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Order parameters and current-phase relations in ^{3}He-B Josephson junctions through a porous layer

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Recent discoveries of the π states in ^{3}He-B Josephson junctions through an array of apertures and a single aperture have aroused much theoretical interest on the mechanism of π states. Both tunneling junction and single orifice junction models were successfully applied to explain the occurrence of π states and their relationship with the texture orientations of $\hat{n}$ vectors in two ^{3}He-B reservoirs. In this paper, we study a model ^{3}He-B Josephson junction through a porous layer. The order parameters and current-phase relations are calculated self-consistently using the quasiclassical theory. In agreement with previous theories, the π state is also observed when the $\hat{n}$'s are aligned antiparallel and normal to the porous layer. In this model, however, the π state exists only when the coupling between two ^{3}He-B reservoirs is strong, and the usual 0 state is present when the coupling diminishes. Being contrary to the single aperture case, the π state in our model is robust only when the magnetic field is aligned either nearly normal to or within the porous layer.

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I. INTRODUCTION

Superconductors and superfluidity ^{3}He are macroscopic quantum coherent states with all Cooper pairs condensed into the same state, in which the phase of order parameters in different spatial locations are interrelated. When two superconductors or two superfluids ^{3}He are brought together and separated by a potential barrier, the phase coherence between the two is gradually established as the coupling between the two gets stronger. In the presence of a phase difference between two superconductors or superfluids, there is, in general, a supercurrent flowing across the barrier. The relationship between superfluid current and phase difference depends on the symmetry of superfluid pairing states and junction properties. For junctions between conventional *s*-wave superconductors, the tunneling Hamiltonian predicts that current-phase relations have a $I(\phi)=I_C\sin(\phi)$ form. The current phase relation remains the same for single-orifice junctions if temperature is near the critical transition temperature T_C, and the sine curve gets slanted towards $\phi=\pi$ as temperature decreases much below T_C.[1] Recently, other types of current phase relations were also investigated for junctions involving *d*-wave superconductors.[2–10] In addition to the nodes at $\phi=0$ and $\phi=\pi$, extra nodes appear depending on the types and orientations of superconductors. Such unique features are extremely important in identifying the Cooper pair symmetry in high T_C superconductors.

In superfluid ^{3}He, Cooper pair states take spin-triplet and orbital *p*-wave functions, whose symmetry properties contain both orbital as well as spin degrees of freedom.[11] Unlike isotropic *s*-wave superconductors which are not sensitive to the presence of defects and surfaces, the superfluid ^{3}He has the anisotropic pairing state, and thus its order parameters in the vicinity of surfaces and interfaces can be completely different from those in bulk. It is well known that the order parameter component normal to a surface is completely suppressed while the parallel components can be suppressed too when surfaces are rough.[12–14] Therefore, the Josephson effect in a superfluid ^{3}He is determined by the pairing state symmetry in bulk as well as the interface scattering properties. Since order parameters in a superfluid ^{3}He contain nine complex components and every component responds to an interface in different manners, the Josephson effect in superfluid ^{3}He can have richer structures than their counterpart in *s*-wave superconductors.

While an early study on Josephson effect on a single aperture revealed the usual current-phase relation $I(\phi)=I_C\sin(\phi)$,[15] recent measurements on a 65×65 array of small apertures,[16,17] and on a single aperture[18] demonstrated the existence of π states which depends on detailed cool-down procedures. It was suggested that different current phase relations be caused by different $\hat{n}$ textures[16,17] and π-state correspond to an antiparallel orientations of $\hat{n}$'s. There is also a suggestion that the π-state is not an intrinsic property of single aperture but rather a collective behavior of many apertures.[19]

Motivated by above experimental observations, Viljas and Thuneberg[20] as well as Yip[21] have independently studied the impact of $\hat{n}$-textures on current-phase relations. Yip emphasized the nature of single aperture and showed analytically how internal phases associated with different momenta results in the π state in the case of antiparallel orientations of $\hat{n}$'s. This gives a very clear physical picture regarding the occurrence of π states, however, the effect of surface pair breaking is not considered. On the other hand, Viljas and Thuneberg demonstrated the importance of gradient energy associated with bending of $\hat{n}$ vectors using the tunneling Hamiltonian, with the interface being assumed to be fully diffusive. They found that a better agreement with the experimental measurement can be reached if an array of 65×65 apertures is modeled by a thin scattering layer.

