Influence of ferromagnetic spin waves on persistent currents in one-dimensional mesoscopic rings

Shen-Shang Wu
Department of Physics, Zhongshan University, Guangzhou 510275, China

Zhong-Shui Ma
*China Center of Advanced Science and Technology (World Laboratory), P. O. Box 8730, Beijing 100080, China and Advanced Research Center, Zhongshan University, Guangzhou 510275, China**

Jian Wang
Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong
(Received 24 October 1995; revised manuscript received 13 February 1996)

The influence of the electron-magnon and the electron-phonon interactions on the persistent current in a one-dimensional mesoscopic ring is studied. We show that, due to the electron-magnon interaction, the amplitude of the persistent current is exponentially reduced compared to the free case. Two features occur in the presence of an electron-phonon interaction. For the normal state of electrons, the persistent current is weakened by the Debye-Waller factor. Considering the so-called Peierls distortions, we show that the effect of the Peierls instability on the amplitude of the persistent current (i.e., the oscillation with respect to the flux) is suppressed significantly and the persistent current will be practically undetectable in the case of a wide-gap Peierls material. [S0163-1829(96)05224-1]

I. INTRODUCTION

Persistent current is an equilibrium property of an electronic system with a ring geometry threaded by a magnetic flux. The prediction of the existence of persistent currents could be traced back to more than three decades ago.[1–3] The renewal of the interest in this topic is due to Büttiker, Imry, and Landauer,[4] who suggested the existence of persistent currents in a one-dimensional ring threaded by a magnetic flux and predicted that any physical property of the ring is a periodic function of the magnetic flux with a fundamental period $\Phi_0 = hc/e$. Since then the persistent current in mesoscopic ring has been the subject of extensive studies both theoretically[5–7] and experimentally.[8] The theoretical studies include the investigation of multichannel rings and disordered rings, the effect of the spin-orbit interaction, the electron-electron interaction, and the electron-phonon interaction on the persistent current.[9] Among these theoretical studies, Loss *et al.*[7] considered a specific model in which the mesoscopic ring was placed in a classical static inhomogeneous magnetic field and illustrated the connection between the persistent current and the Berry phase, as well as the possibility of its experimental verification. In particular, they predicted that the system supports the persistent equilibrium spin and charge currents, even in the absence of conventional magnetic flux through the ring. The applied magnetic flux plays the role of only an adiabatic parameter rather than a trigger of the current.

As pointed out by Loss *et al.*, the texture in the system studied in Ref. 7 could be recognized as an intrinsically ferromagnetic material or an inhomogeneous insulating ferromagnetic substance. Therefore, there will be an elementary excitation of spin waves[10] (ESW) in the texture at finite temperature when applying a small perturbation on the texture. In the study of organic materials, Aoki *et al.*[11] found that the excitation spectrum turns out to comprise an acoustic magnon mode along with an optical magnon mode where the optical magnon involves spatial spin oscillations within a unit cell.[12] Hence it would be interesting to study the effect of the interaction between electrons and magnons on the persistent current. By considering the electron-phonon interaction in the mesoscopic system[13] it has been shown that persistent currents decrease. Such an interaction has been further considered by Nathanson *et al.*[14] in the situation that an energy gap is opened in the electronic spectrum at the Fermi energy. They discussed[14] the Peierls instability in a mesoscopic ring as well as the flux dependence of the Peierls transition and showed that the amplitude of the oscillatory persistent current is suppressed significantly. The Peierls phase transition has been observed in many of organic chains. The fluctuations of a purely one-dimensional Peierls distortion have also been observed below room temperature.[15] Through investigations of the Aharonov-Bohm (AB) effect in Peierls insulators with a charge-density wave (CDW), it has been shown[16] that there is a contribution of thermally activated solitons, in a ring consisting of a commensurate CDW, oscillating with a period $\Phi = \hbar c/2e$. Such an AB effect occurring in insulators is not related to the motion of free carriers, but is due to the polarization of electronic states in the valence band by a magnetic field. The truly intriguing effect of the magnon of the spin wave in texture should appear in the low-lying excitations of electrons in the ring. Therefore, investigations of the suppression on persistent currents by an electron-phonon interaction and the appearance of the Peierls instability in the system considered by Loss *et al.* are certainly necessary.

