Resonant spin Hall conductance in quantum Hall systems lacking bulk and structural inversion symmetry

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A previous work [Shen, Ma, Xie, and Zhang, Phys. Rev. Lett. **92**, 256603 (2004)] on two-dimensional quantum wells with Rashba type spin-orbit interaction under a strong perpendicular magnetic field is generalized to include the Dresselhaus coupling. The Rashba coupling and the Dresselhaus coupling interplay with the Zeeman effect in opposing ways. The former tends to produce a resonant spin Hall effect at certain magnetic fields while the latter suppresses it. Due to the resonant spin Hall effect, the spin Hall current is highly nonohmic at low temperatures. The condition for the resonant spin Hall conductance in the presence of both Rashba and Dresselhaus couplings is derived using a perturbation method. In the presence of disorder, we argue that the resonant spin Hall conductance occurs when the two Zeeman split extended states near the Fermi level become degenerate due to the Rashba coupling, and that the quantized charge Hall conductance changes by $2e^2/h$ instead of e^2/h as the magnetic field changes through the resonant field.

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I. INTRODUCTION

Spintronics, which exploits electron spin rather than charge to develop a new generation of electronic devices, has emerged as an active field in condensed matters because of both the underlying fundamental physics and its potential impact on the information industry.[1–3] One key issue in spintronics is the generation and efficient control of spin current. Spin-orbit interaction of electrons exists extensively in metals and semiconductors and mix spin states. It provides an efficient way to control the coherent motion of electron spins. Recently, it has been proposed theoretically that an electric field may generate a spin current in hole-doped semiconductors and in two-dimensional electron gases (2DEG) in heterostructures with spin-orbit coupling due to the spin helicity and the noncollinearity of the velocity of the single particle wave function.[4–6] Studies of this intrinsic spin Hall effect have evolved into a subject of intense research.[7–13] The spin Hall effect in a paramagnetic metal with magnetic impurities has also been discussed, in which a transverse spin imbalance will be generated when a charge current circulates.[14–17] We also note that the spin chirality in systems with strong spin-orbit interaction may induce a pure spin current.[18]

Over the past two decades, remarkable phenomena have been observed in the 2DEG, most notably, the discovery of integer and fractional quantum Hall effect.[19–21] Research in spin transports provides a good opportunity to explore spin physics in the 2DEG with spin-orbit couplings. The spin-orbit coupling leads to a zero-field spin splitting, and it competes with the Zeeman spin splitting when a perpendicular magnetic field is applied. The result can be detected as beating in Shubnikov-de Haas oscillations.[22,23]

Very recently we have studied the spin Hall effect in the 2DEG with Rashba type spin-orbit coupling in a strong perpendicular magnetic field and predicted a resonant spin Hall effect caused by the Landau level crossing near the Fermi energy.[6] In this paper we present detailed calculations of the problem. The resonance shows up below a characteristic temperature of the order of the Zeeman energy E_Z. The peak of the resonance diverges as $1/\max(k_B T, eEl_b)$ (l_b is the magnetic length and E the electric field), and its weight diverges as $-\ln T$ at low T as $E \to 0$. Near the resonant magnetic field B_0, $G_s \propto 1/|B-B_0|$. The resonance arises from the Fermi level degeneracy of the Zeeman-split Landau levels in the presence of the Rashba coupling. More generally, the spin-orbit interaction present in the 2DEG may be of the Dresselhaus type rather than the Rashba type, or a combination of both. It is thus of interest to extend the analysis of Ref. 6 to beyond the pure Rashba coupling. To do so, it is useful to analyze certain symmetries in systems with the Rashba and/or Dresselhaus couplings. We will show that in contrast to the zero magnetic field case, where two physical systems differing only in a pure Rashba vs a pure Dresselhaus coupling exhibit identical essential physical behavior, this is not the case when a magentic field is present. The difference arises from the way the Rashba coupling vs the Dresselhaus coupling interplays with the Zeeman effect. In particular, the Rashba coupling opposes the Zeeman splitting and causes resonance while the Dresselhaus coupling enhances Zeeman splitting and thereby suppresses the resonance. By using linear response theory, we calculate the spin Hall conductance G_s, including its magnetic field and temperature dependences for realistic parameters of InGaGs/InGaAlGs. For systems possessing both Rashba and Dresselhaus couplings, the resonant condition is derived within a perturbation theory which is accurate for the small ratio of the Zeeman energy to the cyclotron frequency.

The paper is organized as follows. In Sec. II we introduce the Hamiltonian of the system under consideration and analyze its symmetries. In Sec. III we study the spin Hall current for systems with only Rashba or only Dresselhaus coupling. In Sec. IV we consider systems with both Rashba and Dresselhaus couplings. By treating the couplings as small parameters, we develop a perturbation method to derive the resonance condition. The paper is concluded with a summary and a brief discussion on the effects of disorder in Sec. V.

