Xianming Zhou
University of Hong Kong

Peter L. Swan
University of New South Wales

Performance Thresholds in Managerial Incentive Contracts*

Performance thresholds are commonly used in executive compensation contracts. We examine the contractual nonlinearity associated with performance thresholds and its incentive implications. Incorporating a performance threshold into a standard principal-agent model of a linear contract, we show that pay schemes using a performance threshold are optimal. By truncating a linear scheme at poor performance, the threshold mitigates agency costs associated with the downside risk of production. Examining CEO compensation data, we find evidence of the role of performance thresholds. As a consequence of under-threshold performance, the tobit estimator is shown to increase pay-performance sensitivity, notably improving upon the standard OLS estimator.

I. Introduction

Many studies have examined the link between managerial pay and corporate performance. The evidence to date has yielded puzzling observations, the most notable of which is that executive compensation does not seem to have an economically meaningful association with firm performance. Examining the *Forbes* sample of U.S. firms during the years 1974–86, Jensen and Murphy (1990) find that CEO direct pay in the largest companies increases only by about \$.03 for each \$1,000 increase in shareholder value, and CEO total wealth, including indirect pay-related benefits (associated with stock ownership, options, and dismissal), increases by \$3.25. Such small pay-performance sensitivities are often viewed as inconsistent with standard agency theory. While different arguments have been put forth to explain the seemingly small

* We would like to thank Arthur Hosios, Yahoda Kotowitz, and Sudi Sudarsanam for useful suggestions. An anonymous referee made many insightful comments and suggestions that have significantly improved this article. We also benefited from the comments made by attendants in seminars at Hong Kong University of Science and Technology, University of New South Wales, and University of Sydney. An earlier version of this article was presented at the 2000 European Financial Association Meetings. Financial support from the Hong Kong Research Grants Council and the Australian Research Council is gratefully acknowledged.

(*Journal of Business*, 2003, vol. 76, no. 4)

0021-9398/2003/7604-0007\$10.00

magnitude of the sensitivity,[1] there has been no consensus on an explanation.[2]

Departing from this literature, recent studies explore issues beyond the standard broad categorization of compensation contracts that defines whether managerial pay is sensitive, in a linear fashion, to firm performance. From examining the complexity and variety of remuneration arrangements in managerial compensation contracts, Kole (1997) argues that the standard broad categorization ignores many important aspects of compensation contracts and thus tends to understate contractual incentives. Murphy (1999) details how different components of compensation, particularly incentive pay arrangements, are structured. Focusing on management bonus contracts, Murphy (2001) further examines the role of performance standards. This article contributes to this growing literature by examining a significant dimension of managerial compensation contracts: performance thresholds. The role of performance thresholds is highlighted by a common practice in executive compensation contracts: executives are promised a base salary and are entitled to receive performance rewards (an annual bonus or long-term incentive pay) when a prespecified minimum level of performance, or a performance threshold, is met. While this seems to be a prevailing compensation strategy, it has received little attention in previous studies.[3] No evidence has been reported on performance thresholds in compensation contracts and their effect on pay-performance relations.

This article explores this issue by examining a nonlinear contract characterized by the presence of a performance threshold. The contract specifies a linear incentive payment conditional on achievement of the threshold performance. While a simple linear contract is commonly assumed both in principal-agent models and in empirical studies because of its technical simplicity, linear pay schemes are rare in reality. The importance of nonlinear contracts has recently attracted the attention of researchers examining managerial incentives (Gibbons 1997). The search for an explanation for the puzzling evidence presented by Jensen and Murphy (1990) has raised questions concerning the validity of the linear functional form of contracts. Indeed, it is

1. Jensen and Murphy (1990) conjecture that, under disclosure of executive compensation, public and private political forces impose constraints on the type of contracts that are written between shareholders and managers and thus reduce the sensitivity. Joskow, Rose, and Shepard's (1993) and Hubbard and Palia's (1995) findings also support the political pressure argument. Garen (1994), Haubrich (1994), and Aggarwal and Samwick (1999*a*) explain the small sensitivity in terms of agency costs. Given the risk-averse behavior of managers, the compensation scheme must be structured to trade off incentives with insurance. Under this argument, the small pay-performance sensitivity is a response of the pay scheme to high risk in production at the largest U.S. firms, reflecting the shift of the pay scheme from incentives to insurance.

2. Rosen (1992) argues that the arithmetic sensitivity of CEO pay to shareholder value may understate the incentive strength both because of the model specification and because of the market-value-based measure of performance used in the empirical tests. Taking into account option holdings, Hall and Liebman (1998) argue that CEO wealth is strongly correlated to shareholder value, suggesting substantive executive incentives.

3. Lambert and Larcker (1991) and Murphy (1999) note the role of performance thresholds in executive compensation contracts.

theoretically unclear when a linear contract is optimal.[4] The nonlinear contract examined in this article is appealing: while it captures fundamental features of a performance threshold and incentive pay, it avoids complications arising from general nonlinear contracts. Examining such a contract sheds light on the complexity of managerial incentive contracts and provides additional insights into explanations for seemingly weak pay-performance relations.

With a standard principal-agent framework, we show that a finite performance threshold always exists, which is true even when the manager owns a small portion of the firm's common stock. We explain this result as the following. Given the manager's risk-averse behavior, the cost associated with the downside risk in production is "increasingly" high; it is more costly to compensate the manager for his or her reduced utility when firm performance is poorer. In effect, the manager is not held responsible for "bad draws" giving rise to exceptionally poor performance. Hence, penalizing the manager in the case of bad luck does not serve the purpose of providing incentives. Instead, it requires the firm to offer, ex ante, high compensation to attract the manager. While simple linear contracts cannot avoid this problem, a performance threshold truncates a linear scheme at poor performance and as a result directly limits the effect of the downside production risk. This offers an explanation for the common use of performance thresholds in executive compensation contracts. Our model yields other interesting results as well. It shows that, in the presence of a performance threshold, the slope of the pay function may differ substantially from that in a simple linear contract model. For instance, the incentive slope in our model does not necessarily decrease as production becomes more risky.

By examining CEO compensation for a large sample of U.S. firms over the period 1992-97, we find strong evidence in support of performance-threshold-based incentive schemes. The probability of CEOs receiving incentive pay is positively correlated with corporate performance, and this correlation mainly comes from a narrowed range of return performance. The evidence is more pronounced in annual incentive pay and with respect to the firm's accounting performance. We interpret these findings as evidence of performance thresholds in executive compensation contracts. The intensity of incentives in performance thresholds appears to be strong, particularly in the case of annual bonuses. For example, a CEO with a 6% annual return on the firm's total assets has a probability of 0.91 of receiving a bonus. In our sample, this significant component of pay has an average value of $545,000, which is 72% on the median and 104% on the mean of base salary. When the return on assets decreases to −6%, the probability drops to 0.42.

4. Holmstrom and Milgrom (1987) describe a model justifying a linear incentive contract. Mirrlees (1974) demonstrates that nonlinear contracts involving extreme penalties can be superior to the best linear contract so long as utility can be unbounded. Gibbons (1997) notes that "the optimal contract is linear only under very special assumptions about the utility function and the conditional distribution of output" (p. 4).

When contracts are nonlinear in the presence of a performance threshold, incentive pay data are censored at zero. Consequently, the conventionally used OLS estimator is biased. This is verified by our data. Comparing the OLS estimator for a simple linear contract with the tobit estimator that deals with censored data, we find that the former essentially underestimates the pay-performance sensitivity for all components of incentive pay. This underestimation is more serious for components of incentive pay with a larger number of zero observations. For instance, with the tobit estimator, the sensitivity of long-term incentive plan payouts to total shareholder returns is 63% larger than the corresponding OLS estimate. The difference between the two estimators becomes surprisingly large when the comparison is made based on the elasticity of incentive pay with respect to shareholder value. We find that the elasticity of long-term incentive plan payouts is 838% higher with the tobit estimator than with the OLS estimator. Similar findings are obtained on restricted stock awards. Our findings suggest that the standard OLS estimator understates pay-performance relations in two respects. First, it undermines the direct incentive effect of performance thresholds, which, as shown above, appears to be important. A broad, linear categorization of pay-performance relations does not reflect incentives associated with a switch from without to with incentive pay. Second, because of the model misspecification problem, it underestimates the sensitivity for incentive pay when performance surpasses the threshold.

The remainder of this article is organized as follows. In Section II, a principal-agent model is described that employs a nonlinear contract characterized by a performance threshold. In Section III, empirical results are discussed. Conclusions are provided in Section IV.

II. Model

Consider the standard agency problem—a risk-neutral principal, the firm, delegates decision-making or production tasks to a risk-averse individual, the agent or manager, while the principal does not observe the agent's effort in production. Assume a linear production technology,

$$Y = e + \varepsilon, \tag{1}$$

where e is the manager's effort in production and ε is a noise term that is normally distributed with a zero mean and a standard deviation σ. Consider a one-period, piecewise linear contract that takes the following form:

$$W(Y) = \begin{cases} w + \alpha + \beta Y & \text{if } Y \geq Y_0 \\ w & \text{if } Y < Y_0, \end{cases} \tag{2}$$

where Y_0 is a performance threshold. Under this contract, the manager is paid a fixed fee, w, and receives an incentive award, $\alpha + \beta Y$, when the threshold is met; α and βY are the fixed component and variable component, respectively, of incentive pay, where β is the piece rate or incentive slope. When the threshold, Y_0, approaches negative infinity, the contract reduces to the much-discussed simple linear scheme.

Performance-threshold-based contracts are commonly used in corporate executive compensation. The contract defined in (2) reflects two important contractual characteristics of executive compensation. First, executives are typically guaranteed some level of pay, that is, a base salary. While the existence of the base salary may be also due to operational reasons in compensation design (i.e., base salaries are used as a position-related benchmark for determining the level of incentive pay and total pay), base pay provides executives with a certain amount of insurance. Second, annual bonuses and long-term incentive plan payments are nonnegative and are awarded based on achieving a predetermined minimum level of performance. Hence, by using a performance threshold, executives are effectively immunized from the risk of poor performance. While it is true that managers may be fired in the event of poor performance, the expected costs of firing to managers are arguably low. This is because, for one, the link between managerial turnover probability and firm performance appears to be weak,[5] and, for the other, managers actually benefit from the existence of golden parachutes that may be arranged under certain circumstances.

