Fixed-Order Robust H_∞ Filter Design for Markovian Jump Systems With Uncertain Switching Probabilities

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***Abstract*—This paper discusses the fixed-order robust H_∞ filtering problem for a class of Markovian jump linear systems with uncertain switching probabilities. The uncertainties under consideration are assumed to be norm-bounded in the system matrices and to be elementwise bounded in the mode transition rate matrix, respectively. First, a criterion based on linear matrix inequalities is provided for testing the H_∞ filtering level of a filter over all the admissible uncertainties. Then, a sufficient condition for the existence of the fixed-order robust H_∞ filters is established in terms of the solvability of a set of linear matrix inequalities with equality constraints. To determine the filter, a globally convergent algorithm involving convex optimization is suggested. Finally, a numerical example is used to illustrate that the developed theory is more effective than the existing results.**

***Index Terms*—H_∞ filtering, linear matrix inequalities (LMIs), Markovian parameters, robust filtering.**

I. Introduction

ROBUST H_∞ filtering is a technique for estimating unavailable signals in systems. The aim is to design a filter such that 1) the filtering error system is robustly stable and 2) the H_∞-norm of the operator from noise signal to filtering error signal is less than a prescribed level. The system may consist of parameter uncertainties. The noise signal can be arbitrary but needs to be energy bounded. Compared with the classical Kalman filtering technique [1], which is based upon the noisy statistical characteristics, the robust H_∞ filtering technique is insensitive to the noise statistics, and hence is very appropriate to applications where the statistics of the noise signal cannot be known exactly [2]. This feature has motivated the study of robust H_∞ filtering problem for variant systems and a number of results on this topic have been reported in the literature (see [2]–[5] and the references therein).

Markovian jump linear systems (MJLSs) have attracted a great deal of attention since this class of systems is appropriate to describe dynamic systems with structures varying abruptly in a random way [6]. MJLSs can be regarded as a special class of hybrid systems with finite operation modes. An MJLS behaves as a deterministic linear system in each mode and switches from one mode to another at time points governed by a Markov process. A great number of control issues concerning MJLSs have been investigated (see [6]–[14] and the references therein). For filtering problem in continuous-time case, many results are also available. For example, the Kalman filtering problem was considered in [15] and [16] where the system matrices may have norm-bounded uncertainties, and the results are given in terms of coupled Riccati equations. A Kalman filter of reduced order was also suggested based upon algebraic Riccati equation approach in [17] where the system model is described in terms of Itô differential equations and the measured output is assumed to be noise free. On the other hand, the H_∞ filtering problem for MJLSs was tackled in [18] and was extended to the case involving parameter uncertainties in [19] by the same authors. Recently, [20] considered the robust H_∞ filtering problem for MJLSs with time delays. The results of [18]–[20] are given in terms of coupled linear matrix inequalities (LMIs). It is noticed that the robust H_∞ filters designed in [18]–[20] have either an observer structure [18] or a special full-order structure [19], [20] and the uncertainties concerned only exist in the system matrices [15], [16], [19], [20]. More recently, the reduced-order H_∞ filtering problem, where uncertainties do not appear, was studied in [21] and [22] in terms of coupled LMIs with matrix rank constraints. The filtering problem for nonlinear time-delayed Markovian jump systems was studied in [23] as well.

It is worth pointing out that the developed filtering techniques in [15]–[23] for MJLSs require that the mode transition rates are known exactly. However, these values often need to be measured in practice, and hence measurement errors are inevitable. As pointed out in [24], the measurement errors (also referred to as switching probability uncertainties) may lead to instability or at least degrade the performance of MJLSs. Therefore, it is important and necessary to consider the robust H_∞ filtering problem for MJLSs with uncertain switching probabilities. A model about the uncertain switching probabilities has been proposed in an elementwise way [9], [12] such that bounded uncertainties could appear in all the elements of the mode transition rate matrix. This elementwise model has been further studied in [24] for the robust stabilization problem for MJLSs by considering the probability constraints on rows of the mode transition rate matrix. In the present paper, we study the robust H_∞ filtering problem for MJLSs with elementwise switching probability uncertainties and adopt an improved bounding technique for the matrix inequalities which gives less conservative results than those in [9] and [12].

In this paper, we study the fixed-order robust H_∞ filtering problem for uncertain continuous-time Markovian jump linear systems. The uncertainties are assumed to be norm bounded in the system matrices and to be elementwise bounded in the mode transition rate matrix. We aim at designing a general dynamic

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filter of fixed order such that, over all the admissible uncertainties, the obtained filtering error system is quadratically mean square stable and the H_∞-norm of the operator from the exogenous noise signal to the filtering error signal is less than a prescribed level. The solution to the addressed problem is related to a set of coupled LMIs with a set of equality constraints. An effective algorithm involving convex optimization is used to construct the filter. Finally, a comparison with existing results is offered to illustrate the usefulness of the developed approach.

Notation: The notations in this paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $m \times n$ real matrices. $\mathbb{R}^+$ refers to the set of strictly positive real numbers. $\mathbb{S}^{n\times n}$ is the set of all $n \times n$ real symmetric positive definite matrices, and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I_n denotes the $n \times n$ identity matrix, and I refers to the identity matrix with compatible dimensions. All matrices are assumed to be compatible for algebraic operations when their dimensions are not explicitly stated. For a matrix $U \in \mathbb{R}^{m\times n}$ with $\mathrm{rank}(U) = r_u$, the full row rank matrix $U^\perp \in \mathbb{R}^{(m-r_u)\times m}$ denotes the orthogonal complement of U such that $U^\perp U = 0$. The superscript "T" stands for the transpose for vectors or matrices, and $\mathrm{trace}(\cdot)$ is the trace of a square matrix. Moreover, let $(\Omega, \mathcal{F}, \mathrm{P})$ be a complete probability space. $\mathrm{E}(\cdot)$ stands for the mathematical expectation operator. $\mathbb{L}_2^m[0,\infty)$ is the space of square-integrable m-dimensional vector functions over $[0,\infty)$. $\|\cdot\| \triangleq (\rho^T\rho)^{1/2}$ if ρ is a real vector, $\|\rho\|_2 \triangleq (\int_0^\infty \|\rho(t)\|^2 dt)^{1/2}$ if $\rho(\cdot) \in \mathbb{L}_2^m[0,\infty)$ and $\|\rho\|_2 \triangleq [\int_0^\infty \mathrm{E}(\|\rho(t)\|^2) dt]^{1/2}$ if $\rho(\cdot)$ is a stochastic process. In large matrix expressions, we have $(M\star) \triangleq (MM^T)$,

$$\begin{bmatrix} M_1 & M_2 \\ \bullet & M_3 \end{bmatrix} \triangleq \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$$

and $\mathrm{sym}(M) \triangleq M + M^T$.

II. Problem Formulation

Consider the following class of Markovian jump linear systems with uncertain switching probabilities defined on a complete probability space $(\Omega, \mathcal{F}, \mathrm{P})$:

$$\begin{cases} \dot{x}(t) = \hat{A}(\hat{r}(t))\, x(t) + \hat{B}(\hat{r}(t))\, w(t) \\ z(t) = \hat{C}_z(\hat{r}(t))\, x(t) + \hat{D}_z(\hat{r}(t))\, w(t) \\ y(t) = \hat{C}_y(\hat{r}(t))\, x(t) + \hat{D}_y(\hat{r}(t))\, w(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $t \geq 0$, is the system state, $w(t) \in \mathbb{R}^{n_w}$ is the exogenous nonzero noise which belongs to $\mathbb{L}_2^{n_w}[0,\infty)$, $z(t) \in \mathbb{R}^{n_z}$ is the signal to be estimated, and $y(t) \in \mathbb{R}^{n_y}$ is the measured output used to estimate the signal $z(t)$. The mode-jumping process $\{\hat{r}(t) : t \geq 0\}$ is a continuous-time, discrete-state homogeneous Markov process on the probability space, takes values in a finite state space $\mathcal{S} \triangleq \{1, 2, \ldots, s\}$, and has the mode transition probabilities

$$\Pr(\hat{r}(t+\delta t) = j | \hat{r}(t) = i) = \begin{cases} \hat{\pi}_{ij}\delta t + o(\delta t), & \text{if } j \neq i \\ 1 + \hat{\pi}_{ii}\delta t + o(\delta t), & \text{if } j = i \end{cases}$$