II. MODEL HAMILTONIAN AND SYMMETRY

A. Spin-orbit coupling and model Hamiltonian

For the introduction, we start with the three-dimensional (3D) spin-orbit interaction known for III-V compounds such as GaAs and InAs, which is of the form[24,25]

$$V_{so}^{3D} = \alpha_0 \mathbf{K}(\mathbf{p}) \cdot \sigma + \beta_0 \mathcal{E} \cdot (\mathbf{p} \times \sigma), \tag{1}$$

where σ_μ ($\mu = x,y,z$) are the Pauli matrices for spin of electrons, $\mathbf{p}$ is the momentum of the charge carrier, and

$$K_\mu(\mathbf{p}) = \sum_{\nu,\delta} p_\nu p_\mu p_\nu \epsilon_{\mu,\nu,\delta}. \tag{2}$$

In Eq. (1), the first term is the Dresselhaus coupling which originates from the lack of bulk inversion symmetry,[24] while the second term is the Rashba coupling which arises from the lack of structure inversion symmetry.[25] The effective field $\mathcal{E}$ is induced by the asymmetry of the external voltage to the system. In quantum wells, by neglecting the weak interband mixing and retaining the linear contribution of $\mathbf{p}$ parallel to the x-y plane, the spin-orbit interaction in 3D is reduced to an effective one in 2D,

$$V_{so}^{2D} = H_{so}^{D} + H_{so}^{R}, \tag{3a}$$

$$H_{so}^{D}(\alpha) = \frac{\alpha}{\hbar}(\sigma_x p_x - \sigma_y p_y), \tag{3b}$$

$$H_{so}^{R}(\beta) = \frac{\beta}{\hbar}(\sigma_y p_x - \sigma_x p_y), \tag{3c}$$

where $\alpha = -\alpha_0 \hbar \langle p_z^2 \rangle$ and $\beta = \beta_0 \hbar \langle \mathcal{E}_z \rangle$, with the average taken over the lowest energy band of the quasi-2D quantum well. The Rashba coupling can be modulated up to 50% by a gate voltage perpendicular to the plane.[22,26] In some quantum wells such as GaAs the two terms are usually of the same order of magnitude, while in narrow gap compounds like InAs the Rashba coupling dominates.[27–29] Experimentally the relative strength of the Rashba and Dresselhaus couplings can be extracted from photocurrent measurements.[30]

In this paper we consider a spin-$1/2$ particle of charge $-e$ and effective mass m confined by a semiconductor quantum well to a 2D x-y plane of length L_x and width L_y.[31] The particle is subjected to a spin-orbit interaction V_{so}^{2D}. A perpendicular magnetic field $\mathbf{B} = -B\hat{z} = \nabla \times \mathbf{A}$ and an electric field $\mathbf{E} = E\hat{y}$ along the y axis are applied (see Fig. 1 in Ref. 6). Both electron-electron interaction and impurities will be neglected in our study. The Hamiltonian reads

$$H = H_0 + eEy,$$

$$H_0 = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 - \frac{1}{2} g_s \mu_B B \sigma_z + V_{so}^{2D}(\mathbf{A}), \tag{4}$$

where g_s is the Lande g factor, and μ_B is the Bohr magneton. In $V_{so}^{2D}(\mathbf{A})$ the momentum $\mathbf{p}$ is replaced by the canonical momentum, $\Pi = \mathbf{p} + e/c\mathbf{A}$. We choose the Landau gauge $\mathbf{A} = yB\hat{x}$ and consider a periodic boundary condition in the x direction, hence $p_x = k$ is a good quantum number.

Below we rewrite the Hamiltonian in terms of lowering and raising operators. For each k, we introduce the lowering operator

$$a_k = \frac{1}{\sqrt{2} l_b}\left[y + \frac{c}{eB}(k + i p_y) \right]$$

and the corresponding raising operator $a_k^\dagger = (a_k)^\dagger$, with the magnetic length $l_b = \sqrt{\hbar c / eB}$. a and $a^\dagger$ satisfy the commutations $[a_k, a_{k'}^+] = \delta_{kk'}$, and $[a_k, a_{k'}] = 0$. In terms of a_k and a_k^+, we have

$$H_0/\hbar\omega = a_k^+ a_k + \frac{1}{2}(1 - g\sigma_z) + i\sqrt{2}\eta_R(a_k\sigma_- - a_k^+\sigma_+) + \sqrt{2}\eta_D(a_k^+\sigma_- + a_k\sigma_+), \tag{5}$$

where $\omega = eB/mc$ is the cyclotron frequency, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, and $g = g_s m / 2m_e$ is twice the ratio of the Zeeman energy to the cyclotron frequency (m_e is the free electron mass). $\eta_R = \beta m l_b / \hbar^2$ and $\eta_D = \alpha m l_b / \hbar^2$, both inversely proportional to $\sqrt{B}$ are the dimensionless Rashba and Dresselhaus coupling, respectively.

The velocity operator plays an important role in the study of transport properties including the spin Hall conductance. The velocity operator of a single particle is $v_\tau = [\tau, H]/i\hbar$ ($\tau = x, y$), from which we obtain

$$v_x = \frac{\hbar}{\sqrt{2} m l_b}[a_k^+ + a_k + \sqrt{2}\eta_D\sigma_x + \sqrt{2}\eta_R\sigma_y], \tag{6a}$$

$$v_y = \frac{i\hbar}{\sqrt{2} m l_b}[a_k^+ - a_k + i\sqrt{2}\eta_D\sigma_y + i\sqrt{2}\eta_R\sigma_x]. \tag{6b}$$

Comparing this with the standard expression of velocity for a charged particle in a magnetic field, $\mathbf{v} = (\mathbf{p} + e/c\mathbf{A})/m$, the spin-orbit coupling effectively induces a spin-dependent vector potential.

B. Symmetries

We analyze three symmetries of the Hamiltonian in this section, which we will use in our calculations.

