Performance Analysis of Single and Multiuser MIMO Diversity Channels Using Nakagami-m Distribution

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***Abstract*—This letter studies the diversity advantage of single and multiuser systems utilizing multiple-input multiple-output (MIMO) antennas for known channels at the transmitter and receiver(s) by investigating the overall distribution of the resulting channel. For the single-user case, the resultant channel is taken as the largest singular value of the MIMO channel whereas for the multiuser scenario, the resultant channel is obtained by a recently proposed joint-channel diagonalization. The channel distribution is studied using hypothesis testing with the null hypothesis that the distribution follows a Nakagami-m distribution with given parameters. It is concluded from the chi-square goodness-of-fit test that the distribution of the resultant channel for single or multiuser MIMO channel is well matched with a Nakagami-m distribution.**

***Index Terms*—Diversity, joint-channel diagonalization (JCD), multiple-input multiple-output (MIMO), Nakagami distribution.**

I. INTRODUCTION

TO PROFIT from the ever-growing demand for high-quality high-data-rate wireless communications, power-and-spectral efficient techniques are sought. Utilizing multiple antennas at both transmitter and receiver [known as multiple-input multiple-output (MIMO) antenna] is definitely one of the most outstanding candidates. In recent years, many advanced MIMO antenna systems have been demonstrated to provide excellent performance for both single [1]–[4] and multiuser [5], [6]–[8] communications.

In [1] and [2], space-time coding or Bell-Labs layered space-time (BLAST), which does not require prior channel information at the transmitter, was studied. To exploit the full diversity of MIMO antennas, it is, however, advantageous for the transmitter to have prior knowledge of the channel. In [3] and [4], adaptation of antenna weights based on singular value decomposition (SVD) of known random channels was proposed.

Utilizing multiple antennas at the transmitter and all mobile receivers (downlink multiuser MIMO) for performance enhancement has also been considered (e.g., [4]–[8]). In [6] and [7], signal-to-interference plus noise ratio (SINR) enhancement using multielement transmit antenna array was proposed while later in [5], an algorithm was devised for decomposing multiuser MIMO channel matrices into multiple uncoupled single-user systems [i.e., orthogonal space division multiplexing (OSDM)]. More recently, Choi and Murch [8] presented a null space projection method as a transmit preprocessing to decouple multiuser signals.

Motivated by the success of MIMO antenna systems, the focus of this letter is to study the diversity advantage using MIMO antennas by determining the probability density function (PDF) of the resultant channel for single and multiuser MIMO communications, through which the statistical characteristics of general MIMO systems can be completely described. For the single-user case, the resultant channel is taken as the largest singular value of the MIMO channel whereas for multiuser scenario, the resultant channel is obtained by the joint-channel diagonalization (JCD) of the multiuser MIMO channels [5]. For convenience, our consideration of multiuser MIMO will be limited to an N_t-element base station (BS) communicating with $N(=N_t)$ mobile stations (MS) each with two antenna elements.

The letter is organized as follows. In Section II, we review the single-user single-input multiple-output (SIMO) and MIMO systems. Section III presents the OSDM solution we use on multiuser MIMO channels. In Section IV, simulation setup and results are presented. We conclude this letter in Section V.

II. SINGLE-USER SIMO AND MIMO SYSTEMS

A. SIMO With Maximal Ratio Combining Diversity

For a single-user SIMO system where N_r antennas are located at the receiver, the equivalent baseband received signal can be conveniently expressed in vector form as

$$\mathbf{y} = \mathbf{h}z + \mathbf{n} \tag{1}$$

where $\mathbf{y} = [y_1 y_2, \ldots, y_{N_r}]^T$ with y_ℓ denoting the received signal at the ℓth antenna and the superscript T denoting the transpose operation. Vectors $\mathbf{h}$ and $\mathbf{n}$ are defined similarly and they represent, respectively, the complex fading coefficients of the radio channel and the zero-mean complex additive white Gaussian noise (CAWGN) with variance of $N_0/2$ per dimension. Likewise, the scalar z denotes the complex modulated symbol.