where $\delta t > 0$, $\lim_{\delta t\to 0}(o(\delta t)/\delta t) = 0$, and $\hat{\pi}_{ij} \geq 0$ $(i, j \in \mathcal{S}, j \neq i)$ denotes the switching rate from mode i at time t to mode j at time $t + \delta t$ and $\hat{\pi}_{ii} \triangleq -\sum_{j=1,j\neq i}^{s} \hat{\pi}_{ij}$ for all $i \in \mathcal{S}$. In each mode $i \in \mathcal{S}$, the matrices $\hat{A}_i \triangleq \hat{A}(\hat{r}(t) = i)$, $\hat{B}_i \triangleq \hat{B}(\hat{r}(t) = i)$, $\hat{C}_{zi} \triangleq \hat{C}_z(\hat{r}(t) = i)$, $\hat{D}_{zi} \triangleq \hat{D}_z(\hat{r}(t) = i)$, $\hat{C}_{yi} \triangleq \hat{C}_y(\hat{r}(t) = i)$, $\hat{D}_{yi} \triangleq \hat{D}_y(\hat{r}(t) = i)$ and the mode transition rate matrix $\hat{\Pi} \triangleq (\hat{\pi}_{ij}) \in \mathbb{R}^{s\times s}$ are unknown constant real matrices, but of the following forms of uncertainties, respectively:

$$\hat{A}_i = A_i + E_{ai}F_{ai}H_{ai} \quad \text{with} \quad F_{ai}^T F_{ai} \leq I \tag{2a}$$
$$\hat{B}_i = B_i + E_{bi}F_{bi}H_{bi} \quad \text{with} \quad F_{bi}^T F_{bi} \leq I \tag{2b}$$
$$\hat{C}_{zi} = C_{zi} + E_{czi}F_{czi}H_{czi} \quad \text{with} \quad F_{czi}^T F_{czi} \leq I \tag{2c}$$
$$\hat{D}_{zi} = D_{zi} + E_{dzi}F_{dzi}H_{dzi} \quad \text{with} \quad F_{dzi}^T F_{dzi} \leq I \tag{2d}$$
$$\hat{C}_{yi} = C_{yi} + E_{cyi}F_{cyi}H_{cyi} \quad \text{with} \quad F_{cyi}^T F_{cyi} \leq I \tag{2e}$$
$$\hat{D}_{yi} = D_{yi} + E_{dyi}F_{dyi}H_{dyi} \quad \text{with} \quad F_{dyi}^T F_{dyi} \leq I \tag{2f}$$
$$\hat{\Pi} = \Pi + \Delta\Pi \quad \text{with} \quad |\Delta\pi_{ij}| \leq 2\varepsilon_{ij}, \;\; \varepsilon_{ij} \geq 0, \;\; j \neq i \tag{2g}$$

where matrices A_i, B_i, C_{zi}, D_{zi}, C_{yi}, D_{yi}, E_{ai}, H_{ai}, E_{bi}, H_{bi}, E_{czi}, H_{czi}, E_{dzi}, H_{dzi}, E_{cyi}, H_{cyi}, E_{dyi}, H_{dyi} and $\Pi \triangleq (\pi_{ij})$ are known constant real matrices of appropriate dimensions. The matrices F_{ai}, F_{bi}, F_{czi}, F_{dzi}, F_{cyi}, F_{dyi}, $i \in \mathcal{S}$, and $\Delta\Pi = (\Delta\pi_{ij}) \triangleq (\hat{\pi}_{ij} - \pi_{ij})$ denote the uncertainties in the system matrices and in the mode transition rate matrix, respectively. In addition, matrix Π satisfies

$$\begin{cases} \pi_{ij} \geq 0, & \text{if } j \neq i \\ \pi_{ii} \triangleq -\sum_{j=1,j\neq i}^{s} \pi_{ij}, & \text{if } j = i \end{cases}$$

for all $i, j \in \mathcal{S}$ and can be thought of as the measured value of matrix $\hat{\Pi}$ in practice. The uncertainties defined in (2) are called admissible uncertainties for uncertain system (1), and uncertain system (1) is assumed to be robustly mean square stable over all the admissible uncertainties in (2).

We now consider a general fixed-order filter

$$\begin{cases} \dot{x}_f(t) = A_f(\hat{r}(t))\, x_f(t) + B_f(\hat{r}(t))\, y(t) \\ z_f(t) = C_f(\hat{r}(t))\, x_f(t) + D_f(\hat{r}(t))\, y(t) \end{cases} \tag{3}$$

where $x_f(t) \in \mathbb{R}^{n_f}$ is the filter state with $0 \leq n_f \leq n$, the filter output $z_f(t) \in \mathbb{R}^{n_z}$ is an estimation of the signal $z(t)$. Matrices $A_{fi} \triangleq A_f(\hat{r}(t) = i)$, $B_{fi} \triangleq B_f(\hat{r}(t) = i)$, $C_{fi} \triangleq C_f(\hat{r}(t) = i)$ and $D_{fi} \triangleq D_f(\hat{r}(t) = i)$ are to be determined for each mode $i \in \mathcal{S}$, and the filtering error is defined as $e(t) \triangleq z(t) - z_f(t)$.

Connecting filter (3) to uncertain system (1) yields the filtering error system

$$\begin{cases} \dot{x}_e(t) = \hat{A}_e(\hat{r}(t))\, x_e(t) + \hat{B}_e(\hat{r}(t))\, w(t) \\ e(t) = \hat{C}_e(\hat{r}(t))\, x_e(t) + \hat{D}_e(\hat{r}(t))\, w(t) \end{cases} \tag{4}$$

where $x_e(t) = [x^T(t) \;\; x_f^T(t)]^T$ is the state of the error system. For all $i \in \mathcal{S}$, we have

$$\begin{cases} \hat{A}_{ei} = \begin{bmatrix} \hat{A}_i & 0 \\ B_{fi}\hat{C}_{yi} & A_{fi} \end{bmatrix}, \quad \hat{B}_{ei} = \begin{bmatrix} \hat{B}_i \\ B_{fi}\hat{D}_{yi} \end{bmatrix} \\ \hat{C}_{ei} = [\hat{C}_{zi} - D_{fi}\hat{C}_{yi} \quad -C_{fi}] \\ \hat{D}_{ei} = \hat{D}_{zi} - D_{fi}\hat{D}_{yi} \end{cases}. \tag{5}$$

For error system (4), we have the following definitions and proposition.

Definition 1 [10]: The nominal Markovian jump filtering error system of (4) with $w(t) \equiv 0$ is said to be mean square stable if

$$\lim_{t\to\infty} \mathrm{E}\left(\|x_e(t)\|^2\right) = 0$$

for any initial conditions $x_e(0) \in \mathbb{R}^{n+n_f}$ and $\hat{r}(0) \in \mathcal{S}$.

Definition 2 [25]: Consider the nominal Markovian jump filtering error system of (4). Let G_{ew} denote the operator from the exogenous energy-bounded noise $w(t)$ to the stochastic filtering error $e(t)$; the H_∞-norm of the operator G_{ew} is defined as $\|G_{ew}\|_{H_\infty} \triangleq \inf \gamma$ such that

$$\|e\|_2 < \gamma \|w\|_2$$

for all nonzero processes $w(\cdot) \in \mathbb{L}_2^{n_w}[0,\infty)$ for zero initial condition $x_e(0) = 0$ and any initial mode $\hat{r}(0) \in \mathcal{S}$.

Proposition 1 [11]: Given a prescribed scalar $\gamma_{H_\infty} \in \mathbb{R}^+$, the nominal Markovian jump filtering error system of (4) is mean square stable and has H_∞ performance $\|G_{ew}\|_{H_\infty} < \gamma_{H_\infty}$ if there exist matrices $P_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}, i \in \mathcal{S}$, such that, for all $i \in \mathcal{S}$

$$\begin{bmatrix} A_{ei}^T P_i + P_i A_{ei} + \sum_{j=1}^s \pi_{ij} P_j & P_i B_{ei} & C_{ei}^T \\ B_{ei}^T P_i & -\gamma_{H_\infty}^2 I & D_{ei}^T \\ C_{ei} & D_{ei} & -I \end{bmatrix} < 0$$

hold, where A_{ei}, B_{ei}, C_{ei}, D_{ei} and π_{ij} are the nominal values of $\hat{A}_{ei}$, $\hat{B}_{ei}$, $\hat{C}_{ei}$, $\hat{D}_{ei}$ and $\hat{\pi}_{ij}$, respectively, for all $i,j \in \mathcal{S}$.

Based upon Proposition 1, we introduce the following definition.

Definition 3: Given a prescribed scalar $\gamma_{H_\infty} \in \mathbb{R}^+$, filter (3) is said to be a quadratic H_∞ filter of fixed-order n_f with level γ_{H_∞} for uncertain system (1) if there exist matrices $P_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $i \in \mathcal{S}$, such that, for all $i \in \mathcal{S}$

$$\begin{bmatrix} \hat{A}_{ei}^T P_i + P_i \hat{A}_{ei} + \sum_{j=1}^s \hat{\pi}_{ij} P_j & P_i \hat{B}_{ei} & \hat{C}_{ei}^T \\ \hat{B}_{ei}^T P_i & -\gamma_{H_\infty}^2 I & \hat{D}_{ei}^T \\ \hat{C}_{ei} & \hat{D}_{ei} & -I \end{bmatrix} < 0 \quad (6)$$

hold over all the admissible uncertainties in (2).

The objective of the paper is to design a quadratic H_∞ filter of form (3) of order n_f with a prescribed level $\gamma_{H_\infty} \in \mathbb{R}^+$ for uncertain MJLS (1) over the admissible uncertainties in (2).

