Vortex dynamics of a $d+is$-wave superconductor

Qunqing Li and Z. D. Wang
Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong

Qiang-Hua Wang
Department of Physics and National Laboratory of Solid State Microstructure, Institute for Solid State Physics, Nanjing University, Nanjing 210093, People's Republic of China

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The vortex dynamics of a $d+is$-wave superconductor is studied numerically by simulating the time-dependent Ginzburg-Landau equations. The critical fields, the free flux flow, and the flux flow in the presence of twin boundaries are discussed. The relaxation rate of the order parameter turns out to play an important role in the flux flow. We also address briefly the intrinsic Hall effect in d- and $d+is$-wave superconductors. [S0163-1829(99)01642-2]

In recent years the symmetry of the pairing function has been one of the interesting topics in the field of high-temperature superconductors. It is widely accepted that the dominant pairing wave function has a $d_{x^2-y^2}$ symmetry, as supported by experiments using phase-sensitive devices, such as Josephson junctions or superconducting quantum interference devices (SQUID).[1–5] However, the subdominant pairing channels such as s-wave or d_{xy}-wave channels are still possible.[6–8] The mixed state of $s+d$ superconductors was first discussed by Ruckenstein, Hirschfeld, and Appel[9] and that of $d+is$ superconductors by Kotliar.[10] In Ref. 10, it is pointed out that the resonating-valence-bond mechanism can lead to s-wave- and d-wave-like Cooper pairings, and a mixture of s and d waves with a well-defined relative phase close to $\theta=\pi/2$ is energetically favored. Importantly, the superconducting state is time-reversal-symmetry (T hereafter) breaking and the energy gap is nodeless unless $\theta=0$ or π. Moreover, surfaces and interfaces, grain boundaries, and other pair breaking defects have all been shown to enhance the T-breaking states.[11,12]

The vortex structure and vortex dynamics of d-wave superconductor have been studied numerically in detail by several methods.[13–17] Previous simulations for the $d+is$-wave superconductors showed that the vortex structure of $d+is$-wave superconductor is different from that of the d-wave superconductor:[18] The spatial profile of the moduli of s- and d-wave components in the $d+is$-wave state has a twofold symmetry, in contrast to the fourfold symmetry of the magnetic-field-induced s-wave component of d-wave superconductors. However, it turns out that such a $d+is$ state can only be stabilized at extremely low temperatures. With increasing temperature, the amplitude of s-wave component decreases and its symmetry changes from twofold to fourfold. Here we would like to extend our study to the vortex dynamics of $d+is$ superconductors. For conventional superconductors the free-flux-flow (FFF) resistance is linear in the magnetic-field induction B (with $B \ll B_{c2}$). The situation is not clear yet for high-T_c and other unconventional superconductors. Because of the multiple components of the order parameter, this topic is highly nontrivial. We also study the vortex motion in the presence of a twin boundary and the intrinsic Hall effect of $d+is$-wave superconductors.

We start with a model of an isotropic two-dimensional Fermi liquid with attractive interactions in both s and d channels. Obviously, when only one of the two interactions is attractive, the ground state is a pure state with the appropriate pairing symmetry. When both channels are attractive, the competition will lead to either a pure or a mixed pairing function. The Ginzburg-Landau (GL) theory for a superconductor with two attractive channels has been presented by Ren, Xu, and Ting[19] based on Gor'kov equations. Assuming pure dissipative dynamics, the GL equations can be expressed as follows:

$$\left[\eta_s\partial t-\alpha_s+\frac{4}{3}(|S|^2+|D|^2)+\mathbf{\Pi}^2\right]S+\frac{2}{3}D^2S^* + \frac{1}{2}(\mathbf{\Pi}_x^2-\mathbf{\Pi}_y^2)D=0, \tag{1}$$

$$\left[\eta_d\partial t-\alpha_d+\frac{8}{3}|S|^2+|D|^2+\mathbf{\Pi}^2\right]D+\frac{4}{3}S^2D^* + (\mathbf{\Pi}_x^2-\mathbf{\Pi}_y^2)S=0, \tag{2}$$

