Evaluation of the stability of anchor-reinforced slopes

D.Y. Zhu, C.F. Lee, D.H. Chan, and H.D. Jiang

Abstract: The conventional methods of slices are commonly used for the analysis of slope stability. When anchor loads are involved, they are often treated as point loads, which may lead to abrupt changes in the normal stress distribution on the potential slip surface. As such abrupt changes are not reasonable and do not reflect reality in the field, an alternative approach based on the limit equilibrium principle is proposed for the evaluation of the stability of anchor-reinforced slopes. With this approach, the normal stress distribution over the slip surface before the application of the anchor (i.e., σ_0) is computed by the conventional, rigorous methods of slices, and the normal stress on the slip surface purely induced by the anchor load (i.e., $\lambda_p\sigma_p$, where λ_p is the load factor) is taken as the analytical elastic stress distribution in an infinite wedge approximating the slope geometry, with the anchor load acting on the apex. Then the normal stress on the slip surface for the anchor-reinforced slope is assumed to be the linear combination of these two normal stresses involving two auxiliary unknowns, η_1 and η_2; that is, $\sigma = \eta_1\sigma_0 + \eta_2\lambda_p\sigma_p$. Simultaneously solving the horizontal force, the vertical force, and the moment equilibrium equations for the sliding body leads to the explicit expression for the factor of safety (F_s)—or the load factor (λ_p), if the required factor of safety is prescribed. The reasonableness and advantages of the present method in comparison with the conventional procedures are demonstrated with two illustrative examples. The proposed procedure can be readily applied to designs of excavated slopes or remediation of landslides with steel anchors or prestressed cables, as well as with soil nails or geotextile reinforcements.

Key words: slopes, factor of safety, anchors, limit equilibrium method.

Résumé : Les méthodes conventionnelles des tranches sont habituellement utilisées pour l'analyse de la stabilité des talus. Lorsque des charges d'ancrage sont impliquées, elles sont souvent traitées comme des charges ponctuelles, ce qui peut conduire à des changements abruptes dans la distribution de la contrainte normale sur la surface potentielle de glissement. Comme de tels changements abruptes ne sont pas raisonnables et ne reflètent pas la réalité sur le terrain, on propose une approche alternative basée sur le principe d'équilibre limite pour l'évaluation de la stabilité des talus armés par des ancrages. Avec cette approche, la distribution de la contrainte normale sur la surface de glissement avant l'application de l'ancrage, i.e., σ_0, est calculée par les méthodes conventionnelles rigoureuses des tranches, alors que la contrainte normale sur la surface de glissement purement induite par la charge d'ancrage, i.e., $\lambda_p\sigma_p$ (λ_p étant le facteur de charge), est prise comme la distribution de la contrainte analytique élastique en un coin infini qui représente approximativement la géométrie de la pente avec la charge d'ancrage agissant sur le sommet. Alors on suppose que la contrainte normale sur la surface de glissement pour le talus armé d'ancrages est la combinaison linéaire de ces deux contraintes normales impliquant deux inconnues η_1 et η_2, c'est-à-dire, $\sigma = \eta_1\sigma_0 + \eta_2\lambda_p\sigma_p$. La solution simultanée des équations de la force horizontale, de la force verticale et du moment d'équilibre pour le corps en mouvement conduit à l'expression explicite pour le coefficient de sécurité F_s ou pour le facteur de charge λ_p si le coefficient de sécurité requis est prescrit. Le caractère raisonnable et l'avantage de la présente méthode en comparaison avec les procédures conventionnelles sont démontrés ici au moyen de deux exemples explicatifs. La procédure proposée peut être appliquée aisément aux conceptions de pentes excavées ou de comportement de glissements avec des ancrages d'acier ou des câbles précontraints, de même qu'avec des clous dans le sol ou des armatures géotechniques.

Mots clés : talus, coefficient de sécurité, ancrages, méthode d'équilibre limite.

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Introduction

Anchors and soil nails are commonly used to stabilize potentially unstable slopes. The anchor loads not only directly provide the forces and (or) moments counteracting those forces tending to destabilize the slope but also improve the shear resistance along the slip surface by increasing the normal stress on that surface (Hobst and Zajic 1983; Bromhead 1994). Evaluation of slope stability, including the anchor loads, is important for the design of stabilization measures involving anchors.