Optimal reception can be achieved by maximal ratio combining (MRC) so that the array output signal $\hat{z}$ is $\hat{z} = \mathbf{h}^\dagger \mathbf{y}$ where the superscript $\dagger$ denotes the conjugate transposition.

Denoting the resultant channel coefficient after MRC as $\beta \triangleq \|\mathbf{h}\|$, it is well known [9]–[11] that an N_r-branch MRC (with uncorrelated fading) provides an N_r-fold diversity in terms of both receiving power (i.e., the channel gain)

$$\Omega = E[\beta^2] \tag{2}$$

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and the inverse of the normalized variance of the squared channel gain (i.e., the diversity order [11])

$$m = \frac{\Omega^2}{E[(\beta^2 - \Omega)^2]}. \tag{3}$$

In other words, for an N_r-branch MRC, $\Omega = N_r\Omega_0$ and $m = N_r m_0$ where Ω_0 and m_0 are the diversity statistics of a single-user system without antenna diversity. When $m \to \infty$, channel becomes CAWGN.

Given that the fading coefficients h_ℓ are zero-mean complex Gaussian distributed, the PDF of β has been found to be a Nakagami-m distribution [9]–[11] and is given by [12]

$$\mathrm{PDF}(\beta) = \frac{2m^m}{\Gamma(m)\Omega^m}\beta^{2m-1}\exp\left(-\frac{m\beta^2}{\Omega}\right) \tag{4}$$

where Ω and m are defined in (2) and (3), respectively, and $\Gamma(\cdot)$ denotes the gamma function.

B. MIMO With SVD Diversity

When N_t antennas are employed at the transmitter side, the symbol z before transmission will be multiplied by a complex transmit weight vector $\mathbf{t}$. The signal at each receive antenna is a noisy superposition of the N_t transmitted signals perturbed by fading. This can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{t}z + \mathbf{n} \tag{5}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix.

To detect the signal, an array output signal (similar to the case for SIMO system) is produced by $\hat{z} = \mathbf{r}^\dagger\mathbf{y}$ where $\mathbf{r}$ is the receive antenna vector. As found in [4], SVD can be used as the optimal antenna processing. Therefore, the transmit and receive weight vectors can be chosen, respectively, as the right and left singular vectors which correspond to the largest singular value of $\mathbf{H}$. The resulting system is then reduced to a single-input single-output (SISO) system with resultant channel determined by the channel largest singular value, σ_1. Hence, the resultant channel coefficient for a single-user MIMO with SVD diversity is $\beta = \sigma_1$.

In [4] and [13], it was found that for a $(N_t, N_r) = (2, 2)$ system, the PDF of β is given by

$$\mathrm{PDF}(\beta) = 2\beta e^{-\beta^2}(\beta^4 - 2\beta^2 + 2) - 4\beta e^{-2\beta^2}, \quad \text{for } \beta \geq 0. \tag{6}$$

However, the PDF expression for other values of N_t or N_r is unavailable. Inspired by the idea of Nakagami-m distribution for SIMO systems, we would like to re-express (6) in the form of Nakagami-m distribution. In this case, when $(N_t, N_r) = (2, 2)$, it can be shown that $(m, \Omega) \doteq (3.9, 3.5)$. Substituting these values into (4), we obtain the following PDF:

$$\mathrm{PDF}(\beta) \approx 0.5881\beta^{6.7346}e^{-1.11\beta^2}, \quad \text{for } \beta \geq 0. \tag{7}$$

Though (6) and (7) look very different, they are, in fact, extremely close with inappreciable difference (see Fig. 1).

The idea to express the PDF of β using (4) provides an easy way for the procurement of the PDF of β for larger values of N_t or N_r since the PDF can then be completely determined by the two statistics, m and Ω. The details will be discussed in the simulation section (see Section IV).