For this purpose, we study the one-dimensional mesoscopic ring involving both magnons and phonons. In this paper, we first investigate the effect of the quantum fluctuation of the texture due to the magnon upon the persistent current in the ring. We then study the electron-phonon interaction in the system that has been affected by the magnon and discuss the Peierls instability on the persistent current. We will show, in this situation, that the persistent current is suppressed simultaneously due to two nonclassical effects, namely, the fluctuation of ESW due to the electron-magnon and the electron-phonon interactions in the mesoscopic system. The system we are considering is a simple tight-binding one-dimensional mesoscopic ring with the circumference $L=Na$ threaded by a magnetic flux. Here a is the lattice spacing and N is the number of the lattice sites in the ring. In studying the one-dimensional tight-binding mesoscopic ring involving magnons,[7,17] the on-site energy of the electron is assumed to be $\epsilon_0=\alpha\mu_B B$, where $B\hat{n}$ is the magnetic field of the specified texture with an angle χ describing the deviation of the vector $\hat{n}$ from the symmetric axis of the ring, $\mu_B=ge\hbar/4mc$ is the Bohr magneton, and $\alpha=\pm 1$. In the present approach, we will adopt the adiabatic approximation, i.e., the spin aligns along the local magnetic texture.[7] At low temperatures we consider the special situation that the magnetic field is so strong that the spins of the electron are frozen and polarized. Physically, this means that the ring has a ferromagnetic spin order. Hence the spin index will be dropped in the following calculations. Part of the effective spin-orbit coupling will be included in the geometric phase. For simplicity, we take into account only the electron-magnon and electron-phonon interactions. The effect of the magnon-phonon coupling has been neglected, which is believed only to renormalize the magnon energy.

This paper is organized as follows. In Sec. II we use a unitary transformation to diagonalize the Hamiltonian for the one-dimensional tight-binding ring with the electron-magnon and the electron-phonon interactions. Using an orthonormal set of many spin-wave states, we derive the energy spectrum of the system, which can be used to study the persistent current. Section III is devoted to the discussion of the influence of the electron-phonon interaction on persistent currents. The discussion will be divided into two parts, corresponding to with and without a gap in the energy spectrum, respectively. Summary and discussions are given in Sec. IV.

II. TIGHT-BINDING RING WITH ELECTRON-MAGNON INTERACTION

We now consider a one-dimensional tight-binding mesoscopic ring threaded by a magnetic flux in the presence of electron-magnon interaction. The Hamiltonian is

$$H=\sum_{l=1}^{N}[(\epsilon_0-\mu)c_l^\dagger c_l-J(c_{l+1}^\dagger c_l+c^\dagger c_{l+1})]$$

$$+\sum_q \hbar\omega_q b_q^\dagger b_q+\sum_{q,l} M_q e^{ilqa}(b_q+b_q^\dagger)c_l^\dagger c_l, \quad (1)$$

where $b_q^\dagger$ (b_q) is the creation (annihilation) operator of the magnon with wave vector $\mathbf{q}$, $c_l^\dagger$ (c_l) is the creation (annihilation) operator of the electron on the site l, J represents the hopping integral, $\epsilon_0=\mu_B B$ is the on-site energy, and μ is the chemical potential. Equation (1) can be obtained by assuming the validity of the linear spin-wave approximation around the local ferromagnetic order. The first two terms of Eq. (1) are the Hamiltonians of free electrons and magnons, respectively. The electron-magnon interaction is represented by the third term, where we assume that the coupling coefficient M_q satisfies $M_{-q}=M_q^*$. The Hamiltonian Eq. (1) can be rewritten in the following form by separating the hopping term from the other terms:

$$H=H_0+H_1, \quad (2)$$

where

$$H_0=\sum_{l=1}^{N}(\epsilon_0-\mu)c_l^\dagger c_l+\sum_q \hbar\omega_q b_q^\dagger b_q$$

$$+\sum_{q,l} M_q e^{ilqa}(b_q+b_q^\dagger)c_l^\dagger c_l \quad (3)$$

and

$$H_1=-J\sum_l (c_{l+1}^\dagger c_l+c^\dagger c_{l+1}). \quad (4)$$

To facilitate the calculation we introduce a canonical transformation S to diagonalize H_0, where S is given by

$$S=\sum_{q,l}\frac{M_q}{\hbar\omega_q}e^{iqla}c_l^\dagger c_l(b_q+b_q^\dagger). \quad (5)$$

The new Hamiltonian $H'=e^S He^{-S}$ after the transformation then takes the form

$$H'=\sum_l (\epsilon_0-\mu-\triangle_m)c_l^\dagger c_l+\sum_q \hbar\omega_q b_q^\dagger b_q$$

$$-J\sum_l (c_{l+1}^\dagger c_l X_{l+1}^\dagger X_l+c_l^\dagger c_{l+1}X_l^\dagger X_{l+1}), \quad (6)$$

where

$$X_l=\exp\left[\sum_q \frac{M_q}{\hbar\omega_q}e^{iqla}(b_q-b_{-q}^\dagger)\right]$$

satisfies $X_l^\dagger X_l=\mathbf{1}$ and $\triangle_m=2\Sigma_q(|M_q|^2/\hbar\omega_q)$ represents the energy correction to the on-site energy of electrons due to the electron-magnon interaction. From Eq. (6) we can see that the hopping term has the off-diagonal matrix element connecting different spin-wave states. This means that the electron absorbs and emits virtual magnons in its hopping process. Equation (6) can be diagonally averaged by using a vigorously orthonormal set of many spin-wave states,[18] i.e.,