Limit equilibrium methods of slices have been widely used for calculating factors of safety for natural and constructed slopes (Duncan 1996). The commonly used methods include those proposed by Fellenius (1936), Bishop (1955), Morgenstern and Price (1965), Spencer (1967), and Janbu (1973). In principle, all these conventional methods of slices could accommodate anchor loads or other types of concentrated forces acting upon the slope. The most straightforward treatment of concentrated forces is to include them as external forces acting on corresponding slices (Hutchinson 1977; Fredlund and Krahn 1977; Zhu et al. 2001). However, such a treatment will lead to an unreasonably abrupt increase in normal stress on the base of the associated slices (Krahn 2003). This means that the contribution of anchor loads to the increase in shear resistance is solely related to the shear strength of that associated segment on the slip surface, as will be shown later in this paper. This is evidently unreasonable from both theoretical and practical points of view, as the normal stresses on the slip surface induced by an anchor would not be concentrated on a narrow segment. Thus, questions are raised on the reasonableness of directly using conventional methods of slices for analysing the stability of anchor-reinforced slopes.

Notwithstanding the above limitation, the methods of slices are generally accepted as a reliable analytical tool for slope stability, as they have been found to give approximately equal factors of safety (within 15% tolerance) as long as they satisfy the complete equilibrium conditions for the whole sliding body. The commonly used rigorous methods of slices generally assume continuous (and often rather smooth) distribution of the inclinations of interslice forces (Morgenstern and Price 1965; Spencer 1967) or continuous location of the line of thrust across the sliding mass (Janbu 1973), thereby resulting in continuous distribution of normal stresses along the slip surface. Such assumptions of continuity approximately reflect the real characteristics of those slopes subject to gravity, pore-water pressures, and seismic forces. However, when the slope is acted upon by a concentrated load at the ground surface, both the inclinations (and the magnitude) of the interslice forces and the location of the line of thrust are no longer continuous across the sliding body, but the normal stress distribution along the slip surface should still remain continuous. Thus, if the conventional assumptions are made in this case, the resultant characteristics of the interslice forces and the normal stresses on the slip surface would be reversed and contrary to reality. To overcome this inherent shortcoming of the conventional methods, we propose an alternative based on the assumption of continuous normal stress distribution along the slip surface. Before the application of anchor loads, the normal stresses on the slip surface are assumed to be those calculated by the conventional rigorous methods of slices (e.g., the Morgenstern–Price method or the Spencer method). The normal stresses induced by the anchor load are approximately obtained from an elastic solution. The linear combination of these two parts constitutes the distribution of normal stresses on the slip surface of the anchor-reinforced slope. Solving the complete equilibrium equations for the sliding body yields the factor of safety for the slope with given anchor loads or the magnitude of the anchor load required to stabilize the slope with a specified value for the factor of safety.

Basic formulation

A typical slope with anchor loads ($\lambda_p P_3$, $\lambda_p P_2$, with λ_p as the load factor) is shown in Fig. 1*a*. For general purposes, the slip surface is of arbitrary shape. In addition to the anchor loads, the slope body is subject to self-weight (γ), horizontal seismic force ($k_c\gamma$) and pore-water pressure u (not shown in the figure). Without the action of anchor loads, the factor of safety can be calculated by using any method of slices accommodating the general-shaped slip surface. The Morgenstern–Price method (Morgenstern and Price 1965), with an interslice force of constant inclination, is suggested for this purpose. The distribution of normal stresses (σ_0, in terms of total stress) can be obtained as a by-product of the computation process.

In response to the action of anchor loads, an additional normal stress distribution ($\lambda_p\sigma_p$) is induced along the slip surface. Consider a single anchor load, P, acting at point (x_p, y_p) on the slope at an angle of i to the horizontal, as shown in Fig. 1*b*. The induced normal stress on the slip surface is denoted by σ_p. Because the analysis is within the framework of limit equilibrium, the normal stress on the slip surface is not required to be theoretically exact. Thus, for practical purposes, σ_p is assumed to be the elastic stress associated with an infinite wedge with its two edges connecting the point of action of P and the two ends of the slip surface. Fortunately, the analytical solution to σ_p is available from the mechanics of elasticity. As shown in Fig. 2, a pair of forces, P_H (horizontal) and P_V (vertical) act at the apex of an infinite wedge with its symmetrical axis in the horizontal direction and its edges lying at angle of β to the horizontal. According to the mechanics of elasticity (Timoshenko and Goodier 1970), the stresses at a point with polar coordinates (r, θ) in the wedge are

$$[1a] \quad \sigma_r = \frac{P_H \cos\theta}{r(\beta + 0.5 \sin 2\beta)} + \frac{P_V \sin\theta}{r(\beta - 0.5 \sin 2\beta)}$$