Interchange symmetry of the two couplings. Under the unitary transformation, $\sigma_x \to \sigma_y, \sigma_y \to \sigma_x, \sigma_z \to -\sigma_z$, the Rashba and Dresselhaus couplings are interchanged,[7]

$$\alpha(\Pi_x\sigma_x - \Pi_y\sigma_y) \to \alpha(\Pi_x\sigma_y - \Pi_y\sigma_x); \tag{7a}$$

$$\beta(\Pi_x\sigma_y - \Pi_y\sigma_x) \to \beta(\Pi_x\sigma_x - \Pi_y\sigma_y); \tag{7b}$$

$$g_s \to -g_s. \tag{7c}$$

Therefore a system with Rashba coupling β, Dresselhaus coupling α, and Lande g-factor g_s is mapped onto a system with Rashba coupling β, Dresselhaus coupling α, and Lande g-factor $-g_s$. In particular, a system with only Dresselhaus coupling can be mapped onto a system with only Rashba coupling and an opposite sign in g_s. This symmetry will be used in Sec. III. At the symmetric point $\alpha=\beta$, V_{so}^{2D} is invariant under the transformation. $\alpha=-\beta$ is another symmetric point under the transformation, $\sigma_x \to -\sigma_y, \sigma_y \to -\sigma_x, \sigma_z \to -\sigma_z$. For physical parameters, we will always consider $g_s > 0$.

Signs of the couplings. Under the transformation, $\sigma_x \to -\sigma_x, \sigma_y \to -\sigma_y, \sigma_z \to \sigma_z$, we have $\alpha \to -\alpha$ and $\beta \to -\beta$. The eigenenergy spectrum is invariant under the simultaneous sign changes of the two couplings. The eigenenergy spectrum is even in η_R if $\eta_D=0$ and is even in η_D if $\eta_R=0$.

Charge conjugation. Under the charge conjugation transformation, $-e \to e$, the magnetic moment of the carrier also changes its sign, or effectively $g_s \to -g_s$ in Eq. (4). This transformation is equivalent to the flip of the external magnetic field $B \to -B$. Therefore a system of hole carriers has the same physical properties as the corresponding electron system except for possible directional changes in the observables.[31]

H_0 can be solved analytically in the systems with only Rashba or only Dresselhaus coupling. An analytical solution is currently not available for H_0 with both couplings.[32–34] In the next section, we shall discuss the charge and spin Hall conductance of the electron system with a pure Rashba coupling. The results can be mapped easily onto the system with a pure Dresselhaus coupling and to the hole system in semiconductors by using the symmetries discussed above.

III. SYSTEMS WITH PURE RASHBA OR PURE DRESSLHAUS COUPLING

In this section we focus on systems with either Rashba coupling or Dresselhaus only. We will present the calculation with respect to the Rashba case. The Dresselhaus case can then be addressed using the interchange symmetry discussed above. After a brief review of the single particle solution in the absence of an electric field, we will discuss the spin Hall conductance by using linear response theory in Sec. III B, and its nonlinear effect and scaling behavior near the resonance in Sec. III C. Some of the analysis here has been previously reported.[6] For readability purposes we reproduce the highlights here for the linear response section. For the nonlinear effect, we expand the discussion from the previous work to emphasize that the spin Hall effect is not an artifact of perturbation theory.

A. Single particle solution

The single particle problem of H_0 with $\eta_D=0$ can be solved.[25] The Rashba coupling hybridizes a spin down state in the n_0^{th} Landau level with a spin-up state in the $(n_0+1)^{th}$ Landau level, and the eigenenergies are given by

$$\epsilon_{ns}^R = \hbar\omega\left(n + \frac{s}{2}\sqrt{(1-g)^2 + 8n\eta_R^2}\right), \tag{8}$$

with $s=\pm 1$ for positive integer n, and $\epsilon_{0,+}=\hbar\omega(1-g)/2$. There is a large degeneracy $N_\phi = L_x L_y/(2\pi l_b^2)$ to each eigenenergy. The corresponding eigenstates are given by

$$|n,k,s\rangle = \begin{pmatrix} \cos\theta_{ns}\phi_{nk} \\ i\sin\theta_{ns}\phi_{n-1k} \end{pmatrix} \tag{9}$$

where ϕ_{nk} is the eigenstate of the n^{th} Landau level with $p_x = k$ in the absence of the spin-orbit coupling. $\theta_{0+}=0$, and $\tan\theta_{ns} = -u_n + s\sqrt{1+u_n^2}$ for $n \geqslant 1$, with $u_n = (1-g)/\sqrt{8n\eta_R^2}$.

The eigenenergies for the system with Dresselhaus coupling only can be obtained by replacing η_R by η_D and g by $-g$,

$$\epsilon_{ns}^D = \hbar\omega\left(n + \frac{s}{2}\sqrt{(1+g)^2 + 8n\eta_D^2}\right). \tag{10}$$

The energy spectra versus η_R or η_D are plotted in Fig. 1. In the absence of the spin-orbit coupling, the Zeeman energy splits the two degenerate n_0^{th} Landau levels of spin-up and spin-down electron states into two nearby ones with the lower level for spin-up and the higher level for spin-down. As η_R increases from zero, the energy of the n_0^{th} Landau level state of spin-down is lowered because of its hybridization with the spin-up state at the $(n_0+1)^{th}$ Landau level due to the Rashba coupling. The Rashba interaction competes with the Zeeman energy and there is an energy crossing at certain values of η_R or the magnetic fields as we can see in Fig. 1(a). The spin Hall resonance we examine is closely related to this level crossing. The energy level diagram in Fig. 1(b) for the Dresselhaus coupling has different features. In that case, a spin-up state, which is at the lower level due to the Zeeman splitting, mixes with a spin-down state at a higher Landau level, which separates further the Zeeman splitting, thus there is no resonance in the spin Hall current.