To study Josephson effects between anisotropic superfluids ^{3}He, the calculation of an interface structure of order parameters is an important first step since it determines the scattering matrix of quasiparticles near interfaces which, in turn, gives the boundary condition on textures of order parameters, such as the $\hat{n}$ vector in the B phase[22] and $\hat{l}$ vector in the A phase.[23] The interface structure of order parameters and current phase relations can be calculated more easily using the quasiclassical theory of ^{3}He.[24] This theory describes slowly varying phenomena in space and time under the conditions that $\Delta \ll E_F$ and $\xi_0 = \hbar v_F / 2\pi k_B T_c \gg k_F^{-1}$.[24] In this paper, we study a model ^{3}He-B Josephson junction through a porous layer. To imitate a porous layer, we adopt the model devised by Ovchinnikov[25] and Culetto *et al.*[26] for a rough scattering layer. The order parameters and current-phase relations are calculated self-consistently using the quasiclassical theory. In agreement with previous theories, the π state is also observed when the $\hat{n}$'s are aligned antiparallel and normal to the porous layer. However, in this model, the π state exists only when the coupling between two ^{3}He-B reservoirs is strong (roughness parameter of the interface is small) and the usual 0 state is recovered when the coupling diminishes. Being contrary to the single aperture case, the π state in our model is robust only when the magnetic field is aligned either nearly normal to or within the porous layer.

The rest of the paper is organized as follows. In Sec. II, we discuss general aspects of the quasiclassical theory as they apply to the present problem. Also discussed here is the surface model for the rough interface or porous wall. In Sec. III, we present our numerical results on order parameters and current phase relations for different configurations of $\hat{n}$ textures and interface roughness. Section IV is the conclusion.

II. QUASICLASSICAL METHOD

The quasiclassical theory can be formally derived from the Dyson equation of many particle systems.[24] By separating the Green's function into the low and high energy parts, one can absorb the high energy part into the vortex corrections, and yield the quasiclassical propagator by integrating the lower energy Green's function over the magnitude of the momentum. Because the quasiclassical theory eliminates a great deal of fine structure right at the outset, numerical calculations are much easier to carry out in comparison with the Green's function method.

The equilibrium state of superfluid ^{3}He is described by the Matsubara propagator which satisfies the transport like or the Eilenberger-Larkin-Ovchinnikov-Eliashberg (ELOE) equation[24]

$$[i\epsilon_n \hat{\tau}_3 - \hat{\sigma}(\hat{k},\vec{R}), \hat{g}(\hat{k},\vec{R};\epsilon_n)]_- + i\hbar v_F \hat{k}\cdot\nabla \hat{g}(\hat{k},\vec{R};\epsilon_n) = 0, \tag{1a}$$

$$[\hat{g}(\hat{k},\vec{R};\epsilon_n)] = -(\pi\hbar)^2. \tag{1b}$$

The ELOE equation (1a) is an ordinary first order differential equation along "trajectories," lines parallel to $\hat{k}$, and Eq. (1b) is a normalization condition. Here $[\hat{A},\hat{B}]_- = \hat{A}\hat{B} - \hat{B}\hat{A}$ and ϵ_n is the Matsubara frequency $\epsilon_n = \pi k_B T(2n+1)$. A hat denotes (either the unit vector $\hat{k}$ or) a 4×4 matrix, which is a product of spin space and Nambu particle-hole space. The Pauli matrices in these two spaces are denoted by σ_i and $\hat{\tau}_i$, respectively. The quasiparticle propagator $\hat{g}$ and self-energy matrix $\hat{\sigma}$ are parametrized as

$$\hat{g}(\hat{k},\vec{R};\epsilon_n) = \begin{pmatrix} g + \vec{g}\cdot\vec{\sigma} & (f + \vec{f}\cdot\vec{\sigma}) i\sigma_2 \\ i\sigma_2(\underline{f} + \underline{\vec{f}}\cdot\vec{\sigma}) & \underline{g} - \sigma_2 \underline{\vec{g}}\cdot\vec{\sigma}\sigma_2 \end{pmatrix}, \tag{2a}$$

$$\hat{\sigma}(\hat{k},\vec{R}) = \begin{pmatrix} \nu & \vec{\Delta}\cdot\vec{\sigma} i\sigma_2 \\ i\sigma_2 \vec{\Delta}^*\cdot\vec{\sigma} & \underline{\nu} \end{pmatrix}. \tag{2b}$$