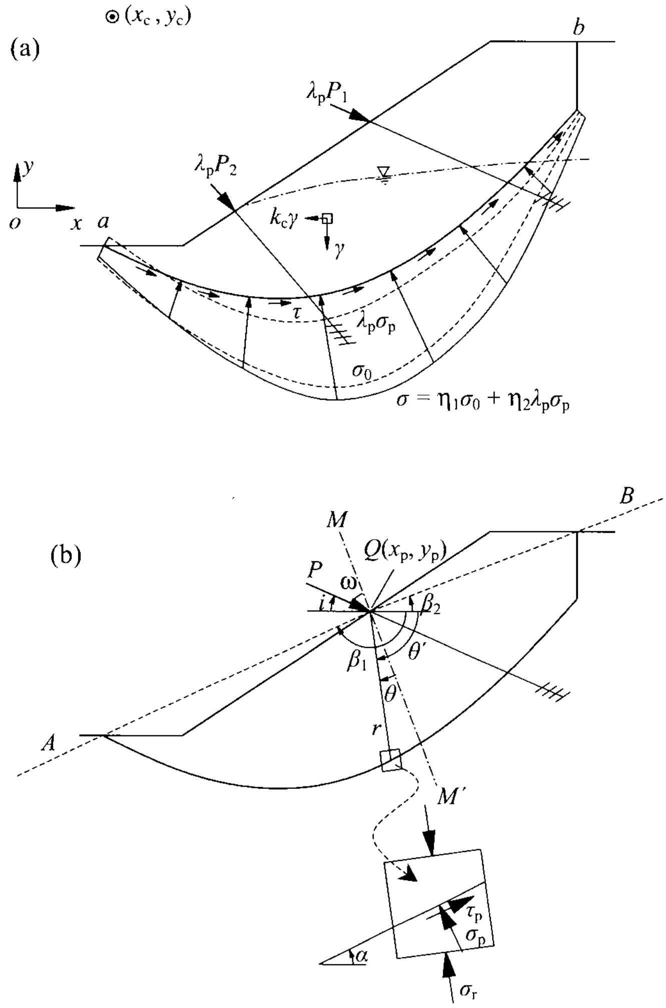
$$[1b] \quad \sigma_\theta = 0$$

$$[1c] \quad \tau_{r\theta} = 0$$

where σ_r is the radial stress; σ_θ is the circumferential stress; and $\tau_{r\theta}$ is the shear stress.

Now consider the corresponding wedge shown in Fig. 1*b*, with its lower and upper edges extending at angles of β_1 and β_2 to the horizontal, respectively. The concentrated force P lies at an angle of ω with the symmetrical axis MM′. The polar coordinates of the point considered are (r, θ'), corresponding to (r, θ) in the coordinate system in Fig. 2. From the geometrical relation, we can see that

Fig. 1. Diagram of an anchor-reinforced slope. (*a*) Slope with normal stresses on the slip surface induced by self-weight and anchor loads, respectively. (*b*) Geometry for computing normal stresses on the slip surface purely induced by an anchor load.



[2*a*] $$\beta = \frac{\beta_1 + \beta_2}{2}$$

[2*b*] $$\theta = \theta' + \beta_2 - \frac{\beta_1 + \beta_2}{2} = \theta' - \frac{\beta_1 - \beta_2}{2}$$