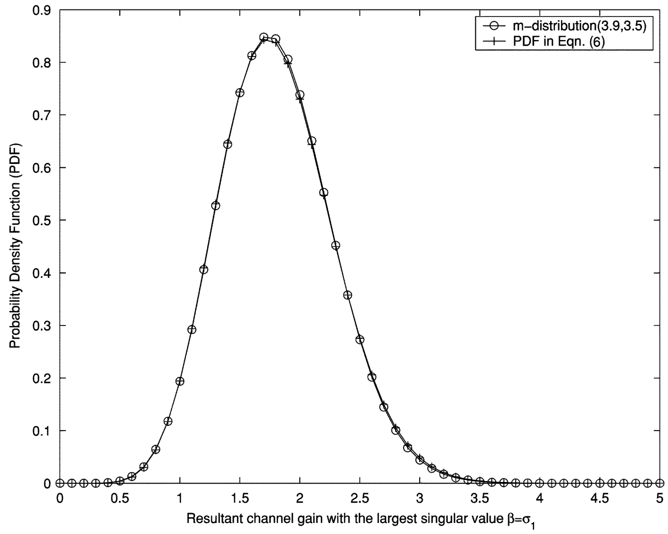

Fig. 1. Comparison for PDFs of Nakagami-m distribution with $(m, \Omega) = (3.9, 3.5)$ and the expression in (6).

III. Multiuser MIMO With JCD Diversity

Consider a multiuser system where an N-element BS communicates simultaneously with N 2-element mobile receivers. Using the system model (5), as described in Section II-B, we can write the multiuser MIMO system as [5], [7]

$$\hat{z}_n = \mathbf{r}_n^\dagger\left(\sum_{k=1}^{N}\mathbf{H}_n\mathbf{t}_k z_k + \mathbf{n}_n\right) \qquad \forall n \tag{8}$$

where $\mathbf{n}_n \in \mathbb{C}^2$ is the noise vector, $z_k \in \mathbb{C}$ denotes the symbols (with time index omitted) transmitted from the kth user, $\mathbf{t}_k \in \mathbb{C}^N$ is the antenna weight vector for transmitting the kth user signal, $\mathbf{H}_n \in \mathbb{C}^{2\times N}$ is the channel matrix from the BS to the nth MS, and $\mathbf{r}_n \in \mathbb{C}^2$ is the antenna weight vector for signal reception by the nth user.

In dealing with multiuser communications, it is advantageous to handle with users in an orthogonal manner, as in conventional systems such as time, frequency or code division multiplexing (T/F/CDM). Through joint adaptation of the multiple antennas at the BS and MS, the idea has been extended in spatial domain [5].

In this letter, we shall adopt the iterative algorithm proposed in [5]. Since the multiuser MIMO system using JCD can be reduced to multiple uncoupled single-user systems, the overall system performance will then be solely determined by the statistics of the resultant channel β_n. To see if Nakagami-m distribution can be used to represent the PDF of the resultant channel coefficient, β_n, in multiuser MIMO systems, simulations were carried out and results in Fig. 2 indicate that the PDF of β_n for a five-user MIMO antenna system is well matched to a Nakagami distribution with $(m, \Omega) = (4.1, 3.4)$. A more rigorous analysis using Chi-square goodness-of-fit test will be discussed in the following section.

IV. Simulation Setup and Results

In this section, we shall provide the simulation results of channel gain and diversity order for various configurations followed by a Chi-square goodness-of-fit test. Perfect channel

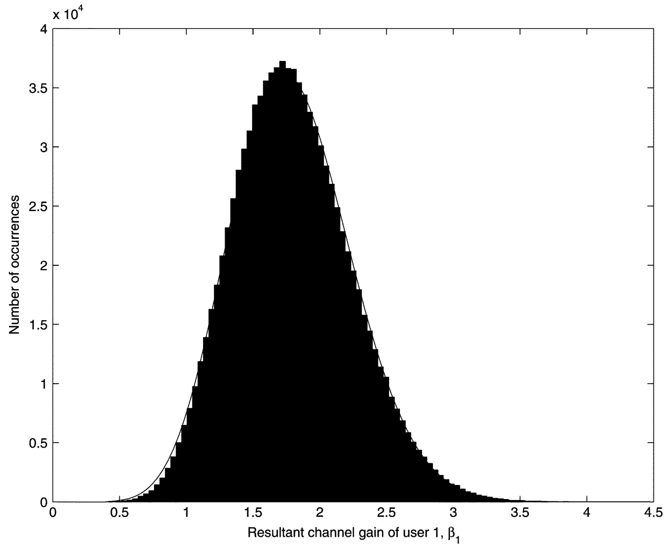

Fig. 2. Comparison between the theoretical values of Nakagami-m distribution with $(m, \Omega) = (4.1, 3.4)$ (solid line) and Monte Carlo simulations of a five-user MIMO using JCD diversity (strips).

