Analytical Study of FFH Systems with Square-Law Diversity Combining in the Presence of Multitone Interference

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***Abstract*—An analytical study of the performance of fast frequency-hopped (FFH), M-ary orthogonal frequency-shift keyed noncoherent modulation with linear combining of square-law envelopes in the presence of multitone interference is presented. The multiple equal-power interference tones are assumed to correspond to some of the possible FFH/M-ary orthogonal signaling tones. It is also assumed that channel fading characteristics of the signal and interference tones are independent. We evaluate the effect of the channel fading on system performance as a function of various parameters, such as the number of hops per symbol, the signal power to multitone interference power ratio, and the number of interference tones. Our numerical results indicate that by use of square-law time diversity combining, a large number of hops per symbol make the bit-error probability of the system more sensitive to the fading of multitone interference. Finally, analysis has been proven valid by simulation.**

***Index Terms*—Fading channel, frequency hopping.**

I. Introduction

FREQUENCY hopping (FH) has been widely studied for various applications [1]–[9]. As VLSI technologies develop, frequency synthesizers for FH are becoming feasible and cost effective. In fact, slow FH (SFH) is an option in the GSM second-generation cellular mobile system.

In FH systems, interference and jamming models include thermal wide-band noise, multitone jamming, partial-band jamming and multiple-access interference. Robertson *et al.* [1] and Levitt [2] analyzed the performance degradation to SFH/M-ary orthogonal frequency-shift keyed (MFSK) from multitone interference for Ricean fading channels, where one or more symbols per hop are transmitted. References [3] and [4] overviewed the error probability performance of noncoherent frequency-shift keyed (FSK) M-ary systems under tone interference, where the channel fading is neglected.

The effectiveness of partial-band noise jamming as an electronic countermeasure against fast frequency-hopping (FFH) binary or M-ary frequency shift keyed (binary FSK or MFSK) modulation has been widely documented; see [5] for static channel results and [6] for Ricean fading channel results. The partial-band noise jammer can jam a portion of the total bandwidth of the FFH spectrum causing the bit-error rate (BER) to be inversely proportional to the signal-to-noise ratio, in contrast to the exponentially decreasing BER as a function of the signal-to-noise ratio in wide-band thermal noise.

The effect of multiple-access interference on FFH/MFSK noncoherent receivers, where one or more hops per symbol are transmitted, has been examined both for Rayleigh and Ricean fading channels in [7].

This paper analytically investigates the performance degradation to orthogonal noncoherent FFH MFSK due to multitone interference, where the effect of thermal wide-band noise is also taken into account. Multiple hops per symbol (constant hop rate) are considered with linear combining of square-law envelopes. The multiple equal-power interference tones are assumed to correspond to some of the possible FFH/MFSK orthogonal signaling tones. Furthermore, the channel for each hop band is modeled as a slowly fading Ricean process [11]. For multitone jamming, a broad range of channel fading is possible. Therefore, it is assumed that both the signal tone and the multiple interference tones are independently affected by channel fading.

The paper is organized as follows. A noncoherent FFH/MFSK system with square-law envelope detection and channel model is presented in Section II. In Section III, the analysis techniques used in this paper are described. Section IV shows the detailed effect of multitone interference fading on the system performance under various conditions. The results are summarized in Section V.

II. Model

A. FFH/MFSK System Model

We consider a communication system whose fundamental requirement is to transmit binary source information over the channel by means of MFSK, where one M-ary symbol is represented by one of $M = 2^n$ orthogonal tones. M is the order of MFSK modulation and n represents the number of bits per transmitted symbol. The binary input data have a period of T_b. The symbol duration is $T_s = nT_b$. Finally, the symbols are mixed with FH tone of frequency f_h for duration T_h. In this FFH/MFSK system, L frequency hops occur for each MFSK symbol; each symbol is partitioned into L independent transmissions of duration $T_h = T_s/L = nT_b/L$. That is, the constant hop rate $R_h = L/T_s$.

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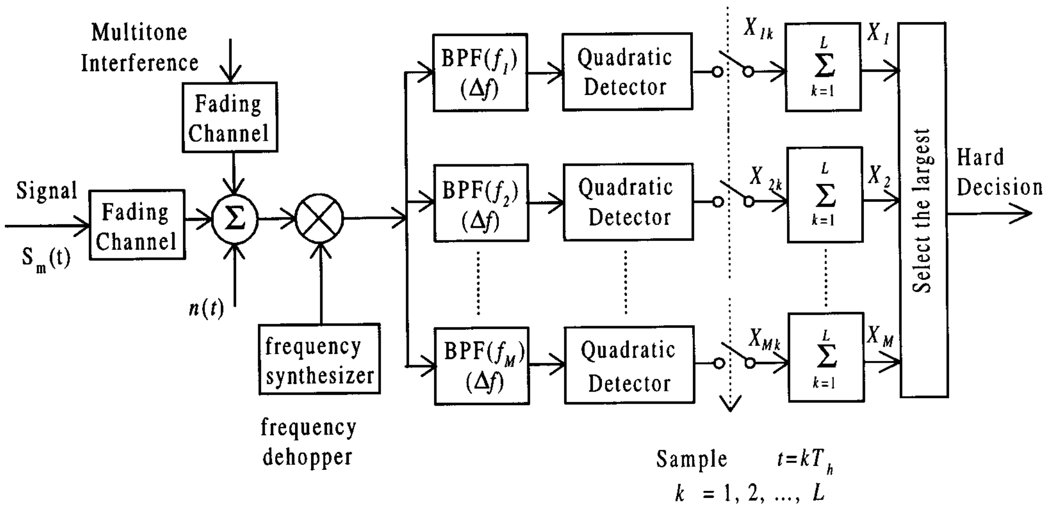

Fig. 1. A detailed model of the frequency-dehopper and the MFSK demodulator.

The transmitted signal is given by

$$S_m(t) = \mathrm{Re}\left\{\sqrt{2P}\,\exp\lfloor j\left\{2\pi\left[f_{h_i}+(m-1)\Delta f\right]t+\theta_m\right\}\rfloor\right\},\quad m = 1, 2, \cdots, M \tag{1}$$

where $\mathrm{Re}\{\bullet\}$ stands for real part and $j=\sqrt{-1}$, P is transmitted signal power, Δf is chosen as the hop rate R_h so that these tones are orthogonal, f_{h_i}, $i = 1, 2, \cdots, N$, designates the possible discrete frequency available for hopping, and θ_m is unknown phase. It is assumed that f_{h_i} and θ_m remain constant during a given hop interval T_h.