Executive stock options present another example of performance thresholds in managerial incentive contracts. Stock options give executives the right to buy a share of the firm's stock at a prespecified ("exercise" or "strike") price for a prespecified term. As executive options are typically granted with an exercise price equal to the grant-date stock price,[6] the options' payoff is zero until the firm's stock price surpasses the current market value. Hence, the payoff function of executive stock options is a special case of the contract, (2), where $w = 0$ and β equals the portion of the firm's shares in options held by executives. The threshold performance in this case is the exercise price.

Piecewise linear contracts are analyzed in some early studies (Weitzman 1976; Holmstrom 1982; Gjesdal 1988). The contracts in those studies often have two linear functions with an assumed kink where the functions meet. This assumption is relaxed in the contract function (2). When $\alpha + \beta Y_0 > 0$, a jump in pay occurs at the threshold performance, which, as Murphy (1999) notes, is the usual case with executive bonus plans. This article focuses on the role of performance thresholds while abstracting from other complexities of incentive contracts. For instance, real-world managerial incentive pay, while

5. While the negative correlation between stock-price performance and subsequent CEO turnover is often interpreted as evidence of dismissal incentives, the correlation is generally weak. A recent finding by Murphy (1999) casts further doubt on the role of dismissal incentives. Examining secular changes in CEO turnover-performance relations, Murphy finds that, when normal retirement is controlled, the relations are weak and have declined over time. He argues that, based on CEO compensation data during the 1970s to 1990s, it is difficult to conclude that the threat of termination provides meaningful incentives.

6. Murphy (1999) documents that the exercise price equals the grant-date fair market value in about 95% of the regular executive option grants.

subsequent to the discontinuity due to performance thresholds, does not necessarily follow a linear schedule. The contract in this model has been assumed to be as simple as possible yet flexible enough to capture important qualitative features of both performance thresholds and continuous incentive pay.

Use $\phi(\varepsilon)$ to denote the probability density function of ε. The probability that the agent's output turns out below the threshold, Y_0, is

$$P(Y < Y_0) = \int_{-\infty}^{Y_0 - e} \phi(\varepsilon) d\varepsilon.$$

Let $V(W, e) = U(W) - C(e)$ be the agent's utility function, which is separable between pay and effort. Variable $C(e)$ is the cost of effort. Both $U(W)$ and $C(e)$ are twice differentiable, and where $U' > 0$, $U'' < 0$, $C' > 0$, and $C'' > 0$. Given (1) and (2), the manager's expected utility is

$$E(V(W, e)) = U(w) \int_{-\infty}^{Y_0 - e} \phi(\varepsilon) d\varepsilon + \int_{Y_0 - e}^{\infty} U(w + \alpha + \beta Y) \phi(\varepsilon) d\varepsilon - C(e).$$

The manager chooses effort to maximize expected utility, yielding the incentive compatibility constraint,

$$\beta \int_{Y_0 - e}^{\infty} U'(w + \alpha + \beta Y) \phi(\varepsilon) d\varepsilon + \phi(Y_0 - e)[U(w + \alpha + \beta Y_0) - U(w)] - C'(e) = 0. \tag{3}$$

The first term in (3) is the marginal utility of effort associated with the incentive parameter, β, which is the only source of incentives in models describing linear contracts. The second term presents the marginal utility of effort due to the presence of the performance threshold. This term is positive as long as incentive pay is positive (i.e., $\alpha + \beta Y_0 > 0$) and approaches zero when $Y_0 \to -\infty$.

The firm's objective is to choose the contract, characterized by $\{w, \alpha, \beta, Y_0\}$, to maximize the expected net profit, $E(\Pi(Y, W)) = E(Y - W)$. Then the firm's problem is

$$\max_{\{w, \alpha, \beta, Y_0\}} E(\Pi(Y, W)) = e - w - \int_{Y_0 - e}^{\infty} (\alpha + \beta Y) \phi(\varepsilon) d\varepsilon, \tag{4}$$

subject to (3) and the manager's participation constraint, $E(V(W, e)) \geq V_r$, where V_r is the reservation utility of the manager.

Here we have implicitly assumed that the distribution of Y is well behaved in the sense that the first-order approach applies.[7] Furthermore, it should be noted that the contract function is exogenously given. In other words, the model does not answer the question of whether a piecewise-linear contract as specified in the function, (2), is generally optimal. This modeling strategy is similar to that of many previous principal-agent models that are confined to a simple linear contract. While it highlights the main features of performance thresholds in incentive contracts, the strategy is useful in avoiding technical complexity.

PROPOSITION 1.

i) A finite performance threshold for incentive pay exists.

ii) Incentive pay is positive (or $\alpha + \beta Y_0 > 0$).

Proof of proposition 1. See appendix A.

The first part of the proposition presents the model's major result and predicts a performance-threshold-based reward scheme. We explain this result as an advantage of performance thresholds in reducing the cost of downside risk in production. Downside random disturbances are costly to the firm due to the manager's risk-averse behavior. On one hand, observed performance below a certain level becomes less relevant to working incentives because it is largely driven by random variations in production. On the other hand, it is more costly to compensate the manager when performance is poor, because utility is more heavily reduced. A threshold truncates the linear scheme at the lower end and directly limits the adverse effect of downside production variations.

The prediction of performance-threshold-based contracts has interesting implications. Because a simple linear contract is no longer optimal, the incentive parameter, β, in this model can be quite different from its value in standard principal-agent models. In the presence of a performance threshold, the expected sensitivity of pay to performance can change greatly with the level of effort, depending on the function of the contract and the distribution of the output. This implication is empirically appealing. Being confined to a simple linear contract, previous studies focus on the global linear sensitivity of pay to performance. The contractual nonlinearity in our model suggests that a global sensitivity may underestimate incentive strength because the sensitivity can be "locally" large.

This is further supported by the second part of the proposition. With incentive pay being positive ($\alpha + \beta Y_0 > 0$), the model predicts a jump in pay at the threshold. Because the incentive parameter at this point approaches positive infinity, the expected pay-performance sensitivity in the neighborhood

7. The first-order approach relaxes the constraint in the general model that the agent chooses a utility-maximizing action to require instead only that the agent choose an action at which his utility is at a stationary point. Being more mathematically tractable, the first-order approach has been the standard method for analyzing the principal-agent problem, though it is not generally correct. For the first-order approach to be valid, two distributional assumptions (sufficient conditions) need to be satisfied, which are known as the monotone likelihood ratio condition (MLRC) and the convexity of distribution function condition (CDFC). For discussions on these conditions, see Mirrlees (1979), Grossman and Hart (1983), and Rogerson (1985).

of the threshold performance can be substantially larger than the global sensitivity. The implication here is that to induce adequate incentives of the manager, the pay-performance sensitivity may only be relevant locally.

Given the nonlinear nature of the first-order derivatives of the model, it is difficult to obtain a closed form solution. The following two propositions characterize the optimal contract for two special cases.

PROPOSITION 2. As production risk becomes sufficiently small, the optimal contract will specify a high threshold together with negligible incentive pay. Specifically, $Y_0 \to e^*$, $\alpha = 0$, and $\beta \to 0$ as $\sigma \to 0$ (where e^* is the first-best level of managerial effort).

Proof of proposition 2. See appendix B.

PROPOSITON 3. As production becomes sufficiently risky, the role of both the performance threshold and the incentive parameter becomes negligible. Specifically, $Y_0 \to -\infty$ and $\beta \to 0$ as $\sigma \to \infty$.

Proof of proposition 3. See appendix C.

Proposition 2 illustrates how far our model can deviate from predictions of standard principal-agent models with a simple linear contract. In these models, the smaller the production risk, the more variable the pay scheme becomes.[8] Therefore, when risk approaches zero, the optimal contract in standard linear-contract models will involve a high incentive parameter and low or even negative fixed pay, essentially letting the agent solely bear production risk. Our model suggests that, when the performance measure is accurate enough to reveal managerial effort, a highly variable pay scheme is unnecessary once there is a performance threshold for awarding incentive pay. As the manager can be disciplined by checking whether or not desired performance is delivered, the cost associated with large pay variations under a performance-sensitive scheme can be reduced by using a performance threshold. Consequently, the likelihood of the firm's use of a highly variable pay scheme is smaller than standard principal-agent models would suggest.

The consequences of proposition 3 are not surprising. When production becomes very risky, the performance measure conveys little information about managerial effort and, hence, the effectiveness of either variable pay or a performance threshold essentially disappears. Proposition 3 together with proposition 2 gives an interesting implication: because β approaches zero when σ approaches either zero or infinity, the relationship between β and risk is nonmonotonic. In other words, the highest β is obtained at some intermediate level of production risk, and thus it is smaller than unity.

In the above discussion, the effect of managerial ownership is ignored. Previous studies have extensively examined the incentive intensity of executive compensation and concluded that stock ownership, including option holdings, accounts for the most important part of management incentives (Hall and Liebman 1998; Murphy 1999). While it remains unclear how effective

8. See, e.g., Gibbons and Murphy (1992), Rosen (1992), Garen (1994), and Aggarwal and Samwick (1999*b*).

ownership is relative to direct compensation, there is little doubt that equity holdings play an important role in managerial incentives. For this reason, we extend the model into the situation where the manager owns a portion of the firm's common stock. We find that, when the manager's equity ownership is small, the results of proposition 1 still hold.[9] This outcome is not surprising, because, as discussed earlier, the efficacy of a performance threshold resides in its ability to avoid the cost of the downside risk in production. While ownership is expected to work as a substitute for the incentive parameter, it should not undermine the role of a performance threshold in mitigating the effect of the downside risk.

III. Empirical Analysis

A. Methodology

We now examine evidence concerning performance thresholds in executive incentive contracts as predicted by proposition 1. Consider the agency relationship between the shareholders and the CEO of a publicly held firm. In the ideal situation, with perfect information on performance measures, we should be able to observe a clear-cut association of CEO incentive pay with a performance threshold: incentive pay is positive when the threshold is met and zero otherwise. However, the real-world situation is much more complex. Given a variety of firm characteristics, different firms are expected to set differing thresholds and to use differing performance measures in compensation contracts. In particular, there likely are multiple performance measures, not all of which are observable to the public. Furthermore, firm characteristics may change from time to time. Hence, given limited information on performance measures, the public would not observe a clear-cut association between incentive pay and the presence of a performance threshold, based on any single, observable measure of performance.

