A New LMI Condition for Delay-Dependent Asymptotic Stability of Delayed Hopfield Neural Networks

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***Abstract*—In this paper, a new delay-dependent asymptotic stability condition for delayed Hopfield neural networks is given in terms of a linear matrix inequality, which is less conservative than existing ones in the literature. This condition guarantees the existence of a unique equilibrium point and its global asymptotic stability of a given delayed Hopfield neural network. Examples are provided to show the reduced conservatism of the proposed condition.**

***Index Terms*—Global asymptotic stability, Hopfield neural networks, linear matrix inequality, time delays.**

I. Introduction

Recently, there has been increasing interest in the study of Hopfield neural networks since Hopfield neural networks have found extensive applications in solving some optimization problems, associative memory, classification of patterns, reconstruction of moving images, and other areas [8]–[10]. It is now well known that applications of neural networks heavily depend on their dynamic behavior. Since stability is one of the most important issues related to such behavior, the problem of stability analysis of Hopfield neural networks has attracted considerable attention in recent years. Based on different assumptions on the network parameters, a great number of results on global asymptotic stability have been proposed; see, e.g., [1], [15], and the references therein.

In the implementation of artificial neural networks, however, time delays are unavoidable due to the finite switching speed of amplifiers. It has been shown that the existence of time delays in a neural network may lead to oscillation, divergence or instability. Therefore, the stability issue of Hopfield neural networks with time delays has been studied. Via different approaches, a great number of stability conditions for delayed Hopfield neural networks have been reported in the literature [4]–[7], [12]–[14], [16], [20]. These stability results can be classified into two types; that is, delay-independent stability and delay-dependent stability; the former does not include any information on the size of delay while the latter employs such information. It is known that delay-dependent stability conditions are generally less conservative than delay-independent ones especially when the size of the delay is small. Although delay-dependent stability results for delayed Hopfield neural networks were proposed in [7], [18], [19], [21], and [22], they are sufficient conditions and have conservatism to some extent, which leaves open room for further improvement.

In this paper, we develop an improved delay-dependent asymptotic stability condition for Hopfield neural networks with time delays by utilizing Lyapunov functionals. Under the proposed condition, both the existence of a unique equilibrium point and the global asymptotic stability of a given delayed Hopfield neural network are guaranteed. The derived condition is expressed in terms of a linear matrix inequality (LMI), which can be checked numerically very efficiently by resorting to recently developed standard algorithms such as interior-point methods, and no tuning of parameters will be involved [3]. Examples are provided to demonstrate the reduced conservatism of the proposed condition.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). The superscript "T" represents the transpose. The notation $|\cdot|$ refers to the Euclidean vector norm. We use $\lambda_{\min}(\cdot)$ to denote the minimum eigenvalue of a real symmetric matrix. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

II. Main Results

Consider a continuous time-delayed Hopfield neural network which is described by the following nonlinear retarded functional differential equations:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} g_j\left(x_j(t-\tau)\right) + b_i, \quad i = 1, 2, \ldots, n \tag{1}$$

or equivalently

$$\dot{x}(t) = -Cx(t) + Ag\left(x(t-\tau)\right) + b \tag{2}$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^T$$

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is the state vector

$$g(x(t-\tau)) = [g_1(x_1(t-\tau)) \quad g_2(x_2(t-\tau)) \quad \cdots g_n(x_n(t-\tau))]^T.$$

In (2), $C = \mathrm{diag}(c_1, c_2, \ldots, c_n) > 0$, $A = \{a_{ij}\}$ are the delayed interconnection matrix representing the weighting coefficients of the neurons, $b = [b_1 \ b_2 \ \cdots \ b_n]^T$ is a constant input vector. The scalar $\tau > 0$ is a constant delay of the system.

Throughout the paper, we make the following assumption.

Assumption 1: [2], [21]: The activation function $g(x)$ is boundless and satisfies

$$0 \le \frac{g_i(\xi_1) - g_i(\xi_2)}{\xi_1 - \xi_2} \le l_i, \qquad i = 1, 2, \ldots, n \tag{3}$$

for any $\xi_1, \xi_2 \in \mathbb{R}$ and $\xi_1 \neq \xi_2$.