The self-consistency equations

$$\nu(\hat{k},\vec{R}) = T\sum_n{}' \int \frac{d\Omega_{\hat{k}}}{4\pi} A_1^S(\hat{k}\cdot\hat{k}') g^M(\hat{k}',\vec{R},\epsilon_n), \tag{3a}$$

$$\frac{k_B T}{\hbar}\sum_n \left[\int \frac{d\Omega'}{4\pi} 3(\hat{k}\cdot\hat{k}')\vec{f}(\hat{k}',\vec{R};\epsilon_n) - \frac{\pi\hbar\vec{\Delta}(\hat{k},\vec{R})}{(\epsilon_n^2 + \Delta^2(T))^{1/2}} \right] = 0 \tag{3b}$$

determine the self-energy matrix in Eq. (1). $A_1^S = F_1^S/(1 + F_1^S/3)$ and F_1^S is the Landau Fermi liquid parameter. The gap equation (3b) is written in a cut-off independent form by introducing $\Delta(T)$ which is the temperature dependent gap in bulk liquid. Furthermore, Matsubara propagators satisfy the basic symmetry relations

$$[\hat{g}(\hat{k},\vec{R};\epsilon_n)]^+ = \hat{\tau}_3 \hat{g}(\hat{k},\vec{R};-\epsilon_n)\hat{\tau}_3, \tag{4a}$$

$$[\hat{g}(\hat{k},\vec{R};\epsilon_n)]^{tr} = \hat{\tau}_2 \hat{g}(-\hat{k},\vec{R};-\epsilon_n)\hat{\tau}_2, \tag{4b}$$

where superscripts + and tr denote Hermitian and transpose matrix. Hence only calculations for positive energies and in a half space of trajectory directions are required. The symmetries in Eq. (4) follow directly from the definition of propagators. In the present study, we have neglected the Landau Fermi liquid correction (except F_1^S) since we are mainly interested in the effect of $\hat{n}$ textures and interface roughness on the particle and spin transports of Josephson junctions. In the absence of boundaries the above equations form a closed set which allows the computation of quasiparticle propagators and self-energies. Out of these one can deduce the particle and spin tunneling currents per area

$$J = \frac{k_B T}{R_0 \hbar}\sum_n{}' \int \frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_z \mathrm{Re}\, g^M(\hat{k},0,\epsilon_n), \tag{5a}$$

$$\vec{J}_{ij}^s = \frac{k_B T}{R_0 \hbar}\sum_n{}' \int \frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_j \mathrm{Re}\, \vec{g}_i^M(\hat{k},0,\epsilon_n), \tag{5b}$$

where $R_0 = [2N(E_F)v_F]^{-1}$ is the modified Sharvin resistance.[27]

At interfaces the quasiclassical equations (1) have to be supplemented with boundary conditions. A fully general boundary condition within the quasiclassical theory of superfluidity was first derived by Buchholtz and Rainer[28] for nonmagnetically scattering surfaces and by Millis *et al.*[29] for magnetically active surfaces. To imitate the porous layer, we adopt the model devised by Ovchinnikov[25] and Culetto *et al.*[26] for a rough scattering layer which reads

$$[\hat{g}^M(\hat{k},\xi;\epsilon_n),\langle\hat{g}^M\rangle(\xi,\epsilon_n)]_- + \frac{2\pi i}{\rho}\hbar k_\perp \frac{d}{d\xi}\hat{g}^M(\hat{k},\xi;\epsilon_n)=0 \tag{6a}$$

with

$$\langle\hat{g}^M\rangle(\xi,\epsilon_n)=\int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{g}^M(\hat{k},\xi;\epsilon_n) \tag{6b}$$