However, we can take the complexities and cross-firm variations in performance measures to be noise and view performance thresholds as a random variable, $Y_0 + \xi$. The noise term, ξ, reflects incomplete public information, which is normally distributed with a zero mean. To econometricians, given any observed performance, Y, the underlying threshold is either met or not met, subject to the realization of ξ. In other words, there exists a probability of awarding incentive pay conditional on observed performance. To derive testable hypotheses, we denote the empirically observed probability of CEOs receiving incentive pay conditional on performance as $H(Y \geq Y_0 + \xi)$. Then,

$$H(Y \geq Y_0 + \xi) = H(\xi \leq Y - Y_0) = \int_{-\infty}^{Y-Y_0} h(\xi)d\xi,$$

9. Executives of large U.S. firms typically own a tiny fraction of their company's stock. In our sample, the median ownership and median option holdings are both smaller than 1% of the firm's total shares outstanding. More detailed discussions on the extended model with managerial ownership are available from the authors upon request.

where $h(\xi)$ is the probability density function of ξ. This probability is different from the theoretically defined probability, $P(Y \geq Y_0)$, which equals either one or zero given Y. Figure 1 illustrates the difference between the two. The upper graph (a) depicts a representative incentive pay scheme with a performance threshold (e.g., an annual bonus plan), and the lower graph (b) illustrates the corresponding relationship between the probability of awarding incentive pay and performance. The dotted lines in the lower graph represent $P(Y \geq Y_0)$, and the solid-line curve represents $H(Y \geq Y_0 + \xi)$. The effect of company performance on the probability $H(Y \geq Y_0 + \xi)$ is obtained by differentiating H with respect to Y:

$$\frac{\partial H}{\partial Y} = \frac{\partial}{\partial Y}\int_{-\infty}^{Y-Y_0} h(\xi)d\xi = h(Y - Y_0).$$

Because $h(Y - Y_0)$ is positive and symmetric and reaches its maximum at $Y = Y_0$, we immediately have the following two hypotheses:

HYPOTHESIS 1. The CEO of a firm is more likely to receive an incentive payment in a good year than in a bad year.

HYPOTHESIS 2. The positive correlation between firm performance and the probability of the CEO receiving an incentive payment becomes stronger as the performance range containing the threshold becomes narrower.

The first hypothesis appears to be very intuitive. It is, however, inconsistent with a simple linear pay scheme. In a linear contract, pay increases with performance; when incentive pay is nonnegative, as is the case with executive compensation, there is no link between the frequency of awarding incentive pay and performance. The second hypothesis focuses on the intensity of the correlation between the frequency of incentive payments and performance (i.e., the slope of H) and further identifies the role of performance thresholds. We test the hypotheses using the following probit model:

$$(\text{Award of incentive pay})_t = a + b(\text{Firm performance})_t + \sum_i c_i X_{it}. \quad (5)$$

The dependent variable is dichotomous, having a value of one when a CEO is awarded incentive pay and a value of zero otherwise. The first term on the right-hand side, a, is a constant. The coefficient on firm performance, b, captures the effect of performance thresholds. Two return variables, market return to common stock and accounting return on total assets, are used in the test. We do not use scale or level performance measures such as the firm's market value or shareholder wealth, because these performance measures (and hence the corresponding performance thresholds) vary hugely across firms. We expect b to be positive (hypothesis 1) and locally large in a narrowed range of performance containing a threshold (hypothesis 2). The third term, $\sum c_i X_{it}$, denotes control variables, which is detailed below. The model estimates the latent variable of the probability of CEOs receiving incentive pay, $\Omega_t[a + b(\text{firm performance})_t + \sum c_i X_{it}]$, which is the standard normal distribution function.

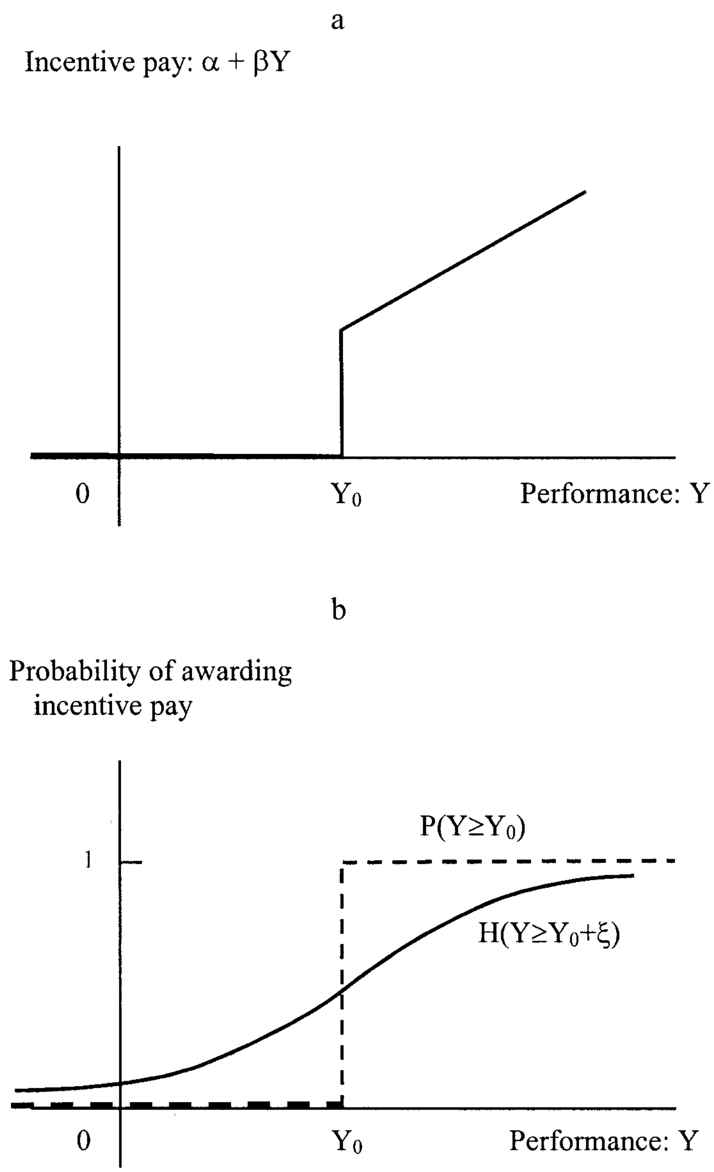

FIG. 1.—The relationship between awarding incentive pay and performance. *a,* A representative incentive pay scheme with a performance threshold; *b,* Corresponding relationship between the probability of awarding incentive pay and performance. Dotted lines in the lower graph represent $P(Y \geq Y_0)$, and the solid-line curve represents $H(Y \geq Y_0 + \xi)$.

Several variables of firm characteristics are controlled in the model. Firm size is a potentially important factor that affects the likelihood of rewarding executive incentive payments. Several previous studies find that firm size is strongly negatively correlated with the pay-performance relationship (e.g., Garen 1994; Schaefer 1998). The implication of this finding is that incentive payments are more likely to be made in large firms regardless of company performance. Note also that large firms tend to have a lower expected return due to, arguably, low risk or low stock illiquidity. Hence, based on return performance, thresholds in large firms are expected to be smaller than in small firms, all else being equal. This means that, for the same performance, there is likely to be a higher frequency of CEOs receiving incentive pay in large firms. We use the log value of total assets as the proxy for firm size. Noticing that total revenue or sales are also often used to measure firm size, we also examine the model using the log value of sales as the proxy, although we find little qualitative difference in the results.

Return risk is another factor affecting incentive contracts. Standard principal-agent theory posits an economic trade-off between inducing managerial effort and minimizing the cost borne by risk-averse managers.[10] Propositions 2 and 3 in the prior section show that this trade-off becomes more complex in the presence of a performance threshold. As in standard principal-agent models, the incentive contract in our model depends on output variations. It is then appropriate to take the firm's return variability as the measure of risk. We use monthly stock return data to estimate return variability. More specifically, the risk measure in a fiscal year is calculated as the standard deviation of monthly stock returns over the 60-month period preceding the fiscal year.

The essence of incentive contracts is to tie managerial pay with shareholder wealth such that the interests of managers and shareholders are aligned. To the extent that CEO involvement in the board of directors affects the monitoring role of the board, the firm's compensation committee's decision on incentive pay depends on whether or how CEOs influence their boards. To account for this potential effect, we use a dummy variable to control for CEOs who also chair their boards. Furthermore, other firm characteristics such as asset intangibility, capital structure, capital constraints, and dividend policy are also important factors that contribute to the firm's contractual environment. We use market-to-book ratios, debt ratios, cash flow to assets, and dividend yields, respectively, to control for each of these factors.[11]

10. Holmstrom (1979) explores this issue theoretically in standard principal-agent models. Garen (1994) and Aggarwal and Samwick (1999*a*) use executive compensation data to examine evidence of this trade-off.

11. Smith and Watts (1992) discuss the investment opportunity set and firm characteristics related to the market-to-book ratio, and John and John (1993) motivate the use of the debt ratio and Core and Guay (1999) the use of normalized cash flow and the payment of dividends as proxies for cash constraints. Bryant, Hwang, and Lilien (2000) provide comprehensive sensitivity estimates incorporating these controls.

B. Data

The CEO compensation data are taken from the *ExecuComp* database distributed by Standard and Poor's for the 6-year period 1992–97. Firm financial data are obtained from the Standard and Poor's *Compustat*, which also contains monthly data on stock returns. We confine our sample to firms that include at least 5 years of information on CEO pay and firm financial data. The final sample used in this study contains 1,198 U.S. companies.

There are various components of incentive pay. In a broad sense, these components can be grouped into short-term incentive pay and long-term incentive pay. Short-term incentive pay consists of annual bonuses, which are commonly used in executive compensation and, according to the firms' reports in their corporate proxy statements, are usually paid upon achievement of some prespecified threshold performance. Long-term incentive pay includes three main components: long-term incentive plan payouts (LTIP), restricted stock awards, and stock option grants. Like annual bonuses, LTIP is usually paid upon achieving a certain minimum level of performance. It is more difficult to determine empirically the performance thresholds for LTIP and their effects than for annual bonuses because LTIP is defined over a longer time frame. The role of performance thresholds in granting restricted stock and stock options appears to be less obvious. All these components of direct incentive pay will be examined in our test.

Stock ownership and prior (or previously granted, unexercised) stock options provide important indirect pay-related incentives. However, as performance thresholds do not directly apply to managerial wealth that is beyond the firm's control, we confine our test with model (5) to direct incentive pay, including current option grants. In this article, we focus on the role of performance thresholds in incentive pay rather than CEO total incentives. Hence, ignoring privately held equities should not pose a problem in our discussions.