state information is assumed and known to both the transmitter and receiver so that SVD or JCD can be performed. The radio channel is assumed to be in flat Rayleigh fading and the channels among transmit and receive antenna pairs are independent and identically distributed. For each simulation, $10^4 \leq N_s \leq 10^6$ independent channel realizations are simulated and for each channel realization, the resultant channel gain(s), β or β_n, is (are) computed.

A. Diversity Analysis

We measure the diversity advantage of a system in terms of Ω and m [defined earlier in (2) and (3)] relative to a system without diversity. The benchmark system without diversity we use is a single-user system with a single transmit and receive antenna, where the diversity statistics $\Omega_0 = m_0 = 1$.

Results in Table I provide the values of Ω and m for single-user MIMO systems with various number of transmit and receive antennas. For any given number of transmit (or receive) antennas, the diversity statistics grow up linearly as the number of receive (or transmit) antennas. Therefore, we can use the following multiple linear regression model:

$$m_{\text{SVD}}(N_{t,i}, N_{r,j}) = \beta_t N_{t,i} + \beta_r N_{r,j} + \beta_{t,r} N_{t,i} N_{r,j} + \beta_0 \quad (9)$$

where $\beta_t, \beta_r, \beta_{t,r}$, and β_0 are the regression coefficients, $N_{t,i} = i$, and $N_{r,j} = j$ for $i = 2, \ldots, 5$, and $2 \leq j \leq i$. The regression coefficients are determined using least-squares criterion such that the sum of squared deviations between the observations and the regression surface is minimized. After solving this from our results, m_{SVD} can be conveniently approximated by

$$m_{\text{SVD}} \approx 0.9469 N_t + 1.1961 N_r + 0.3698 N_t N_r - 1.9746 \quad (10)$$

with the sum of squared error equals 0.1344. Similarly, we have also

$$\Omega_{\text{SVD}} \approx 1.1907 N_t + 1.4908 N_r + 0.0787 N_t N_r - 2.1797 \quad (11)$$

TABLE I
RESULTS OF THE CHI-SQUARE TESTS FOR SINGLE-USER MIMO SYSTEMS

N_t	N_r	m	Ω	$\chi^2_{\text{statistic}}$
2	2	3.8673	3.4840	5.9880*
3	2	5.4542	4.8915	6.3360*
	3	7.7263	6.5092	3.2580*
4	2	7.1314	6.1872	7.3020*
	3	9.8662	8.0616	14.3360*
	4	12.5216	9.7758	11.8760*
5	2	8.6438	7.4672	4.1840*
	3	12.0152	9.5100	17.0000*
	4	15.0209	11.3860	23.5140
	5	17.9214	13.1835	19.2420
6	2	10.4650	8.7255	16.3580*
	3	14.1357	10.9511	8.7600*
	4	17.3895	12.9186	19.9980
	5	20.8344	14.8464	9.5560*
	6	24.0821	16.5881	20.7200

with the sum of squared deviations equals 0.1049. Note from these equations that the number of receive antennas (N_r) plays a more important role on diversity gain than the number of transmit antennas (N_t). This comes to a similar conclusion as in [14] that a SIMO antenna system will always outperform a MISO (multiple transmit antennas and a single receive antenna) antenna system with fixed numbers of total antenna elements. For example, as we can see in Table I, $(N_t, N_r) = (4, 4)$ obtains higher diversity advantages compared to $(N_t, N_r) = (5, 3)$, which is also better than a $(N_t, N_r) = (6, 2)$ system.