$$|\ldots,n_q,\ldots\rangle=\Pi_q\frac{1}{\sqrt{n!}}(b_q^\dagger)^{n_q}|0\rangle, \quad (7)$$

where $|0\rangle$ represents the vacuum state. After averaging over the magnon part of Hamiltonian Eq. (6) under the orthonormal set of many spin-wave states given by Eq. (7), the effective Hamiltonian of electron becomes

$$H_{\mathrm{eff}}=\sum_{l=1}[(\epsilon_0-\mu-\triangle_m)c_l^\dagger c_l-Je^{-W_m}(c_{l+1}^\dagger c_l+c_l^\dagger c_{l+1})]$$

$$+\sum_q \hbar\omega_q\langle n_q\rangle, \quad (8)$$

where $W_m=\Sigma_q|f_q|^2(\langle n_q\rangle+\frac{1}{2})$, $f_q=(M_q/\hbar\omega_q)(1-e^{iqa})$, and $\langle n_q\rangle$ is the average number of magnons. The factor $\exp(-W_m)$ in Eq. (8) comes from the electron-magnon interaction. The effective hopping energy is $J\exp(-W_m)$, where W_m is given by

$$W_m=\sum_q\left\{2\frac{|M_q|^2}{(\hbar\omega_q)^2}\sin^2\frac{qa}{2}\coth\left(\frac{\hbar\omega_q}{2k_BT}\right)\right\}. \tag{9}$$

At $T=0$, $W_m=\Sigma_q[2\sin^2(qa/2)]|M_q|^2/(\hbar\omega_q)^2\approx\Sigma_q|M_q|^2/(\hbar\omega_q)^2\equiv g_m$. Here g_m is the effective coupling strength. In deriving Eq. (8), we have made two approximations: (1), in which we have eliminated the magnon's degrees of freedom using an orthonormal set of many spin-wave states, and (2), in which we have used the adiabatic approximation for the spin.[7] In general, we can decouple the spin and orbital degrees of freedom in the sense that the spin evolves in the presence of an external magnetic field, which depends parametrically on the path of the orbital motion. The wave function takes the form as $|\Psi\rangle=|\psi\rangle\otimes|\vec{n}(\theta)\rangle$, where $|\vec{n}(\theta)\rangle=\exp(i\chi/2)[\alpha\cos(\chi/2)|\hat{e}_z,\alpha\rangle+\exp(i2\pi l/N)\sin(\chi/2)|\hat{e}_z,-\alpha\rangle]$ is the wave function in the intrinsic space of the spin. The spin-dependent part acquires a geometrical phase during the evolution of the spin in a varying magnetic field. Such a geometrical phase (Berry phase) is given by $\Gamma(2\pi)=-\mathrm{Im}\oint\langle\vec{n}|d/d\vec{n}|\vec{n}\rangle d\vec{n}$ which has been shown to be $\pi(\cos\chi-1)$,[7] where the parameter χ is an angle measuring the deviation of the texture from the symmetric axis of the system. We now use this effective Hamiltonian Eq. (8) to find the eigenspectrum. The corresponding Schrödinger equation is $H_{\mathrm{eff}}|\psi\rangle=E|\psi\rangle$. Let the wave function be

$$|\psi\rangle=\frac{1}{\sqrt{N}}\sum_{l=1}c_l^\dagger\exp\left[\frac{2\pi}{N}i\left(n+\frac{\Phi}{\Phi_0}\right)l\right]|0\rangle, \tag{10}$$

subjected to the twisted boundary condition $c_{l+N}=c_l\exp(2\pi i\Phi/\Phi_0)$. Here $\Phi=\Phi_{\mathrm{e.m.}}+\Phi_g\Phi_0$ is the spin-dependent flux that combines the usual electromagnetic contribution $\Phi_{\mathrm{e.m.}}$ and the purely geometrical Berry phase $\Phi_g=(\cos\chi-1)/2$. After solving the Schrödinger equation, the energy of the system is found to be

$$E_n=\epsilon_0-\mu-\triangle_m-2Je^{-W_m}\cos\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\mathrm{e.m.}}}{\Phi_0}+\Phi_g\right)\right], \tag{11}$$

where $n=0,\pm1,\pm2,\ldots$, $\Phi_0=hc/e$ is the elementary flux quantum, and $\Phi_{\mathrm{e.m.}}$ is the magnetic flux threading the ring (measured in units of Φ_0). From the role Φ_g played in the energy spectrum, the geometrical phase can be viewed as a spin-dependent gauge potential. We see from Eq. (11) that the oscillation with respect to the flux in the energy spectrum is exponentially weakened by a factor e^{-W_m} compared to the free case. Since the persistent current is obtained from the Φ dependence of the eigenspectrum, we then expect that the electron-magnon interaction suppresses the persistent current.