Selected statistics are presented in table 1. To give a brief picture of the frequency and magnitude of different incentive payments, we report the four components of incentive pay separately for zero observations (no incentive pay) and nonzero observations (when incentive payments were made). Total pay is the sum of all components of pay, including base salary, incentive pay, and various fringe benefits or other payments. It is worth noting that base salary, other payments, and total pay are all reported as an unconditional variable including both zero and nonzero observations. Because the means (and also the medians) for nonzero incentive pay do not take into account zero observations, the total of the individual means is much larger than the mean of total pay. During the sample period, a majority of the CEOs received annual bonuses; in only about 19% of the observations, CEOs did not receive a bonus. Similarly, in the years under study, approximately two-thirds of the CEOs were granted stock options. The situation with LTIP and restricted stock,

TABLE 1 Selected Statistics

	Median	Mean	Standard Deviation	Observations
CEO pay variables ($1,000):				
Bonus:				
Zero				1,331
Nonzero	330	545	796	5,711
LTIP:				
Zero				5,910
Nonzero	359	716	1,200	1,132
Restricted stock:				
Zero				5,733
Nonzero	371	856	1,435	1,309
Stock options:				
Zero				2,373
Nonzero	559	1,448	4,513	4,669
Base salary	453	502	276	7,042
Other payments	24	127	1,130	7,042
Total pay	1,237	2,306	4,587	7,042
Firm variables:				
Total assets ($1,000,000)	1,021	6,829	22,524	7,106
Sales ($1,000,000)	904	3,404	9,057	7,108
Market return (%)	14.4	21.2	50.2	7,004
Return on assets (%)	4.4	3.9	10.8	7,104

NOTE.—The sample is taken from Standard and Poor's *ExecuComp* database for the years 1992–97. Companies with less than 5 years of data have been excluded. The sample contains 1,198 firms. The term LTIP denotes long-term incentive plan payouts. Stock options are the values of currently granted options estimated by the Black-Scholes formula. Other payments include various fringe benefits. Total pay is the sum of all components of pay, including currently granted stock options. For the four components of incentive pay, zero and nonzero observations are separately reported. All variables are in 1991 dollars.

however, seems to be the opposite, as these were awarded in less than 20% of the CEO-years. This difference reflects the fact that LTIP and restricted stock, though common in large companies, are not so widely used as annual bonuses and options in executive compensation.

C. Evidence of Performance Thresholds

We start by testing the first hypothesis. Table 2 presents basic probit regressions of the four components of incentive pay against the firm's market performance. The first two columns present the regressions for annual bonuses. In the first column, the coefficient on the performance measure, return to stock, is positive and statistically highly significant. When the firm-characteristic variables are controlled in the second column, the coefficient on stock returns is still positive and highly significant though the magnitude is slightly smaller and the t-ratio is reduced. Consistent with hypothesis 1, the coefficients show that CEOs are more likely to receive a bonus when the firm performs better.

The control variables in the second regression capture the effect of firm characteristics on the probability of CEOs receiving bonuses. The coefficient on the log value of total assets is positive and statistically significant; for the same performance, large firms are more likely than small firms to grant an

TABLE 2 **Probit Regressions of Incentive Pay on Stock Performance: The Basic Model**

Independent Variable	Dependent Variable							
	Bonus		LTIP		Restricted Stock		Options	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	.752	−.370	−1.006	−1.973	−.890	−1.520	.435	−.642
	(40)	(2.5)	(51)	(11)	(47)	(10)	(26)	(5.1)
Return to stock	.921	.762	.081	.189	.024	.050	−.002	−.027
	(18)	(13)	(2.3)	(3.9)	(.7)	(1.1)	(.1)	(.7)
Log of total assets		.168		.202		.141		.189
		(11)		(13)		(10)		(15)
Standard deviation of stock return		1.111		−8.705		−4.099		1.291
		(1.9)		(10)		(6.1)		(2.7)
Chairman of the board		.045		.159		.029		−.135
		(1.0)		(3.0)		(.6)		(3.4)
Market-to-book ratio		−.003		−.007		−.003		−.006
		(.8)		(1.2)		(.7)		(1.4)
Debt ratio		−1.507		−.152		.005		−.853
		(12)		(1.0)		(.0)		(7.6)
Cash flow to assets		1.270		1.052		−.617		−.553
		(6.1)		(2.8)		(2.7)		(3.0)
Dividend yield		2.323		1.507		1.325		−2.615
		(1.9)		(1.5)		(1.4)		(3.3)
R^2	.073	.121	4.3×10^{-4}	.135	4.1×10^{-5}	.052	9.1×10^{-7}	.047
Observations	6,910	5,712	6,910	5,712	6,910	5,712	6,910	5,712

NOTE.—The dependent variable equals one when a CEO receives a performance award and zero otherwise. The term LTIP denotes long-term incentive plan payouts. Options are current year grants. The standard deviation of stock return is calculated for the 60-month period preceding the current fiscal year using Standard and Poor's *Compustat* monthly data file. The chairman of the board is a dummy variable for CEOs who also chair the board. The *t*-statistics are in parentheses.

annual bonus to their CEOs. This finding is consistent with the notion that large firms usually have lower stock returns and, hence, based on return performance measures, are likely to set lower performance thresholds. More frequent bonus payments unrelated to performance in large firms are also consistent with the view that pay-performance relations are weaker in large firms. This firm size effect, also shown in table 3, is very robust to components of incentive pay and to performance measures. On the other hand, the coefficients on other control variables are not robust and are mostly mixed. In the following discussions, we will ignore these coefficients.

The results for LTIP, presented in columns 3 and 4 of table 2, are similar though weaker. While the coefficient on stock returns is still significantly positive, both its magnitude and *t*-ratio become much smaller. This is not surprising because, as a typical long-term incentive scheme, LTIP is awarded based on the firm's performance over a few or several years. The role of a threshold of long-term performance necessarily becomes less evident when it is examined based on a single year's performance. This problem is more obvious with restricted stock, in which awards are less likely to follow a clear pattern. Not surprisingly, the coefficient on stock returns, given in columns 5 and 6 of table 2, is not statistically different from zero. For currently granted stock options, shown in columns 7 and 8, the coefficients even become negative, though none is significant.

The corresponding regressions with respect to the firm's accounting performance, return on assets, are reported in table 3. While the results are largely similar to those in table 2, there are two notable differences. The coefficients show a stronger link between the probability of bonus grants and accounting performance. This is consistent with the evidence documented by Murphy (1999) on performance measures for executive bonus plans: while companies use a variety of financial and nonfinancial performance measures, most companies rely on some measure of accounting profits.

The second difference is that there is a strong negative correlation between option grants and the firm's accounting performance. This result confirms the impression that poorly performing firms are more likely to award executives stock options. This observation, though seemingly puzzling, may be explained by the nature of executive options in contrast to traditional incentive schemes. With traditional schemes such as annual bonuses and LTIP, one examines the ex post payoffs to CEOs and shareholders. The payoff of stock options to CEOs, however, is not realized until the options are either exercised or have expired, subject to restrictions on trading and exercising the options. As incentives derive mainly from the postgrants link between payoff and performance, the main objective of stock options is inducing incentives for future performance rather than playing a role in rewarding realized performance. Therefore, performance thresholds may not apply to whether or when stock options are granted.

To test hypothesis 2, we examine a spline specification that divides perform-

TABLE 3 Probit Regressions of Incentive Pay on Accounting Performance: The Basic Model

	Dependent Variable							
	Bonus		LTIP		Restricted Stock		Options	
Independent Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	.809	−.094	−1.022	−2.019	−.893	−1.560	.447	−.955
	(45)	(.6)	(50)	(11)	(48)	(10)	(27)	(7.2)
Return on assets	2.412	3.880	.714	−1.533	−.006	−.589	−.617	−4.927
	(15)	(5.4)	(3.4)	(1.7)	(.0)	(.8)	(3.9)	(7.3)
Log of total assets		.170		.209		.144		.205
		(11)		(14)		(10)		(16)
Standard deviation of stock return		.942		−8.470		−4.047		1.359
		(1.7)		(10)		(6.0)		(2.9)
Chairman of the board		.046		.155		.028		−.136
		(1.0)		(3.0)		(.6)		(3.4)
Market-to-book ratio		.003		−.005		−.003		−.006
		(.7)		(.8)		(.6)		(1.5)
Debt ratio		−1.544		−.251		−.013		−.935
		(13)		(1.6)		(.1)		(8.3)
Cash flow to assets		−2.065		2.277		−.047		4.066
		(2.9)		(2.8)		(.1)		(6.2)
Dividend yield		.319		1.218		1.254		−2.304
		(.4)		(1.2)		(1.3)		(2.9)
R^2	.038	.087	.010	.133	1.1×10^{-7}	.052	.002	.056
Observations	7,034	5,712	7,034	5,712	7,034	5,712	7,034	5,712

NOTE.—The dependent variable equals one when a CEO receives a performance award and zero otherwise. The term LTIP denotes long-term incentive plan payouts. Options are current-year grants. The standard deviation of stock return is calculated for the 60-month period preceding the current fiscal year using Standard and Poor's *Compustat* monthly data file. The chairman of the board is a dummy variable for CEOs who also chair the board. The *t*-statistics are in parentheses.

ance into three sections: low, intermediate, and high performance. Based on stock returns, the spline model derived from the basic model (5) is

$$\begin{aligned}(\text{Award of incentive pay})_t = a + b_1(\text{RTS1}_t) + b_2(\text{RTS2}_t)\\ + b_3(\text{RTS3}_t) + \sum_i c_i X_{it},\end{aligned} \tag{6}$$

where RTS1, RTS2, and RTS3 are the returns to stock for the low-, intermediate-, and high-performance sections, respectively. Specifically,

$$\text{RTS1} = \begin{cases}\text{return to stock} & \text{if return to stock} < R_L\\ R_L & \text{if return to stock} \geq R_{L,}\end{cases}$$

$$\text{RTS2} = \begin{cases}0 & \text{if return to stock} < R_L\\ \text{return to stock} - R_L & \text{if } R_L \leq \text{return to stock} < R_H\\ R_H - R_L & \text{if return to stock} \geq R_H,\end{cases}$$

$$\text{RTS3} = \begin{cases}0 & \text{if return to stock} < R_H\\ \text{return to stock} - R_H & \text{if return to stock} \geq R_H.\end{cases}$$

The dependent variable is also dichotomous, having a value of one when a CEO is awarded incentive pay and zero otherwise. Variables R_L and R_H are two critical levels (low and high) of stock returns, which determine the three sections of performance. The coefficients b_1, b_2, and b_3 estimate the sensitivity of the probability of incentive pay to performance for the three sections, respectively. When the span of intermediate performance, (R_L, R_H), contains a threshold, proposition 2 predicts b_2 to be larger than b_1 and b_3 and larger than the corresponding coefficient from the basic model reported in tables 2 and 3. The spline model with respect to accounting performance is similarly defined.