[2*c*] $$\omega = \theta' - \theta - i = \frac{\beta_1 - \beta_2}{2} - i$$

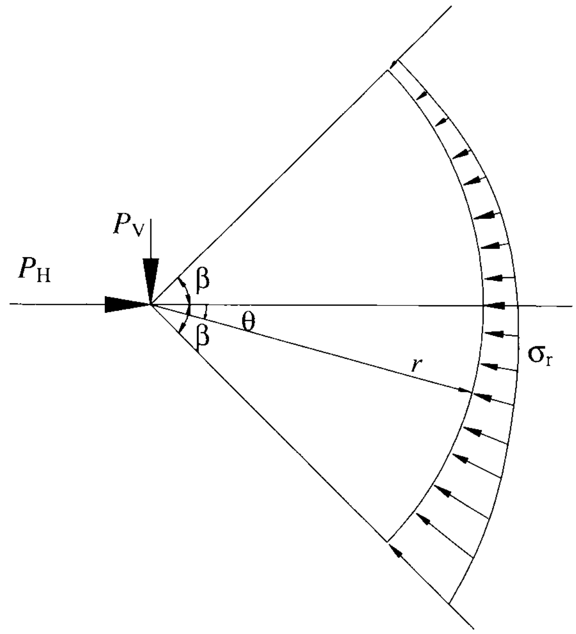
Thus

[3*a*] $$P_H = P\cos\omega = P\cos\left(\frac{\beta_1 - \beta_2}{2} - i\right)$$

[3*b*] $$P_V = -P\sin\omega = -P\sin\left(\frac{\beta_1 - \beta_2}{2} - i\right)$$

Substituting eqs. [3*a*] and [3*b*] into eq. [1*a*] gives

Fig. 2. Stress distribution in a wedge with concentrated forces at its apex.



[4] $$\sigma_r = \frac{2P}{r}\left[\frac{\cos\left(\frac{\beta_1 - \beta_2}{2} - i\right)\cos\left(\theta' - \frac{\beta_1 - \beta_2}{2}\right)}{\beta_1 + \beta_2 + \sin(\beta_1 + \beta_2)} - \frac{\sin\left(\frac{\beta_1 - \beta_2}{2} - i\right)\sin\left(\theta' - \frac{\beta_1 - \beta_2}{2}\right)}{\beta_1 + \beta_2 - \sin(\beta_1 + \beta_2)}\right]$$

The circumferential and shear stresses are still zero.

The normal stress σ_p on the slip surface with an inclination of α to the horizontal is obtained from static analysis as

[5] $$\sigma_p = \sigma_r \sin^2(\theta' + \alpha)$$

If more than one anchor load is acting on the slope, σ_p is taken as the sum of their individual contributions.

Usually, the prestressing of an anchor is accomplished over a short duration, and some cohesive soils are, to some degree, in an undrained condition. This will lead to a change in pore-water pressure (Δu) within the sliding mass. According to Skempton (1954), Δu is related to changes in the principal stresses in the soil by the following relationship:

[6] $$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

where A and B are pore pressure parameters; and $\Delta\sigma_1$ and $\Delta\sigma_3$ are changes in major and minor principal stresses respectively.

The pore pressure parameters A and B can determined by laboratory tests. For saturated soils, B approaches unity. The value of A varies with the degree of overconsolidation of the soil, being positive for normally consolidated soils (in the range of 0.5–1.0), and in contrast, being negative for heavily consolidated soils (in the range of –0.5 to 0.0).

As is shown in eqs. [1a]–[1c], if only one anchor load acts, the changes in major and minor principal stresses would be

[7a] $\Delta\sigma_1 = \lambda_p\sigma_r$

[7b] $\Delta\sigma_3 = 0$

Thus

[8] $\Delta u = \overline{B}\lambda_p\sigma_r$

where $\overline{B} = BA$.

However, if two or more anchor loads act, the soil in the slope will no longer be under uniaxial stress. Because we are attempting to only approximately evaluate the effect of the degree of drainage on slope stability, $\Delta\sigma_1$ is herein assumed to be the simple algebraic sum of σ_r caused by individual anchor loads.

Before the application of anchor loads, we can calculate the factor of safety for the slope with existing methods of slices and obtain the normal stress on the slip surface σ_0. After the anchor loads are applied, the factor of safety for the slope would change and would need to be recalculated. Rather than using the conventional methods of slices, which make assumptions about the interslice forces, we apply the principle of the newly proposed procedure (Zhu et al. 2003) by modifying the normal stress on the slip surface and using it to compute the stability of the anchor-reinforced slopes.

Because there are three equilibrium conditions for the whole sliding body, and one unknown (i.e., the factor of safety, F_s) is to be determined, we can assume normal stress (σ) on the slip surface, with two auxiliary unknowns. Naturally, the normal stress (σ) is contributed by two parts: σ_0 and $\lambda_p\sigma_p$. To render the problem determinate, we assume that

[9] $\sigma = \eta_1\sigma_0 + \eta_2\lambda_p\sigma_p$

where η_1 and η_2 are the auxiliary unknowns.