The persistent current carried by electrons with energy E_n is

$$I_n=-c\frac{\partial E_n}{\partial\Phi_{\mathrm{e.m.}}}=-\frac{2eJ}{N\hbar}\sin\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\mathrm{e.m.}}}{\Phi_0}+\Phi_g\right)\right]e^{-W_m}. \tag{12}$$

At $T=0$ the total current is obtained by summing over all lowest-lying occupied states up to the Fermi level and is given by $I=\Sigma_{n=0}I_n(T=0)$. At finite temperatures, one can calculate the persistent current with the help of the free energy F of the system and obtain

$$I(\Phi,T)=-c\frac{\partial F}{\partial\Phi_{\mathrm{e.m.}}}=\sum_n I_nf(E_n), \tag{13}$$

where $f(E_n)$ is the Fermi distribution function for the electron of energy E_n and I_n is given by Eq. (12).

According to the formula given by Ref. 7 in the low-temperature limit, the spin current is obtained by taking the derivative with respect to the geometrical phase Φ_g,

$$I^s=V-\frac{1}{h}\frac{\partial F}{\partial\Phi_g}M, \tag{14}$$

with

$$\vec{V}=-\tfrac{1}{2}\sin^2\chi\hat{e}_z \tag{15}$$

and

$$\vec{M}=\cos\chi\hat{e}_z, \tag{16}$$

where $\vec{V}$ is a geometrical vector that represents the quantum-mechanical correlation between the spin and orbital degrees of freedom and $\vec{M}$ is the expectation value of the magnetization. Substituting the expression of free energy into the above equations, we obtain the spin current

$$I^s=-\tfrac{1}{2}\sin^2\chi+\sum_n I_n^sf(E_n), \tag{17}$$

where

$$I_n^s=-\frac{2J}{\hbar N}e^{-W_m}\sin\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\mathrm{e.m.}}}{\Phi_0}+\Phi_g\right)\right]. \tag{18}$$

Equations (12) and (18) show that when the electron-magnon interaction is taken into account, the amplitude of persistent currents (charge current I_n and spin current I_n^s) is decreased exponentially with increasing coupling strength $g_m=\Sigma_q|M_q|^2/(\hbar\omega_q)^2$.

III. FLUCTUATION DUE TO THE LATTICE VIBRATION AND LATTICE DISTORTION ON PERSISTENT CURRENTS

Historically, the first microscopic calculation of the effect of the phonon on electrical conductivity is contained in Bloch's pioneering work[19] on the theory of metals. It is well known that the phonon plays an important role in the BCS theory of superconductivity. Similar to the case of superconductivity, the phonon may take place in the mesoscopic system and affect the persistent current in the mesoscopic ring. Recently there are many studies considering the electron-phonon interaction on the mesoscopic system[13] as well as the effect of the appearance of the Peierls instability on the persistent current.[14–16] The effect of the Peierls instability on the mesoscopic system is expected because an average energy gap does exist and modifies the energy spectrum of the electron drastically.[14] In the following subsection we will treat a one-dimensional tight-binding mesoscopic ring in the presence of both electron-magnon and electron-phonon interactions and discuss the suppression of the persistent current.

A. Influence of lattice vibration on the persistent current

Including the electron-phonon interaction, the Hamiltonian Eq. (6) can be written as

$$H_{e\text{-}p}=H_{\text{eff}}+\sum_p \hbar\omega_p(d_p^\dagger d_p+\tfrac{1}{2})+\sum_{p,l}\widetilde{M}_p e^{ipla}(d_p+d_p^\dagger)c_l^\dagger c_l, \tag{19}$$

where H_{eff} is the effective Hamiltonian for the electron given by Eq. (8). Here the corrections for both the on-site energy $\triangle_m$ and the hopping energy due to the electron-magnon interaction Je^{-W_m} have been included. The new terms in Eq. (19) are the free phonon Hamiltonian and the electron-phonon interaction term. The coefficient $\widetilde{M}_p$ for the electron-phonon interaction satisfies the relation $\widetilde{M}_{-p}=\widetilde{M}_p^*$ and $d_p^\dagger$ (d_p) is the creation (annihilation) operator of the phonon with wave vector p. For simplicity of the calculation we have not considered the interaction between the phonon and the magnon. We will also neglect the magnon energy $\Sigma_q\hbar\omega_q\langle n_q\rangle$ in the following calculations.

Now we will solve the equation $H_{e\text{-}p}|\overline{\psi}\rangle=E_{e\text{-}p}|\overline{\psi}\rangle$ by using the variational method. The variational wave function can be chosen as[18]

$$|\overline{\psi}\rangle=\frac{1}{\sqrt{N}}\sum_{l=1}c_l^\dagger\exp\left[\frac{2\pi}{N}i\left(n+\frac{\Phi}{\phi_0}\right)l\right]S_l|\ldots,N_p,\ldots\rangle, \tag{20}$$

where Φ is again the total flux $\Phi_{\text{e.m.}}+\Phi_g\Phi_0$ and the phonon states are

$$|\ldots,N_p,\ldots\rangle=\Pi_p\frac{1}{\sqrt{N_p!}}(d_p^\dagger)^{N_p}|0\rangle \tag{21}$$