III. Fixed-Order Robust H_∞ Filter

In this section, we first present a result for analyzing the robust H_∞ filtering level when filter (3) is given, then establish a sufficient condition for constructing the desired robust H_∞ filter (3). Finally, an effective algorithm is suggested to solve the proposed problem.

The following result gives us a criterion for testing the robust H_∞ filtering level of filter (3) for uncertain MJLS (1) over all the admissible uncertainties in (2) in terms of coupled LMIs.

Theorem 1: Given a prescribed scalar $\gamma_{H_\infty} \in \mathbb{R}^+$, filter (3) is a quadratic H_∞ filter of order n_f with level γ_{H_∞} for uncertain system (1) if there exist matrices $P_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $T_{ij} \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, and scalars $\lambda_{ai} \in \mathbb{R}^+$, $\lambda_{bi} \in \mathbb{R}^+$, $\lambda_{czi} \in \mathbb{R}^+$, $\lambda_{dzi} \in \mathbb{R}^+$, $\lambda_{cyi} \in \mathbb{R}^+$, $\lambda_{dyi} \in \mathbb{R}^+$, $i,j \in \mathcal{S}$, $j \neq i$, satisfying the coupled LMIs

$$\begin{bmatrix} Q_{1i} & P_i\tilde{B}_i & P_i M_{1i} & M_{2i} \\ \tilde{B}_i^T P_i & Q_{2i} & 0 & 0 \\ M_{1i}^T P_i & 0 & -\Lambda_{1i} & 0 \\ M_{2i}^T & 0 & 0 & -\Lambda_{2i} \end{bmatrix} + \left(\begin{bmatrix} \tilde{C}_i^T \\ \tilde{D}_i^T \\ M_{3i}^T \\ 0 \end{bmatrix} \star \right) < 0 \quad (7)$$

for all $i \in \mathcal{S}$, where

$$\begin{aligned}
Q_{1i} &= \tilde{A}_i^T P_i + P_i \tilde{A}_i + \sum_{j=1}^s \pi_{ij} P_j + \sum_{j=1, j\neq i}^s \varepsilon_{ij}^2 T_{ij} \\
&\quad + N_1\big(\lambda_{ai} H_{ai}^T H_{ai} + \lambda_{czi} H_{czi}^T H_{czi} + \lambda_{cyi} H_{cyi}^T H_{cyi}\big) N_1^T \\
Q_{2i} &= -\gamma_{H_\infty}^2 I + \lambda_{bi} H_{bi}^T H_{bi} + \lambda_{dzi} H_{dzi}^T H_{dzi} \\
&\quad + \lambda_{dyi} H_{dyi}^T H_{dyi} \\
M_{1i} &= [N_1 E_{ai} \quad N_1 E_{bi} \quad N_2 B_{fi} E_{cyi} \quad N_2 B_{fi} E_{dyi} \quad 0 \quad 0] \\
M_{2i} &= [P_i - P_1 \ \cdots \ P_i - P_{i-1} \ P_i - P_{i+1} \ \cdots \ P_i - P_s] \\
M_{3i} &= [0 \quad 0 \quad -D_{fi}E_{cyi} \quad -D_{fi}E_{dyi} \quad E_{czi} \quad E_{dzi}] \\
\Lambda_{1i} &= \mathrm{diag}(\lambda_{ai} I, \lambda_{bi} I, \lambda_{cyi} I, \lambda_{dyi} I, \lambda_{czi} I, \lambda_{dzi} I) \\
\Lambda_{2i} &= \mathrm{diag}\left(T_{i1}, \ldots, T_{i(i-1)}, T_{i(i+1)}, \ldots, T_{is}\right) \\
\tilde{A}_i &= \begin{bmatrix} A_i & 0 \\ B_{fi} C_{yi} & A_{fi} \end{bmatrix} \\
\tilde{B}_i &= \begin{bmatrix} B_i \\ B_{fi} D_{yi} \end{bmatrix} \\
\tilde{C}_i &= [C_{zi} - D_{fi} C_{yi} \quad -C_{fi}] \\
\tilde{D}_i &= D_{zi} - D_{fi} D_{yi} \\
N_1 &= \begin{bmatrix} I_n \\ 0_{n_f \times n} \end{bmatrix} \\
N_2 &= \begin{bmatrix} 0_{n \times n_f} \\ I_{n_f} \end{bmatrix}.
\end{aligned}$$

Proof: According to Definition 3 and Schur complement equivalence, given a prescribed scalar $\gamma_{H_\infty} \in \mathbb{R}^+$, filter (3) is a quadratic H_∞ filter of order n_f with level γ_{H_∞} for uncertain system (1) if, and only if, there exist matrices $P_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $i \in \mathcal{S}$, such that inequality (6) holds for all $i \in \mathcal{S}$ over all the admissible uncertainties in (2). Note that matrices $\hat{A}_{ei}$, $\hat{B}_{ei}$, $\hat{C}_{ei}$ and $\hat{D}_{ei}$ given in (5) can be written as

$$\begin{aligned}
\hat{A}_{ei} &= \tilde{A}_i + N_1 E_{ai} F_{ai} H_{ai} N_1^T + N_2 B_{fi} E_{cyi} F_{cyi} H_{cyi} N_1^T \\
\hat{B}_{ei} &= \tilde{B}_i + N_1 E_{bi} F_{bi} H_{bi} + N_2 B_{fi} E_{dyi} F_{dyi} H_{dyi} \\
\hat{C}_{ei} &= \tilde{C}_i + E_{czi} F_{czi} H_{czi} N_1^T - D_{fi} E_{cyi} F_{cyi} H_{cyi} N_1^T \\
\hat{D}_{ei} &= \tilde{D}_i + E_{dzi} F_{dzi} H_{dzi} - D_{fi} E_{dyi} F_{dyi} H_{dyi}.
\end{aligned}$$

Therefore, through expanding the uncertainty terms, inequality (6) can be rewritten as (8), shown at the bottom of the next page. First, note that for any matrix $T_{ij} \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, the inequality

$$\left(\left[\frac{1}{2} \Delta\pi_{ij} T_{ij}^{\frac{1}{2}} - (P_j - P_i) T_{ij}^{-\frac{1}{2}} \right] \star \right) \geq 0$$

yields

$$\Delta\pi_{ij}(P_j - P_i) \leq \left(\frac{1}{2}\Delta\pi_{ij}\right)^2 T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j).$$

Also note that $\Delta\pi_{ii} = -\sum_{j=1,j\neq i}^{s}\Delta\pi_{ij}$ and $|\Delta\pi_{ij}| \leq 2\varepsilon_{ij}$, $i,j \in \mathcal{S}$, $j \neq i$, and we have

$$\begin{aligned}
&\sum_{j=1}^{s}\Delta\pi_{ij}P_j \\
&\quad = \sum_{j=1,j\neq i}^{s}\Delta\pi_{ij}(P_j - P_i) \\
&\quad \leq \sum_{j=1,j\neq i}^{s}\left[\left(\frac{1}{2}\Delta\pi_{ij}\right)^2 T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j)\right] \\
&\quad \leq \sum_{j=1,j\neq i}^{s}\left[\varepsilon_{ij}^2 T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j)\right]. \qquad (9)
\end{aligned}$$

Then in view of [15, Lemma 2.2], inequality (8) holds over all the admissible uncertainties in (2) if there exist real numbers $\lambda_{ai} \in \mathbb{R}^+$, $\lambda_{bi} \in \mathbb{R}^+$, $\lambda_{czi} \in \mathbb{R}^+$, $\lambda_{dzi} \in \mathbb{R}^+$, $\lambda_{cyi} \in \mathbb{R}^+$, $\lambda_{dyi} \in \mathbb{R}^+$, and matrices $T_{ij} \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $j \in \mathcal{S}$, $j \neq i$, such that

$$\begin{bmatrix} L_{1i} & P_i\tilde{B}_i & \tilde{C}_i^T \\ \tilde{B}_i^T P_i & Q_{2i} & \tilde{D}_i^T \\ \tilde{C}_i & \tilde{D}_i & L_{2i} \end{bmatrix} + \frac{1}{\lambda_{cyi}}\left(\begin{bmatrix} P_i N_2 B_{fi} E_{cyi} \\ 0 \\ -D_{fi}E_{cyi} \end{bmatrix}\star\right) + \frac{1}{\lambda_{dyi}}\left(\begin{bmatrix} P_i N_2 B_{fi} E_{dyi} \\ 0 \\ -D_{fi}E_{dyi} \end{bmatrix}\star\right) < 0$$

where

$$\begin{aligned}
L_{1i} =& \tilde{A}_i^T P_i + P_i\tilde{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j \\
&+ \sum_{j=1,j\neq i}^{s}\left[\varepsilon_{ij}^2 T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j)\right] \\
&+ N_1\left(\lambda_{ai}H_{ai}^T H_{ai} + \lambda_{czi}H_{czi}^T H_{czi} + \lambda_{cyi}H_{cyi}^T H_{cyi}\right)N_1^T \\
&+ P_i N_1\left(\frac{1}{\lambda_{ai}}E_{ai}E_{ai}^T + \frac{1}{\lambda_{bi}}E_{bi}E_{bi}^T\right)N_1^T P_i \\
L_{2i} =& -I + \frac{1}{\lambda_{czi}}E_{czi}E_{czi}^T + \frac{1}{\lambda_{dzi}}E_{dzi}E_{dzi}^T.
\end{aligned}$$

Applying Schur complement equivalence and doing a congruence transformation, we have the above inequality is equivalent to (7). ∎

In the following, we provide a comparison of the results in [9], [12], and the current paper.