denoting the impurity self-energy, where $k_\perp$ is the projection of trajectory perpendicular to an interface, ρ is the roughness parameter of an interface and is related to the conventional diffusivity parameter p (Ref. 30) through the relation $p=1-4\int_0^{\pi/2}d\theta\cos\theta\sin^3\theta\exp(-\rho/\cos\theta)$. With $p(\rho=0)=0$ standing for the transparent interface and $p(\rho=\infty)=1$ for the fully diffuse interface. $\xi=\pm 1/2$ correspond to $\vec{R}=\vec{R}_{\mathrm{surf}}\pm 0^+$, where $\vec{R}_{\mathrm{surf}}$ is the coordinate of interface layer.

Throughout this paper, we consider only planar Superfluid-Porous-Superfluid junctions. The Cartesian coordinate is chosen such that the xy plane is within the interface of junctions and z is the axis normal to interfaces. Superfluids He^3-B are on both sides, but they may have different orientation texture vectors $\hat{n}$. To calculate the current phase relation, the phase difference of order parameters ϕ between right and left bulk superfluids is fixed and the order parameters in bulk are given by

$$\hat{\Delta}(\hat{k},z)=\begin{cases}\hat{A}(\hat{n}_r,\theta_r)\hat{\Delta}(\hat{k})\exp(\phi/2), & z\gg 0,\\ \hat{A}(\hat{n}_l,\theta_l)\hat{\Delta}(\hat{k})\exp(-\phi/2), & z\ll 0.\end{cases} \tag{7}$$

$\hat{A}(\hat{n}_{r,l},\theta_{r,l})$ is the rotation matrix along texture vector $\hat{n}_{r,l}$ for an angle $\theta_{r,l}$, $\hat{\Delta}(\hat{k})$ is the order parameter for bulk superfluid B phase. The selfconsistent solution is achieved via the iteration scheme for order parameters, with the accuracy better than 1% being required for convergent solutions.

III. RESULTS AND DISCUSSIONS

The above equations are solved selfconsistently for different combinations of $\hat{n}_{l,r}$ vectors using iteration scheme. The rotation angles $\theta_{r,l}$ are fixed at Leggett's angle $\theta_L=\arccos(-1/4)$ so that dipolar energies are minimized. The Landau parameter is set as $F_1^S=9.27$. The current phase relations are calculated for different roughness parameter ρ of porous interfaces and temperature is fixed at $T/T_C=0.4$. We use the roughness parameter to adjust the coupling strength between two superfluids, and the π state is observed only when coupling strength is strong or ρ is small.

In the absence of magnetic field, the $\hat{n}$ vector is mainly determined by the surface effect[22] and is normal to the interface. Thus, $\hat{n}_{r,l}$ are either parallel or antiparallel to each other. In Fig. 1 the current phase relations are presented for the interface roughness $\rho=0.64$ and temperature $T/T_C=0.4$. Similar to the analytic calculation by Yip for single aperture,[21] the planar junction also yields the usual 0-state when $\hat{n}_{r,l}$ are parallel [(Fig. 1(a)], or π state if $\hat{n}_{r,l}$ are antiparallel [Fig. 1(b)]. The distinct current phase relationship is caused by the different order parameter profiles along the junction since order parameters have to connect continously from one side to another. As we will see later, the order parameter profile has a monotonic behavior in space in the parallel case while it has nonmonotonic behavior near the interface in the antiparallel case; the π state is closely related to such nonmonotonic character. Note that the critical current for the parallel configuration is more than twice as larger as the antiparallel configuration; recall that the critical current of H state is also more than twice that of L state.