The receiver (Fig. 1) consists of a frequency dehopper, a bank of M matched filters and quadratic detectors. The output of each filter/detector is synchronously sampled every T_h and random variables X_{mk}, $k = 1, 2, \cdots, L$, are obtained. The L random variables at each branch are summed to obtain decision variables $X_m = \sum_{k=1}^{L} X_{mk}$, $m = 1, \cdots, M$. Finally, which is the largest will be decided.

B. Channel Model

The total bandwidth W_T of the FFH/MFSK system can be divided into N FH bands (W_{band}) and each FH band can be further divided into M bins. Therefore, $W_T = N \times W_{\mathrm{band}} = M \times N \times \Delta f$. It is assumed that the signal fades independently during each hop. In making this assumption, we are implying that the total hopping band is much wider than the coherence bandwidth of the channel and the fading channel is characterized by frequency-selective fading. Thus, similar to [8], we may wish to design the hopping pattern to satisfy that the smallest spacing between the FH bands used by the hops of one symbol is larger than the coherence bandwidth of the channel. In addition, it is assumed that the instantaneous bandwidth on a single hop is much less than the coherence bandwidth of the channel. That is, the channel for each hop is modeled as a frequency-nonselective, slowly fading Ricean process. "Slowly" means that the fading is constant during a hop interval. As a result, the signal amplitude can be modeled as a Ricean random variable that remains constant at least for the duration of a single hop.

The signal and jamming tones may have different fading statistics because they have different transmission paths. Therefore, it is assumed that channel fading characteristics are independent for the signal tone and interference tone. By modeling the channel as a Ricean fading channel, a general result is obtained.

C. Interference Model

It is assumed that the channel is jammed by an intentional multitone jammer. The jammer's strategy is to choose the number and the distribution of jamming tones that will cause maximum degradation to the communicator's performance. It is possible to distribute the tone jamming noncontiguously or contiguously over W_T in [4, Fig. 2]. From the jammer's viewpoint, the contiguous distribution is not an effective strategy. A more effective jamming strategy is to distribute the jamming tones noncontiguously at random with separation that is not smaller than the FH bandwidth over the total NW_{band}. That implies that at most one of M channels of the receiver is jammed by an interference tone during one hop. Consequently, the maximum number of interfering tones corresponds to the total number of FH bands, and $N \geq q \geq 1$, where q is the number of interference tones. In fact, [9] has shown that this multitone jammer distribution is related to the worst-case multitone jamming for a static channel.

The jammer is assumed to have a total jamming power P_{JT}, which is transmitted in a total of q equal power interfering tones spread randomly over the spread spectrum bandwidth of the FFH/MFSK system. Hence, the power of a single interfering tone is $P_J = P_{JT}/q$. It is also assumed that the multiple interfering tones are transmitted at frequencies corresponding exactly to the $M \times N$ FFH/MFSK signal tones. Fig. 2 shows typical signal and multitone interference spectrum. The thermal noise is modeled as additive white Gaussian noise with two-sided spectral density $N_0/2$.

III. Analysis

In this section, the bit-error probability for orthogonal, noncoherent FFH/MFSK communication systems with multitone in-

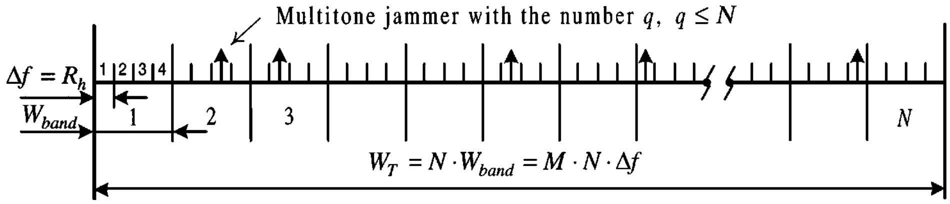

Fig. 2. Typical signal and multitone interference spectrum. A 4-ary signal is used so that the signal will fall in one of the four signal bins shown on each line.

terference is calculated. The bit-error probability is dependent on the signal-to-multitone power ratio P_S/P_{JT}, the bit energy to thermal noise density ratio E_b/N_0, the number of interfering tones q, the number of hops per symbol L, the modulation order M, the channel fading characteristics of the information signal, and multitone interference.

To obtain the symbol-error probability P_{SE}, we need to average the conditional symbol-error probability over all of possible multitone jamming pattern combinations. Let l_0 denote the number of hops which are not jammed by any interference tone during the observed symbol interval and let l_s denote the number of hops during which the channel containing the signal is jammed. In addition, for the other $M-1$ channels (i.e., channels not containing the signaling tone), m_0 is defined as the number of channels, which are not jammed over all L hops and $m_1, m_2, \cdots, m_L$ are defined as the number of channels, which have one hop, two hops,$\cdots$, L hops, respectively, jammed. Each hop is jammed at most by one interference tone. Hence

$$l_0 + l_S + \sum_{l=1}^{L} (m_l \times l) = L \tag{2a}$$

and

$$\sum_{l=0}^{L} m_l = M - 1 \tag{2b}$$

where all of l_0, l_S, and m_l are positive integers. It is assumed the vector

$$K = (l_0,\, l_S,\, m_0,\, m_1,\, \cdots m_L) \tag{2c}$$

satisfies the relation (2a) and (2b). Therefore, any possible jamming pattern in the M channels over L hops can be described by a corresponding vector K. Each vector K results in a different conditional error probability and we average the conditional symbol-error probability over all the possible combinations to obtain the unconditional symbol-error probability, which is given by

$$P_{\mathrm{SE}} = \sum_{K} P_r(K) \times P_{SE|K} \tag{3}$$

where $P_r(K)$ is the probability of the jamming pattern K, and $P_{SE|K}$ is the symbol-error probability conditioned on the jamming pattern K. $P_r(K)$ and $P_{SE|K}$ will be presented in the following.