Variables R_L and R_H need to be specified such that performance thresholds are most likely to fall within (R_L, R_H). Without theoretical guidance for a benchmark, we determine the two values by comparing different pairs of R_L and R_H and choosing the one that best fits the data (i.e., with the highest R^2 in the regression). Table 4 presents the regressions for the spline model, with the upper portion of the table reporting the results for stock performance and the lower panel of the table reporting those for accounting performance. The regressions of annual bonuses are reported in the first column. In the stock performance portion, the coefficient on RTS2 or stock returns within (−25%, 11%) increases dramatically. It becomes two times larger than the corresponding coefficient in table 2. On the other hand, that coefficient for the low- and high-performance sections is statistically or economically insignificant. In the accounting performance portion, the result with respect to return on assets is even stronger: the coefficient on ROA2, accounting returns on total assets within (−4%, 5%), is more than three times larger than the corresponding number with the basic model in table 3, and it becomes statistically or economically insignificant for returns outside (−4%, 5%). These

TABLE 4 **Probit Regressions of Incentive Pay on Performance: The Spline Model**

	Dependent Variable			
Independent Variable	Bonus	LTIP	Restricted Stock	Options
Stock performance:				
Low performance (RTS1)	.565	−.299	.441	.090
	(1.5)	(.8)	(1.5)	(.4)
Intermediate performance (RTS2)	2.253	1.440	.980	.229
	(11)	(3.4)	(2.0)	(.8)
High performance (RTS3)	.189	.103	−.094	−.069
	(2.5)	(1.6)	(1.4)	(1.5)
R^2	.133	.137	.054	.047
Accounting performance:				
Low performance (ROA1)	1.920	−3.135	−.543	−4.717
	(2.6)	(3.6)	(.7)	(6.4)
Intermediate performance (ROA2)	16.470	10.278	2.836	−4.863
	(15)	(5.4)	(2.0)	(5.9)
High performance (ROA3)	−.110	−2.951	−2.326	−5.620
	(.1)	(2.6)	(2.3)	(6.1)
R^2	.133	.142	.053	.056

NOTE.—The dependent variable equals one when a CEO receives a performance award and zero otherwise. The term LTIP denotes long-term incentive plan payouts. Options are current-year grants. Return performance is divided into three sections corresponding to the three-pieces spline model. The terms RTS1, RTS2, and RTS3 denote, respectively, the stock return for the low, intermediate, and high performance sections, and ROA1, ROA2, and ROA3 denote, respectively, the low, intermediate, and high sections of return on assets. The two critical values of returns determining the performance division are chosen such that the regression obtains the highest R^2. The resulting intermediate performance sections of stock returns and return on assets for the four components of incentive pay are as follows: for return to stock, bonus (−25%, 11%), LTIP (−7%, 10%), restricted stock (−4%, 9%), options (−10%, 10%); for return on assets, bonus (−4%, 5%), LTIP (−3%, 4%), restricted stock (−2%, 5%), options (−10%, 10%). The coefficients on the control variables are not reported. The *t*-statistics are in parentheses.

observations are consistent with hypothesis 2. While the probability of CEOs receiving bonuses is strongly positively associated with performance within a narrowed range of performance that probably contains a threshold, it is uncorrelated or only weakly correlated with performance either below or above this range.

In column 2, the regressions for LTIP are similar, indicating a stronger positive correlation of the long-term incentive awards to return performance for RTS2 and ROA2. In column 3, however, the regressions for restricted stock lend only weak support for hypothesis 2. The regressions for option grants in column 4 again do not show any meaningful effect of performance thresholds. Instead, in the lower panel of column 4, the coefficients indicate a strong, uniformly negative association between option grants and the accounting return. This result is very robust to the choice of the values for R_L and R_H. In the table, we report the coefficients for (−10%, 10%) purely for convenience.

To show explicitly the intensity and nonlinearity of the relationship, table

TABLE 5 **Point Estimates of the Probability of the CEO's Receiving Incentive Payments**

Firm Performance	Probability: Bonus	LTIP	Restricted Stock	Options
Return to stock:				
−12%	.7449	.1033	.1559	.6594
−9%	.7661	.1017	.1591	.6609
−6%	.7863	.1032	.1623	.6635
−3%	.8055	.1112	.1670	.6660
0%	.8235	.1196	.1744	.6685
3%	.8405	.1284	.1821	.6709
6%	.8564	.1377	.1899	.6734
9%	.8711	.1475	.1980	.6759
Return on assets:				
−8%	.4026	.0689	.1580	.8519
−6%	.4175	.0609	.1553	.8283
−4%	.4325	.0537	.1528	.8024
−2%	.5634	.0620	.1502	.7742
0%	.6875	.0913	.1639	.7440
2%	.7934	.1298	.1783	.7117
4%	.8745	.1783	.1934	.6776
6%	.9051	.1634	.1948	.6420

NOTE.—The estimates are obtained from the spline regressions (table 4). The term LTIP denotes long-term incentive plan payouts. Options are current year grants. In the estimation, median values are used for the control variables.

5 presents illustrative point estimates of the probability of CEOs receiving different components of incentive pay, using the spline model regressions. The lower portion of the table shows the effect of the accounting returns. A CEO with an annual return on the firm's total assets of 6% is very likely (with a probability of about 0.91) to receive an annual bonus, a component of pay of $545,000 on average, which, in our sample, is 72% on the median and 104% on the mean of base salary. When the return on assets decreases to −6%, however, the probability drops to 0.42. These numbers suggest a strong link between CEO annual incentive pay and corporate accounting performance, which is not revealed by standard linear pay-performance sensitivities. A probability of 0.42 of receiving a bonus for an assets return of −6% seems high. This may possibly result from noise in the data and, in particular, from the complexity in performance measures such as multi-measures of performance and cross-firm variations in performance thresholds. On the other hand, the performance effect on LTIP and restricted stock is much weaker, and the probability of granting options is notably negatively associated with the accounting measure of performance.

The upper portion of table 5 presents the probability for the effect of the firm's stock returns. The probability of awarding a bonus is positively associated with stock returns, but the association is substantially weaker than the association with the accounting return. Except for this, there seems to be no meaningful effect of the firm's market performance on the probability of granting any component of long-term incentive pay. These results suggest that market performance plays a weak role in a firm's decision to grant incentive pay to its CEO. Perhaps because a stock price is subject to many factors that are beyond the manager's control, firms may be reluctant to set a threshold in terms of market performance.[12] The estimates also show that, while the role of performance thresholds is empirically strong in executive bonus plans, it is economically less significant in long-term incentive pay.

Previous studies focus on the broad linear relationship between executive pay and firm performance to argue that the pay-performance link is weak.[13] Our findings suggest that a simple linear sensitivity underestimates the incentive intensity by not taking into account the effect of performance thresholds. In the presence of a threshold, the pay-performance relationship in a real pay scheme can change substantially with the level of performance. While a broad sensitivity summarizes a general (or average) relation between pay and performance, it does not necessarily deliver an accurate message on incentives as it ignores the structural details of the compensation system that, while complicated, contain rich incentive implications. For instance, because variable pay for performance far below a certain expectation becomes costly to the firm and less relevant to working incentives, a contract should provide adequate incentives within a narrowed but relevant performance range. This is achieved by utilizing a performance threshold.

D. Downward Bias in Pay-Performance Sensitivities under OLS

Having established that performance thresholds represent a significant dimension of incentive contracts, a further question that arises is how they affect pay-performance relations that are empirically obtained with standard OLS estimators assuming a simple linear contract. A fact of executive compensation is that incentive pay data are censored at zero. In the presence of a performance threshold, CEOs do not receive incentive pay unless the threshold, whether or not observable to econometricians, is met. The implication of this feature of incentive-pay data is obvious: the pay-performance relationship estimated with OLS is biased. To verify this bias, we compare the sensitivity of CEO pay to firm performance between a standard OLS estimator and a tobit estimator, the latter being a standard approach to censored data.

We first examine the following specification for this comparison:

$$(\text{Incentive pay})_t = a + b(\text{Total shareholder return})_t + \sum_i c_i X_{it}. \qquad (7)$$

The dependent variable is CEO incentive pay in dollar values. The first term on the right-hand side is a constant. Total shareholder returns are defined as the product of the rate of return on common stock in a year and the firm's market value at the beginning of the year. The coefficient, b, estimates the

12. Lambert and Larcker (1987) and Sloan (1993) discuss the use of security price measures versus accounting measures in executive compensation.

13. See Rosen (1992) and Murphy (1999) for a survey of the literature.

arithmetic pay-performance sensitivity in the spirit of Jensen and Murphy (1990). Unlike in Jensen and Murphy, however, the dependent variable is not in first-difference form. This is because the tobit estimator needs to identify censored observations (zero incentive pay). To minimize the effect of heterogeneity in firm characteristics, the control variables examined above are also included in this model.

The model is run separately for different components of CEO incentive pay. Option incentives are relatively complex and are thus examined in two parts: one for currently granted options and the other for prior options (i.e., option holdings due to previously granted, unexercised options). As a component of CEO pay for current or realized performance, currently granted options are valued using the Black-Scholes formula and then, as with other components of incentive pay, are regressed against total shareholder return. Simple linear pay-performance sensitivities are obtained from OLS regressions; the corresponding sensitivities taking into account the censored data are obtained from tobit regressions.

Prior options provide indirect pay-related incentives through the link between the payoff of options to CEOs and the firm's market value. As in Aggarwal and Samwick (1999*a*), we estimate the simple linear sensitivity of prior options by using the *ExecuComp* data on existing options.[14] Regressing the value of CEO option holdings on the firm's market value yields the OLS estimate of the simple linear sensitivity. On the other hand, since most options are issued at the money, the option's exercise price presents a natural performance threshold. For performance above the threshold, CEO wealth in option holdings changes dollar for dollar with the firm's market value due to the nontradable feature of executive options. In other words, the incentive slope for performance surpassing the threshold is equivalent to that of stock ownership. Hence, we estimate the sensitivity by treating prior options as stock holdings and take it to be the sensitivity for performance above the threshold, which is the tobit estimate equivalent.