B. Linear response theory: Spin Hall conductance

We consider the charge and spin Hall currents along the x axis induced by an electric field along the y axis. In terms of the velocity operator, the charge and spin-z component current operators are defined by

$$j_c = -ev_x, \tag{11}$$

$$j_s = \frac{\hbar}{4}(\sigma_z v_x + v_x \sigma_z), \tag{12}$$

respectively. We refer readers to Ref. 6 for the discussions on the other spin components. The symmeterized form of the spin current operator guarantees that it is Hermitian. Each single particle state $|\phi_{nks}\rangle$ carries a current $\langle\phi_{nks}|j_{c,s}|\phi_{nks}\rangle$. The charge and spin Hall conductance are then given by

$$G_{c,s} = \frac{1}{L_x L_y}\sum_{nks} f_{nks}\langle\phi_{nks}|j_{c,s}|\phi_{nks}\rangle/E, \tag{13}$$

where f_{nks} is the Fermi-Dirac distribution function. Note that since spin is not a conserved quantity in the presence of

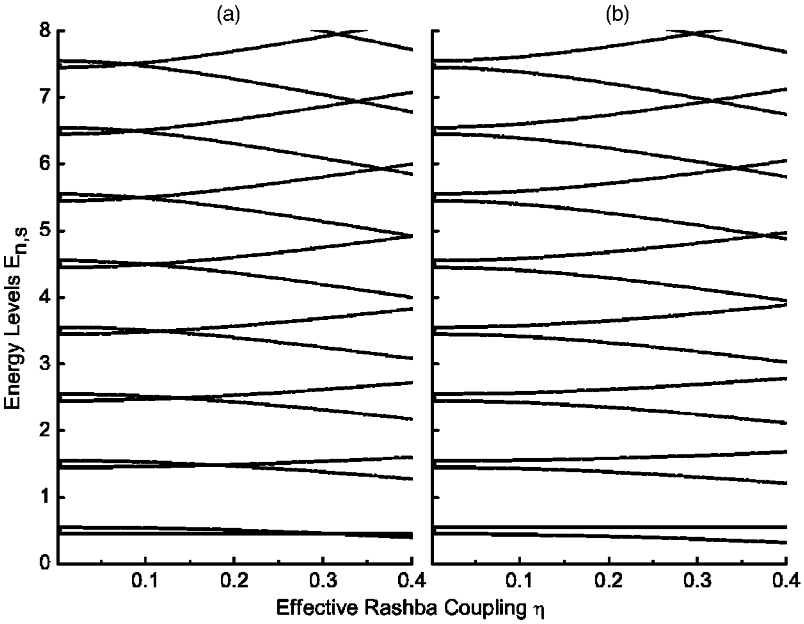

FIG. 1. (a) Energy levels in units of $\hbar\omega$ as a function of the dimensionless Rashba coupling η_R. The parameters are $\beta=0.9\times10^{-11}$ eV m, $n_e=1.9\times10^{16}/\mathrm{m}^2$, $m=0.05m_e$, and $g_s=4$, taken from Ref. 22 for the inversion heterostructures $\mathrm{In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As}$. (b) Same as in (a), but for the Dresselhaus coupling η_D (with the same strength of the Rashba coupling).

spin-orbit couplings, the spin current defined above and the spin density do not satisfy a continuity equation. Nevertheless, the expectation values of the spin density and the spin current are well-defined. Unlike a free electron in a uniform magnetic field, the single particle problem with the spin-orbit coupling in the presence of an electric field $E\hat{y}$ is not analytically solvable, since the Landau levels mixing no longer truncate. After a replacement of $y\rightarrow y+eE/m\omega^2$ in the operator a_k by $\tilde{a}_k=a_k+eEl_b/\sqrt{2}\hbar\omega$, the Hamiltonian of the system in the presence of the electric field reads, apart from a constant, $H=H_0(E)+H'$, where $H_0(E)$ is the one in Eq. (5) by replacing a_k with $\tilde{a}_k$ and $H'=-eEl_b\eta_R\sigma_y$. We now consider H' as a perturbative Hamiltonian to study the charge and spin Hall currents. Up to the first order in E, we obtain

$$G_{c,s}=G_{c,s}^{(0)}+G_{c,s}^{(1)} \tag{14}$$

where the superscript refers to the zeroth order and first order in the perturbation in H'. The charge Hall conductance is found to be independent of the spin-orbit coupling, $G_c=\nu e^2/h$, with $\nu=N_e/N_\phi$ being the filling factor. Within the perturbation theory, the spin Hall conductance G_s can be divided into two parts. The part arising from the zeroth order in H' is found to be the product of the spin polarization $\langle S^z\rangle$ per electron and the Hall conductance G_c, divided by the electron charge $(-e)$,

$$G_s^{(0)}=-\langle S^z\rangle G_c/e. \tag{15}$$

The expectation value of the spin polarization per electron is

$$\langle S^z\rangle=\frac{1}{N_e}\frac{\hbar}{2}\sum_{nks}\langle n,k,s|\sigma_z|n,k,s\rangle f_{nks}=\frac{1}{N_e}\frac{\hbar}{2}\sum_{nks}\cos 2\theta_{ns}f_{nks}. \tag{16}$$

$\langle S^z\rangle$ at $T=0$ is plotted in Fig. 2(a). The oscillation is due to the alternate filling by electrons of the energy levels with mainly spin-up and spin-down. A jump is visible at $\nu=12.6$ (corresponding to inverse magnetic field $0.162T^{-1}$) because of the energy crossing. Conversely, the lack of such an energy crossing in the Dresselhaus case (see later) implies no such jump, as can be seen in Fig. 2(c).