A. Probability of the Jamming Pattern K

Since the FFH/MFSK receiver observes the output of the M receiver channels over L hops for the symbol decision process, all possible two dimensional assignments of the jamming patterns in the M channels over L hops must be considered.

Note that there are

$$\binom{M-1}{m_0,\, m_1,\, \cdots m_L}$$

ways to form vector K over the M channels and

$$\binom{L}{l_0,\, l_s,\, \underbrace{1, \cdots 1}_{m_1},\, \underbrace{2, \cdots 2}_{m_2},\, \cdots \underbrace{L, \cdots L}_{m_L}}$$

ways to form vector K over the L hops. In addition, the probability that all of M channels are not jammed by interference tone during one hop is $1-q/N$, and the probability that the specific one of M channels during one hop is jammed is $q/(NM)$. Therefore, $P_r(K)$ is given by

$$\begin{aligned} P_r(K) &= \binom{L}{l_0,\, l_s,\, \underbrace{1, \cdots 1}_{m_1},\, \underbrace{2, \cdots 2}_{m_2},\, \cdots \underbrace{L, \cdots L}_{m_L}} \\ &\quad \cdot \binom{M-1}{m_0,\, m_1,\, \cdots m_L} \cdot \left(1 - \frac{q}{N}\right)^{l_0} \cdot \left(\frac{q}{MN}\right)^{L-l_0} \\ &= \frac{L!}{l_0! \cdot l_s! \cdot \prod_{l=1}^{L} (l!)^{m_l}} \cdot \frac{(M-1)!}{\prod_{l=0}^{L} m_l!} \cdot \left(1 - \frac{q}{N}\right)^{l_0} \\ &\quad \cdot \left(\frac{q}{MN}\right)^{L-l_0}. \end{aligned} \tag{4}$$

B. Symbol-Error Probability, Conditioned on the Jamming Pattern K

As shown in Fig. 1, $\{X_m; 1 \le m \le M\}$ is the set of decision random variables, where m means the mth receiver channel. The symbol-error probability conditioned on the jamming pattern K can be obtained by assuming that the desired signal is present in channel 1 of the receiver and the channel m, $2 + \sum_{i=0}^{l} m_i - m_l \le m \le 1 + \sum_{i=0}^{l} m_i$, is jammed by the interference tone over l out of L hops during the observed symbol interval. For convenience in the analysis we have numbered the nondesired-signal channels in groups having a common number of jammed hops. For example, if $m_1 = 3$ and $m_0 = 0$ then the three channels with one hop jammed are numbered 2, 3, and 4. Thus, $X_{m|K}$ can be rewritten as

$$X_{1|K} = X_{1|l_S} \tag{5a}$$

and

$$X_{m|K} = X_{m|l}, \quad 2 + \sum_{i=0}^{l} m_i - m_l \le m \le 1 + \sum_{i=0}^{l} m_i. \quad (5b)$$

Note that $X_m, m \neq 1$, does not contain the desired signal. Therefore, the symbol-error probability conditioned on the jamming pattern K is given by

$$\begin{aligned} P_{SE|K} &= P_r\left[\bigcup_{m=2}^{M} (X_{1|K} < X_{m|K})\right] \\ &= 1 - P_r\left[\bigcap_{m=2} M(X_{1|K} > X_{m|K})\right] \\ &= 1 - P_r\left\{\bigcap_{l=0}^{L}\left[\bigcap_{m=2+\sum_{i=0}^{l} m_i - m_l}^{1+\sum_{i=0}^{l} m_i} \right.\right. \\ &\qquad \left.\left. \cdot\ (X_{1|l_S} > X_{m|l})\right]\right\} \end{aligned} \quad (6)$$

where $\bigcup$ represents "or" and $\bigcap$ represents "and."

It is assumed that $\{f_{X_m}(x_m)\}$ are the probability density functions of the random variables $\{X_m\}$. Since the outputs of each channel of the FFH/MFSK receiver are assumed to be independent and since each $X_{m|l}$, $2 + \sum_{i=0}^{l} m_i - m_l \le m \le 1 + \sum_{i=0}^{l} m_i$, is assumed to be identical, (6) can be rewritten as

$$\begin{aligned} P_{SE|K} &= 1 - \int_0^\infty f_{X_1}(x_1|l_S) \prod_{l=0}^{L} \\ &\quad \cdot \left[\int_0^{x_1} f_{X_m}(x_m|l)dx_m\right]^{m_l} dx_1 \\ &= 1 - \int_0^\infty f_{X_1}(x_1|l_S) \prod_{l=0}^{L} [1 - z_l]^{m_l} dx_1, \end{aligned} \quad (7)$$

where z_l is defined as

$$z_l = \int_{x_1}^{\infty} f_{X_m}(x_m|l)dx_m. \quad (8)$$

Since most often, $0 < z_l \ll 1$, $P_{\mathrm{SE}}|_K$can be simplified, using the following inequality:

$$\prod_{l=0}^{L} (1 - z_l)^{m_l} \ge \prod_{l=0}^{L} (1 - m_l z_l) \ge 1 - \sum_{l=0}^{L} m_l z_l. \quad (9)$$

Therefore, the upper bound of $P_{SE|K}$is given by

$$\begin{aligned} P_{SE|K} &\le 1 - \int_0^\infty f_{X_1}(x_1|l_S) \\ &\quad \cdot \left[1 - \sum_{l=0}^{L} m_l \int_{x_1}^{\infty} f_{X_m}(x_m|l)dx_m\right] dx_1 \\ &= \sum_{l=0}^{L} \left[m_l \int_0^\infty \int_0^\infty f_{X_1} \right. \\ &\quad \left. \cdot (x_1|l_S) f_{X_m}(x_1 + x_m|l) dx_1 dx_m\right]. \end{aligned} \quad (10)$$