and

$$S_l=\exp\left[\sum_p f_p e^{ipla}(d_p-d_{-p}^\dagger)\right], \tag{22}$$

with variational parameter satisfying $f_{-p}=f_p^*$. Such a variational wave function is similar to that of Fröhlich's small polaron theory, but here we apply it to a mesoscopic ring satisfying a twisted boundary condition $c_{l+N}=\exp(i2\pi\Phi/\Phi_0)c_l$. Using the relation $S_l^\dagger d_p S_l=d_p-f_pe^{-ipla}$, we obtain the average energy

$$\overline{E}=\langle\overline{\psi}|H_{e\text{-}p}|\overline{\psi}\rangle=(\epsilon_0-\mu-\triangle_m)-2Je^{-W_m-W_{\text{ph}}}\cos\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\text{e.m.}}}{\Phi_0}+\Phi_g\right)\right]+\sum_p\hbar\omega_p(N_p+\tfrac{1}{2}+|f_p|^2)-\sum_p 2\widetilde{M}_pf_p^*, \tag{23}$$

where $W_{\text{ph}}=\Sigma_p2|f_p|^2(N_p+\frac{1}{2})[1-\cos(pa)]$, ω_p is the frequency of the phonon with the wave vector p, and N_p is the average number of phonons. The Debye-Waller factor can be calculated by the formula $\exp(-W_{\text{ph}})=\langle\ldots,N_p,\ldots|S_{l+1}^\dagger S_l|\underline{\ldots},N_p,\ldots\rangle$. The parameter f_p^* can be determined by $(\delta E/\delta f_p^*)=0$. We found that

$$f_p=\widetilde{M}_p\left\{\hbar\omega_p-4Je^{-W_m-W_{\text{ph}}}\cos\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\text{e.m.}}}{\Phi_0}+\Phi_g\right)\right]\times(N_p+\tfrac{1}{2})[1-\cos(pa)]\right\}^{-1}. \tag{24}$$

For the case of a narrow energy band (small J) and optical phonons $\hbar\omega_p=\hbar\omega_0$, f_p can be simplified to $f_p=\widetilde{M}_p/\hbar\omega_0$. So we have

$$W_{\text{ph}}=\sum_p\frac{|\widetilde{M}_p|^2}{(\hbar\omega_0)^2}\coth\left(\frac{\hbar\omega_0}{k_BT}\right)[1-\cos(pa)]. \tag{25}$$

The energy of the system is

$$\overline{E_n}=(\epsilon_0-\mu-\triangle_m-\triangle_{\text{ph}})-2Je^{-W_m-W_{\text{ph}}}\cos\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\text{e.m.}}}{\Phi_0}+\Phi_g\right)\right], \tag{26}$$

where $n=0,\pm1,\pm2,\ldots$ and $\triangle_{\text{ph}}=\Sigma_p|\widetilde{M}_p|^2/\hbar\omega_0$, which represents the energy correction to the on-site energy of the electron due to phonons. In Eq. (26) we have neglected the phonon energy. From this expression we see that the influence of the phonon is similar to that of the magnon. It gives a further suppression on the oscillation with respect to the flux in the energy. Using the formula $I(\Phi,T)=-c(\partial F/\partial\Phi_{\text{e.m.}})$, where F is the free energy, we obtain the persistent current of the system

$$I(\Phi,T)=\sum_n I_nf(\overline{E_n}), \tag{27}$$

where the current I_n corresponding to the nth eigenstate is given by

$$I_n=-\frac{2eJ}{N\hbar}e^{-W_m-W_{\text{ph}}}\sin\left[\frac{2\pi}{N}\left(n+\frac{\Phi_{\text{e.m.}}}{\Phi_0}+\Phi_g\right)\right]. \tag{28}$$

For the spin current, we obtain expressions similar to Eqs. (17) and (18). This shows that in comparison to that of the electron-magnon interaction, the amplitude of persistent currents (both charge current and spin current) are decreased by another exponential factor (the Debye-Waller factor) $e^{-W_{\text{ph}}}$ due to the electron-phonon interaction.

B. Peierls instability

In this subsection we discuss the Peierls instability on the persistent current in the mesoscopic ring. A theory of the Peierls instability for a finite one-dimensional ring with a discrete energy spectrum has been presented by Nathanson *et al.*[14] For the case of a half-filled band conductor (hence N is even), the one-dimensional system is unstable with respect to a static distortion of the lattice. The distortion opens a gap of magnitude 2Δ in the electron spectrum at the Fermi energy where Δ can be determined through a self-consistent mean-field equation. As shown in Ref. 14 for a finite system, there is always a nontrivial solution Δ for the case $N=4j$, where j is an integer. For the case $N=4j+2$, however, the electron-phonon coupling has to be small in order to give a nontrivial solution. In the following discussion, instead of working in real space we will work in k space. The effective Hamiltonian of the electron Eq. (8) that considers the electron-magnon interaction can be written as