Now, let $x^* = [x_1^* \ x_2^* \ \cdots \ x_n^*]^T$ be an equilibrium point of (2), and set

$$y(t) = x(t) - x^*. \tag{4}$$

Then, it is easy to see that (2) can be transformed to

$$\dot{y}(t) = -Cy(t) + Af(y(t-\tau)) \tag{5}$$

where

$$y(t) = [y_1(t) \quad y_2(t) \quad \cdots \quad y_n(t)]^T$$

is the state vector of the transformed system, and

$$f(y(t)) = [f_1(y_1(t)) \quad f_2(y_2(t)) \quad \cdots \quad f_n(y_n(t))]^T$$

with

$$f_i(y_i(t)) = g_i(y_i(t) + x_i^*) - g_i(x_i^*), \qquad i = 1, 2, \ldots, n$$

and $f_i(0) = 0$, for $i = 1, 2, \ldots, n$. It is noted that $f_i(\cdot)$ satisfies (3); that is

$$0 \le \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \le l_i, \quad i = 1, 2, \ldots, n \tag{6}$$

for any $\xi_1, \xi_2 \in \mathbb{R}$ and $\xi_1 \neq \xi_2$.

It can be seen that the global asymptotic stability of x^* of (2) is equivalent to that of the trivial solution of (5). Therefore, in the next, attention will be focused on the study of the global asymptotic stability of (5).

Before presenting the main results, we give the following lemma, which will be used in the sequel.

Lemma 1: [17]: Let $\mathcal{D}$, $\mathcal{S}$ and $\mathcal{P}$ be real matrices of appropriate dimensions with $\mathcal{P} > 0$. Then for any vectors x, y with appropriate dimensions

$$2x^T \mathcal{D}\mathcal{S}y \le x^T \mathcal{D}\mathcal{P}\mathcal{D}^T x + y^T \mathcal{S}^T \mathcal{P}^{-1} \mathcal{S} y.$$

Now, we are in a position to present a new asymptotic stability condition for system (5), which is dependent on the size of the delay.

Theorem 1: The origin of the delayed Hopfield neural network in (5) is the unique equilibrium point and it is globally asymptotically stable for any delay $0 < \tau \le \bar{\tau}$ if there exist matrices $P > 0$, $Q > 0$, $S > 0$, Y, Z, and two diagonal matrices $W > 0$ and $H > 0$ such that the LMI shown in (7) at the bottom of the page, holds, where

$$L = \mathrm{diag}(l_1, l_2, \ldots, l_n) \tag{8}$$

$$\Psi_{11} = -PC - CP + Q - Y - Y^T \tag{9}$$

$$\Psi_{12} = \frac{1}{2}PAL + Y - Z^T \tag{10}$$

$$\Psi_{22} = Z + Z^T - Q + \frac{1}{4}LHL. \tag{11}$$

Proof: We will first prove the uniqueness of the equilibrium point by contradiction. Let $\bar{y}$ be the equilibrium point of the delayed Hopfield neural network in (5). Then, we have

$$-C\bar{y} + Af(\bar{y}) = 0. \tag{12}$$

Suppose $\bar{y} \neq 0$. Then, from (12), we have

$$2\bar{y}^T P[-C\bar{y} + Af(\bar{y})] = 0 \tag{13}$$

which can be re-written as

$$2\bar{y}^T P\left(-C + \frac{1}{2}AL\right)\bar{y} + 2\bar{y}^T PA\left[\bar{F}(\bar{y}) - \frac{1}{2}L\right]\bar{y} = 0 \tag{14}$$

where

$$\bar{F}(y) = \mathrm{diag}\left(\bar{F}_1(y_1), \bar{F}_2(y_2), \cdots \bar{F}_n(y_n)\right) \tag{15}$$

$$\bar{F}_i(y_i) = \begin{cases} \frac{f_i(y_i)}{y_i}, & y_i \neq 0 \\ 0, & y_i = 0. \end{cases} \tag{16}$$

By (6), we have that for $i = 1, 2, \ldots, n$

$$-\frac{1}{2}l_i \le \bar{F}_i(y_i) - \frac{1}{2}l_i \le \frac{1}{2}l_i.$$

That is

$$-\frac{1}{2}L \le \bar{F}(\bar{y}) - \frac{1}{2}L \le \frac{1}{2}L. \tag{17}$$

$$\begin{bmatrix} \Psi_{11} + \bar{\tau}C^T SC & \Psi_{12} - \frac{1}{2}\bar{\tau}C^T SAL & \bar{\tau}Y & \bar{\tau}C^T SA & PA \\ \Psi_{12}^T - \frac{1}{2}\bar{\tau}LA^T SC & \Psi_{22} + \frac{1}{4}\bar{\tau}LA^T SAL + \frac{1}{4}\bar{\tau}LWL & \bar{\tau}Z & -\frac{1}{2}\bar{\tau}LA^T SA & 0 \\ \bar{\tau}Y^T & \bar{\tau}Z^T & -\bar{\tau}S & 0 & 0 \\ \bar{\tau}A^T SC & -\frac{1}{2}\bar{\tau}A^T SAL & 0 & \bar{\tau}A^T SA - \bar{\tau}W & 0 \\ A^T P & 0 & 0 & 0 & -H \end{bmatrix} < 0 \tag{7}$$