Similar results for multiuser MIMO using JCD diversity are provided in Table II. Ω_n and m_n denote, respectively, the channel gain and diversity order of the resultant channel for user n. Results in Table II reveal that for a two-user MIMO system, it has about 2.4 diversity gain of receiving power Ω_n and 2.6 diversity gain for reducing the effect of fading m_n, compared with a single-user system without diversity. It is important to note that the diversity orders obtained can be more than the number of receive antennas at the MS, meaning that in addition to support of multiple users, the transmit antennas at the BS can provide diversity to the MS's. More diversity can be achieved for a larger value of $N = N_t$. Generally speaking, both Ω_n and m_n will increase as the number of transmit antennas increases even the number of users increases correspondingly.

It should also be noted that for a particular N, all users perform similarly and have nearly the same diversity statistics (i.e., $\Omega_1 \approx \cdots \approx \Omega_N$ and $m_1 \approx \cdots \approx m_N$). This agrees with the fact that, on average, all users are processed in the same way. Thus, to describe the diversity performances of the system, we use

$$\bar{\Omega} \triangleq \frac{1}{N} \sum_{n=1}^{N} \Omega_n \quad (12)$$

and

$$\bar{m} \triangleq \frac{1}{N} \sum_{n=1}^{N} m_n \quad (13)$$

as the diversity statistics of a N-user MIMO system using JCD diversity. The results for $\bar{m}$ and $\bar{\Omega}$ against the number of users N are illustrated in Fig. 3. Results in this figure indicate that the diversity advantages for both $\bar{m}$ and $\bar{\Omega}$ grow almost linearly

TABLE II
RESULTS OF THE CHI-SQUARE TESTS FOR MULTIUSER MIMO SYSTEMS

N	User n	m_n	Ω_n	$\chi^2_{\text{statistic}}$	$\overline{m}$	$\overline{\Omega}$
2	1	2.5946	2.3988	4.7740*	2.6001	2.3958
	2	2.6057	2.3928	1.4980*		
3	1	3.1072	2.7686	18.8320	3.0762	2.7578
	2	3.1048	2.7613	9.1900*		
	3	3.0167	2.7436	19.1120		
4	1	3.6065	3.1206	14.2360*	3.6709	3.1094
	2	3.7361	3.1109	4.5160*		
	3	3.6580	3.0966	13.6400*		
	4	3.6831	3.1095	17.2460*		
5	1	4.1412	3.4526	16.37*	4.1413	3.4226
	2	4.1747	3.4162	9.4240*		
	3	4.1078	3.4349	10.2820*		
	4	4.1349	3.4169	11.6120*		
	5	4.1480	3.3925	9.7300*		
6	1	4.4858	3.7079	31.004	4.5771	3.7230
	2	4.6294	3.7456	11.3120*		
	3	4.5033	3.7180	13.6760*		
	4	4.7190	3.7212	15.8720*		
	5	4.6308	3.7254	13.2760*		
	6	4.4943	3.7197	16.6320*		
7	1	5.0400	4.0233	14.406*	5.0713	4.0169
	2	5.0974	4.0046	24.1000		
	3	4.9712	4.0124	22.3100		
	4	5.1266	4.0076	12.8060*		
	5	5.1557	4.0108	21.678*		
	6	4.9769	4.0194	13.3900*		
	7	5.1312	4.0402	12.368*		
8	1	5.6589	4.2735	4.8700*	5.5680	4.2884
	2	5.5658	4.2853	13.3660*		
	3	5.5075	4.2977	16.1020*		
	4	5.4698	4.2800	13.642*		
	5	5.6627	4.2939	7.8280*		
	6	5.7046	4.2929	7.1060*		
	7	5.3986	4.3092	33.8840		
	8	5.5762	4.2745	16.0740*		
9	1	5.7715	4.5675	17.264*	5.9704	4.5597
	2	6.0568	4.5644	17.3440*		
	3	6.0435	4.5588	18.27200*		
	4	5.8665	4.5591	23.2800		
	5	5.9517	4.5744	16.9500*		
	6	5.8837	4.5667	14.5160*		
	7	6.1012	4.5578	9.9100*		
	8	5.9052	4.5412	10.8280*		
	9	6.1533	4.5473	6.3580*		

with N and the gain does not seem to diminish as N keeps increasing. Results demonstrate that about $0.4844(\approx 0.5)$ diversity gain for $\bar{m}$ and $0.3074(\approx 0.3)$ diversity gain for $\bar{\Omega}$ can be benefited for all users from every additional accommodated user. This can be explained by recognizing that in support of additional users via space, JCD gains additional diversity by better use of multiple channels with increased degrees of freedom. Similar to single-user MIMO systems, we can also have approximate formulas for computing the diversity order and channel gain of multiuser MIMO systems. The formulas are given by

$$m_{\text{JCD}} \approx 0.4844N + 1.6703 \tag{14}$$

and

$$\Omega_{\text{JCD}} \approx 0.3074N + 1.8434. \tag{15}$$

However, it should be noted that they are only useful for a system with N-element transmit antennas to communicate with N simultaneous users, each with two antenna elements.