Table 6 summarizes incentive-pay sensitivities to total shareholder returns for the OLS estimator and the tobit estimator, together with the percentage changes from the OLS to the tobit estimates. For comparison purposes, the results are reported for regressions both with and without the control variables. All sensitivities are positive, statistically highly significant, and with a magnitude mostly consistent with previous studies. The OLS estimator is shown to underestimate the pay-performance sensitivity for all five components of incentive pay, though the extent of underestimation varies. With firm char-

14. *ExecuComp* reports the value of existing options only for those that are currently in the money. While this data drawback is unavoidably a source of potential bias in estimation, the bias may be limited. As Aggarwal and Samwick (1999*a*) argue, there are two offsetting effects of this data problem. On the one hand, the sensitivity of option values to stock prices may be overstated because of a jump in reported values surrounding the option's exercise price due to the reporting convention. On the other hand, the sensitivity tends to be downward biased for price movements in either direction for out-of-the-money options that remain out of the money.

TABLE 6 **Pay-Performance Sensitivities: The OLS Estimator versus the Tobit Estimator**

Components of Incentive Pay	Without Control Variables			With Control Variables		
	OLS (¢)	Tobit (¢)	Change (%)	OLS (¢)	Tobit (¢)	Change (%)
Bonus	7.8	8.6	10	3.1	3.3	6
	(27)	(25)		(10)	(9.4)	
LTIP	4.1	10.4	154	2.7	4.4	63
	(19)	(13)		(10)	(4.7)	
Restricted stock	4.3	9.5	121	2.6	4.2	62
	(15)	(9.8)		(7.8)	(3.6)	
Currently granted options	24.3	30.0	23	16.1	19.2	19
	(16)	(15)		(9.1)	(8.4)	
Prior options	74.1	123	66	72.6	107	47
	(30)	(33)		(24)	(23)	

NOTE.—The estimates are obtained from the arithmetic specification, eq. (7), which regresses CEO incentive pay against total shareholder return. The term LTIP denotes long-term incentive plan payouts. All components of incentive pay, including current and prior options, are in dollar values. The pay-performance sensitivities are the change in CEO incentive pay for every $1,000 change in shareholder return. The *t*-statistics are in parentheses.

acteristics controlled, the OLS estimator understates the sensitivity of annual bonuses by just 6%. This is not surprising because, on average, only about 19% of CEOs did not receive a bonus each year, and so the effect of ignoring performance thresholds in estimating the sensitivity is limited. This does not mean, however, that the role of performance thresholds in bonus schemes can be ignored. As discussed earlier, there is a strong link between the probability of CEOs receiving bonuses and performance. The incentive intensity of such a switching scheme is not reflected in simple linear sensitivities.

For LTIP and restricted stock, the difference in the sensitivity between the two estimators is large. Note that these two components of incentive pay were made in only about 15% of the CEO years, or about 85% of the observations were censored at zero. The difference between the two estimators is thus expected to be substantial. The downward bias with prior options is also notable. Because executive options are not tradable, the payoff schedule of options highlighted by the exercise price as the performance threshold presents a more complicated incentive scheme than a simple linear sensitivity implies.

To further compare the two estimators, we also examine the following elasticity-form specification:

$$\ln(\text{Incentive pay})_t = a + b[\ln(1 + \text{Stock return})]_t + \sum_i c_i X_{it}. \qquad (8)$$

The performance variable, $\ln(1 + \text{stock return})$, approximates changes in $\ln(\text{shareholder value})$ when the firm's total shares outstanding change slowly from year to year. Hence, the coefficient, b, is an approximate estimate of the pay-performance elasticity. Whenever a CEO does not receive incentive pay (i.e., when incentive pay is zero), $1 is assumed for the dependent variable to preserve the censored observations. The two specifications, (7) and (8), are both commonly used in examination of pay-performance relations. While the arithmetic specification is appealing for an intuitive interpretation of the

TABLE 7 Pay-Performance Elasticities: The OLS Estimator versus the Tobit Estimator

Components of Incentive Pay	Without Control Variables			With Control Variables		
	OLS	Tobit	Change (%)	OLS	Tobit	Change (%)
Bonus	1.927	2.398	24	1.616	2.009	24
	(25)	(25)		(18)	(18)	
LTIP	.476	3.066	544	.276	2.590	838
	(6.3)	(6.1)		(3.2)	(4.2)	
Restricted stock	.363	1.867	414	.210	1.169	457
	(4.5)	(4.1)		(2.2)	(2.2)	
Currently granted options	.432	.559	29	.024	.011	−54
	(4.0)	(3.4)		(.2)	(.1)	
Prior options	.893	.608	−32	1.187	.833	−30
	(37)	(32)		(18)	(15)	

NOTE.— The estimates are obtained from the elasticity specification, eq. (8), which regresses the log of CEO incentive pay against the log of (1 + stock return). The term LTIP denotes long-term incentive plan payouts. All components of incentive pay including current and prior options are in dollar values. The pay-performance elasticities are the approximate percentage change in CEO incentive pay for every 1% change in shareholder value. The *t*-statistics are in parentheses.

incentive parameter, the elasticity specification has an advantage in that it fits compensation data better and the elasticity measure is relatively invariant to firm size.

Table 7 shows the comparison with the elasticity of incentive pay to shareholder value. Except for options, the elasticities indicate a much more severe underestimation with the OLS estimator. In tobit estimation and when firm characteristics are controlled, the elasticity increases by 457% for restricted stock and jumps 838% for LTIP. The elasticity of prior options, however, indicates an upward bias in the OLS regression, which contrasts with the results for other components of incentive pay and appears to be inconsistent with the comparison for the arithmetic sensitivities of prior options (table 6). One possible explanation is that the elasticity of option holdings with respect to shareholder value is not an adequate measure of incentive strength and so the difference in the elasticities does not reflect the difference in incentives between the two estimates. To understand this point, consider two CEOs of comparable firms, assuming that one CEO has an ownership of 10% of the firm's stock and the other has an ownership of 0.1%. If the firm's total shares and CEO ownership do not change or change slowly from year to year, as is the usual case in the real world, one would obtain an elasticity close to one for both CEOs. This problem applies similarly to option holdings.

IV. Conclusion

In this article, we examine performance thresholds as a significant dimension of incentive contracts. We show that a performance threshold is efficient in the sense that it mitigates agency costs associated with the downside risk in production. After examining CEO compensation data, we find empirical support for the role of performance thresholds in incentive pay. Contributing to the literature exploring the complexity and variety of executive compensation contracts, we show that standard linear pay-performance sensitivities, by ignoring the nonlinearity associated with performance thresholds, understate the underlying incentive strength. This threshold-related underestimation is twofold. First, a simple linear sensitivity does not reflect incentives associated with a performance-based switch from zero incentive pay to (often substantial) incentive pay and, second, with the standard OLS estimator, the sensitivity is downward biased due to the fact that incentive pay data are censored at zero.

Examining the role of performance thresholds undoubtedly enriches our understanding of the structural complexity of incentive contracts. In this article, we do not address the much debated issue of whether executive pay-performance relations are sufficiently strong to align the interests of shareholders and managers. Indeed, we do not examine another important component of managerial wealth: stock ownership. Because ownership is not directly controlled by the board's compensation committee and executive wealth in ownership is not subject to a share price threshold, issues concerning performance thresholds do not apply to ownership incentives. Furthermore, in this article, we ignore a related phenomenon in executive compensation: firms may also set a cap on incentive pay together with a performance threshold, which is typical in executive bonus plans (Lambert and Larcker 1991; Murphy 1999, 2001). It is theoretically straightforward to analyze the role of a pay cap in a contract model similar to that for a performance threshold. However, it is difficult to identify empirically the effect of a pay cap, because there is no publicly available information on if an incentive payment hits a pay cap that is expected to alter both across firms and over time.

Appendix A

Proof of Proposition 1

(i) Y_0 is finite.

Obviously, Y_0 would not be infinitely positive because otherwise the pay scheme reduces to a fixed fee. We need to show only that Y_0 is larger than some finite value. The Lagrange of the optimization problem, (4), is

$$\begin{aligned} L = {} & e - w - \int_{Y_0-e}^{\infty} (\alpha + \beta Y)\phi(\varepsilon)d\varepsilon \\ & + \lambda\left[U(w)\int_{-\infty}^{Y_0-e}\phi(\varepsilon)d\varepsilon + \int_{Y_0-e}^{\infty} U(w+\alpha+\beta Y)\phi(\varepsilon)d\varepsilon - C(e) - V_r\right] \\ & + \mu\left[(U(w+\alpha+\beta Y_0) - U(w))\phi(Y_0 - e) + \beta\int_{Y_0-e}^{\infty} U'(w+\alpha+\beta Y)\phi(\varepsilon)d\varepsilon - C'(e)\right]. \end{aligned} \tag{A1}$$

The first-order derivative with respect to Y_0 is

$$\frac{\partial L}{\partial Y_0} = (\alpha + \beta Y_0)\phi(Y_0 - e) - \lambda[U(w + \alpha + \beta Y_0) - U(w)]\phi(Y_0 - e)$$

$$+\mu[U(w + \alpha + \beta Y_0) - U(w)]\phi'(Y_0 - e).$$

With $\phi'(Y_0 - e) = [(e - Y_0)/\sigma^2]\phi(Y_0 - e)$ for a normal distribution, this condition is rewritten as

$$\frac{\partial L}{\partial Y_0} = \left[1 - \left(\lambda - \frac{e - Y_0}{\sigma^2}\mu\right)\left(\frac{U(w + \alpha + \beta Y_0) - U(w)}{\alpha + \beta Y_0}\right)\right](\alpha + \beta Y_0)\phi(Y_0 - e). \quad \text{(A2)}$$

Optimality requires $\partial L/\partial Y_0 = 0$, which means that either the expression in the square bracket equals zero or that $\alpha + \beta Y_0 = 0$. In the former case, because $[U(w + \alpha + \beta Y_0) - U(w)]/(\alpha + \beta Y_0) > 0$, then

$$\lambda - \frac{e - Y_0}{\sigma^2}\mu > 0 \qquad \text{or} \qquad Y_0 > e - \frac{\lambda\sigma^2}{\mu}. \quad \text{(A3)}$$

As λ, μ, and σ are finite, there exists a finite lower bound for Y_0. In the latter case, $\alpha + \beta Y_0 = 0$. Use (A2) to obtain the second-order derivative with respect to Y_0,

$$\frac{\partial^2 L}{\partial Y_0^2} = \beta\phi(Y_0 - e)\left[1 - \left(\lambda - \frac{e - Y_0}{\sigma^2}\mu\right)U'(w + \alpha + \beta Y_0)\right]$$

$$- \frac{\mu}{\sigma^2}\phi(Y_0 - e)[U(w + \alpha + \beta Y_0) - U(w)] + \left(\frac{e - Y_0}{\sigma^2}\right)\frac{\partial L}{\partial Y_0}.$$

Optimality requires that $\partial^2 L/\partial Y_0^2 \leq 0$. Because the second and third terms are both zero, then $1 - [\lambda - (e - Y_0)\mu/\sigma^2]U'(w) \leq 0$ or

$$Y_0 \geq \frac{\sigma^2}{\mu}\left(\frac{1}{U'(w)} - \lambda + \frac{e\mu}{\sigma^2}\right), \quad \text{(A4)}$$

where the right-hand side is finite. Equations (A3) and (A4) jointly prove that Y_0 is finite.