Remark 1: The model of the uncertain mode transition rate matrix considered in [9] and [12] is of the form

$$\hat{\Pi} = \Pi + \Delta\Pi \text{ with } |\Delta\pi_{ij}| \leq 2\varepsilon_{ij}, \qquad \varepsilon_{ij} \geq 0 \qquad (10)$$

for all $i,j \in \mathcal{S}$. A crucial difference between (10) and (2g) is that ε_{ii} is not defined in (2g) for all $i \in \mathcal{S}$. The reason is that the probability constraint $\sum_{j=1}^{s}\Delta\pi_{ij} = 0$, which ensures $\sum_{j=1}^{s}(\pi_{ij} + \Delta\pi_{ij}) = 0$, implies $\varepsilon_{ii} = \sum_{j=1,j\neq i}^{s}\varepsilon_{ij}$.

Based upon Remark 1, we can prove that our bounding technique in (9) gives less conservative results than those in [9] and [12] for dealing with the switching probability uncertainties.

Remark 2: Suppose there do exist uncertainties, that is, at least one $\varepsilon_{ij} > 0$, $j \neq i$. The bounding technique for the matrix inequalities used in [9] and [12] is

$$\sum_{j=1}^{s}\Delta\pi_{ij}P_j \leq \sum_{j=1}^{s}2\varepsilon_{ij}P_j.$$

The bounding technique used in this paper is in (9). For those $\varepsilon_{ij} > 0$, $j \neq i$, we can choose $T_{ij} = (1/\varepsilon_{ij})(P_i + P_j)$, then

$$\begin{aligned}
&\varepsilon_{ij}^2 T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j) \\
&\quad = \varepsilon_{ij}(P_i + P_j) + \varepsilon_{ij}\left[P_i + P_j + 4P_j(P_i + P_j)^{-1}P_j - 4P_j\right] \\
&\quad = 2\varepsilon_{ij}(P_i + P_j) + 4\varepsilon_{ij}P_j\left[(P_i + P_j)^{-1} - P_j^{-1}\right]P_j \\
&\quad < 2\varepsilon_{ij}(P_i + P_j).
\end{aligned}$$

For those $\varepsilon_{ij} = 0$, $j \neq i$, we choose $T_{ij} = (1/\alpha)I$ with $\alpha \in \mathbb{R}^+$ sufficiently small such that

$$\begin{aligned}
&\sum_{j=1,j\neq i}^{s}\left[\varepsilon_{ij}^2 T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j)\right] \\
&\quad < \sum_{j=1,j\neq i}^{s}2\varepsilon_{ij}(P_i + P_j) \\
&\quad = \sum_{j=1}^{s}2\varepsilon_{ij}P_j.
\end{aligned}$$

$$\begin{aligned}
&\begin{bmatrix} \left(\tilde{A}_i^T P_i + P_i\tilde{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j + \sum_{j=1}^{s}\Delta\pi_{ij}P_j\right) & P_i\tilde{B}_i & \tilde{C}_i^T \\ \tilde{B}_i^T P_i & -\gamma_{H_\infty}^2 I & \tilde{D}_i^T \\ \tilde{C}_i & \tilde{D}_i & -I \end{bmatrix} \\
&+ \mathrm{sym}\left(\begin{bmatrix} P_i N_1 E_{ai} \\ 0 \\ 0 \end{bmatrix} F_{ai}\begin{bmatrix} N_1 H_{ai}^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} P_i N_1 E_{bi} \\ 0 \\ 0 \end{bmatrix} F_{bi}\begin{bmatrix} 0 \\ H_{bi}^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ E_{czi} \end{bmatrix} F_{czi}\begin{bmatrix} N_1 H_{czi}^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ E_{dzi} \end{bmatrix} F_{dzi}\begin{bmatrix} 0 \\ H_{dzi}^T \\ 0 \end{bmatrix}^T\right. \\
&\qquad \left. + \begin{bmatrix} P_i N_2 B_{fi} E_{cyi} \\ 0 \\ -D_{fi}E_{cyi} \end{bmatrix} F_{cyi}\begin{bmatrix} N_1 H_{cyi}^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} P_i N_2 B_{fi} E_{dyi} \\ 0 \\ -D_{fi}E_{dyi} \end{bmatrix} F_{dyi}\begin{bmatrix} 0 \\ H_{dyi}^T \\ 0 \end{bmatrix}^T\right) < 0. \qquad (8)
\end{aligned}$$

That is, our result is less conservative than the one in [9] and [12] as long as there exist uncertainties.

The following theorem provides us with a solution to the *fixed-order robust H_∞ filtering problem* (FRFP) for MJLSs with uncertain switching probabilities in terms of coupled LMIs and equality constraints.

Theorem 2: Given a prescribed scalar $\gamma_{H_\infty} \in \mathbb{R}^+$, there exists a quadratic H_∞ filter (3) of order n_f with level γ_{H_∞} for uncertain system (1) if there exist matrices $P_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $X_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $V_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $Z_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $T_{ij} \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, and scalars $\lambda_{ai} \in \mathbb{R}^+$, $\alpha_{ai} \in \mathbb{R}^+$, $\lambda_{bi} \in \mathbb{R}^+$, $\lambda_{czi} \in \mathbb{R}^+$, $\alpha_{czi} \in \mathbb{R}^+$, $\lambda_{dzi} \in \mathbb{R}^+$, $\lambda_{cyi} \in \mathbb{R}^+$, $\alpha_{cyi} \in \mathbb{R}^+$, $\lambda_{dyi} \in \mathbb{R}^+$, $i, j \in \mathcal{S}$, $j \neq i$, such that the coupled LMIs

$$\begin{bmatrix} Q_{3i} & B_i & M_{4i} & N_1^T X_i \\ B_i^T & Q_{2i} & 0 & 0 \\ M_{4i}^T & 0 & -\Lambda_{3i} & 0 \\ X_i N_1 & 0 & 0 & -Z_i \end{bmatrix} < 0 \quad (11)$$

$$\begin{bmatrix} -V_i + \sum_{j=1, j\neq i}^{s} \left(\pi_{ij} P_j + \varepsilon_{ij}^2 T_{ij}\right) & M_{2i} \\ M_{2i}^T & -\Lambda_{2i} \end{bmatrix} \le 0 \quad (12)$$

$$\begin{bmatrix} \Xi_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_{4i} & M_{5i} \\ M_{5i}^T & Q_{5i} \end{bmatrix} \begin{bmatrix} \Xi_i^T & 0 \\ 0 & I \end{bmatrix} < 0 \quad (13)$$

with equality constraints

$$\begin{cases} P_i X_i = I, \ V_i Z_i = I \\ \lambda_{ai}\alpha_{ai} = 1, \ \lambda_{czi}\alpha_{czi} = 1, \ \lambda_{cyi}\alpha_{cyi} = 1 \end{cases} \quad (14)$$

holding for all $i \in \mathcal{S}$, where

$$\begin{aligned} Q_{3i} &= N_1^T X_i N_1 A_i^T + A_i N_1^T X_i N_1 \\ &\quad + \pi_{ii} N_1^T X_i N_1 + \alpha_{ai} E_{ai} E_{ai}^T \\ M_{4i} &= \begin{bmatrix} E_{bi} & N_1^T X_i N_1 H_{ai}^T & N_1^T X_i N_1 H_{cyi}^T & N_1^T X_i N_1 H_{czi}^T \end{bmatrix} \\ \Lambda_{3i} &= \mathrm{diag}(\lambda_{bi} I, \alpha_{ai} I, \alpha_{cyi} I, \alpha_{czi} I) \\ \Xi_i &= \left([C_{yi} \quad D_{yi} \quad E_{cyi} \quad E_{dyi}]^T\right)^\perp \end{aligned}$$

$$Q_{4i} = \begin{bmatrix} Q_{6i} & N_1^T P_i N_1 B_i + C_{zi}^T D_{zi} & 0 & 0 \\ \bullet & Q_{2i} + D_{zi}^T D_{zi} & 0 & 0 \\ \bullet & \bullet & -\lambda_{cyi} I & 0 \\ \bullet & \bullet & \bullet & -\lambda_{dyi} I \end{bmatrix}$$