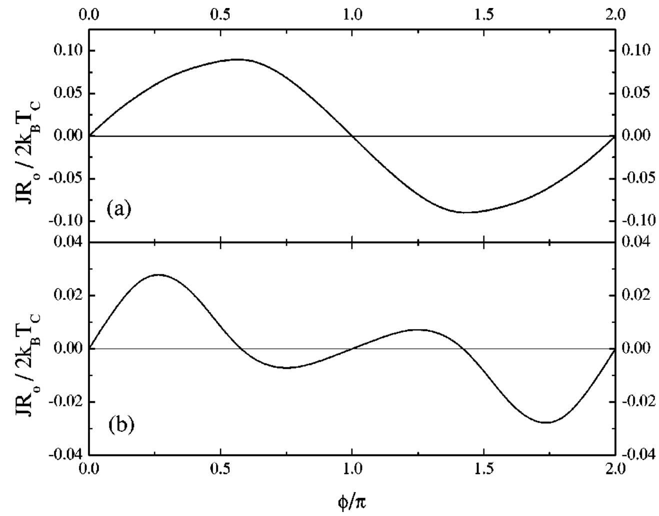

FIG. 1. The particle current phase relation for roughness parameter $\rho=0.64$ and temperature $T=0.4T_C$. (a) Parallel, (b) antiparallel configuration for $\hat{n}_{r,l}$, respectively.

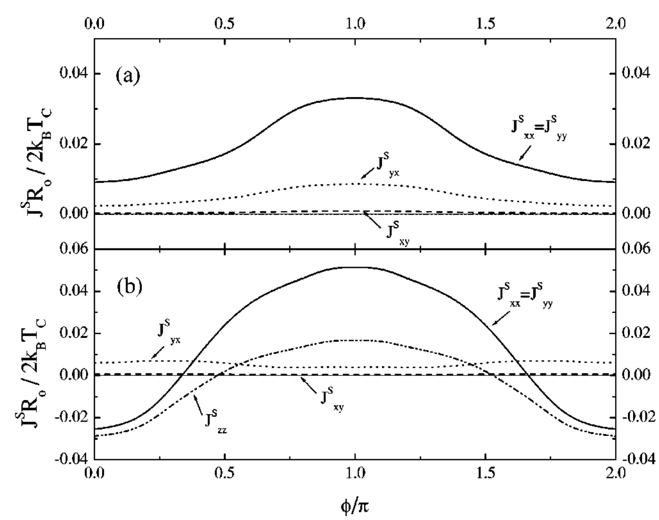

FIG. 2. The spin current phase relation for roughness parameter $\rho=0.64$ and temperature $T=0.4T_C$. (a) Parallel, (b) antiparallel configuration for $\hat{n}_{r,l}$, respectively.

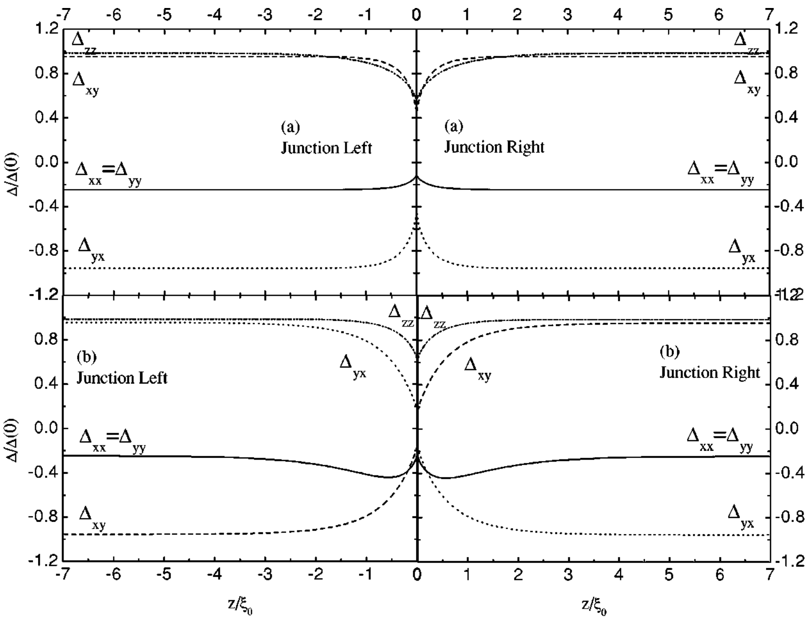

FIG. 3. The order parameter profiles for roughness parameter $\rho=0.64$ and temperature $T=0.4T_C$. (a) Parallel, (b) antiparallel configuration for $\hat{n}_{r,l}$, respectively.