(ii) $\alpha + \beta Y_0 > 0$.

We start with the solution under the constraint $\beta = 0$. In this solution, $\alpha > 0$ or $\alpha + \beta Y_0 > 0$ (the contract becomes a fixed fee otherwise). The optimality requires that (i) $\partial L/\partial Y_0 = 0$ and that (ii) $\partial^2 L/\partial Y_0^2 < 0$ in the neighborhood of the optimum. This means that the expression in the square bracket in (A2) is zero at the optimum and is negatively sloped in the neighborhood of the optimum.

We then relax β such that $\beta \geq 0$ and $\beta < \delta_1$ where δ_1 is arbitrarily small. Since β can only be trivially positive, there is little change in the solution, and then $\alpha + \beta Y_0 > 0$ still holds. We further relax β such that $\beta \geq 0$ and $\beta < \delta_1 + \delta_2$ where δ_2 is arbitrarily small. With a similar argument, we again have $\alpha + \beta Y_0 > 0$. Using this strategy repeatedly, we argue that $\alpha + \beta Y_0 > 0$ holds for any $\beta \geq 0$. The reasoning is the following. For any $\alpha + \beta Y_0 \leq 0$ to be optimal, there must exist some value of β such that $\alpha + \beta Y_0 = 0$ is optimal. For $\alpha + \beta Y_0 = 0$ to be optimal, the expression in the square bracket in (A2) must be negative in the neighborhood of $\alpha + \beta Y_0 = 0$. But this cannot happen, because, as we relax β from any solution with $\alpha + \beta Y_0 = \varepsilon > 0$

(no matter how small ε is), the square bracket must be zero and it must be negatively sloped in the neighborhood of the optimum. Therefore, at the optimum, $\alpha + \beta Y_0 \neq 0$. Assuming continuity for the solution, the situation with $\alpha + \beta Y_0 \leq 0$ at optimum would not occur. Q.E.D.

Appendix B

Proof of Proposition 2

(i) $Y_0 \to e$ and $\alpha = 0$ as $\sigma \to 0$.

(1) The first-order derivatives with respect to α and β are

$$\frac{\partial L}{\partial \alpha} = -\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon + \lambda \int_{Y_0-e}^{\infty} U'(w + \alpha + \beta Y)\phi(\varepsilon)d\varepsilon$$

$$+\mu\left[U'(w + \alpha + \beta Y_0)\phi(Y_0 - e) + \beta \int_{Y_0-e}^{\infty} U''(w + \alpha + \beta Y)\phi(\varepsilon)d\varepsilon\right], \quad \text{(B1)}$$

$$\frac{\partial L}{\partial \beta} = -\int_{Y_0-e}^{\infty} Y\phi(\varepsilon)d\varepsilon + \lambda \int_{Y_0-e}^{\infty} U'(w + \alpha + \beta Y)Y\phi(\varepsilon)d\varepsilon$$

$$+ \mu[U'(w + \alpha + \beta Y_0)Y_0\phi(Y_0 - e) + \int_{Y_0-e}^{\infty} U'(w + \alpha + \beta Y)\phi(\varepsilon)d\varepsilon \quad \text{(B2)}$$

$$+ \beta \int_{Y_0-e}^{\infty} U''(w + \alpha + \beta Y)Y\phi(\varepsilon)d\varepsilon].$$

From (A2) we know that $Y_0 \to e$ as $\sigma \to 0$ because $\partial L/\partial Y_0 \neq 0$ otherwise. Then (B1) and (B2) can be approximated as follows:

$$\frac{\partial L}{\partial \alpha} = -\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon + \lambda U'(w + \alpha + \beta e)\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon$$

$$+\mu\left[U'(w + \alpha + \beta Y_0)\phi(Y_0 - e) + \beta U''(w + \alpha + \beta e)\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon\right], \quad \text{(B3)}$$

$$\frac{\partial L}{\partial \beta} = e\Bigg\{-\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon + \lambda U'(w + \alpha + \beta e)\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon$$

$$+\mu\left[U'(w + \alpha + \beta Y_0)\phi(Y_0 - e) + \beta U''(w + \alpha + \beta e)\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon\right]\Bigg\} \quad \text{(B4)}$$

$$+\mu U'(w + \alpha + \beta e)\int_{Y_0-e}^{\infty} \phi(\varepsilon)d\varepsilon.$$

From (B3) and (B4),

$$\frac{\partial L}{\partial \beta} = e\frac{\partial L}{\partial \alpha} + \mu U'(w + \alpha + \beta e)\int_{Y_0 - e}^{\infty} \phi(\varepsilon)d\varepsilon.$$

As $\partial L/\partial\beta \leq 0$ at the optimum, then $\partial L/\partial\alpha < 0$. In other words, $\alpha = 0$ when σ becomes sufficiently small.

(ii) $\beta \to 0$ as $\sigma \to 0$.

As σ approaches zero, $\phi(Y_0 - e)$ approaches either ∞ or 0. We compare these two cases. In the first case, $\phi(Y_0 - e) \to \infty$. Clearly, $Y_0 \to e$. Obtaining $\partial L/\partial e = 0$ from (A1) and solving for μ gives

$$\begin{aligned}\mu = &\left[1 - (\alpha + \beta Y_0)\phi(Y_0 - e) - \beta\int_{Y_0 - e}^{\infty}\phi(\varepsilon)d\varepsilon\right]\\ &\times\Big[C''(e) - \beta U'(w + \beta Y_0)\phi(Y_0 - e) - \beta^2 U''(w + \alpha + \beta e)\int_{Y_0 - e}^{\infty}\phi(\varepsilon)d\varepsilon\\ &- (U(w + \alpha + \beta Y_0) - U(w))\phi'(Y_0 - e)\Big]^{-1}.\end{aligned}$$

At the optimum, the denominator (the second factor) must be positive to satisfy the optimality condition (the second-order condition for the agent's choice of effort). This requires the numerator (the first factor) to be positive, which in turn requires $(\alpha + \beta Y_0)\phi(Y_0 - e) < 1$. Because $\phi(Y_0 - e) \to \infty$ and $Y_0 \to e$, this inequality means $\beta \to 0$. Therefore, as $\sigma \to 0$, we have

$$Y_0 \to e \qquad \text{and} \qquad \beta \to 0. \tag{B5}$$

In the second case, $\phi(Y_0 - e) \to 0$. If $Y_0 - e > 0$, then $\int_{Y_0 - e}^{\infty}\phi(\varepsilon)d\varepsilon \to 0$, which means $W \to w$ (the contract becomes a fixed fee). But this would not happen. There must be $Y_0 - e < 0$, and so, $\int_{Y_0 - e}^{\infty}\phi(\varepsilon)d\varepsilon \to 1$. This means that the contract approaches a simple linear scheme where

$$\beta \to 1 \qquad \text{and} \qquad \int_{Y_0 - e}^{\infty}\phi(\varepsilon)d\varepsilon \to 1 \tag{B6}$$

because $\sigma \to 0$.

We further show that, as $\sigma \to 0$, the scheme of (B5) dominates that of (B6). When $\sigma = 0$, the two schemes are equivalent. In the first scheme, $\beta_1 \to 0$. Define $\delta_1 = e - Y_0$. Then, $\delta_1 \to 0$ as $\sigma \to 0$. Noticing that $\alpha = 0$, we have

$$EU_a = U(w)\int_{-\infty}^{-\delta_1}\phi(\varepsilon)d\varepsilon + \int_{-\delta_1}^{\infty}U(w + \beta_1 Y)\phi(\varepsilon)d\varepsilon.$$

The distortion because of production riskiness and unobservable effort can be explained by

$$\int_{-\delta_1}^{\infty}U(w + \beta_1 Y)\phi(\varepsilon)d\varepsilon < U\left(\int_{-\delta_1}^{\infty}(w + \beta_1 Y)\phi(\varepsilon)d\varepsilon\right). \tag{B7}$$

What matters to the principal is $\int_{-\delta_1}^{\infty}(w+\beta_1 Y)\phi(\varepsilon)d\varepsilon$, while what matters to the agent is $\int_{-\delta_1}^{\infty}U(w+\beta_1 Y)\phi(\varepsilon)d\varepsilon$. In the second scheme, $\beta_2 \to 1$. Defining $\delta_2 = e - Y_0$, then

$$EU_a = U(w)\int_{-\infty}^{-\delta_2}\phi(\varepsilon)d\varepsilon + \int_{-\delta_2}^{\infty}U(w+\beta_2 Y)\phi(\varepsilon)d\varepsilon.$$

Similarly, the distortion can be explained by

$$\int_{-\delta_2}^{\infty}U(w+\beta_2 Y)\phi(\varepsilon)d\varepsilon < U\left(\int_{-\delta_2}^{\infty}(w+\beta_2 Y)\phi(\varepsilon)d\varepsilon\right). \tag{B8}$$

Since $\beta_1 < \beta_2$ and $\delta_1 < \delta_2$, the distortion in (B8) must be larger than that in (B7). That is, scheme (B5) dominates scheme (B6). When $\sigma = 0$, both (B7) and (B8) become equality and the two schemes are equivalent. Q.E.D.

Appendix C

Proof of Proposition 3

(i) $\beta \to 0$ as $\sigma \to \infty$.