$$\frac{\partial\mathbf{A}}{\partial t}+\kappa^2(\nabla\times\nabla\times\mathbf{A}-\nabla\times\mathbf{H}_e)+\left\{S^*\mathbf{\Pi}S+\frac{1}{2}D^*\mathbf{\Pi}D + \frac{1}{2}[S^*(\mathbf{\Pi}_x-\mathbf{\Pi}_y)D+D^*(\mathbf{\Pi}_x-\mathbf{\Pi}_y)S]+\text{H.c.}\right\}=0. \tag{3}$$

In these equations, the two order parameters, S and D, are normalized by $\Delta_0=\sqrt{4/3\alpha\ln(T_d/T)}$ with $\alpha \approx 7\zeta(3)/8(\pi T_c)^2$, the space by the coherence length ξ, and the vector potential $\mathbf{A}$ by $\Phi_0/2\pi\xi$ with $\Phi_0=h/2e$ being the flux quantum, respectively. In Eq. (1), α_s may be expressed as a function of temperature T:[19,20]

$$\alpha_s=\ln(T_s/T)/\ln(T_d/T), \tag{4}$$

where T_s and T_d may be viewed as the apparent superconducting transition temperatures for the s wave and the d wave, respectively, with $T_s \propto e^{-1/N(0)V_s}$ and $T_d \propto e^{-2/N(0)V_d}$.

$\alpha_d = 1$ unless specified otherwise. Here $N(0)$ is the density of states at the Fermi surface, V_s and V_d are the effective attractive interaction strengths in the s- and d-wave channels, respectively. $\mathbf{\Pi} = i\nabla + \mathbf{A}$, $\mathbf{\Pi}_k = \hat{x}_k \Pi_k$. κ is the GL parameter. The time t is normalized by $\tau = \sigma_n \xi^2$ with σ_n the normal-state conductivity of the superconductor. $\eta_{s,d} = \tau_{s,d}/\tau$, with $\tau_{s,d}$ being the relaxation time of the s- and d-wave order parameters, respectively. For gapless conventional superconductors, the relaxation time of the order parameter near T_c is $\tau_s = \pi\hbar/8k(T_c - T) \propto \xi^2$.[21] For unconventional superconductors, it is plausible to assume the general scaling law $\tau_{s,d} \sim \xi^z$ with z as the dynamical exponent, which should hold at least near the critical temperature. In this case, it is clear that $\eta_{s,d} \sim \xi^{z-2}$ would depend on temperature if $z \neq 2$. However, in the absence of a microscopic theory on the relaxation times, we shall content ourselves to treat them as phenomenological parameters, the effect of which on the vortex dynamics is examined in this paper.

The scale for the magnetic field is set by the upper critical field H_{c2}. Because of its nontrivial nature in the present system, we would like to discuss it before moving to the dynamical properties of the vortices. Following the same method described by Sigrist and Ueda,[22] and using the relation $[\Pi_x, \Pi_y] = iB$, we introduce a pair of operators $a = (1/\sqrt{2B})(\Pi_x + i\Pi_y)$, $a^+ = (1/\sqrt{2B})(\Pi_x - i\Pi_y)$, and have the commutative relation $[a, a^+] = 1$. Substituting a, a^+ into the linearized Eq. (1) and Eq. (2) in the static case, we obtain

$$[-2\alpha_s + 2B(1 + 2\hat{n})]S + B(aa + a^+a^+)D = 0,$$

$$[-1 + B(1 + 2\hat{n})]D + B(aa + a^+a^+)S = 0, \qquad (5)$$

where $\hat{n} = a^+ a$. Therefore the order parameter can be expanded in this occupation representation as $\Psi = \Sigma_{m=0}^{\infty} \binom{s_m|m\rangle_s}{d_m|m\rangle_d}$, where $\hat{n}|m\rangle = m|m\rangle$. Eq. (5) cannot be solved exactly, so we treat it variationally by assuming $\Psi = \binom{s_0|0\rangle_s}{d_0|0\rangle_d} + \binom{s_2|2\rangle_s}{d_2|2\rangle_d} + \binom{s_4|4\rangle_s}{d_4|4\rangle_d}$. H_{c2} is determined by the condition that the ground-state energy of the eigenvalue problem Eq. (5) is zero. It turns out that the variational ground state is $\Psi = \binom{0}{d_0|0\rangle_d} + \binom{s_2|2\rangle_s}{0} + \binom{0}{d_4|4\rangle_d}$, and the corresponding H_{c2} is given by the root of the equation