Therefore, using Lemma 1, we obtain

$$\begin{aligned}
&2\bar{y}^T PA\left[\bar{F}(\bar{y}) - \frac{1}{2}L\right]\bar{y}\\
&\quad\le \bar{y}^T PAH^{-1}A^T P\bar{y} + \bar{y}^T\left[\bar{F}(\bar{y}) - \frac{1}{2}L\right]^T\\
&\qquad\times H\left[\bar{F}(\bar{y}) - \frac{1}{2}L\right]\bar{y}\\
&\quad\le \bar{y}^T PAH^{-1}A^T P\bar{y} + \frac{1}{4}\bar{y}^T LHL\bar{y}.
\end{aligned}$$

This together with (14) gives

$$\bar{y}^T\left[P\left(-C + \frac{1}{2}AL\right) + \left(-C + \frac{1}{2}AL\right)^T P + \frac{1}{4}LHL + PAH^{-1}A^T P\right]\bar{y} \ge 0. \quad (18)$$

On the other hand, considering (7), it can be shown that

$$\begin{bmatrix} \Psi_{11} + PAH^{-1}A^T P & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22} \end{bmatrix} < 0.$$

That is, we get the inequality shown at the bottom of the page. Pre- and post-multiplying this inequality by

$$[I \quad I]$$

and its transpose, respectively, we obtain

$$P\left(-C + \frac{1}{2}AL\right) + \left(-C + \frac{1}{2}AL\right)^T P + \frac{1}{4}LHL + PAH^{-1}A^T P < 0$$

which contradicts with (18). Therefore, we have $\bar{y} = 0$. That is, (5) has a unique equilibrium point.

Next, we show that the unique equilibrium point of (5) is globally asymptotically stable. To this end, we define a Lyapunov–Krasovskii functional candidate as

$$V(y_t) = V_1(y_t) + V_2(y_t) + V_3(y_t)$$

where

$$y_t = y(t+\theta), \quad -2\tau \le \theta \le 0$$

and

$$V_1(y_t) = y(t)^T Py(t) \quad (19)$$

$$V_2(y_t) = \int_{t-\tau}^{t} y(\alpha)^T Qy(\alpha)d\alpha \quad (20)$$

$$V_3(y_t) = \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{y}(\alpha)^T S\dot{y}(\alpha)d\alpha d\beta. \quad (21)$$

Noting (15)–(17) and applying Lemma 1, we have that the time-derivative of $V_i(x_t)$, $i = 1, 2, 3$, along the solution of (5) gives

$$\begin{aligned}
\dot{V}_1(y_t) &= 2y(t)^T P\left[-Cy(t) + A\bar{F}(y(t-\tau))\,y(t-\tau)\right]\\
&\le 2y(t)^T P\left[-Cy(t) + \frac{1}{2}ALy(t-\tau)\right]\\
&\quad + y(t)^T PAH^{-1}A^T Py(t)\\
&\quad + y(t-\tau)^T\left[\bar{F}(y(t-\tau)) - \frac{1}{2}L\right]\\
&\quad \times H\left[\bar{F}(y(t-\tau)) - \frac{1}{2}L\right]y(t-\tau)\\
&\le 2y(t)^T P\left[-Cy(t) + \frac{1}{2}ALy(t-\tau)\right]\\
&\quad + y(t)^T PAH^{-1}A^T Py(t)\\
&\quad + \frac{1}{4}y(t-\tau)^T LHLy(t-\tau) \quad (22)\\
\dot{V}_2(y_t) &= y(t)^T Qy(t) - y(t-\tau)^T Qy(t-\tau) \quad (23)\\
\dot{V}_3(y_t) &= \int_{-\tau}^{0}\left[\dot{y}(t)^T S\dot{y}(t) - \dot{y}(t+\beta)^T S\dot{y}(t+\beta)\right]d\beta\\
&= \int_{t-\tau}^{t}\left[\dot{y}(t)^T S\dot{y}(t) - \dot{y}(\beta)^T S\dot{y}(\beta)\right]d\beta. \quad (24)
\end{aligned}$$