The second part in G_s arises from the first order in H',

$$G_s^{(1)}=\frac{e\eta_R}{8\pi\sqrt{2}}\sum_{n,s,n'=n+1,s'}\frac{f_{ns}-f_{n's'}}{\epsilon_{ns}^R-\epsilon_{n's'}^R}\times(\sqrt{n}\sin 2\theta_{ns}\sin^2\theta_{n's'} - \sqrt{n'}\cos^2\theta_{ns}\sin 2\theta_{n's'}). \tag{17}$$

At $T=0$, if the two degenerate energy levels are partially occupied, G_s^z may become divergent. Mathematically, the resonance is given by the condition $2n<\nu<2n+1$ for the electron filling factor ν, with n an integer satisfying the equation

$$\sqrt{(1-g)^2+8n\eta_R^2}+\sqrt{(1-g)^2+8(n+1)\eta_R^2}=2. \tag{18}$$

From the above condition, for a system with any $\eta_R\neq0$, $\eta_D=0$, and $g_s>0$, there is a unique resonant magnetic field B_0 such that the resonant condition is satisfied. By symmetry, we obtain the resonance condition for the system with a pure Dresselhaus coupling, which is given by the solution for n of the equation,

$$\sqrt{(1+g)^2+8n\eta_D^2}+\sqrt{(1+g)^2+8(n+1)\eta_D^2}=2. \tag{19}$$

Unlike the pure Rashba coupling case, there is no solution for any $g_s>0$ in the pure Dresselhaus coupling system. This is because the energy levels ϵ_{ns}^D and $\epsilon_{n's'}^D$ with $n'=n\pm1$ do not cross over, so the pairs of the crossing levels in the Dresselhaus coupling system correspond to $n'\neq n\pm1$ and do not contribute to the spin Hall conductance.

We have calculated the spin Hall conductance numerically. G_s^z at $T=0$ is shown in Fig. 2(b). In addition to the oscillation in $1/B$ similar to that of σ_z, there is a pronounced resonant peak at the filling $\nu=12.6$. No such resonant peak occurs for the Dresselhaus case, as is shown in Fig. 2(d). In Fig. 3, we show G_s^z at several temperatures for the Rashba