$$60B^3 - 86B^2 + 9\alpha_s B^2 + (10 - 10\alpha_s)B + \alpha_s = 0. \qquad (6)$$

The temperature dependence of H_{c2} calculated in this way is shown as the solid line in Fig. 1. Here we use Eq. (4) for the temperature dependence of α_s, and assume $T_d = 100$ K and $T_s = 90$ K. In order to check the reliability of the variational treatment, we also simulate the upper critical field of such a $d + is$ superconductor numerically by solving Eq. (1) ~ Eq. (3) with the finite element method. In practice, we fix the magnetic field induction B by specifying one or two vortices in a square unit cell with periodic boundary conditions. The side length is varied so as to change the magnetic induction B. The technical details of the simulation have been given elsewhere.[17] The external magnetic field H can be derived from the Virial relation.[23] With H the Gibbs free-energy density can be constructed, as shown in Fig. 2. We can read off H_{c1} from the intersection of Meissner state line and the mixed-state line, and H_{c2} from the intersection of the mixed-state line and the normal-state line. The simulation result for the upper critical field is shown as black squares in Fig. 1. The variational result (solid line) appears to be in excellent agreement with the simulation result, indicating that expanding the trial wave function up to the occupation state of $|4\rangle$ is already rather reliable.[24] Note that H_{c2} is larger than that of the pure d-wave superconductor, which is always $B_0 = \Phi_0/2\pi\xi^2$ in our case. From Fig. 2, it is obvious that the magnetic-field-induced transitions at $H = H_{c1}$ and $H = H_{c2}$ are both of the usual second-order transition, which should be compared to the unusual first-order transitions found numerically in some p-wave superconductors.[25]

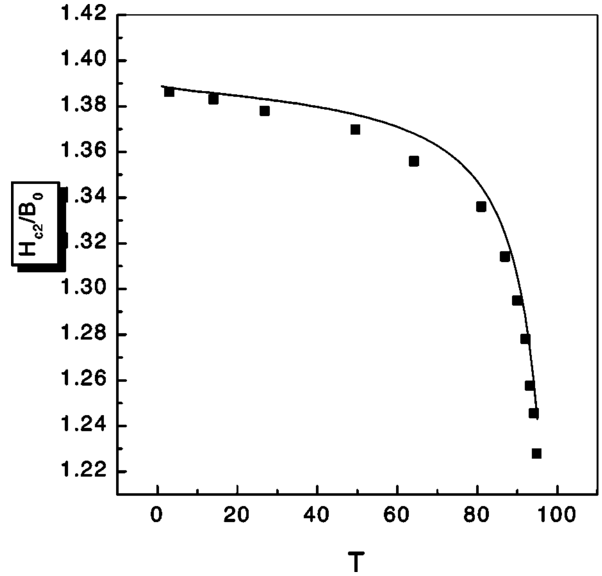

FIG. 1. The upper critical field H_{c2}/B_0 as a function of temperature T with B_0 as the upper critical field of pure d wave. The solid line is the approximation result determined from Eq. (6). The scatter point is the numerical result. Here T_s is set to 90 K, T_d is set to 100 K.