C. Probability Density Functions $f_{X_1}(x_1|l_S)$ and $f_{X_m}(x_m|l)$

As shown from Fig. 1, the random variable X_m that represents the output of the channel after diversity combining is given as

$$X_m = \sum_{k=1}^{L} X_{mk} \quad (11)$$

where X_{mk}, $1 \le k \le L$, is the mth channel output of the receiver from hop k of a symbol. This channel either contains the signal or not and either is jammed or not by an interference tone during the hop. The former case will be considered first. When the channel containing the signal is jammed by a interference tone during the hop k, it will be assumed that the frequencies of the signal tone and interference tone are identical and their phase difference is a random variable, uniformly distributed in $[0, 2\pi)$. Thus, the sum of the signal tone and interference tone can be expressed as a single composite sine wave. The power of the composite signal at the channel output has the value

$$a^2 = a_S^2 + a_J^2 + 2a_S a_J \cos\theta \quad (12)$$

where a_S^2 and a_J^2 are the powers of the signal and interference tone, respectively, at the output of the detector and θ is the phase difference between the signal tone and interference tone. Since the random variable X_{1k} is the sum of composite signal and the receiver background thermal noise (modeled by a zero-mean, Gaussian process) at the channel output for hop k of a symbol, X_{1k} has the probability density function of the square of the envelope of a sine wave with random phase plus a narrowband Gaussian process. The result is a special case of the noncentral chi-squared distribution. For a value of composite signal amplitude $\sqrt{2}\,a$, the probability density function, conditioned on a, is given by

$$\begin{aligned} f_{X_{1k}}(x_{1k}|\text{hop jammed}, a) &= \frac{1}{2\sigma_N^2} \exp\left(-\frac{x_{1k} + 2a^2}{2\sigma_N^2}\right) I_0 \\ &\quad \cdot \left(\frac{a\sqrt{2x_{1k}}}{\sigma_N^2}\right), \qquad x_{1k} \ge 0 \end{aligned} \quad (13)$$

where $I_0(\bullet)$ is the modified Bessel function of the first kind and zero order, and $\sigma_N^2 = N_0 \Delta f = N_0 R_h$ is the thermal noise power. The conditional characteristic function, conditioned on a, corresponding to this pdf is

$$\begin{aligned} &\psi_{X_{1k}}(jv|\text{hop jammed}, a) \\ &\quad = \frac{1}{1 - j2\sigma_N^2 v} \exp\left(\frac{j2a^2 v}{1 - j2\sigma_N^2 v}\right) \\ &\quad = \frac{1}{1 - j2\sigma_N^2 v} \exp\left(\frac{j2(a_S^2 + a_J^2 + 2a_S a_J \cos\theta)v}{1 - j2\sigma_N^2 v}\right). \end{aligned} \quad (14)$$

In order to obtain the unconditional characteristic function for X_{1k}, (14) is averaged over a, given by (12). Averaging (14) over a implies averaging it over a_J, a_S and θ, respectively. Thus,

$$\begin{aligned} &\psi_{X_{1k}}(jv|\text{hop jammed}) \\ &\quad = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty f_{A_S}(a_S) \int_0^\infty f_{A_J}(a_J) \\ &\qquad \cdot \psi_{X_{1k}}(jv|\text{hop jammed}, a)\, da_J da_S d\theta. \end{aligned} \quad (15)$$

Since the Ricean fading is assumed for both signal and multitone interference, the amplitude probability density functions of the signal tone and the interference tones are given by

$$f_{A_S}(a_s) = \frac{a_S}{\sigma_S^2} \exp\left(-\frac{a_S^2+\alpha_S^2}{2\sigma_S^2}\right) I_0\left(\frac{a_S\alpha_S}{\sigma_S^2}\right), \quad a_S \geq 0 \tag{16}$$

$$f_{A_J}(a_J) = \frac{a_J}{\sigma_J^2} \exp\left(-\frac{a_J^2+\alpha_J^2}{2\sigma_J^2}\right) I_0\left(\frac{a_J\alpha_J}{\sigma_J^2}\right), \quad a_J \geq 0 \tag{17}$$

where α_{S^2} and $2\sigma_{S^2}$ are the powers of the direct and diffuse component of the signal, respectively. α_{J^2} and $2\sigma_{J^2}$ are the powers of the direct and diffuse component of the interference tone, respectively. The total average power of signal is $P_S = \alpha_{S^2} + 2\sigma_{S^2}$ and the total average power of one interference tone is $P_J = P_{JT}/q = \alpha_{J^2} + 2\sigma_{J^2}$. Both of the average powers are assumed to remain constant from hop to hop. Therefore, from the above, (15) becomes

$$\begin{aligned}
&\psi_{X_{1k}}(jv|\text{hop jammed}) \\
&= \frac{1}{1-j2\sigma_N^2 v} \int_0^\infty f_{A_S}(a_S) \int_0^\infty f_{A_J}(a_J) \\
&\quad \cdot \exp\left(\frac{j2(a_S^2+a_J^2)v}{1-j2\sigma_N^2 v}\right) \\
&\quad \cdot \left\{\int_0^{2\pi} \frac{1}{2\pi} \exp\left(\frac{j4a_S a_J v}{1-j2\sigma_N^2 v} \cos\theta\right) d\theta\right\} da_J da_S \\
&= \frac{1}{1-j2\sigma_N^2 v} \int_0^\infty f_{A_S}(a_S) \int_0^\infty f_{A_J}(a_J) \\
&\quad \cdot \exp\left(\frac{j2(a_S^2+a_J^2)v}{1-j2\sigma_N^2 v}\right) J_0\left(\frac{4a_S a_J v}{1-j2\sigma_N^2 v}\right) da_J da_S
\end{aligned} \tag{18}$$

where $J_0(\bullet)$ is the zeroth order Bessel function, given by

$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp(jx \cos(\theta)) d\theta. \tag{19}$$

Substituting (17) into (18), we have

$$\begin{aligned}
&\psi_{X_{1k}}(jv|\text{hop jammed}) \\
&= \frac{1}{1-j2\sigma_N^2 v} \int_0^\infty f_{A_S}(a_S) \\
&\quad \cdot \left\{\int_0^\infty \frac{a_J}{\sigma_J^2} \exp\left(\frac{j2(a_S^2+a_J^2)v}{1-j2\sigma_N^2 v} - \frac{a_J^2+\alpha_J^2}{2\sigma_J^2}\right)\right. \\
&\quad \left. \cdot I_0\left(\frac{a_J\alpha_J}{\sigma_J^2}\right) J_0\left(\frac{4a_S a_J v}{1-j2\sigma_N^2 v}\right) da_J\right\} da_S.
\end{aligned} \tag{20}$$