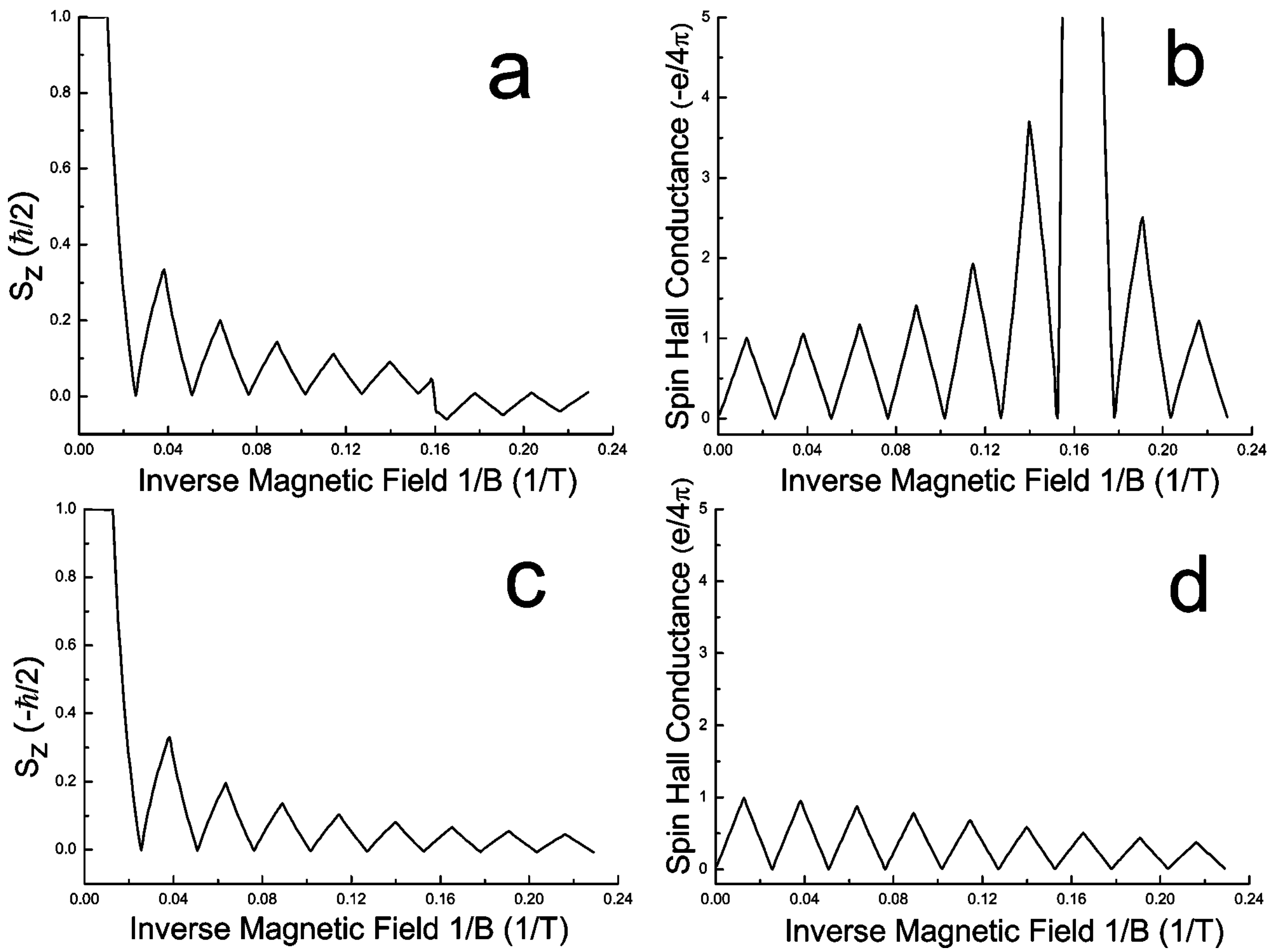

FIG. 2. Average spin S_z and spin Hall conductance as a function of $1/B$ at $T=0$. The parameters are the same as in Fig. 1.

case. The height of the resonance peak increases drastically as the temperature decreases below a few kelvin. In the inset of Fig. 3, we show the T-dependence of the height of the resonant peak and the two nearby side peaks. The characteristic temperature for the occurrence of the peak can be estimated to be the Zeeman energy E_Z, which is about 10 K at the resonant field for the parameters in the caption. More explicit derivation of this will be given in the next section.

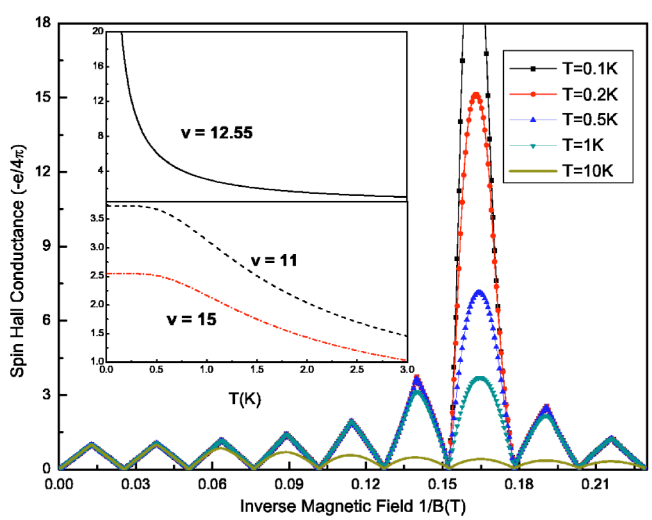

FIG. 3. Spin Hall conductance vs $1/B$ at several temperatures for Rashba coupling systems. The parameters are the same as those in Fig. 1. In the inset, temperature dependence of the height of the resonance peak and two side peaks is plotted.

C. Nonohmic spin Hall current and scaling behavior

In this section we study the nonlinear effect of the electric field to the resonant spin Hall current and the scaling behavior. Since the resonance originates from the interference of two degenerate levels near the Fermi energy, we will focus on those two levels to examine the problem. As an example, we shall consider $In_{0.53}Ga_{0.47}As/In_{0.52}Ga_{0.48}As$ with the parameters given in Fig. 1, in which case the resonance occurs at the filling factor $\nu=12.6$ [see Fig. 1(b)] and the relevant two levels are $|1\rangle=|n=6,k,s=+1\rangle$ and $|2\rangle=|n+1=7,k,s=-1\rangle$. The energy levels below the two levels are assumed to be fully filled, and all levels above the two to be empty. This is valid if $\hbar\omega \gg k_B T$. The Hamiltonian is then, up to a constant, reduced to a 2×2 matrix,

$$H_{\text{reduced}} = \begin{pmatrix} \Delta\epsilon & v_0 \\ v_0 & -\Delta\epsilon \end{pmatrix}, \qquad (20)$$

where $\Delta\epsilon=(\epsilon^R_{6,+1}-\epsilon^R_{7,-1})/2$, and

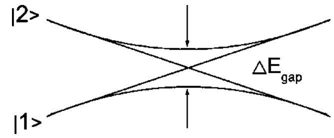

FIG. 4. Schematic illustration of the energy shift due to the electric field in the two degenerate levels near the resonant point.

$$v_0 = \langle 2|H'|1\rangle = -eEl_b\eta_R \cos\theta_{6,+1}\sin\theta_{7,-1}.$$

As we can see from the reduced Hamiltonian and from Fig. 4, the electric field breaks the level degeneracy and opens an energy gap $\Delta E_{gap}=2|v_0|$. Denoting the two eigenstates of the reduced Hamiltonian by $|\Phi_\pm\rangle$, the spin Hall current density is given by

$$I_s = \frac{1}{2\pi l_b^2}(i_- f_- + i_+ f_+), \tag{21}$$

where the Fermi-Dirac distribution $f_\pm=\{\exp([\pm\sqrt{(\Delta\epsilon)^2+v_0^2}-\mu]/k_BT)+1\}^{-1}$, with $f_+ + f_- = \delta_\nu = \nu - 2n$, μ the chemical potential, and $i_\pm = \langle\Phi_\pm|j_x^z|\Phi_\pm\rangle$. The electric field and temperature dependences of the spin current I_s near the resonance point is plotted in Fig. 5. At low temperatures the resonant spin current approaches to a constant in a weak electric field.