$$M_{5i} = \begin{bmatrix} N_1^T P_i N_1 E_{ai} & N_1^T P_i N_1 E_{bi} & C_{zi}^T E_{czi} & C_{zi}^T E_{dzi} \\ 0 & 0 & D_{zi}^T E_{czi} & D_{zi}^T E_{dzi} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_{5i} = \begin{bmatrix} -\lambda_{ai} I & 0 & 0 & 0 \\ 0 & -\lambda_{bi} I & 0 & 0 \\ 0 & 0 & -\lambda_{czi} I & 0 \\ 0 & 0 & 0 & -\lambda_{dzi} I \end{bmatrix} + \left(\begin{bmatrix} 0 \\ 0 \\ E_{czi}^T \\ E_{dzi}^T \end{bmatrix} \star \right)$$

$$\begin{aligned} Q_{6i} &= A_i^T N_1^T P_i N_1 + N_1^T P_i N_1 A_i + \pi_{ii} N_1^T P_i N_1 + C_{zi}^T C_{zi} \\ &\quad + N_1^T V_i N_1 + \lambda_{ai} H_{ai}^T H_{ai} + \lambda_{czi} H_{czi}^T H_{czi} \\ &\quad + \lambda_{cyi} H_{cyi}^T H_{cyi} \end{aligned}$$

and Q_{2i}, M_{2i}, Λ_{2i}, and N_1 are given in Theorem 1. In this case, by substituting the solution of (11)–(14) into

$$\Phi_i + \Psi_{Li} J_i \Psi_{Ri} + (\Psi_{Li} J_i \Psi_{Ri})^T < 0 \quad (15)$$

for all $i \in \mathcal{S}$, where

$$\Phi_i = \begin{bmatrix} Q_{7i} & P_i N_1 B_i & N_1 C_{zi}^T & 0 & 0 \\ B_i^T N_1^T P_i & Q_{2i} & D_{zi}^T & 0 & 0 \\ C_{zi} N_1^T & D_{zi} & Q_{8i} & 0 & 0 \\ 0 & 0 & 0 & -\lambda_{cyi} I & 0 \\ 0 & 0 & 0 & 0 & -\lambda_{dyi} I \end{bmatrix}$$

$$\Psi_{Li} = \begin{bmatrix} P_i N_2 & 0 \\ 0 & 0 \\ 0 & -I \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_i = \begin{bmatrix} A_{fi} & B_{fi} \\ C_{fi} & D_{fi} \end{bmatrix}$$

$$\Psi_{Ri} = \begin{bmatrix} N_2^T & 0 & 0 & 0 & 0 \\ C_{yi} N_1^T & D_{yi} & 0 & E_{cyi} & E_{dyi} \end{bmatrix}$$

$$\begin{aligned} Q_{7i} &= N_1 A_i^T N_1^T P_i + P_i N_1 A_i N_1^T + \sum_{j=1}^{s} \pi_{ij} P_j \\ &\quad + \sum_{j=1, j\neq i}^{s} \left[\varepsilon_{ij}^2 T_{ij} + (P_i - P_j) T_{ij}^{-1} (P_i - P_j)\right] \\ &\quad + P_i N_1 \left(\frac{1}{\lambda_{ai}} E_{ai} E_{ai}^T + \frac{1}{\lambda_{bi}} E_{bi} E_{bi}^T \right) N_1^T P_i \\ &\quad + N_1 \left(\lambda_{ai} H_{ai}^T H_{ai} + \lambda_{czi} H_{czi}^T H_{czi} + \lambda_{cyi} H_{cyi}^T H_{cyi} \right) N_1^T \\ Q_{8i} &= -I + \frac{1}{\lambda_{czi}} E_{czi} E_{czi}^T + \frac{1}{\lambda_{dzi}} E_{dzi} E_{dzi}^T \end{aligned}$$

and N_2 is given in Theorem 1, one filter of form (3) can be obtained by solving the coupled LMIs (15).

Proof: Note that the matrices $\hat{A}_{ei}$, $\hat{B}_{ei}$, $\hat{C}_{ei}$, and $\hat{D}_{ei}$ given in (5) can also be expressed as

$$\begin{aligned} \hat{A}_{ei} &= N_1 A_i N_1^T + N_1 E_{ai} F_{ai} H_{ai} N_1^T + N_5 J_i \bar{C}_{yi} \\ &\quad + N_5 J_i N_3 E_{cyi} F_{cyi} H_{cyi} N_1^T \\ \hat{B}_{ei} &= N_1 B_i + N_1 E_{bi} F_{bi} H_{bi} + N_5 J_i N_3 D_{yi} \\ &\quad + N_5 J_i N_3 E_{dyi} F_{dyi} H_{dyi} \\ \hat{C}_{ei} &= C_{zi} N_1^T + E_{czi} F_{czi} H_{czi} N_1^T + N_4 J_i \bar{C}_{yi} \\ &\quad + N_4 J_i N_3 E_{cyi} F_{cyi} H_{cyi} N_1^T \\ \hat{D}_{ei} &= D_{zi} + E_{dzi} F_{dzi} H_{dzi} + N_4 J_i N_3 D_{yi} \\ &\quad + N_4 J_i N_3 E_{dyi} F_{dyi} H_{dyi} \end{aligned}$$

where

$$N_3 = \begin{bmatrix} 0_{n\times n_y} \\ I_{n_y} \end{bmatrix}, \quad N_5 = \begin{bmatrix} 0 & 0_{n\times n_z} \\ I_{n_f} & 0 \end{bmatrix} = [N_2 \quad 0]$$

$$N_4 = \begin{bmatrix} 0_{n_z\times n_f} & -I_{n_z} \end{bmatrix}, \quad \bar{C}_{yi} = \begin{bmatrix} 0 & I_{n_f} \\ C_{yi} & 0 \end{bmatrix} = \begin{bmatrix} N_2^T \\ C_{yi} N_1^T \end{bmatrix}.$$

Now, inequality (6) can be expanded as the equation shown at the bottom of the next page. Applying [15, Lemma 2.2] and the bounding technique in (9), the above inequality holds over all the admissible uncertainties in (2) if there exist real numbers $\lambda_{ai} \in \mathbb{R}^+$, $\lambda_{bi} \in \mathbb{R}^+$, $\lambda_{czi} \in \mathbb{R}^+$, $\lambda_{dzi} \in \mathbb{R}^+$, $\lambda_{cyi} \in \mathbb{R}^+$,

$\lambda_{dyi} \in \mathbb{R}^+$, and matrices $T_{ij} \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $j \in \mathcal{S}$, $j \neq i$, such that

$$\begin{bmatrix} Q_{7i} & P_iN_1B_i & N_1C_{zi}^T \\ B_i^TN_1^TP_i & Q_{2i} & D_{zi}^T \\ C_{zi}N_1^T & D_{zi} & Q_{8i} \end{bmatrix} + \mathrm{sym}\left(\begin{bmatrix} P_iN_2 & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix} J_i \begin{bmatrix} N_2^T & 0 & 0 \\ C_{yi}N_1^T & D_{yi} & 0 \end{bmatrix}\right) + \frac{1}{\lambda_{cyi}}\left(\left(\begin{bmatrix} P_iN_2 & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix} J_i \begin{bmatrix} 0 \\ E_{cyi} \end{bmatrix}\right)\star\right) + \frac{1}{\lambda_{dyi}}\left(\left(\begin{bmatrix} P_iN_2 & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix} J_i \begin{bmatrix} 0 \\ E_{dyi} \end{bmatrix}\right)\star\right) < 0.$$

In view of Schur complement equivalence, the above inequality is equivalent to (15). Now, according to the Projection Lemma [26], [27], inequality (15) is solvable for J_i if, and only if, the following two matrix inequalities hold:

$$\Psi_{Li}^{\perp}\Phi_i\left(\Psi_{Li}^{\perp}\right)^T < 0 \tag{16}$$

$$\left(\Psi_{Ri}^T\right)^{\perp}\Phi_i\left[\left(\Psi_{Ri}^T\right)^{\perp}\right]^T < 0. \tag{17}$$

Note that matrix inequalities (16) and (17) cannot be solved easily since they are not LMIs. Therefore, in the sequel, we try to translate (16) and (17) into the form of LMIs with equality constraints, which can be solved easily using algorithms developed in [28] and [29]. To end this, we first have

$$\Psi_{Li}^{\perp} = \begin{bmatrix} N_1^TP_i^{-1} & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}.$$

Note that $N_1^TN_1 = I_n$, $\lambda_{cyi} \in \mathbb{R}^+$, $\lambda_{dyi} \in \mathbb{R}^+$ and define $X_i \triangleq P_i^{-1}$, then (16) is equivalent to

$$\begin{bmatrix} L_{3i} & B_i \\ B_i^T & Q_{2i} \end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned} L_{3i} = {} & N_1^TX_iN_1A_i^T + A_iN_1^TX_iN_1 + \pi_{ii}N_1^TX_iN_1 + N_1^TX_i \\ & \times \sum_{j=1,j\neq i}^{s}\left[\pi_{ij}P_j + \varepsilon_{ij}^2T_{ij} + (P_i-P_j)T_{ij}^{-1}(P_i-P_j)\right]X_iN_1 \\ & + N_1^TX_iN_1\left(\lambda_{ai}H_{ai}^TH_{ai} + \lambda_{cyi}H_{cyi}^TH_{cyi} + \lambda_{czi}H_{czi}^TH_{czi}\right)N_1^TX_iN_1 \\ & + \left(\frac{1}{\lambda_{ai}}E_{ai}E_{ai}^T + \frac{1}{\lambda_{bi}}E_{bi}E_{bi}^T\right). \end{aligned}$$

Now, let $V_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$ such that

$$\sum_{j=1,j\neq i}^{s}\left[\pi_{ij}P_j + \varepsilon_{ij}^2T_{ij} + (P_i-P_j)T_{ij}^{-1}(P_i-P_j)\right] \le V_i$$

and define $\alpha_{ai} \triangleq (1/\lambda_{ai})$, $\alpha_{czi} \triangleq (1/\lambda_{czi})$, $\alpha_{cyi} \triangleq (1/\lambda_{cyi})$ and $Z_i \triangleq V_i^{-1}$. Then, inequality (18) is equivalent to (11) and (12) with equality constraints (14).