According to [12, eq. (6.633.4)], the following result is quoted:

$$\begin{aligned}
&\int_0^\infty x \exp\left(-\lambda x^2\right) I_0(\beta x) J_0(\gamma x) dx \\
&= \frac{1}{2\lambda} \exp\left(\frac{\beta^2-\gamma^2}{4\lambda}\right) J_0\left(\frac{\beta\gamma}{2\lambda}\right), \qquad \mathrm{Re}\{\lambda\} > 0.
\end{aligned} \tag{21}$$

Letting

$$\lambda = \frac{1-j2(2\sigma_J^2+\sigma_N^2)v}{2\sigma_J^2(1-j2\sigma_N^2 v)} \tag{22a}$$

$$\beta = \frac{\alpha_J}{\sigma_J^2} \tag{22b}$$

and

$$\gamma = \frac{4a_S v}{1-j2\sigma_N^2 v} \tag{22c}$$

(20) is simplified by means of (21)

$$\begin{aligned}
\psi_{X_{1k}}(jv|\text{hop jammed}) &= \frac{1}{1-j2\left(2\sigma_J^2+\sigma_N^2\right)v} \int_0^\infty f_{A_S}(a_S) \\
&\quad \cdot \exp\left(\frac{j2(a_S^2+\alpha_J^2)v}{1-j2(2\sigma_J^2+\sigma_N^2)}\right) \\
&\quad \cdot J_0\left(\frac{4a_S\alpha_J v}{1-j2(2\sigma_J^2+\sigma_N^2)v}\right) da_S.
\end{aligned} \tag{23}$$

Similarly, Substituting (16) into (23) and integrating, we have

$$\begin{aligned}
\psi_{X_{1k}}(jv|\text{hop jammed}) &= \frac{1}{1-j2(2\sigma_S^2+2\sigma_J^2+\sigma_N^2)v} \\
&\quad \cdot \exp\left[\frac{j2(\alpha_S^2+\alpha_J^2)v}{1-j2(2\sigma_S^2+2\sigma_J^2+\sigma_N^2)v}\right] \\
&\quad \cdot J_0\left[\frac{4\alpha_S\alpha_J v}{1-j2(2\sigma_S^2+2\sigma_J^2+\sigma_N^2)v}\right].
\end{aligned} \tag{24}$$

When the channel containing the signal is not jammed by interference tone during hop k, substituting $\alpha_J = \sigma_J = 0$ into (24), one obtains

$$\begin{aligned}
\psi_{X_{1k}}(jv|\text{hop not jammed}) &= \frac{1}{1-j2(2\sigma_S^2+\sigma_N^2)v} \\
&\quad \cdot \exp\left[\frac{j2\alpha_S^2 v}{1-j2(2\sigma_S^2+\sigma_N^2)v}\right].
\end{aligned} \tag{25}$$

Since the interference tone and noise are assumed independent from hop to hop, the random variables are independent from hop to hop. Therefore, when the channel containing the signal is jammed over l_S hops and is not jammed over $L - l_S$ hops during the observed symbol interval, the conditional characteristic function of the random variable X_1, defined by (11) is

$$\begin{aligned}
\psi_{X_1}(jv|l_S) &= \left(\frac{1}{1-j2(2\sigma_S^2+\sigma_N^2)v}\right)^{L-l_S} \\
&\quad \cdot \left(\frac{1}{1-j2(2\sigma_S^2+2\sigma_J^2+\sigma_N^2)v}\right)^{l_S} \\
&\quad \cdot \exp\left(\frac{j2\alpha_S^2(L-l_S)v}{1-j2(2\sigma_S^2+\sigma_N^2)v}\right. \\
&\qquad \left. + \frac{j2(\alpha_S^2+\alpha_J^2)l_S v}{1-j2(2\sigma_S^2+2\sigma_J^2+\sigma_N^2)v}\right) \\
&\quad \cdot \left[J_0\left[\frac{4\alpha_S\alpha_J}{1-j2(2\sigma_S^2+2\sigma_J^2+\sigma_N^2)v}\right]\right]^{l_S}.
\end{aligned} \tag{26}$$

Similarly, using the above result, the conditional characteristic function of the random variable $X_m|_l$, $m \neq 1$, can be obtained. Since $X_m|_l$, $m \neq 1$, does not contain the signal and is jammed over l hops during the observed symbol interval, substituting $\alpha_S = \sigma_S = 0$ and l_S with l in (26), the conditional characteristic function of the random variable $X_m|_l$ is given by

$$\psi_{X_m}(jv|l) = \left(\frac{1}{1 - j2\sigma_N^2 v}\right)^{L-l} \left(\frac{1}{1 - j2(2\sigma_J^2 + \sigma_N^2)v}\right)^{l} \cdot \exp\left(\frac{j2\alpha_J^2 lv}{1 - j2(2\sigma_J^2 + \sigma_N^2)v}\right). \quad (27)$$

Finally, the conditional symbol-error probability in (10), conditioned on the jamming pattern K, can be obtained by means of the convolution function

$$\int_0^{\infty} f_{X_1}(x_1|l_S) f_{X_m}(x_1 + x_m|l)dx_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_{X_1}(-jv|l_S)\psi_{X_m}(jv|l)\exp(-jvx_m)dv. \quad (28)$$

Substituting (28) into (10), $P_{\mathrm{SE}}|_K$ is given by

$$P_{SE|K} = \frac{1}{2\pi}\sum_{l=0}^{L}\left[m_l \int_0^{\infty}\int_{-\infty}^{\infty} \psi_{X_1}(-jv|l_S) \cdot \psi_{X_m}(jv|l)\exp(-jvx_m)dv dx_m\right]. \quad (29)$$

By means of the following equation in the table of [12, eq. (17.23)]

$$\int_0^{\infty} \exp(-jvx)dx = \pi\delta(v) + \frac{1}{jv} \quad (30)$$

where $\delta(v)$ represents the conventional delta function, we have

$$P_{SE|K} = \frac{1}{2\pi}\sum_{l=0}^{L}\left\{m_l \left[\pi + \int_{-\infty}^{\infty} \frac{1}{jv}\psi_{X_1} \cdot (-jv|l_S)\psi_{X_m}(jv|l)dv\right]\right\}. \quad (31)$$

Substituting (31) into (3), the symbol-error probability is given by

$$P_{\mathrm{SE}} = \frac{1}{2\pi}\sum_{K}\sum_{l=0}^{L}\left\{m_l P_r(K)\left[\pi + \int_{-\infty}^{\infty}\frac{1}{jv}\psi_{X_1} \cdot(-jv|l_S)\psi_{X_m}(jv|l)dv\right]\right\}. \quad (32)$$

For numerical computation of (32), the following two points should be considered.