Now we analyze the scaling behavior of the spin conductance near the resonance point. For simplicity we limit our discussion to the case of $\delta_\nu < 1$ and $g \ll 1$. Near the resonant point, $\Delta\epsilon \approx -E_Z b$ where $E_Z = g\hbar\omega_0/2$ is the Zeeman energy and $b=(B-B_0)/B_0$ is the reduced dimensionless magnetic field. Using the identity

$$f_- - f_+ \equiv f_-(1-f_+)[1-e^{-2\sqrt{(\Delta\epsilon)^2+v_0^2}/k_BT}], \tag{22}$$

we obtain the singular part of the spin Hall conductance to be

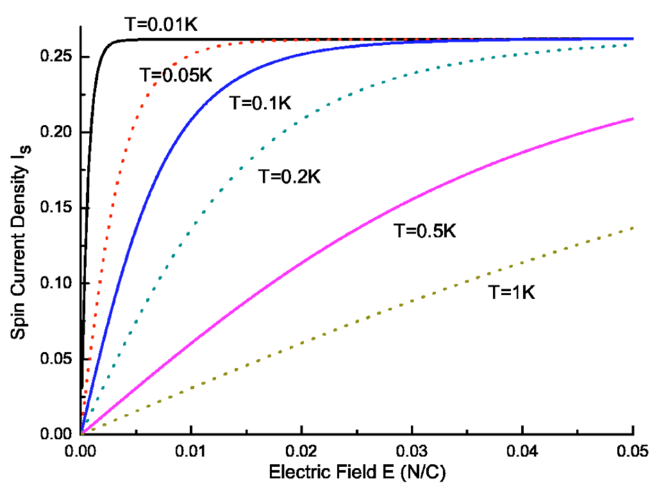

FIG. 5. Resonant spin current density as a function of the electric field at different temperatures. The spin current unit is $(-e/4\pi)(N/C)$. The filling factor at the resonance is $\nu=12.6$.

$$G_s \simeq -\frac{\delta_\nu e}{4\pi}\frac{E_Z}{\sqrt{(\Delta\epsilon)^2+v_0^2}}\frac{f_-(1-f_+)}{(f_-+f_+)} \times [1-e^{-2\sqrt{(\Delta\epsilon)^2+v_0^2}/k_BT}] \tag{23}$$

where the factor $f_-(1-f_+)/(f_-+f_+)$ is a slowly varying function of T ranging from 1 at low temperatures to $(1-\delta_\nu/2)/2$ at high temperatures. At low temperatures G_s is given by

$$G_s \simeq -\frac{e}{4\pi}\frac{\delta_\nu}{|b|}. \tag{24}$$

It is only a function of the reduced magnetic field and the excess part of the filling factor from $2n$. At the resonant magnetic field, i.e., $b=0$, the spin Hall current approaches with lowering temperature to a constant, $I_s = -e/4\pi\delta_\nu E_Z/(el_b\eta_R\cos\theta_{n,+1}\sin\theta_{n+1,-1})$ as can be seen in Fig. 5. Using the resonance condition in Eq. (18), $B_0 \approx 4nm^2c\beta^2/g\hbar^3$ $(n=6)$ and using the fact that for large n, n is proportional to $1/B_0$, the resonant magnetic field $B_0 \propto \beta/\sqrt{g}$ approximately. Thus the resonant spin current is proportional to

$$I_s = -\frac{\delta_\nu e^2}{8\pi m^2c^2}\frac{gB_0^2}{\beta} \propto \delta_\nu\beta. \tag{25}$$

Therefore for a given filling factor, the larger the spin-orbit coupling β is, the stronger the spin Hall resonance. The resulted spin Hall conductance diverges at $T=0$ as

$$G_s \simeq -\frac{\delta_\nu e}{4\pi}\frac{E_Z}{|v_0|} = -\frac{\delta_\nu e^2}{8\pi m^2c^2}\frac{gB_0^2}{\beta}\frac{1}{E}. \tag{26}$$

At temperatures $k_BT > \sqrt{(\Delta\epsilon)^2+v_0^2}$,

$$G_s \simeq -\frac{\delta_\nu(1-\delta_\nu/2)e}{4\pi}\frac{2E_Z}{k_BT} \tag{27}$$

and the integral

$$\int G_s db \to -\frac{\delta_\nu e}{2\pi}\left(\ln\frac{2E_Z}{k_BT}\right).$$

This integral reflects the weight of the resonant peak of the spin Hall conductance.

Since the method used in this section is beyond perturbation theory, we conclude that the resonance spin Hall conductance we predict is not an artifact of the perturbation method. Instead, the resonance is caused by the interference between the two degenerate energy levels at the Fermi energy.

IV. SYSTEMS WITH BOTH RASHBA AND DRESSELHAUS COUPLINGS

In this section we briefly discuss the resonance in the spin Hall conductance in systems with both Rashba and Dresselhaus couplings. The Hamiltonian including the electric potential reads

$$H = H_0(E) + H' \tag{28}$$

with $H' = -eEl_b(\eta_D\sigma_x + \eta_R\sigma_y)$. In this case H_0 is not solvable analytically. A state $|n_0,\downarrow\rangle$ (in the basis of the Landau levels with $\eta_D = \eta_R = 0$) is coupled to $|n_0+1,\uparrow\rangle$ via the Rashba coupling, which is further coupled to $|n_0+2,\downarrow\rangle$ due to the Dresselhaus coupling. In this way, a Landau level is coupled to an infinite number of other Landau levels, and the analytic solution is not available. The problem, however, may be approximately solved by using perturbation theory to treat η_R and η_D as small parameters. This is equivalent to the limit $B \to \infty$, since $\eta_{D,R} \propto 1/\sqrt{B}$. For parameter values given in Fig. 1, $\eta_R^2 = 0.004 \ll 1$ at the resonant field $B \approx 6.1$ T. In the absence of the electric field, the single particle energy, up to the second order in η_R and η_D, is given by

$$\frac{\epsilon_{n_0\uparrow}}{\hbar\omega} = n_0 + \frac{1-g}{2} + \frac{2n_0\eta_R^2}{1-g} - \frac{2(n_0+1)\eta_D^2}{1+g}, \tag{29a}$$

$$\frac{\epsilon_{n_0\downarrow}}{\hbar\omega} = n_0 + \frac{1+g}{2} + \frac{2n_0\eta_D^2}{1+g} - \frac{2(n_0+1)\eta_R^2}{1-g}. \tag{29b}$$