Furthermore

$$\begin{aligned}
\dot{y}(t)^T S\dot{y}(t) \le{}& \left[-Cy(t) + \frac{1}{2}ALy(t-\tau)\right]^T\\
&\times S\left[-Cy(t) + \frac{1}{2}ALy(t-\tau)\right]\\
&+ \left[-Cy(t) + \frac{1}{2}ALy(t-\tau)\right]^T\\
&\times SA(W - A^T SA)^{-1}A^T S\\
&\times\left[-Cy(t) + \frac{1}{2}ALy(t-\tau)\right]\\
&+ \frac{1}{4}y(t-\tau)^T LWLy(t-\tau). \quad (25)
\end{aligned}$$

$$\begin{bmatrix} -PC - CP + Q - Y - Y^T + PAH^{-1}A^T P & \frac{1}{2}PAL + Y - Z^T \\ \frac{1}{2}LA^T P + Y^T - Z & Z + Z^T - Q + \frac{1}{4}LHL \end{bmatrix} < 0.$$

Now, by the Leibniz–Newton formula, it is easy to show that

$$y(t-\tau) = y(t) - \int_{t-\tau}^{t} \dot{y}(\alpha)d\alpha.$$

This together with (22)–(25) gives

$$\begin{aligned}\dot{V}(y_t) \le & 2y(t)^T P\left[-Cy(t)+\frac{1}{2}ALy(t-\tau)\right] \\ & + y(t)^T PAH^{-1}A^T Py(t) \\ & + \frac{1}{4}y(t-\tau)^T LHLy(t-\tau)+y(t)^T Qy(t) \\ & - y(t-\tau)^T Qy(t-\tau) \\ & + \int_{t-\tau}^{t}\left\{\left[-Cy(t)+\frac{1}{2}ALy(t-\tau)\right]^T\right. \\ & \quad \times S\left[-Cy(t)+\frac{1}{2}ALy(t-\tau)\right] \\ & \quad +\left[-Cy(t)+\frac{1}{2}ALy(t-\tau)\right]^T \\ & \quad \times SA(W-A^T SA)^{-1}A^T S \\ & \quad \times\left[-Cy(t)+\frac{1}{2}ALy(t-\tau)\right]+\frac{1}{4}y(t-\tau)^T \\ & \quad \left.\times LWLy(t-\tau)-\dot{y}(\beta)^T S\dot{y}(\beta)\right\}d\beta \\ & + 2y(t)^T Y\int_{t-\tau}^{t}\dot{y}(\alpha)d\alpha-2y(t)^T Y\left[y(t)-y(t-\tau)\right] \\ & + 2y(t-\tau)^T Z\int_{t-\tau}^{t}\dot{y}(\alpha)d\alpha-2y(t-\tau)^T \\ & \times Z\left[y(t)-y(t-\tau)\right] \\ = & \frac{1}{\tau}\int_{t-\tau}^{t}\delta(t,\beta)^T\Pi(\tau)\delta(t,\beta)d\beta \end{aligned} \tag{26}$$

where

$$\begin{aligned}\Pi(\tau) = & \begin{bmatrix}\Psi_{11}+PAH^{-1}A^T P & \Psi_{12} & \tau Y \\ \Psi_{12}^T & \Psi_{22}+\frac{1}{4}\tau LWL & \tau Z \\ \tau Y^T & \tau Z^T & -\tau S\end{bmatrix} \\ & + \begin{bmatrix} C^T \\ -\frac{1}{2}LA^T \\ 0\end{bmatrix}\left[\tau S+\tau SA(W-A^T SA)^{-1}A^T S\right] \\ & \times\begin{bmatrix} C^T \\ -\frac{1}{2}LA^T \\ 0\end{bmatrix}^T\end{aligned}$$

$$\delta(t,\beta) = \begin{bmatrix} y(t)^T & y(t-\tau)^T & \dot{y}(\beta)^T\end{bmatrix}^T$$

where Ψ_{11}, Ψ_{12} and Ψ_{22} are given in (9)–(11), respectively. Now, applying the Schur complement formula to the LMI in (7) results in

$$\begin{aligned}&\begin{bmatrix}\Psi_{11}+PAH^{-1}A^T P & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22}+\frac{1}{4}\bar{\tau}LWL\end{bmatrix}+\begin{bmatrix}-C^T \\ \frac{1}{2}LA^T\end{bmatrix} \\ &\quad\times\left[\bar{\tau}S+\bar{\tau}SA(W-A^T SA)^{-1}A^T S\right]\begin{bmatrix}-C^T \\ \frac{1}{2}LA^T\end{bmatrix}^T \\ &\quad+\begin{bmatrix}Y \\ Z\end{bmatrix}\bar{\tau}S^{-1}\begin{bmatrix}Y \\ Z\end{bmatrix}^T < 0.\end{aligned}$$