Unlike s-wave superconductors, the superfluid ^{3}He has both spin and orbital degrees of freedom. The tunneling effect involves not only particle flow J, but also spin flows J^s_{ij} as well. We have also calculated spin current phase relations for the same interface roughness and temperature (see Fig. 2). For the parallel configuration of $\hat{n}$ vectors, five of the spin current components are zero due to symmetry requirement, and the rest ones are all positive with their maximum values at $\phi=\pi$. $J^s_{xx}=J^s_{yy}$, and J^s_{xy} is not the same as J^s_{yx} because the spin-orbit rotation minimizes dipolar energies. For the antiparallel configuration, in addition to the above four spin current components, J^s_{zz} also becomes nonzero. All diagonal spin current components change sign around the phases $\phi \approx \pi/2$ and $3\pi/2$, which are very close to the value $\cos^{-1}(1-15/8*\langle \sin^2\beta_{\hat{p}}\rangle)=\cos^{-1}(1/16)$ obtained by Yip.[21]

Since the particle and spin current phase relations are essentially determined by order parameter profiles in space, it is of interest to see how order parameter profiles differ in the parallel and antiparallel configurations for $\hat{n}$. In our program, the selfconsistency condition is imposed on the order parameter profile in space and better than 1% of accuracy is secured for convergent solutions. The accuracy for the particle and spin currents is even higher and better than 0.1% is easily achieved. As an example, the order parameter profiles without phase difference is illustrated in Fig. 3, in which the left and right panels correspond to the left and right superfluids. Although there are nine complex components for order parameters, they are chosen as real numbers in the absence of bulk flow. Furthermore, some of components are zero due to the symmetry of the geometry setup, and thus only nonzero components are plotted. One can see that for the parallel configuration the order parameters have an overall symmetrical and monotonic behaviors with respect to the interface and all components are depleted near the interface

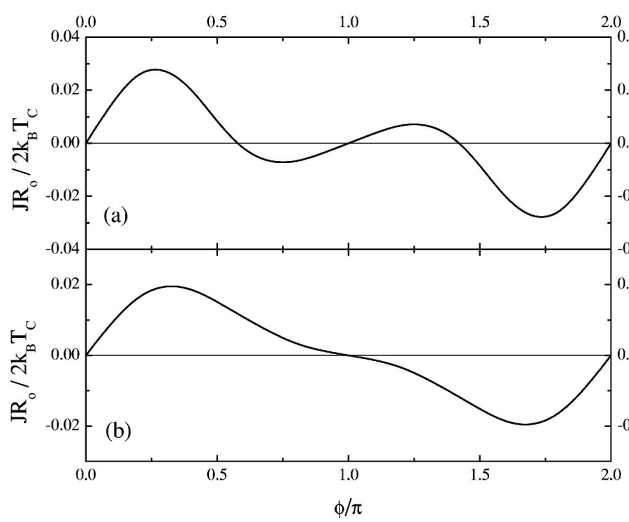

FIG. 4. The particle current phase relation for antiparallel configuration at temperature $T=0.4T_C$. (a) $\rho=0.64$, (b) $\rho=1.27$.

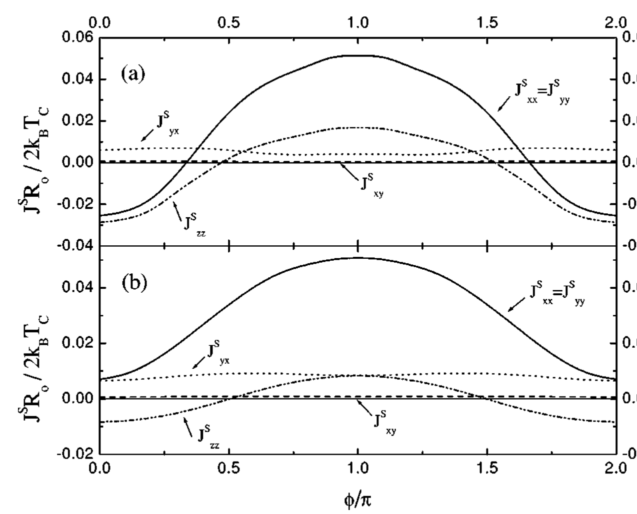

FIG. 5. The spin current phase relation for antiparallel configuration at temperature $T=0.4T_C$. (a) $\rho=0.64$, (b) $\rho=1.27$.