Next, we have

$$\left(\Psi_{Ri}^T\right)^{\perp} = \begin{bmatrix} \begin{bmatrix} C_{yi}^T \\ D_{yi}^T \\ E_{cyi}^T \\ E_{dyi}^T \end{bmatrix}^{\perp} & 0 \\ 0 & I \end{bmatrix} \left[\begin{array}{ccccc} N_1^T & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ \hline 0 & 0 & I & 0 & 0 \end{array}\right].$$

Then, inequality (17) is equivalent to the equation shown at bottom of the next page. In view of Schur complement equivalence and a congruence transformation, the above inequality is equivalent to (12) and (13). This completes the first part of the proof. To end the proof, note that (11)–(14) being solvable means that (16) and (17) hold, which further implies that a filter of form (3) can be obtained by solving (15) after substituting the solution of (11)–(14) into (15) in view of the Projection Lemma [26], [27]. ■

In the case when the mode transition rate matrix is known exactly, we can obtain a simplified result for constructing fixed-

$$\begin{bmatrix} \left(N_1A_i^TN_1^TP_i + P_iN_1A_iN_1^T + \sum_{j=1}^{s}(\pi_{ij}+\Delta\pi_{ij})P_j\right) & P_iN_1B_i & N_1C_{zi}^T \\ B_i^TN_1^TP_i & -\gamma_{H_\infty}^2 I & D_{zi}^T \\ C_{zi}N_1^T & D_{zi} & -I \end{bmatrix}$$
$$+\,\mathrm{sym}\Bigg(\begin{bmatrix} P_iN_1E_{ai} \\ 0 \\ 0 \end{bmatrix} F_{ai} \begin{bmatrix} N_1H_{ai}^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ E_{czi} \end{bmatrix} F_{czi} \begin{bmatrix} N_1H_{czi}^T \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} P_iN_1E_{bi} \\ 0 \\ 0 \end{bmatrix} F_{bi} \begin{bmatrix} 0 \\ H_{bi}^T \\ 0 \end{bmatrix}^T$$
$$+ \begin{bmatrix} 0 \\ 0 \\ E_{dzi} \end{bmatrix} F_{dzi} \begin{bmatrix} 0 \\ H_{dzi}^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} P_iN_5 \\ 0 \\ N_4 \end{bmatrix} J_i \begin{bmatrix} \bar{C}_{yi}^T \\ D_{yi}^TN_3^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} P_iN_5 \\ 0 \\ N_4 \end{bmatrix} J_iN_3E_{cyi}F_{cyi} \begin{bmatrix} N_1H_{cyi}^T \\ 0 \\ 0 \end{bmatrix}^T$$
$$+ \begin{bmatrix} P_iN_5 \\ 0 \\ N_4 \end{bmatrix} J_iN_3E_{dyi}F_{dyi} \begin{bmatrix} 0 \\ H_{dyi}^T \\ 0 \end{bmatrix}^T\Bigg) < 0.$$

order robust H_∞ filter (3), which is stated in the following corollary and can be proved similarly to that of Theorem 2. However, the condition is necessary and sufficient since the bounding technique in (9) is no longer needed in the proof.

Corollary 1: Consider uncertain Markovian jump linear system (1) with mode transition rate matrix known exactly; for a prescribed scalar $\gamma_{H_\infty} \in \mathbb{R}^+$, there exists a quadratic H_∞ filter (3) of order n_f with level γ_{H_∞} if, and only if, there exist matrices $P_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$, $X_i \in \mathbb{S}^{(n+n_f)\times(n+n_f)}$ and scalars $\lambda_{ai} \in \mathbb{R}^+$, $\alpha_{ai} \in \mathbb{R}^+$, $\lambda_{bi} \in \mathbb{R}^+$, $\lambda_{czi} \in \mathbb{R}^+$, $\alpha_{czi} \in \mathbb{R}^+$, $\lambda_{dzi} \in \mathbb{R}^+$, $\lambda_{cyi} \in \mathbb{R}^+$, $\alpha_{cyi} \in \mathbb{R}^+$, $\lambda_{dyi} \in \mathbb{R}^+$, $i \in \mathcal{S}$, such that the coupled LMIs

$$\begin{bmatrix} Q_{3i} & B_i & M_{4i} & M_{6i} \\ B_i^T & Q_{2i} & 0 & 0 \\ M_{4i}^T & 0 & -\Lambda_{3i} & 0 \\ M_{6i}^T & 0 & 0 & -\Lambda_{4i} \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} \Xi_i & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \bar{Q}_{4i} & M_{5i} \\ M_{5i}^T & Q_{5i} \end{bmatrix}\begin{bmatrix} \Xi_i^T & 0 \\ 0 & I \end{bmatrix} < 0 \tag{20}$$

with equality constraints

$$P_iX_i = I, \quad \lambda_{ai}\alpha_{ai} = 1, \quad \lambda_{czi}\alpha_{czi} = 1, \quad \lambda_{cyi}\alpha_{cyi} = 1 \tag{21}$$

holding for all $i \in \mathcal{S}$, where

$$\begin{aligned} M_{6i} &= \big[\sqrt{\pi_{i1}}N_1^TX_i \ \sqrt{\pi_{i2}}N_1^TX_i \ \cdots \ \sqrt{\pi_{i(i-1)}}N_1^TX_i \\ &\qquad \sqrt{\pi_{i(i+1)}}N_1^TX_i \ \cdots \ \sqrt{\pi_{is}}N_1^TX_i\big] \\ \Lambda_{4i} &= \mathrm{diag}(X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_s) \\ \bar{Q}_{4i} &= \begin{bmatrix} \bar{Q}_{6i} & N_1^TP_iN_1B_i + C_{zi}^TD_{zi} & 0 & 0 \\ \bullet & Q_{2i} + D_{zi}^TD_{zi} & 0 & 0 \\ \bullet & \bullet & -\lambda_{cyi}I & 0 \\ \bullet & \bullet & \bullet & -\lambda_{dyi}I \end{bmatrix} \\ \bar{Q}_{6i} &= Q_{6i} - N_1^TV_iN_1 + \sum_{j=1,j\neq i}^{s} \pi_{ij}N_1^TP_jN_1 \end{aligned}$$

and $Q_{2i}, Q_{5i}, \Lambda_{3i}, M_{4i}, M_{5i}$, and Ξ_i are given in Theorem 2. In this case, one filter of form (3) can be obtained by solving the LMIs (15) with Q_{7i} replaced by $\bar{Q}_{7i}$ for all $i \in \mathcal{S}$, where

$$\bar{Q}_{7i} = Q_{7i} - \sum_{j=1,j\neq i}^{s} \left[\varepsilon_{ij}^2T_{ij} + (P_i - P_j)T_{ij}^{-1}(P_i - P_j)\right].$$

To solve the coupled LMIs in (11)–(13) with equality constraints in (14) effectively, we first choose a sufficiently small number $\beta \in \mathbb{R}^+$, then replace Q_{3i} in (11) with $Q_{3i} + \beta I$, Q_{6i} in (13) with $Q_{6i} + \beta I$, respectively, and change "$<$" to "$\leq$" in both (11) and (13). The modified versions of (11) and (13) will be denoted by $(\overline{11})$ and $(\overline{13})$, respectively. Finally, the equality constraints in (14) are relaxed to

$$\begin{cases} \begin{bmatrix} P_i & I \\ I & X_i \end{bmatrix} \geq 0, \ \begin{bmatrix} V_i & I \\ I & Z_i \end{bmatrix} \geq 0, \ \begin{bmatrix} \lambda_{ai} & 1 \\ 1 & \alpha_{ai} \end{bmatrix} \geq 0 \\ \begin{bmatrix} \lambda_{czi} & 1 \\ 1 & \alpha_{czi} \end{bmatrix} \geq 0, \ \begin{bmatrix} \lambda_{cyi} & 1 \\ 1 & \alpha_{cyi} \end{bmatrix} \geq 0 \end{cases}. \tag{22}$$

Now, the optimization algorithms developed in [28] and [29] can be employed to solve this nonlinear problem. The solution of FRFP is summarized below.