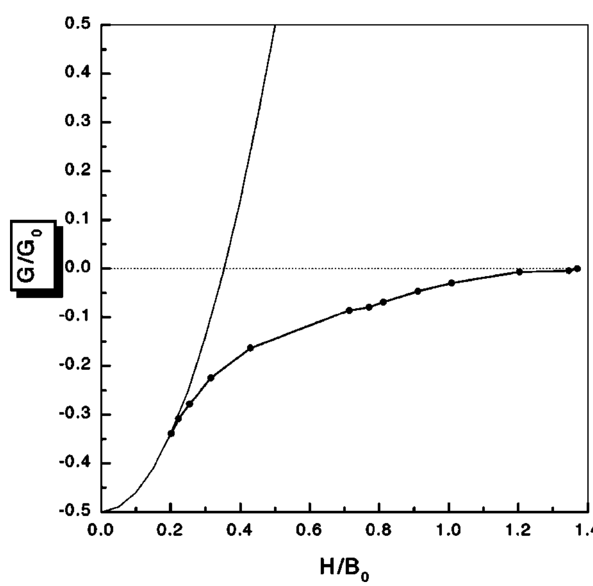

FIG. 2. Gibbs free-energy density as a function of applied field. Here $\alpha_s = 0.85$, $\kappa = 3$. The solid line is the Messiner-state line, dashed line is the normal-state line.

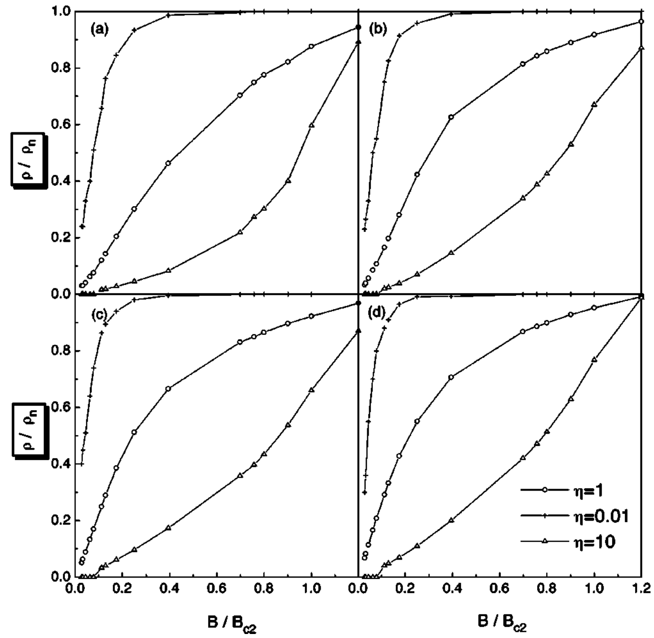

FIG. 3. Free vortex flow resistivity as a function of the applied field for $\eta_s=1$, 10, and 0.01. Here $\alpha_s=0.97$, 0.85, 0.67, and -1 in (a), (b), (c), and (d), respectively.

We now discuss the vortex flow driven by a transport current **J** in the mixed state (i.e., with $B<H_{c2}$). This is realized by requiring $\nabla\times\mathbf{H_e}=\mathbf{J}$ in Eq. (3). Here we have chosen a gauge in which the electrostatic potential does not appear, so that the local electric field is simply $\mathbf{E}=-\partial_t\mathbf{A}$. To investigate the relaxation effects of the order parameter on the FFF resistivity, we chose three values of the relaxation coefficient of the order parameter S and D as (1) $\eta_s=2\eta_d=1$, (2) $\eta_s=2\eta_d=10$ ($\gg 1$), (3) $\eta_s=2\eta_d=0.01$ ($\ll 1$). The field dependence of the FFF resistivity is shown in Fig. 3. Noticeably it evolves from a convex to a concave with increasing η_s, albeit slight difference exists at the four temperatures shown in the four panels. This is the correct trend by general reasoning: The motion of vortices is equivalent to the phase slipping of the order parameter,[26] so that a small relaxation time means a quick rate of phase slipping, and thus a large resistance. Thus with increasing magnetic field the vortex flow resistivity approaches the normal-state resistivity more quickly. It follows from Fig. 3 that the effect of the relaxation time is more prominent at lower temperatures (or larger α_s).