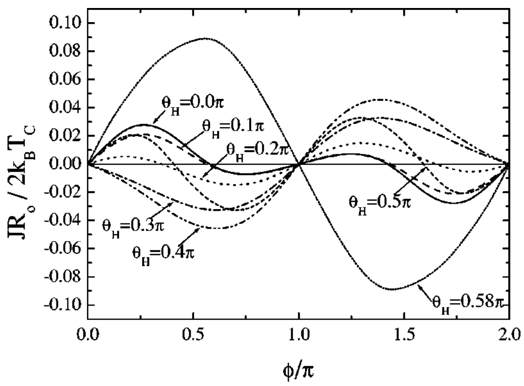

FIG. 6. The particle current phase relation for various angles of magnetic field in the AB case (Ref. 21), where the temperature is $T=0.4T_C$ and roughness parameter is $\rho=0.64$.

because of the interface rough scattering. For the antiparallel configuration of the $\hat{n}$ vectors, spin rotation along $\hat{n}$ has opposite sense and there are severe twists in the order parameter profile in space. The nonmonotonic behavior takes place near the interface, reflecting the enhanced gradient energy.[20]

To mimic different interface scattering properties, we plot the current phase relations for two sets of roughness parameter $\rho=0.64$ and 1.27 in Fig. 4. Since the roughness parameter does not bring any significant change to the shape of current phase relations for the parallel configuration except the reduction of critical currents, only the antiparallel case is plotted. We found that the interesting π-state exists only when the roughness parameter is small or the coupling between two superfluids is strong. In fact, when ρ approaches 1.27, the intervening cross points disappear and the usual 0-state is recovered. This suggests that π-state is very sensitive to interface scattering properties and only low roughness or strong coupling favors its formation. Since the roughness parameter simulates the coupling strength between two superfluids, we speculate that the π state may disappear when either the size of apertures or the density of apertures is reduced to a certain threshold value.

We show the comparative effect of the rough scattering on the spin current phase relation for the antiparallel configuration in Fig. 5. Corresponding to the fundamental change of the particle current phase relation from the π-state to the 0-state, the negative parts of J^s_{xx}, J^s_{yy}, and J^s_{zz} are greatly reduced, while their positive parts are less affected. J^s_{yx} is very much enhanced while J^s_{xy} is almost unchanged. This brings the overall spin current phase relation closer to that in the parallel case.

In the above discussions, we concentrated on the various aspects of Josephson effects in the absence of magnetic field. In the presence of magnetic field, the situation is more complicated since the magnetic field and interface effects competes with each other on the orientation of the $\hat{n}$ vector. However, as shown by Yip,[21] a simple analytic formulas for $\hat{n}$ can be obtained if magnetic field is relatively strong. Here we repeat the calculation of Fig. 5 in Ref. 21 for various directions of magnetic field, but for our planar porous junction. The current phase relations depicted in Fig. 6 show drastic difference between the orifice junction and planar junction either due to the difference in geometry or due to the selfconsistency in order parameters. In Yip's calculation,[21] the portion of current phase relations near $\phi=0$ and $\phi=2\pi$ expands and the π state disappears only when the direction of magnetic field approaches $\theta_H\approx\pi/2$; our self-consistent calculation for the planar junction suggests that the π state is stable when magnetic field is either nearly normal to or within the interface, but unstable in between. Thus, our study indicates that while the physical mechanism for the π state is quite clear from the works in Refs. 20,21 its dependence on magnetic field as well as on interface scattering properties may need the information of selfconsistently determined order parameters.

IV. CONCLUSION

In this paper, we have studied order parameters, the particle and spin current phase relations as functions of interface roughness, orientations $\hat{n}_{r,l}$, and the direction of magnetic field. Our results show that for planar porous junction, the π state exists only when the coupling between superfluids is strong, and it becomes the usual 0 state if the coupling strength diminishes. Furthermore, our selfconsistent calculation in the presence of magnetic field suggests that the π state is stable only when magnetic field is either nearly normal to or within the interface.

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