1) Since the imaginary component of the integrated function in (32) is an odd function, the integration of its imaginary component is zero.
2) $v = 0$ is a removable singular point of the integrated function, since

$$\lim_{v\to 0}\left\{\mathrm{Re}\left[\frac{1}{jv}\psi_{X_1}(-jv|l_S)\psi_{X_m}(jv|l)\right]\right\}$$

can be shown to be a limited value.

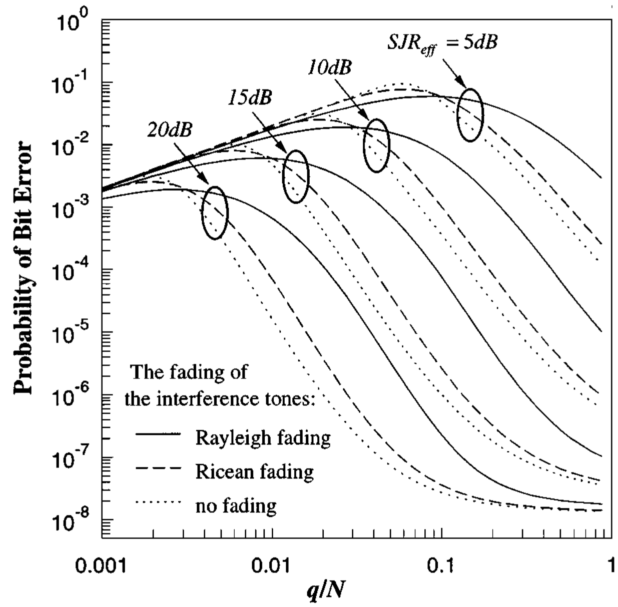

Fig. 3. Probability of bit error versus the ratio of the number of interference tones to the total FH bands (q/N) for $L = 4$, $M = 4$ and $E_b/N_0 = 16$ when the desired signal experiences Ricean fading.

Therefore, P_{SE} can be further simplified by

$$P_{\mathrm{SE}} = \frac{1}{2\pi}\sum_{K}\sum_{l=0}^{L}\left\{m_l P_r(K)\left[\pi + 2 \cdot \int_{0^+}^{\infty}\mathrm{Re}\left\{\frac{1}{jv}\psi_{X_1}(-jv|l_S)\psi_{X_m}(jv|l)\right\}dv\right]\right\} \quad (33)$$

where 0^+ stands for an extremely small positive value. In the numerical integration of (33), 0^+ should be set as small as possible, for example, $0^+ = 10^{-30}$.

Once the symbol-error probability P_{SE} after hard decision has been obtained, assuming symbol errors are random when interleaving is used, the corresponding BER, P_{BE}, after hard decision is given by

$$P_{\mathrm{BE}} = \frac{M/2}{M-1}P_{\mathrm{SE}}. \quad (34)$$

IV. Numerical Results

The effect of the fading of multitone interference on FFH/MFSK system performance is numerically investigated for various jamming tone power levels and numbers. The effective signal power to multitone interference power ratio SJR_{eff} is defined as $SJR_{\mathrm{eff}} = (P_S/(P_{JT}/N)) = (P_S/P_j)\cdot(N/q)$. With this artifice, the numerical results will be shown without using a particular number of hopping bands N or a particular number of interference tones q.

Firstly, the performance of FFH/MFSK system with multitone interference as a function of the ratio (q/N) of the number of the interference tones to the total FH bands is shown in Fig. 3 for various values of the effective signal power to multitone interference power ratio ($SJR_{\mathrm{eff}} = 5, 10, 15,$ and 20 20 dB), respectively. The number of hops per symbol $L = 4$, the modulation order $M = 4$ and $E_b/N_0 = 16$ dB. The signal experiences

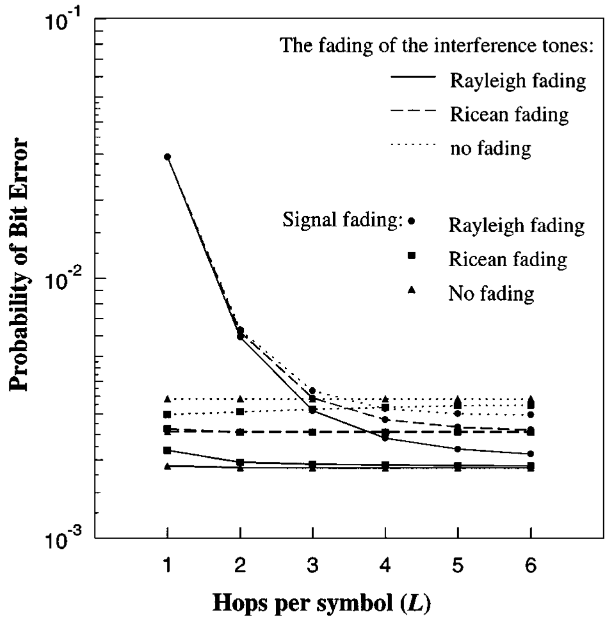

Fig. 4. Probability of bit error versus the number of hops/symbol under conditions of worst-case multitone interference for $SJR_{\rm eff} = 20$ dB, $E_b/N_0 = 16$ dB and $M = 4$.