Obtaining $\partial L/\partial\beta$ from (A1), setting $\partial L/\partial\beta = 0$, and approximating $U(w+\alpha+\beta Y)$ by the second-order Taylor expansion gives

$$\begin{aligned}
&-\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon + \lambda\Big[U'(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon \\
&+\beta U''(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}\varepsilon(e+\varepsilon)\phi(\varepsilon)d\varepsilon\Big] + \mu\Big[U'(w+\alpha+\beta Y_0)Y_0\phi(Y_0-e) \\
&+U'(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}\phi(\varepsilon)d\varepsilon + \beta U''(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}\varepsilon\phi(\varepsilon)d\varepsilon \\
&+\beta U''(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon\Big] = 0.
\end{aligned}$$

Solving for β gives

$$\begin{aligned}
\beta = {} & \frac{-\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon + \lambda U'(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon}{-U''(w+\alpha+\beta e)\left[\lambda\int_{Y_0-e}^{\infty}\varepsilon(e+\varepsilon)\phi(\varepsilon)d\varepsilon + \mu\int_{Y_0-e}^{\infty}(e+2\varepsilon)\phi(\varepsilon)d\varepsilon\right]} \\
&+\frac{\mu\left[U'(w+\alpha+\beta Y_0)Y_0\phi(Y_0-e) + U'(w+\alpha+\beta e)\int_{Y_0-e}^{\infty}\phi(\varepsilon)d\varepsilon\right]}{-U''(w+\alpha+\beta e)\left[\lambda\int_{Y_0-e}^{\infty}\varepsilon(e+\varepsilon)\phi(\varepsilon)d\varepsilon + \mu\int_{Y_0-e}^{\infty}(e+2\varepsilon)\phi(\varepsilon)d\varepsilon\right]}.
\end{aligned}$$

With $-\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon<0$ and $-U''(w+\alpha+\beta e)>0$, we have

$$\begin{aligned}\beta &< \frac{U'(w+\alpha+\beta e)\left[\lambda\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon+\mu Y_0\phi(Y_0-e)+\mu\int_{Y_0-e}^{\infty}\phi(\varepsilon)d\varepsilon\right]}{-U''(w+\alpha+\beta e)\left[\lambda\int_{Y_0-e}^{\infty}\varepsilon(e+\varepsilon)\phi(\varepsilon)d\varepsilon+\mu\int_{Y_0-e}^{\infty}(e+2\varepsilon)\phi(\varepsilon)d\varepsilon\right]}\\
&< \left[-\frac{U'(w+\alpha+\beta e)}{U''(w+\alpha+\beta e)}\right]\left[\frac{\lambda\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon+\mu Y_0\phi(Y_0-e)+\mu\int_{Y_0-e}^{\infty}\phi(\varepsilon)d\varepsilon}{\lambda\int_{Y_0-e}^{\infty}\varepsilon^2\phi(\varepsilon)d\varepsilon}\right]\\
&= \left[-\frac{U'(w+\alpha+\beta e)}{U''(w+\alpha+\beta e)}\cdot\frac{1}{(w+\alpha+\beta e)}\right]\times(w+\alpha+\beta e)\\
&\quad\times\left[\frac{\int_{Y_0-e}^{\infty}(e+\varepsilon)\phi(\varepsilon)d\varepsilon}{\int_{Y_0-e}^{\infty}\varepsilon^2\phi(\varepsilon)d\varepsilon}+\frac{\mu Y_0\phi(Y_0-e)+\mu\int_{Y_0-e}^{\infty}\phi(\varepsilon)d\varepsilon}{\lambda\int_{Y_0-e}^{\infty}\varepsilon^2\phi(\varepsilon)d\varepsilon}\right].\end{aligned}$$

Since the coefficient of relative risk aversion, $R(w+\alpha+\beta e)\equiv-[(w+\alpha+\beta e)U''(w+\alpha+\beta e)]/U'(w+\alpha+\beta e)$, can be roughly taken to be constant, $(w+\alpha+\beta e)$ is limited, $Y_0\phi(Y_0-e)$ does not explode as Y_0 changes, and $\int_{Y_0-e}^{\infty}\varepsilon^2\phi(\varepsilon)d\varepsilon\to\infty$ as $\sigma\to\infty$, then $\beta\to0$ as $\sigma\to0$.

(ii) $Y_0\to-\infty$ as $\sigma\to\infty$.

Obtain $\partial L/\partial w$ from (A1). Set $\partial L/\partial w=0$ and $\partial L/\partial\alpha=0$ in (B1). Then, we have

$$1-\int_{Y_0-e}^{\infty}\phi(\varepsilon)d\varepsilon-\lambda U'(w)\int_{-\infty}^{Y_0-e}\phi(\varepsilon)d\varepsilon+\mu U'(w)\phi(Y_0-e)=0,$$

or

$$\mu U'(w)\phi(Y_0-e)=[\lambda U'(w)-1]\int_{-\infty}^{Y_0-e}\phi(\varepsilon)d\varepsilon.$$

When $\sigma\to\infty$, the left-hand side approaches zero as does the right-hand side, which requires either $\int_{-\infty}^{Y_0-e}\phi(\varepsilon)d\varepsilon\to0$ or $\lambda U'(w)\to1$. When $\int_{-\infty}^{Y_0-e}\phi(\varepsilon)d\varepsilon\to0$, clearly $Y_0\to-\infty$. Below, we show that $\lambda U'(w)\to1$ also implies that $Y_0\to-\infty$. Let $\partial L/\partial Y_0=0$ in (A2). Then,

$$\frac{e-Y_0}{\sigma^2}\mu=\lambda[U(w+\alpha+\beta Y_0)-U(w)]-(\alpha+\beta Y_0).$$

When $\sigma\to\infty$, the left-hand side approaches zero unless $Y_0\to-\infty$. As the right-hand side approaches zero,

$$\lambda\frac{U(w+\alpha+\beta Y_0)-U(w)}{\alpha+\beta Y_0}\to1. \tag{C1}$$

Then, equation (C1) and $\lambda U'(w)\to1$ jointly means that $\alpha+\beta Y_0\to0$ or $Y_0\to-\alpha/\beta$. With $\beta\to0$, this means that $Y_0\to-\infty$. Q.E.D.

References

Aggarwal, R. K., and Samwick, A. A. 1999*a*. The other side of the trade-off: The impact of risk on executive compensation. *Journal of Political Economy* 107 (February): 65–105.

Aggarwal, R. K., and Samwick, A. A. 1999*b*. Performance incentives within firms: The effect of managerial responsibility. Working paper. Cambridge, Mass.: National Bureau of Economic Research.

Bryant, S.; Hwang, L.; and Lilien, S. 2000. CEO stock-based compensation: An empirical analysis of incentive-intensity, relative mix, and economic determinants. *Journal of Business* 73 (October): 661–94.

Core, J., and Guay, W. 1999. The use of equity grants to manage optimal equity incentive levels. *Journal of Accounting and Economics* 28 (December): 151–84.

Garen, J. E. 1994. Executive compensation and principal-agent theory. *Journal of Political Economy* 102 (December): 1175–99.

Gibbons, R. 1997. Incentives and careers in organizations. Working paper. Ithaca, N.Y., and Cambridge, Mass.: Cornell University and National Bureau of Economic Research.

Gibbons, R., and Murphy, K. J. 1992. Optimal incentive contracts in the presence of career concerns: Theory and evidence. *Journal of Political Economy* 100 (June): 468–505.

Gjesdal, F. 1988. Piecewise linear incentive schemes. *Scandinavian Journal of Economics* 90(3): 305–28.

Grossman, S., and Hart, O. 1983. An analysis of the principal-agent problem. *Econometrica* 51 (January): 7–45.

Hall, B. J., and Liebman, J. B. 1998. Are CEOs really paid like bureaucrats? *Quarterly Journal of Economics* 113 (August): 653–91.

Haubrich, J. G. 1994. Risk aversion, performance pay, and the principal-agent problem. *Journal of Political Economy* 102 (April): 258–76.

Holmstrom, B. 1979. Moral hazard and observability. *Bell Journal of Economics* 10 (Spring): 74–91.

Holmstrom, B. 1982. Design of incentive schemes and the new Soviet incentive model. *European Economic Review* 17 (January): 127–48.

Holmstrom, B., and Milgrom, P. 1987. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica* 55 (March): 303–28.

Hubbard, R. G., and Palia, D. 1995. Executive pay and performance: Evidence from the U.S. banking industry. *Journal of Financial Economics* 39 (September): 105–30.

Jensen, M. C., and Murphy, K. J. 1990. Performance pay and top-management incentives. *Journal of Political Economy* 98 (April): 225–64.

John, T. A., and John, K. 1993. Top management compensation and capital structure. *Journal of Finance* 48 (July): 949–74.

Joskow, P.; Rose, N.; and Shepard, A. 1993. Regulatory constraints on CEO compensation. *Brookings Papers on Economic Activity: Microeconomics*, no. 1, pp. 1–58.

Kole, S. R. 1997. The complexity of compensation contracts. *Journal of Financial Economics* 43 (January): 79–104.

Lambert, R., and Larcker, D. 1987. An analysis of the use of accounting and market measures of performance in executive compensation contracts. *Journal of Accounting Research* 25, suppl.:85–129.

Lambert, R., and Larcker, D. 1991. Executive compensation, corporate decision making, and shareholder wealth: A review of the evidence. In F. K. Foulkes (ed.), *Executive Compensation.* Cambridge, Mass.: Harvard Business School Press.

Mirrlees, J. 1974. Notes on welfare economics, information, and uncertainty. In M. Balch, D. McFadden, and S. Wu (eds.), *Essays on Economic Behavior under Uncertainty*. Amsterdam: North Holland.

Mirrlees, J. 1979. The implication of moral hazard for optimal insurance. Mimeographed. Seminar given at conference held in honor of Karl Borch, Bergen, Norway.

Murphy, K. J. 1999. Executive compensation. In O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics,* vol. 3. Armsterdam: North Holland.

Murphy, K. J. 2001. Performance standards in incentive contracts. *Journal of Accounting and Economics* 30 (December): 245–78.

Rogerson, W. P. 1985. The first-order approach to principal-agent problems. *Econometrica* 53 (November): 1357–67.

Rosen, S. 1992. Contracts and the market for executives. In L. Werin and H. Wijkander (eds.), *Contract Economics.* Oxford: Blackwell.

Schaefer, S. 1998. The dependence of CEO pay-performance sensitivity on the value of the firm. *Review of Economics and Statistics* 80 (August): 436–43.

Sloan, R. 1993. Accounting earnings and top executive compensation. *Journal of Accounting and Economics* 16 (January/April/July): 55–100.

Smith, C. W., Jr., and Watts, R. 1992. The investment opportunity set and corporate financing, dividend, and compensation policies. *Journal of Financial Economics* 32 (December): 263–92.
Weitzman, M. L. 1976. The new Soviet incentive model. *Bell Journal of Economics and Management Sciences* 7 (Spring): 251–57.