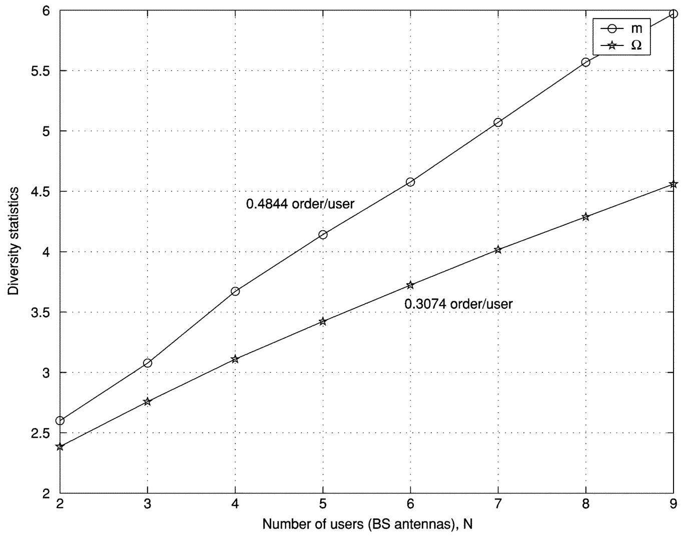

Fig. 3. Diversity gains of N-user MIMO systems using JCD diversity.

B. Chi-Square Goodness-of-Fit Test

In Fig. 2, we have seen that the histogram of simulated multiuser MIMO channels matches very well with the Nakagami-m distribution with the same m and Ω. To demonstrate this more rigorously, we use the Chi-square goodness-of-fit test to check if the resultant channel coefficient for single or multiuser MIMO system is Nakagami-m distributed, under significance level of 1%.

To perform the test, we first calculate Ω and m from the data according to (2) and (3) and the following hypothesis is set up:

H_0: follow a Nakagami distribution with (m, Ω).
H_a: do not follow the given distribution.

Then, the data (N_s samples of β) are divided into ten bins and the test statistic $\chi^2_{\text{statistic}}$ is computed as

$$\chi^2_{\text{statistic}} = \sum_{k=1}^{10} \frac{(O_k - E_k)^2}{E_k} \tag{16}$$

where O_k is the observed number of outcomes and E_k is the expected number of outcomes that fall in the kth interval. The expected frequency E_k is calculated by

$$E_k = N_s(F(U_k) - F(L_k)) \tag{17}$$

where U_k is the upper limit for class k, L_k is the lower limit for class k, and $F(\cdot)$ is the cumulative distribution function (CDF) for the distribution, which is now given by [12]

$$F(\beta) = \Gamma_{\text{inc}}\left(\frac{m\beta^2}{\Omega}, m\right) \tag{18}$$

where Γ_{inc} is the incomplete Gamma function. The test statistic, in principle, follows a Chi-square distribution with seven degrees of freedoms [15]. Let $\chi^2_{\text{df},1-\alpha}$ be the Chi-square percent point function where df is the degrees of freedom and α is the significance level. In our case, $\text{df} = 7$ and we have set $\alpha = 1\%$. Therefore, $\chi^2_{\text{df},1-\alpha} = \chi^2_{7,0.99} = 18.48$. If $\chi^2_{\text{statistic}} < \chi^2_{\text{df},1-\alpha}$, then we accept H_0; otherwise H_0 is rejected.