Next, we investigate the vortex motion of the $d+is$ superconductor in the presence of twin boundaries and then look into the pinning effect. A periodic array of twin boundaries (with a transverse spacing of $L=10.8\xi$) are assumed and described by $\alpha_i=\alpha_{i,0}-u_i\Sigma_k\delta(y-y_0-kL)$, where the subscript i stands for s or d. Here, u_i describes the variation of $\alpha_{s,d}$ across the twin boundary along the line $y=y_0$ due to the local misorientation or chemical contamination. Along the twin boundary, we apply a transport current $\mathbf{J}=J\hat{x}$. The vortex motion will be pinned by the twin boundary up to a depinning current $J=J_c$, which we shall determine. In the following simulation we fix $\eta_s=2\eta_d=1$. Figure 4 gives the current dependence of the flux-flow resistivity, which turns out to be highly nonlinear. In the present simulation, we may expect a simple result of the overdamped vortex motion in a

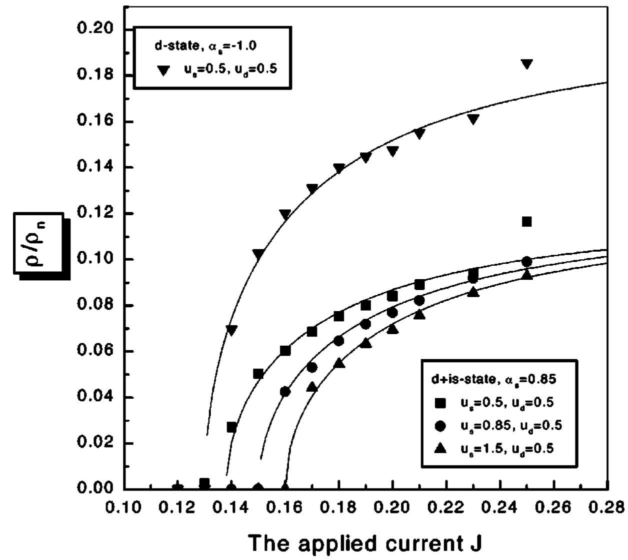

FIG. 4. The current dependence of the flux-flow resistivity in the presence of a twin boundary. The solid lines represent the fitting at low J (see the text).

periodic pinning potential: $\rho/\rho_n=a\sqrt{1-(J_c/J)^2}$ at $J\geq J_c$ (the solid lines in Fig. 4),[17] where a is the asymptotic reduced resistivity and J_c can be thus determined. We see that a higher depinning current arises from higher values of u_s (or u_d). This is because of the increasing suppression of the the amplitudes of s-wave and d-wave components at the boundary. In fact, with a suppression of the order parameter at the twin, the vortices lose less condensation energy, and thus have a lower energy than they would in the bulk. For a

