a PSR B1509−58 glitch is

$$\left(\frac{6}{25}\right)\frac{(\Omega\dot{\Omega})_{\rm B1509}}{(\Omega\dot{\Omega})_{\rm Crab}} \times 11 = 0.1 .$$

Thus is may take a few decades of timing observations of PSR B1509−58 before a glitch is observed.

Finally, we note that there are aspects of the vortex current, and variants and alternatives to vortex creep theory which have not been included in the model described here, but may be relevant in extensions of the model. Vortex motion with respect to the lattice at temperatures too low for thermal activation, as well as the rapid scattering of unpinned vortices during glitches (Epstein & Baym 1992) are aspects of vortex motion that are not treated in vortex creep theory. These aspects have been discussed in terms of vortex-phonon scattering involving excitations of the vortex line (Bildsten & Epstein 1989; Jones 1990a, b, c; Jones 1991, 1993). The exponentially relaxing part of postglitch behavior, interpreted as linear response in the vortex creep theory, has been explained in terms of vortex-phonon scattering (Jones 1990a, b, c; Jones 1991, 1992, 1993). Recently Link, Epstein, & Baym (1993) showed that the Kim-Anderson thermal creep formula still holds in case the temperature T of the star is low enough that quantum mechanical tunneling becomes important, provided that T is replaced by an effective temperature $T_{\rm eff}$ which tends to T for sufficiently high temperature. Extensions of the pinning model to take the effects of vortex tension into account have also been carried out (Chau & Cheng 1991, 1993; Baym, Epstein, & Link 1992); these approaches yield effective pinning energies within the same range as the initial estimates. Moreover, the microscopic physics of vortex pinning continues to be a difficult problem with many uncertainties. In this paper we have used estimates of the pinning energy based on the comparison of superfluid condensation energies inside and outside the nuclei (Anderson et al. 1982; Alpar et al. 1984), but the problem deserves further study.

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