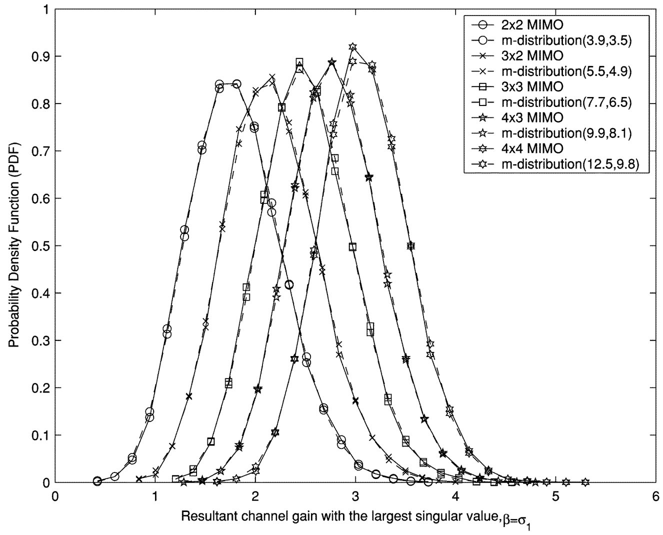

Fig. 4. PDFs of single-user MIMO systems using SVD diversity (solid lines) and Nakagami-m distributions (dashed lines).

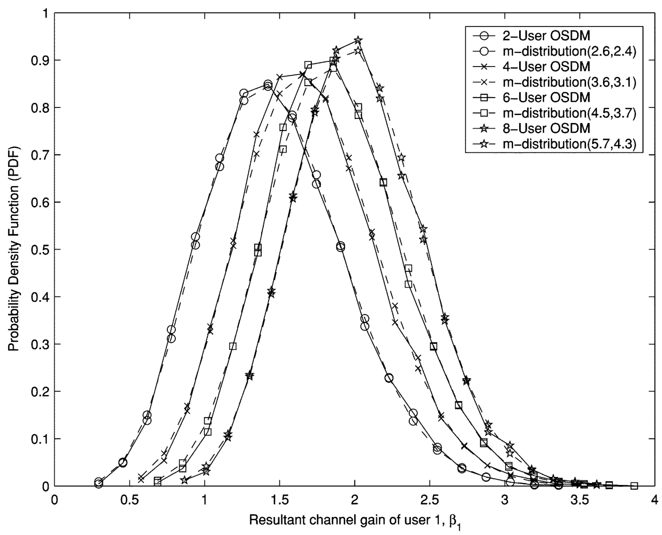

Fig. 5. PDFs of multiuser MIMO systems using JCD diversity (solid lines) and Nakagami-m distributions (dashed lines).

The test statistics were calculated and listed in Tables I and II. The numbers marked with the superscript $\star$ indicate that the null hypothesis is accepted. Results in Table I show that 11 out of 15 configurations accept the null hypothesis. Similarly for multiuser channels, it is illustrated in Table II that 38 out of 44 configurations accept the null hypothesis. As a result, we can conclude that the PDF of single and multiuser MIMO channels can be well expressed (or approximated) as a Nakagami-m distribution (4) with the diversity statistics that can be estimated by (10) and (11) for single-user MIMO or (14) and (15) for multiuser MIMO channels.

In Figs. 4 and 5, the PDFs of the simulated channels are compared to the PDF's of Nakagami-m distribution with the same diversity statistics of the simulated channels. Results in Fig. 4 are provided for single-user MIMO channels using SVD diversity. As we can see, the PDFs of the simulated channels almost overlap with the PDFs of the Nakagami-m distribution. The same is also true for multiuser MIMO channels concluded by the results in Fig. 5.

V. Conclusion

It is well understood that if prior channel knowledge is available at the transmitter, SVD or more generally JCD can be used to improve the system capacity and performance using MIMO channels. The focus of this letter is to quantify the diversity advantages achievable by the antennas and to investigate the overall distribution of the resultant channel of MIMO systems. Through Monte Carlo simulations, we end up with formulas to generally describe the channel gain and diversity order of single and multiuser MIMO systems. We have also concluded by Chi-square goodness-of-fit test that the distribution of the resultant channel gain can be well approximated by a Nakagami-m distribution with corresponding Ω and m for single and multiuser MIMO antenna systems.

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