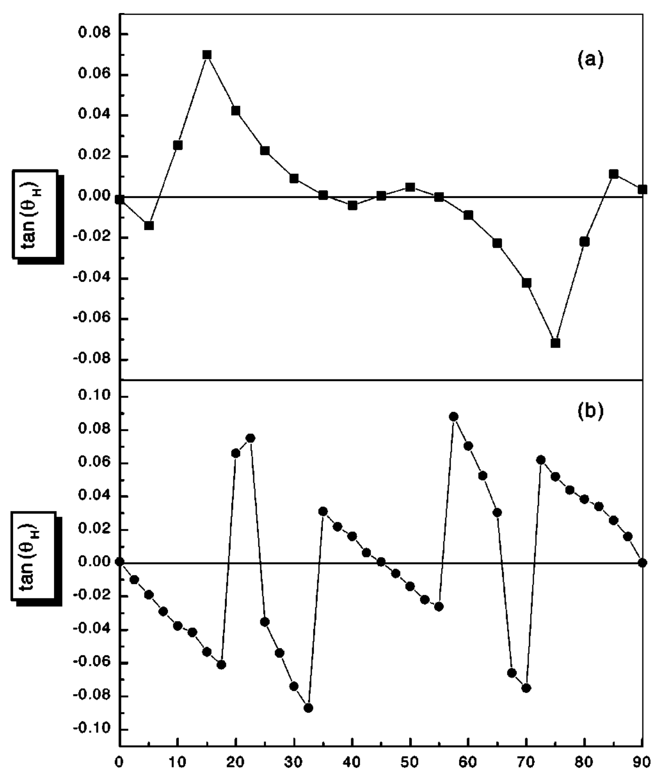

FIG. 5. tan θ_H as function of the current orientation angle φ at $|\mathbf{J}|=0.08$ and $B=0.034B_{c2}$. (a) $\alpha_s=-1.0$ and $\alpha_d=1.0$; (b) $\alpha_s=0.85$, $\alpha_d=1.0$. The lines are guides for the eye.

d-wave superconductor, u_s is irrelevant because $\alpha_s<0$, only u_d takes effect, so it is reasonable that a *d*-wave superconductor has a lower depinning current than a $d+is$ superconductor does with the same twin boundary.

Finally, we turn our attention to the so called intrinsic Hall effect, which occurs for anisotropic vortex structures even without considering normal Hall conductivity σ_H^n. Alvarez, Dominguez, and Balseiro have found that in *d*-wave superconductors the intrinsic Hall effect depends on the orientation angle, φ, of the driving current roughly as $\sim\sin(4\varphi)$, and increases nonlinearly with $\mathbf{J}$.[27] It is believed that this effect is due to the four-lobe structure of the *d*-wave vortices. So here we would like to explore the effect of the twofold symmetry of *s* and *d* components on the intrinsic Hall effect. As usual, the intrinsic Hall effect is measured by $\tan\theta_H=E_{AV,\perp}/E_{AV,\parallel}$, where $\mathbf{E}_{AV}$ and $E_{AV,\perp}$ are the components of $\mathbf{E}_{AV}$ perpendicular and parallel to the applied current, respectively. Figure 5 presents $\tan\theta_H$ versus φ for (a) a *d*-wave superconductor with $\alpha_s=-1.0$ and $\alpha_d=1.0$; (b) a $d+is$-wave superconductor with $\alpha_s=0.85$ and $\alpha_d=1.0$. Here $B=0.034B_{c2}$, and $|\mathbf{J}|=0.08$ at which $\overline{\rho}/\rho_n\approx0.6$ in both cases (a) and (b). It can be seen that for the *d*-wave superconductor [Fig. 5(a)], $\tan\theta_H$ has two peaks (at $\varphi_p=15°$ and $\varphi_p=75°$), and is identically zero at $\varphi=0°$, 45°, and 90°. For the $d+is$ superconductor, $\tan\theta_H$ is still zero at $\varphi=0°$, 45°, and 90°, but there are new sign reversals at approximately $\varphi=22.5°$ and 67.5°. Clearly, the vanishing of $\tan\theta_H$ at $\varphi=0°$, 45°, and 90° can be attributed to the fact that the vortex (and thus the supercurrent around it) can adjust its symmetry axis to the the direction of the driving force from the the applied current. At other directions of the driving force, the vortex can only partially adjust its symmetry axis. The new sign reversal for the $d+is$ superconductors is more subtle. In this case, the vortex profile is twofold symmetric. Therefore there are two nonequivalent configurations for the vortex to have its (reflection) symmetry axis along the *x*, *y*, or the diagonal directions. We suspect that the abnormal sign reversal results from a switching between these two metastable configurations as the direction of the driving force changes.

In summary, the dynamics of the vortices in both $d+is$-wave and *d*-wave superconductors is studied. The upper critical field of the $d+is$-wave superconductors is studied analytically and numerically. From simulation results, the curvature of the FFF resistivity as a function of the magnetic field strongly depends on the relaxation rate of the order parameter. The flux flow in the presence of a twin boundary is also addressed. Finally, the intrinsic Hall effect of *d*- and $d+is$-wave superconductors are studied numerically. We find the orientation dependence of the intrinsic Hall effect in these two types of superconductors are rather different.

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