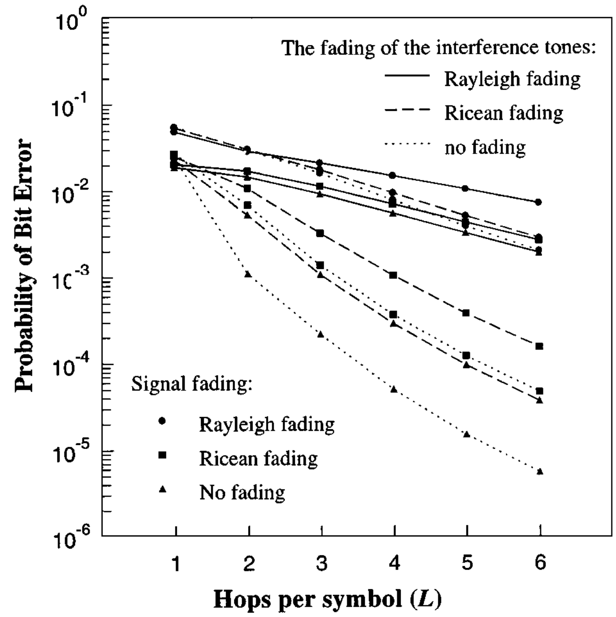

Fig. 5. Probability of bit error versus the number of hops/symbol for $SJR_{\rm eff} = 10$ dB, $E_b/N_0 = 16$ dB and $M = 4$ when $q/N = 0.1$.

Ricean fading with $\alpha_S^2/2\sigma_S^2 = 10$ dB. The multitone interference experiences either nearly no fading with $\alpha_J^2/2\sigma_J^2 = 40$ dB, or Ricean fading with $\alpha_J^2/2\sigma_J^2 = 10$ dB, or nearly Rayleigh fading with $\alpha_J^2/2\sigma_J^2 = -20$ dB. It is shown from this figure that for a given $SJR_{\rm eff}$ there is a worst-case value ($q_{\rm worst}/N$) of q/N, which maximizes the bit-error probability. $q_{\rm worst}/N$ is a function of $SJR_{\rm eff}$ and specifically, it is about 0.1, 0.03, 0.01, and 0.003 for $SJR_{\rm eff} = 5, 10, 15,$ and 20 dB, respectively, which shows that $q_{\rm worst}/N$ is inversely proportional to $SJR_{\rm eff}$. It can also be seen that the worst case performance ($q/N = q_{\rm worst}/N$) of the FFH/MFSK receiver is very similar whatever fading the interference tones experience. However, when $q/N > q_{\rm worst}/N$, the performance of FFH/MFSK receiver is much better with no fading of interference tones than with Rayleigh fading of interference tones.

The effects of channel fading on the system performance as a function of the number of hops per symbol L are illustrated in Fig. 4 for $SJR_{\rm eff} = 20$ dB under the conditions of worst-case multitone interference ($q = q_{\rm worst}$) and in Fig. 5 for $SJR_{\rm eff} = 10$ dB and a fixed value of q ($q/N = 0.1$), respectively. It can be seen from Fig. 4 that under the conditions of worst-case multitone interference, a larger L improves the system performance when the desired signal experiences Rayleigh fading. This is because $q_{\rm worst}/N$ is large when L is small (i.e., $q_{\rm worst}/N$ approaches one when $L = 1$). In this case, there are a large number of multitones with small power each. A larger L is helpful in reducing the fluctuation of the desired signal at the outputs of the square law summers and makes $q_{\rm worst}/N$ decrease significantly. Thus, the performance is improved with L increasing. However, when the desired signal experiences Ricean fading, a higher diversity is not helpful. Even when the multitone is not fading, the performance becomes slightly worse with L increasing. The reason is that $q_{\rm worst}/N$ is very small whatever L is (i.e., $q_{\rm worst}/N = 0.003$ shown in Fig. 3). For given total jamming power, it means that the power of each tone is slightly larger than that of the desired signal. In this case, the fluctuation of the desired signal is helpful to reduce the probability of error when the signal is hit by an interference tone without fading. Since a larger L reduces the fluctuation of the desired signal, the BER's increases slightly when L increases. It is also seen from Fig. 4 that a larger L does not affect the worst case performance when the desired signal is no fading. It is shown from Fig. 5 that for a fixed value of q ($q/N = 0.1$), a larger value L always improves performance no matter what fading the desired signal experiences. Furthermore, when the interference tones experience channel fading, the improvement in system performance due to larger value of L is reduced significantly. This trend is more pronounced with no fading of the signal than with Rayleigh fading of the signal.

The FFH/MFSK system performance as a function of the direct-to-diffuse component power ratio ($R_J = \alpha_J^2/2\sigma_J^2$) of the interference tone is shown in Fig. 6 when the desired signal experiences Ricean fading ($a_s^2/2\sigma_s^2 = 10$ dB). It can be seen that when R_J is less than 0 dB or larger than 20 dB, the system performance is smooth for given values of $SJR_{\rm eff}$. However, when 0 dB $< R_J <$ 20 dB, the performance is sensitive to the fading of interference tones. In addition, the reduction of probability of error due to larger value of R_J is more significant for larger $SJR_{\rm eff}$than for small $SJR_{\rm eff}$.

In Fig. 7, P_e is plotted versus $SJR_{\rm eff}$ for $q/N = 0.1$ and 0.01, respectively, when both of the signal and interference tone experience Ricean fading ($\alpha_S^2/2\sigma_S^2 = \alpha_J^2/2\sigma_J^2 = 10$ dB). For comparison, computer simulation results are provided with the same parameters. It is seen that when $SJR_{\rm eff}$ is small, there is no difference between simulation and numerical results and the upper bound (10) is very tight. However, when $SJR_{\rm eff} \geq$ 25 dB, numerical results are slightly larger than simulation results. Therefore, our analytical results are quite accurate. On the other hand, since analytical results are involved with only

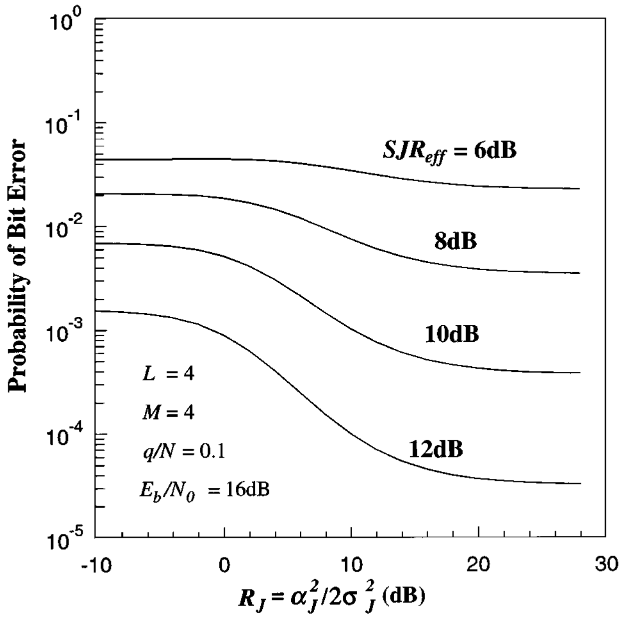

Fig. 6. Probability of bit error versus the direct-to-diffuse component power ratio of the interference tone with Ricean fading of the signal.