Algorithm FRFP: For a desired precision $\delta \in \mathbb{R}^+$, let $k_{\max}$ be the maximum number of iterations, and a sufficiently small number $\beta \in \mathbb{R}^+$ be given. Define two decision variables $W_i \triangleq \mathrm{diag}(P_i, V_i, \lambda_{ai}, \lambda_{czi}, \lambda_{cyi})$ and $U_i \triangleq \mathrm{diag}(X_i, Z_i, \alpha_{ai}, \alpha_{czi}, \alpha_{cyi})$.

1) Determine $W_i^0, U_i^0, \lambda_{bi}^0, \lambda_{dzi}^0, \lambda_{dyi}^0, T_{ij}^0, i, j \in \mathcal{S}, j \neq i$, satisfying $(\overline{11})$, (12), $(\overline{13})$, and (22), and let $k := 0$.
2) Solve the following convex optimization problem for the decision variables $W_i, U_i, \lambda_{bi}, \lambda_{dzi}, \lambda_{dyi}, T_{ij}, i, j \in \mathcal{S}$, $j \neq i$:
$$\min \sum_{i=1}^{s} \mathrm{trace}\left(W_iU_i^k + W_i^kU_i\right)$$
subject to $(\overline{11})$, (12), $(\overline{13})$, and (22) for all $i \in \mathcal{S}$.
3) Let $L_i^k := W_i$ and $R_i^k := U_i$ for all $i \in \mathcal{S}$.
4) If $|\sum_{i=1}^{s} \mathrm{trace}(L_i^kU_i^k + W_i^kR_i^k) - 2\sum_{i=1}^{s} \mathrm{trace}(W_i^kU_i^k)| < \delta$, then go to Step 7), else go to Step 5).
5) Compute $\theta^* \in [0, 1]$ by solving
$$\min_{\theta\in[0,1]} \sum_{i=1}^{s} \mathrm{trace}\left(\left[W_i^k + \theta\left(L_i^k - W_i^k\right)\right] \times \left[U_i^k + \theta\left(R_i^k - U_i^k\right)\right]\right).$$
6) Let $W_i^{k+1} := W_i^k + \theta^*(L_i^k - W_i^k)$, $U_i^{k+1} := U_i^k + \theta^*(R_i^k - U_i^k)$ for all $i \in \mathcal{S}$, and $k := k+1$. If $k < k_{\max}$, then go to Step 2), else go to Step 7).
7) Stop. If $\sum_{i=1}^{s} \mathrm{trace}(W_i^kU_i^k) = 3s + 2s(n + n_f)$, a solution is found successfully and a desired fixed-order filter of form (3) can be obtained by solving (15), else a solution cannot be found.

Remark 3: As explained in [29], Algorithm FRFP is globally convergent since the sequence of the function

$$f(k) \triangleq \sum_{i=1}^{s} \mathrm{trace}\left(W_i^kU_i^k\right), \quad k = 0, 1, 2, \ldots$$

$$\begin{bmatrix} \begin{bmatrix} C_{yi}^T \\ D_{yi}^T \\ E_{cyi}^T \\ E_{dyi}^T \end{bmatrix}^{\perp} & 0 \\ 0 & I \end{bmatrix} \left[\begin{array}{cccc|c} N_1^TQ_{7i}N_1 & N_1^TP_iN_1B_i & 0 & 0 & C_{zi}^T \\ B_i^TN_1^TP_iN_1 & Q_{2i} & 0 & 0 & D_{zi}^T \\ 0 & 0 & -\lambda_{cyi}I & 0 & 0 \\ 0 & 0 & 0 & -\lambda_{dyi}I & 0 \\ \hline C_{zi} & D_{zi} & 0 & 0 & Q_{8i} \end{array}\right] \begin{bmatrix} \begin{bmatrix} C_{yi}^T \\ D_{yi}^T \\ E_{cyi}^T \\ E_{dyi}^T \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix} < 0.$$

generated by Algorithm FRFP always converges to some $f^* \geq 3s+2s(n+n_f)$, while the alternating projection method, which was used to construct the reduced-order filters for MJLSs in [21] and [22] is guaranteed to converge only locally [29], [30]. Algorithm FRFP can be applied to Corollary 1 similarly.

IV. Numerical Example

In this section, to illustrate the usefulness and flexibility of the theory developed in this paper, we present a comparison with existing results [18]–[20] using a numerical example. Attention is focused on designing robust H_∞ filters for MJLSs with uncertain switching probabilities. It is assumed that the system under consideration has two operation modes, and the uncertainties only exist in the mode transition rate matrix. The system data of (1) are as follows:

$$A_1 = \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0 & -0.1 & 0 \\ 0.2 & 0 & -3 \end{bmatrix}, \ A_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0.2 & -3 & -0.2 \\ 0 & 0 & -4 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 2 & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \ C_{z_1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \end{bmatrix}$$
$$C_{z_2} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \ D_{z_1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ D_{z_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$C_{y_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ C_{y_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$
$$D_{y_1} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} \ D_{y_2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$
$$\Pi = \begin{bmatrix} -2 & 2 \\ 7 & -7 \end{bmatrix}, \ \varepsilon_{12} = 0.7, \quad \varepsilon_{21} = 3.$$

The nominal system of this uncertain system is robustly mean square stable. Suppose that a robust H_∞ filter with level $\gamma_{H_\infty} \triangleq 0.4$ is desired over the switching probability uncertainties $\Delta\pi_{12} \in [-1.4, 1.4]$ and $\Delta\pi_{21} \in [-6, 6]$.

A. Filter Design Ignoring Switching Probability Uncertainties

Observer-Structured Filter: We can construct an observer-structured filter $\mathrm{OF}_{\Delta=0}$

$$\begin{cases} \dot{x}_f(t) = A\left(r(t)\right)x_f(t) + K\left(r(t)\right)\left[y(t) - C_y\left(r(t)\right)x_f(t)\right] \\ z_f(t) = C_z\left(r(t)\right)x_{f(t)} \end{cases}$$

with

$$K_1 = \begin{bmatrix} 27.4126 & -16.7995 \\ -11.5524 & 13.9000 \\ 29.3859 & -7.9428 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 3.1135 & -0.0749 \\ 6.0657 & -3.8481 \\ 3.1646 & 7.3039 \end{bmatrix}$$

based on Theorem 3.1 and Corollary 3.1 of [18]. The H_∞ filtering level of this filter for the nominal system is $\gamma^*_{H_\infty} = 0.2324$.

Special Structured Full-Order Filter: This method comes from Theorem 3.2 and Corollary 3.1 of [19] (also [20, Th. 1]). We can obtain a special structured full-order filter $\mathrm{SFF}_{3\Delta=0}$ of the form

$$\begin{cases} \dot{x}_f(t) = A_f\left(r(t)\right)x_f(t) + B_f\left(r(t)\right)y(t) \\ z_f(t) = C_z\left(r(t)\right)x_f(t) \end{cases}$$

with

$$A_{f_1} = \begin{bmatrix} -32.4597 & -13.3938 & 0.5624 \\ 11.8860 & 0.2883 & -0.2643 \\ -43.7030 & -23.3530 & -2.0224 \end{bmatrix}$$
$$A_{f_2} = \begin{bmatrix} -2.3653 & -0.0363 & -2.0025 \\ -1.3190 & -1.9069 & -4.5147 \\ -8.2833 & 0.3496 & -5.4904 \end{bmatrix}$$
$$B_{f_1} = \begin{bmatrix} 33.4598 & -18.6459 \\ -12.3721 & 11.4281 \\ 45.5602 & -20.2060 \end{bmatrix}$$
$$B_{f_2} = \begin{bmatrix} 1.7548 & -0.3383 \\ 4.9297 & -3.6057 \\ 1.2722 & 7.0161 \end{bmatrix}.$$

We say this filter is special structured since it is a special case of the filter (3) with $n_f \triangleq n$, $C_{fi} \triangleq C_{zi}$ and $D_{fi} \triangleq 0$ for all $i \in \mathcal{S}$. The H_∞ filtering level of this filter for the nominal system is $\gamma^*_{H_\infty} = 0.2570$.

General Fixed-Order Filter: Based on Corollary 1 and Algorithm FRFP in this paper, both full-order $(n_f = 3)$ filters and reduced-order $(n_f = 2)$ filters of form (3) can be obtained. To compute with Algorithm FRFP for this problem, it is chosen that $\delta = 10^{-10}$, $k_{\max} = 100$ and $\beta = 0.01$.