Then, it is easy to show that for all $0 < \tau \le \bar{\tau}$, we have

$$\begin{aligned}&\begin{bmatrix}\Psi_{11}+PAH^{-1}A^T P & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22}+\frac{1}{4}\tau LWL\end{bmatrix} \\ &\quad+\begin{bmatrix}C^T \\ -\frac{1}{2}LA^T\end{bmatrix}\left[\tau S+\tau SA(W-A^T SA)^{-1}A^T S\right] \\ &\quad\times\begin{bmatrix}C^T \\ -\frac{1}{2}LA^T\end{bmatrix}^T+\begin{bmatrix}Y \\ Z\end{bmatrix}\tau S^{-1}\begin{bmatrix}Y \\ Z\end{bmatrix}^T < 0\end{aligned}$$

which, by the Schur complement formula, implies $\Pi(\tau) < 0$. This together with (26) gives

$$\dot{V}(y_t) \le -k\left\|y(t)\right\|^2 \tag{27}$$

where $k = (1/\tau)\lambda_{\min}(-\Upsilon(\tau)) > 0$. Finally, by [11], it follows from (27) that the delayed Hopfield neural network in (5) is asymptotically stable for any delay $0 < \tau \le \bar{\tau}$. This completes the proof. □

Remark 1: Theorem 1 provides a new delay-dependent global asymptotic stability condition for delayed Hopfield neural networks in terms of an LMI; this is derived by choosing an appropriate Lyapunov functional different from those in [7], [19], [21], which leads to a less conservative stability condition in (7).

III. Examples

In this section, we provide examples to illustrate the reduced conservatism of Theorem 1 by comparing it with recently reported delay-dependent asymptotic stability results in the literature.

Example 1: [21]: Consider a delayed Hopfield network described by

$$\begin{aligned}\dot{x}_1(t) &= -0.7x_1(t)+0.1g_1(x_1(t-\tau))+0.1g_2(x_2(t-\tau))-2 \\ \dot{x}_2(t) &= -0.7x_1(t)+0.3g_1(x_1(t-\tau))+0.3g_2(x_2(t-\tau))+1\end{aligned}$$

where the activation function is given as

$$g_1(x) = g_2(x) = 0.5\left(|x+1|-|x-1|\right).$$

Then, it is easy to see that

$$C = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix} \quad A = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

By the methods in [7], [19] and [21], the maximum delay $\bar{\tau}$ that guarantees the delayed Hopfield network to be asymptotically stable are calculated to be 0.3636, 2.2361 and 0.4926, respectively. However, by Theorem 1 in this paper, it is found that LMI (7) is feasible for any arbitrarily large $\bar{\tau}$ (as long as the numerical computation is reliable). Therefore, it can be seen that Theorem 1 provides a less conservative condition for this example.

Example 2: Consider a delayed Hopfield network in (5) with parameters

$$C = \begin{bmatrix} 4.1989 & 0 & 0 \\ 0 & 0.7160 & 0 \\ 0 & 0 & 1.9985 \end{bmatrix}$$
$$A = \begin{bmatrix} -0.1052 & -0.5069 & -0.1121 \\ -0.0257 & -0.2808 & 0.0212 \\ 0.1205 & -0.2153 & 0.1315 \end{bmatrix}$$
$$L = \begin{bmatrix} 0.4219 & 0 & 0 \\ 0 & 3.8993 & 0 \\ 0 & 0 & 1.0160 \end{bmatrix}.$$

For this example, both of the delay-dependent conditions in [7], [21], [22] cannot be satisfied for any $\tau > 0$. Therefore, they cannot provide any results on the maximum allowed delay $\bar{\tau}$. However, by the method in [18] and [19], we obtain $\bar{\tau}$ as 1.7484 and 0.4121, respectively, while by Theorem 1 in this paper, we have $\bar{\tau} = 1.7644$, which also shows that the condition given in Theorem 1 is less conservative than those in [7], [18], [19], [21], and [22].

IV. Conclusion

An improved delay-dependent asymptotic stability condition for Hopfield neural networks with time delays has been developed in this paper. This condition is expressed in terms of an LMI, which can be checked easily. Examples have been provided to demonstrate the less conservatism of the proposed results.

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