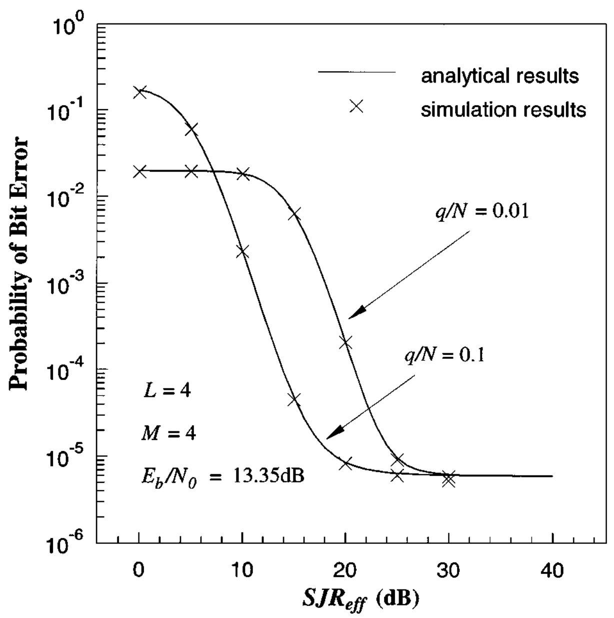

Fig. 7. Probability of bit error versus the effective signal power to multitone interference power ratio.

one level of integration in (33), numerical computation is much faster than simulation. For example, to get error probability of 10^{-5}, roughly 10^7 data symbols (or simulations) should be involved, which takes at least 100 times longer than numerical calculation.

V. Conclusions

In this paper, the performance of the multiple hops per symbol FFH MFSK noncoherent receiver with square-law envelope detection in the presence of multitone interference over a wide range of channel conditions has analytically been studied. The following conclusions have been drawn.

1) The worst case performance ($q/N = q_{\mathrm{worst}}/N$) of the FFH/MFSK receiver is very similar whatever fading the interference tones experience. However, when $q/N > q_{\mathrm{worst}}/N$, the performance of FFH/MFSK receiver is much better with no fading of interference tones than with Rayleigh fading of interference tones.
2) Under the conditions of worst-case multitone interference, a larger value of L improves the system performance when the desired signal experiences Rayleigh fading but is not helpful when the desired signal experiences Ricean fading or no fading. However, for a fixed value of q and $q/N > q_{\mathrm{worst}}/N$, a large L always improves performance no matter what fading the desired signal experiences. Furthermore, when the interference tones experience channel fading, the improvement in system performance due to larger value of L is reduced significantly.
3) When the direct-to-diffuse component power ratio $(R_J)$of the interference tone is larger than 0 dB and less than 20 dB, the FFH/MFSK system performance is sensitive to the fading of interference tones. In addition, the reduction of probability of error due to larger value of R_J is more significant for larger SJR_{eff} than for small SJR_{eff}.
4) Numerical computation of BER is much faster than simulation, since analytical results are involved only one level of integration.

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References

[1] R. C. Robertson and J. F. Sheltry, "Multiple tone interference of frequency-hopped noncoherent MFSK signals transmitted over Ricean fading channels," *IEEE Trans. Commun.*, vol. 44, pp. 867–875, July 1996.

[2] B. K. Levitt, "FH/MFSK performance in Multitone jamming," *IEEE J. Select. Areas Commun.*, vol. SAC-3, pp. 627–643, Sept. 1985.

[3] M. J. Massaro, "Error performance of M-ary noncoherent FSK in the presence of CW tone interference," *IEEE Trans. Commun.*, vol. COM-23, pp. 1367–1369, Nov. 1975.

[4] J. S. Bird and E. B. Felsted, "Antijam performance of fast frequency-hopped M-ary NCFSK-An overview," *IEEE J. Select. Areas Commun.*, vol. SAC-4, pp. 216–233, Mar. 1986.

[5] J. S. Lee, R. H. French, and L. E. Miller, "Probability of error analyzes of a BFSK frequency-hopping system with diversity under partial-band jamming interference—Part I: Performance of square-law linear combining soft decision receiver," *IEEE Trans. Commun.*, vol. COM-32, pp. 645–653, June 1984.

[6] C. D. Chung and P. C. Huang, "Effect of fading and partial-band noise jamming on a fast FH/BFSK acquisition receiver with noise-normalization combination," *IEEE Trans. Commun.*, vol. 44, pp. 94–104, Jan. 1996.

[7] J. Wang and M. Moeneclaey, "Multiple hopes/symbol FFH-SSMA with MFSK modulation and Reed–Solomon coding for indoor radio," *IEEE Trans. Commun.*, vol. 41, pp. 793–801, May 1993.

[8] C. M. Keller and M. B. Pursley, "Diversity combining for channels with fading and partial-band interference," *IEEE J. Select. Areas Commun.*, vol. SAC-5, pp. 248–260, Feb. 1987.

[9] S. W. Houston, "Modulation techniques for communications—Part 1: Tone and noise jamming performance of spread spectrum M-ary FSK and 2,4-ary DPSK waveforms," in *IEEE NAECON'75 Rec.*, 1975, pp. 51–58.

[10] T. A. Gulliver and E. B. Felstead, "Anti-jam by fast FH NCFSK-myths and realities," in *Conf. Rec. of the IEEE MILCOM'93*, Boston, MA, Oct. 1993, pp. 187–191.
[11] J. G. Proakis, *Digital Communication*, 2nd ed. New York: McGraw-Hill, 1989.
[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integration, Series and Productions*, 5nd ed. New York: Academic, 1996.

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