First, we can find a general full-order filter (GFF) of form (3) with

$$A_{f_1} = \begin{bmatrix} -3.7888 & -0.6949 & 1.9856 \\ -1.1536 & -0.3740 & 0.6688 \\ 3.5505 & 0.4978 & -1.8045 \end{bmatrix} \times 10^3$$
$$A_{f_2} = \begin{bmatrix} -2.0386 & -1.2090 & 0.8649 \\ -1.3772 & -1.6331 & 0.4890 \\ 1.1990 & 0.3026 & -0.5684 \end{bmatrix} \times 10^3$$
$$B_{f_1} = \begin{bmatrix} -103.0831 & -200.2568 \\ -75.2408 & -47.0061 \\ 56.1931 & 200.3621 \end{bmatrix}$$
$$B_{f_2} = \begin{bmatrix} -156.3161 & 12.6782 \\ -156.6466 & -75.3309 \\ 66.1130 & -46.6642 \end{bmatrix}$$
$$C_{f_1} = \begin{bmatrix} -2.3414 & 2.9495 & 1.8712 \\ -8.6909 & -3.0911 & 5.3367 \end{bmatrix}$$
$$C_{f_2} = \begin{bmatrix} -7.4588 & 2.2201 & 2.3564 \\ -20.2044 & 2.2240 & 11.3131 \end{bmatrix}$$
$$D_{f_1} = \begin{bmatrix} 0.0360 & 0.1632 \\ 0.2647 & 0.6462 \end{bmatrix}, \quad D_{f_2} = \begin{bmatrix} 0.2017 & -0.0804 \\ 0.6995 & -0.1865 \end{bmatrix}.$$

This filter is denoted by $\mathrm{GFF}_{3\Delta=0}$ in Table I. The H_∞ filtering level of this filter for the nominal system is $\gamma^*_{H_\infty} = 0.3743$.

TABLE I
FILTERING PERFORMANCE FOR DIFFERENT SWITCHING PROBABILITY UNCERTAINTIES

$\Delta\Pi_{12}$	$\Delta\Pi_{21}$	$\mathrm{OF}_{\Delta=0}$	$\mathrm{SFF}_{3\Delta=0}$	$\mathrm{GFF}_{3\Delta=0}$	$\mathrm{GFF}_{2\Delta=0}$	$\mathrm{GFF}_{3\Delta\neq0}$	$\mathrm{GFF}_{2\Delta\neq0}$
−1.4	6	0.2102	0.2337	**0.5493**	**0.6410**	0.3378	0.3471
0	6	0.2125	0.2382	0.3935	**0.4004**	0.3346	0.3385
1.4	6	0.2169	0.2426	0.3846	0.3386	0.3316	0.3322
−1.4	0	0.2168	0.2406	**0.4830**	**0.5498**	0.3388	0.3412
0	0	0.2324	0.2570	0.3743	0.3383	0.3326	0.3289
1.4	0	0.2604	0.2827	0.3669	0.3219	0.3279	0.3217
−1.4	−6	**3.7899**‡	**0.9929**	0.3475	0.3266	0.3116	0.3099
0	−6	**54.3409**	**1.1534**	0.3192	0.2996	0.3066	0.2949
1.4	−6	**U**§	**1.3129**	0.3104	0.2982	0.3038	0.2893

‡ Boldface figure indicates that the filter has violated the specified performance.
§ **U** indicates that the filtering error system is unstable.

Also, a general reduced-order filter of form (3) can be found with

$$A_{f_1} = \begin{bmatrix} -73.8423 & -64.0607 \\ -97.3595 & -97.5365 \end{bmatrix}$$
$$A_{f_2} = \begin{bmatrix} -4.2696 & -0.8801 \\ -2.7695 & -50.3559 \end{bmatrix}$$
$$B_{f_1} = \begin{bmatrix} -32.7018 & 6.6411 \\ -43.1893 & 6.5372 \end{bmatrix}$$
$$B_{f_2} = \begin{bmatrix} -0.0830 & -1.6061 \\ -13.5549 & 18.0562 \end{bmatrix}$$
$$C_{f_1} = \begin{bmatrix} 0.5608 & 4.8390 \\ -1.4354 & -0.6276 \end{bmatrix}$$
$$C_{f_2} = \begin{bmatrix} 2.1690 & -1.3958 \\ -1.2647 & -3.4868 \end{bmatrix}$$
$$D_{f_1} = \begin{bmatrix} -0.1064 & 0.7384 \\ 0.3251 & 1.1509 \end{bmatrix}$$
$$D_{f_2} = \begin{bmatrix} 0.1673 & 0.0268 \\ -0.0801 & -0.0977 \end{bmatrix}.$$

This filter is denoted by $\mathrm{GFF}_{2\Delta=0}$ in Table I. The H_∞ filtering level of this filter for the nominal system is $\gamma^*_{H_\infty} = 0.3383$.

B. Filter Design Considering Switching Probability Uncertainties

Based on Theorem 2 and Algorithm FRFP, general fixed-order filters of form (3) can be constructed which take into consideration of the uncertain switching probabilities.

A general full-order filter of form (3) is obtained with

$$A_{f_1} = \begin{bmatrix} -3586.5831 & -1892.4432 & 542.6471 \\ -2449.2871 & -1539.9287 & 301.0376 \\ -312.2163 & -383.4127 & -19.9566 \end{bmatrix}$$
$$A_{f_2} = \begin{bmatrix} -4471.6879 & -3327.3988 & 607.0330 \\ -3390.9467 & -2534.0061 & 415.8533 \\ -787.9683 & -595.7455 & 48.5043 \end{bmatrix}$$
$$B_{f_1} = \begin{bmatrix} 70.5386 & -407.6835 \\ 155.3934 & -271.8032 \\ 100.3480 & -28.2886 \end{bmatrix}$$
$$B_{f_2} = \begin{bmatrix} 252.2152 & 161.7097 \\ 210.9103 & 97.5028 \\ 68.0593 & -3.4918 \end{bmatrix}$$
$$C_{f_1} = \begin{bmatrix} -4.9557 & -3.4530 & -2.1800 \\ -5.6549 & -0.5205 & 1.2402 \end{bmatrix}$$
$$C_{f_2} = \begin{bmatrix} 0.1162 & -0.7361 & 2.3490 \\ -0.7206 & 0.8872 & 2.4016 \end{bmatrix}$$
$$D_{f_1} = \begin{bmatrix} -0.0784 & 0.3491 \\ -0.0191 & 0.3495 \end{bmatrix}$$
$$D_{f_2} = \begin{bmatrix} -0.0708 & -0.0655 \\ -0.0268 & -0.0346 \end{bmatrix}$$

This filter is denoted by $\mathrm{GFF}_{3\Delta\neq0}$ in Table I. The performance of this filter for the nominal system is $\gamma^*_{H_\infty} = 0.3326$.

A general reduced-order filter of form (3) can also be constructed with

$$A_{f_1} = \begin{bmatrix} -41.4956 & 195.5587 \\ 386.2257 & -2071.566 \end{bmatrix}$$
$$A_{f_2} = \begin{bmatrix} -8.3801 & 17.7585 \\ 12.2964 & -138.497 \end{bmatrix}$$
$$B_{f_1} = \begin{bmatrix} 13.4273 & 23.8642 \\ -111.7457 & -284.3378 \end{bmatrix}$$
$$B_{f_2} = \begin{bmatrix} -1.0129 & 5.1328 \\ -10.9206 & 13.4662 \end{bmatrix}$$
$$C_{f_1} = \begin{bmatrix} 3.0783 & -4.7947 \\ 1.7707 & -7.8267 \end{bmatrix}$$
$$C_{f_2} = \begin{bmatrix} -2.0323 & 0.3450 \\ 0.2520 & -5.7626 \end{bmatrix}$$
$$D_{f_1} = \begin{bmatrix} -1.8045 & 0.7760 \\ 0.3670 & 0.0759 \end{bmatrix}$$
$$D_{f_2} = \begin{bmatrix} 0.2217 & 0.0907 \\ 0.2323 & 0.0937 \end{bmatrix}.$$

This filter is denoted by $\mathrm{GFF}_{2\Delta\neq0}$ in Table I. The performance of this filter for the nominal system is $\gamma^*_{H_\infty} = 0.3289$.

Table I gives a comparison of the performance of these filters on some points in the uncertainty domain (including the no uncertainty case, and the vertices of the uncertainty domain). From this table, we can see that the filters designed by ignoring switching probability uncertainties do not always guarantee the desired H_∞ filtering level, while the filters designed by considering switching probability uncertainties can. Table I also shows us that these uncertainties can degrade the performance of filters and even destabilize the filtering error system in some cases.

Therefore, it is important and necessary to consider the effect of uncertain switching probabilities for MJLSs when designing filters. Fortunately, the developed theory in this paper provides us with a powerful design procedure for such problems.

V. Conclusion

In this paper, we have investigated the fixed-order robust H_∞ filter design problem for a class of Markovian jump linear systems with uncertain switching probabilities. Attention was focused on dealing with the uncertainties in the switching probabilities. This led to a nonlinear problem consisting of a set of coupled linear matrix inequalities with equality constraints. To solve such a problem, an effective algorithm involving convex optimization was addressed. Once the nonlinear problem is solved, a robust H_∞ filter can be constructed by solving a set of linear matrix inequalities. The developed theory was illustrated by a comparison and presented powerful utility and flexibility.

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