Note that the mixed term of $\eta_R\eta_D$ does not appear in the perturbation to the second order. The two levels become degenerate if the following equation is satisfied:

$$\frac{g}{2(2n_0+1)} = \frac{\eta_R^2}{1-g} - \frac{\eta_D^2}{1+g}. \tag{30}$$

It follows that a necessary condition for the resonant spin Hall current is $\eta_R^2/\eta_D^2 > (1-g)/(1+g) \approx 1$, for $g \ll 1$. At $\eta_D = 0$ and in the limit $\eta_R \ll 1$, Eq. (30) is consistent with Eq. (18) for the resonant condition we derived for the pure Rashba system. Alternatively the resonant magnetic field is

$$B_0 \approx \frac{2(2n_0+1)}{g}\frac{m^2c}{e\hbar^3}(\beta^2 - \alpha^2). \tag{31}$$

The large number n_0 increases with $1/B_0$ for a specific density of particles. Thus for a certain Rashba coupling the increasing of Dresselhaus coupling will decrease the resonant magnetic field B_0. The singular part of the spin Hall conductance can be studied by examining the two level system in the presence of an electric field as we described in Sec. III C. At the resonant point and at low temperature,

$$G_s = -\frac{\delta_\nu e^2\hbar^2}{8\pi m^2c^2}\frac{gB_0^2}{\sqrt{\alpha^2+\beta^2}}\frac{1}{E}. \tag{32}$$

As the Dresselhaus coupling increases from zero, the resonance is shifted to lower magnetic fields and occurs at higher Landau levels with a weaker resonant strength.

V. SUMMARY AND DISCUSSIONS

In summary, we have studied the spin Hall effect in a two-dimensional electron system with spin-orbit couplings in a strong perpendicular magnetic field. In systems with the Rashba coupling dominating over the Dresselhaus coupling, there is a resonant magnetic field at which the spin Hall conductance diverges at low temperature and low electric field. The physics for this resonance is the energy level crossing of the two Landau levels due to the competition of the Zeeman splitting and the Rashba coupling. For a given system, there is a unique resonant magnetic field, at which the two Landau levels become degenerate at the Fermi energy. In this case, some physical properties may show singularity. As studied earlier, the spin polarization will change its sign as the magnetic field is varied passing through the resonant field. Namely the magnetic susceptibility is divergent. The spin Hall conductance is another singular response due to this level crossing. When an infinitesimally weak dc electric field is applied in the plane, the two degenerate Landau levels are split accordingly and a finite spin Hall current is induced. The resonance is macroscopic in the sense that a huge number of the states in the same Landau level are involved in the process. We have calculated the temperature and electric field dependences of the resonance. The characteristic temperature for the resonant spin Hall current is of order of the Zeeman energy. As the temperature decreases, the height of the resonance peak diverges like $\propto 1/T$ and the weight diverges like $\propto \ln T$. While the spin orbit coupling has a dramatic effect on the spin Hall conductance, the charge Hall conductance is not affected and remains quantized. The spin Hall current is nonlinear with the electric field at the resonant field. At low temperatures, the spin Hall current rapidly rises linearly with the electric field and saturates at higher electric fields. At $T=0$, the spin Hall conductance diverges as $1/E$ at resonance. Near the resonant magnetic field B_0, it is $\propto 1/|B - B_0|$. Contrary to the Rashba coupling, the Dresselhaus coupling further increases the Zeeman energy splitting to suppress the effect of the Rashba coupling. The strength of the Rashba coupling necessary to surpass the Dresselhaus coupling, in order to have the resonant spin Hall current, was estimated by using a perturbation method treating the couplings as small parameters. This is accurate as long as the Zeeman energy is much smaller than the cyclotron frequency.

We have assumed no potential disorder in our theory. The effects of disorder in 2DEG with Rashba coupling, especially in a strong magnetic field, is not well understood at this point.[35] Nevertheless, it seems reasonable to assume that the spin-orbit coupling does not change the effects of disorder qualitatively. This is likely to be the case in the presence of a strong magnetic field, which ensures extended states in the Landau levels when the disorder is not sufficiently strong as evidenced by the experimentally observed quantization of the Hall conductance. We then assume that the disorder gives rise to broadening of the Landau level and localization so that the extended states in a Landau levels are separate in energy from those in the next one by localized states. Inspection of the spin-orbit coupling shows that Laughlin's gauge argument still holds,[36,37] and each Landau level with its extended states completely filled contribute e^2/h to the charge Hall conductance. Thus we conclude that the quantum Hall conductance remains intact with the spin-orbit interaction, except at the special degeneracy point. As the Fermi energy varies across this degenerate extended state, the charge Hall conductance G_c is expected to change by $2e^2/h$, instead of e^2/h for the other extended levels. This fact can be used experimentally to determine the Rashba interaction induced degeneracy discussed in this paper.

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