$$H_{\mathrm{eff}}=\sum_k \left\{\mu_B B-\triangle_m-\mu-2Je^{-W_m}\times\cos\left[\left(k+\frac{e}{\hbar c}\frac{\Phi_{\mathrm{e.m.}}}{L}\frac{2\pi}{L}\Phi_g\right)a\right]\right\}c_k^{\dagger}c_k=\sum_k(\epsilon_0-\mu+\epsilon_k)c_k^{\dagger}c_k, \tag{29}$$

where $\epsilon_0=\mu_B B-\triangle_m$ and

$$\epsilon_k=-2Je^{-W_m}\cos\left[\left(k+\frac{e}{\hbar c}\frac{\Phi_{\mathrm{e.m.}}}{L}+\frac{2\pi}{L}\Phi_g\right)a\right].$$

Taking into account the electron-phonon interaction, the Hamiltonian can be written as

$$H_{e\text{-}p}=H_{\mathrm{eff}}+H_{\mathrm{ph}}+H_I, \tag{30}$$

where

$$H_{\mathrm{ph}}=\sum_p \hbar\omega_p(d_p^{\dagger}d_p+\tfrac{1}{2}) \tag{31}$$

and

$$H_I=\frac{1}{\sqrt{N}}\sum_{k,p}\widetilde{M}_p c_{k-p}^{\dagger}c_k(d_p^{\dagger}+d_{-p}). \tag{32}$$

In the following we will confine ourselves to the case of a half-filled band for the mesoscopic ring. Thus the chemical potential coincides with ϵ_0. At low enough temperature, the electronic system is unstable at the nesting vector Q, where $Q=\pm 2k_F$ is the soft phonon mode for the case of a perfect nesting. The Fermi wave vector takes the value $k_F=\pi/2a$. The ring is now placed in a texture. Therefore, the nesting condition is given by $\epsilon(k+Q)+\epsilon(k)=2\mu_B B$. We consider the so-called Peierls distortion in which the corresponding displacement of the lattice depends on the magnetic field. Under the static distortion of the lattice, the condensation of a phonon mode with wave vector $Q=\pm 2k_F$ would open a gap of magnitude $2\triangle_Q$ in the electronic energy spectrum at the Fermi energy. Following the argument in Ref. 14, for a small finite system with the circumference of the ring being much smaller than the wavelength of the variation of the lattice distortion around the mean-field value, the discussion can be restricted to a single mode for a lattice distortion. The corresponding Hamiltonian of the system is then given by

$$H=2N_0\hbar\omega_Q+\sum_k\{\epsilon_{k+Q/2}c_{k+Q/2}^{\dagger}c_{k+Q/2}+\epsilon_{k-Q/2}c_{k-Q/2}^{\dagger}c_{k-Q/2}+\triangle_Q(c_{k-Q/2}^{\dagger}c_{k+Q/2}+c_{k+Q/2}^{\dagger}c_{k-Q/2})\}, \tag{33}$$

where $|k|<Q/2$, $\triangle_Q=\sqrt{N_0/N}\widetilde{M}_Q$ is the energy gap, and $N_0=\langle d_Q^{\dagger}d_Q\rangle$ is the condensation phonon number. Using the Bogoliubov transformation

$$\alpha_k=u_k c_{k+Q/2}-v_k c_{k-Q/2},\quad \alpha_k^{\dagger}=u_k c_{k+Q/2}^{\dagger}-v_k c_{k-Q/2}^{\dagger}, \tag{34}$$

$$\beta_k=v_k c_{k+Q/2}+u_k c_{k-Q/2},\quad \beta_k^{\dagger}=v_k c_{k+Q/2}^{\dagger}+u_k c_{k-Q/2}^{\dagger}, \tag{35}$$

with $u_k^2+v_k^2=1$, we can write the Hamiltonian Eq. (33) as

$$H=\sum_k\{[\epsilon_{k+Q/2}u_k^2+\epsilon_{k-Q/2}v_k^2-2\triangle_Q u_k v_k]\alpha_k^{\dagger}\alpha_k+[\epsilon_{k+Q/2}v_k^2+\epsilon_{k-Q/2}u_k^2+2\triangle_Q u_k v_k]\beta_k^{\dagger}\beta_k+[\epsilon_{k+Q/2}u_k v_k-\epsilon_{k-Q/2}u_k v_k+\triangle_Q(u_k^2-v_k^2)]\times(\alpha_k^{\dagger}\beta_k+\beta_k^{\dagger}\alpha_k)\}. \tag{36}$$

By setting $[\epsilon_{k+Q/2}u_k v_k-\epsilon_{k-Q/2}u_k v_k+\triangle_Q(u_k^2-v_k^2)]=0$, we find

$$H=2N_0\hbar\omega_Q+\sum_k[\mathcal{E}_+(k)\alpha_k^{\dagger}\alpha_k+\mathcal{E}_-(k)\beta_k^{\dagger}\beta_k], \tag{37}$$

where $\mathcal{E}_{\pm}(k)=\pm\sqrt{\epsilon_{k+Q/2}^2+\triangle_Q^2}$ is the electronic spectrum in consideration of the Peierls instability. We see that the corresponding energy gap modifies the energy spectrum of the system. The nonmonotonic dependence on $\triangle_Q$ will give the correction to the persistent current in Peierls materials. By a straightforward calculation we obtain the free energy of the system

$$F=\frac{2N\hbar\omega_Q}{\widetilde{M}_Q^2}\triangle_Q^2-\frac{1}{\beta}\sum_k\{\ln[1+\exp(\beta\sqrt{\epsilon_k^2+\triangle_Q^2})]+\ln[1+\exp(-\beta\sqrt{\epsilon_k^2+\triangle_Q^2})]\}. \tag{38}$$

The equilibrium value of the gap $\triangle_Q$ can be found by minimizing the free energy, i.e., $\delta F/\delta\triangle=0$, from which we obtain the gap equation

$$\frac{N\hbar\omega_Q}{\widetilde{M}_Q^2}=\sum_k\frac{1}{\sqrt{\epsilon_k^2+\triangle_Q^2}}\frac{\sinh\beta\sqrt{\epsilon_k^2+\triangle_Q^2}}{1+\cosh(\beta\sqrt{\epsilon_k^2+\triangle_Q^2})}. \tag{39}$$

In the limit $\triangle_Q\rightarrow 0$, we obtain the equation that determines the inverse transition temperature β_c for the Peierls transition.

At finite temperatures the total persistent current is given by

$$I(\Phi,T)=-c\frac{\partial F}{\partial \Phi_{\text{e.m.}}}=-c\left(\frac{\partial F}{\partial \epsilon_k}\frac{\partial \epsilon_k}{\partial \Phi_{\text{e.m.}}}+\frac{\partial F}{\partial \triangle}\frac{\partial \triangle}{\partial \Phi_{\text{e.m.}}}\right). \tag{40}$$

Therefore we obtain the persistent current of a Peierls insulating mesoscopic ring

$$I(\Phi,T)=-\sum_k \frac{2eJ}{N\hbar}e^{-W_m}\sin\left[ka+\frac{2\pi}{N}\frac{\Phi}{\Phi_0}\right]\frac{\epsilon_k}{\sqrt{\epsilon_k^2+\triangle_Q^2}} \times\frac{\sinh\beta\sqrt{\epsilon_k^2+\triangle_Q^2}}{1+\cosh(\beta\sqrt{\epsilon_k^2+\triangle_Q^2})}. \tag{41}$$

Our result is consistent with that given by Nathanson *et al.*,[14] i.e., the gap parameter $\triangle_Q$ strongly affects the persistent current. When the gap is large enough, all of the electrons are condensed into the ground state and the current vanishes. In fact, for a wide-gap insulator, the amplitude of the persistent current is so small that it is practically undetectable. In the case of a narrow-gap insulator, there is no true phase transition. While the Peierls distortion drastically reduces the amplitude of the persistent current, in our case the ring is also affected by the texture, which suppresses the persistent current by magnons. The influence due to magnons on the persistent current in the Peierls insulating states is stronger than that of in the normal states. Equation (41) reproduces the expressions for the persistent currents given by Eqs. (13) and (27) with $W_{\text{ph}}=0$ when the electron-phonon interaction is much weaker. Generally, at low temperatures the persistent current in a Peierls material is reduced by an order of magnitude compared to the other materials.

IV. SUMMARY AND DISCUSSIONS

So far, we have discussed the persistent current under the influence of both electron-magnon and electron-phonon interactions in a one-dimensional tight-binding mesoscopic ring. In our approach we have used the adiabatic approximation and neglected the magnon-phonon interaction since it makes no contribution to the oscillatory part of the energy with respect to Φ (which is the sum of the electromagnetic flux and the geometrical Berry phase) and hence persistent currents.

Loss *et al.*[7] have discussed the role of quantum fluctuation in the persistent current and shown that there is an additive correction to the energy spectrum due to the zero-point energy. However, this additive correction is irrelevant for the calculation of the persistent current. Motivated by the work of Loss *et al.*,[7] we consider the quantum fluctuation that arises from emitted and absorbed magnons. The result shows that the persistent current is decreased by an exponential factor, which depends on the electron-magnon coupling coefficient. This can be understood as follows. In the mesoscopic texture, the ferromagnetic spin wave propagates coherently and makes an effective action on the motion of electrons. Such an exponential factor decreases the probability for preserving the phase coherence of electrons and in turn reduces the amplitude of the persistent current. In the low-temperature limit, the effect of emitting and absorbing virtual magnons does not vanish. When the temperature is high the excitations of the texture become the Stoner excitations. This result is no longer correct.

In summary, we have studied a model considering the electron-magnon and the electron-phonon interactions and discussed the persistent current with and without Peierls instability. Two results are obtained for the normal state and the Peierls distortion state. (a) In the normal state the amplitude of the persistent current is reduced by the Debye-Waller factor $\exp(-W_{\text{ph}})$, which describes the probability of the elastic and the coherent scattering. Both magnons and phonons have a similar behavior in suppressing the amplitude of persistent currents. In fact, these quasiparticles are all bosons and therefore have the same statistics. Their interactions with electrons produce a similar result when the spin of the electron is not included explicitly. (b) In the Peierls distortion state, near half-filling, the electron-phonon interaction leads to a soft phonon mode. As a consequence, the appearance of the Peierls transition can change a metal into an insulator. However, in our case, there is no true phase transition. The order parameter is not a real quantity. Therefore, the persistent current is not caused by the charge density wave. On the other hand, according to the previous result,[16] the increase of the texture magnetic field strength will decrease the transition temperature.

ACKNOWLEDGMENTS

We are grateful to Professor H.Z. Li and Professor Y.C. Zhou for stimulating conversations and also to Dr. L. Hu for important remarks. S.S.W. and Z.S.M. were supported by the NNSF-China, the Foundation of Advanced Research Center of Zhongshan University, CCAST, and ITP-CAS. J.W. was supported by a RGC research grant from the Hong Kong government under Grant No. HKU 261/95P.

*Permanent address.

[1] F. Hund, Ann. Phys. (Leipzig) **32**, 102 (1938).

[2] N. Byers and C.N. Yang, Phys. Rev. Lett. **46**, 7 (1961); F. Bloch, Phys. Rev. **137**, A787 (1965).

[3] F. London, J. Phys. Radium **8**, 397 (1937); J.A. Pople and K.G. Untch, J. Am. Chem. Soc. **88**, 4811 (1966).

[4] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).

[5] H.F. Cheung, Y. Gefen, E.K. Riedel, and W.H. Shih, Phys. Rev. B **37**, 6050 (1988); H.F. Cheung, E.K. Riedel, and Y. Gefen, Phys. Rev. Lett. **62**, 587 (1989).

[6] V. Ambegaoker and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990); B.L. Altshuler, Y. Gefen, and Y. Imry, *ibid*. **66**, 88 (1991).

[7] D. Loss, P. Goldbart, and A.V. Balatsky, Phys. Rev. Lett. **65**, 1455 (1990); D. Loss and P. Goldbart, Phys. Rev. B **43**, 13 762 (1991); **45**, 13 544 (1992).

[8] L.P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990); V. Chandraseker, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Gallagher, and A. Kleinsasser, *ibid.* **67**, 3578 (1991); D. Mailly, C. Chapelier, and A. Benoit, *ibid.* **70**, 2120 (1993).

[9] V. Ambegaoker and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990); R.A. Smith and V. Ambegaokar, Europhys. Lett. **20**, 161 (1992); G. Bouzerar, D. Poilblanc, and G. Montambaux, Phys. Rev. B **49**, 8258 (1994); H. Kato and D. Yoshioka, *ibid*. **50**, 4943 (1994); M. Abraham and R. Berkovits, Phys. Rev. Lett. **70**, 1509 (1993); R. Berkovits and Y. Avishai, Europhys. Lett. **29**, 475 (1995); T. Giamarchi and B. S. Shastry, Phys. Rev. B **51**, 10 915 (1995); J. Wang and Z. S. Ma, *ibid.* **52**, 14 829 (1995).
[10] G. Grüner, Rev. Mod. Phys. **66**, 1 (1994).
[11] K. Kusakabe and H. Aoki, Phys. Rev. Lett. **72**, 144 (1994); N. Shima and H. Aoki (unpublished); S. Fujmoto and N. Kawakami, Phys. Rev. B **48**, 17 406 (1993).
[12] V.I. Ozhogin, R.M. Farzetdinova, and A.S. Mikhailor, Zh. Éksp. Teor. Fiz. **97**, 863 (1990) [Sov. Phys. JETP **70**, 483 (1990)].
[13] P. Vasilopoulos and C.M. Van Vliet, Phys. Rev. B **34**, 5217 (1986); F.V. Kusmartsov, *ibid.* **46**, 7674 (1992); Y.C. Zhou, X.E. Yang, and H.Z. Li, Phys. Lett. A **190**, 123 (1994).
[14] B. Nathanson, O. Entin-Wohlman, and B. Mühlechlegei, Phys. Rev. B **45**, 3499 (1992).
[15] S.K. Khanna, J.P. Pouget, R. Comes, A.F. Garito, and J. Heeger, Phys. Rev. B **16**, 1468 (1977).
[16] E.N. Bogachek, S.V. Krive, I.O. Kulik, and A.S. Kozhavskii, Zh. Éksp. Teor. Fiz. **97**, 603 (1990) [Sov. Phys. JETP **70**, 336 (1990)].
[17] Y. Meir, Y. Gefen, and O. Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1988).
[18] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981).
[19] F. Bloch, Z. Phys. **52**, 55 (1928); **59**, 208 (1930).