A constant factor of safety (F_s) is assigned to the whole slip surface. The shear resistance along the slip surface is determined by the Mohr–Coulomb failure criterion and the principle of effective stress:

[10] $$\tau = \frac{1}{F_s}[(\sigma - u - \Delta u)\tan\phi' + c']$$

where ϕ' and c' are the effective internal friction angle and cohesion, respectively.

For simplicity, suppose that

[11] $\psi = \tan\phi'$; $c = c'$

From eqs. [10] and [11], it follows that

[12] $$\tau = \frac{1}{F_s}[(\sigma - u)\psi + c] - \frac{1}{F_s}\overline{B}\lambda_p\Delta\sigma_1\psi$$

From the horizontal and vertical force equilibrium and the moment equilibrium with respect to an arbitrarily specified point (x_c, y_c), one obtains

[13a] $$\int_a^b(-\sigma s' + \tau - k_c w)\,dx = -\lambda_p\sum P_x$$

[13b] $$\int_a^b(\sigma + \tau s' - w)\,dx = \lambda_p\sum P_y$$

[13c] $$\int_a^b[(-\sigma s' + \tau)(y_c - s) + (\sigma + \tau s' - w)(x - x_c) - k_c w(y_c - 0.5s - 0.5g)]\,dx = \lambda_p\sum P_x(y_p - y_c) + \lambda_p\sum P_y(x_p - x_c)$$

where P_x (positive to the right) and P_y (positive downwards) are horizontal and vertical components of anchor load P (the suffix identification is omitted for simplicity); $s(x)$ and $g(x)$ denote the curves of the slip surface and the ground, respectively; $w(x)$ denotes the self-weight of a slice of unit width; and $s'(x)$ is the inclination of the slip surface (i.e., $s' = \tan\alpha$).

Assuming that

[14a] $F_x = \int_a^b k_c w\,dx$

[14b] $F_y = \int_a^b w\,dx$

[14c] $M_c = \int_a^b[k_c w(y_c - 0.5s - 0.5g) + w(x - x_c)]\,dx$

[14d] $\sum M_p = \sum P_x(y_p - y_c) + \sum P_y(x_p - x_c)$

[14e] $r_\sigma(x) = -s'(y_c - s) + x - x_c$

[14f] $r_\tau(x) = y_c - s + s'(x - x_c)$

and considering eqs. [9] and [12], eqs. [13a]–[13c] are rewritten as

[15a] $$\int_a^b(\eta_1\sigma_0 + \eta_2\lambda_p\sigma_p)\left(-s' + \psi\frac{1}{F_s}\right)dx = F_x - \lambda_p\sum P_x + \frac{1}{F_s}\int_a^b(u\psi - c)\,dx + \frac{\lambda_p}{F_s}\int_a^b\overline{B}\Delta\sigma_1\psi\,dx$$

[15b] $$\int_a^b(\eta_1\sigma_0 + \eta_2\lambda_p\sigma_p)\left(1 + s'\psi\frac{1}{F_s}\right)dx = F_y + \lambda_p\sum P_y + \frac{1}{F_s}\int_a^b s'(u\psi - c)\,dx + \frac{\lambda_p}{F_s}\int_a^b s'\overline{B}\Delta\sigma_1\psi\,dx$$

[15c] $$\int_a^b(\eta_1\sigma_0 + \eta_2\lambda_p\sigma_p)\left(r_\sigma + r_\tau\psi\frac{1}{F_s}\right)dx = M_c + \lambda_p\sum M_p + \frac{1}{F_s}\int_a^b r_\tau(u\psi - c)\,dx + \frac{\lambda_p}{F_s}\int_a^b r_\tau\overline{B}\Delta\sigma_1\psi\,dx$$

Solving eqs. [15a]–[15c] simultaneously will yield solutions to the factor of safety (F_s)—or the load factor (λ_p) if F_s is prescribed—and the auxiliary unknowns (λ_1 and λ_2) as well.

Solution to the factor of safety

If the magnitude of the anchor loads is given, the solution to the factor of safety of the reinforced slope is derived in this section. Assuming

$$[16]\quad \omega_c = u\psi - c + \lambda_p \bar{B}\Delta\sigma_1\psi$$

eqs. [15a]–[15c] are rewritten as

$$[17a]\quad \eta_1\int_a^b \sigma_0\left(-s' + \psi\frac{1}{F_s}\right)dx + \eta_2\int_a^b \lambda_p\sigma_p\left(-s' + \psi\frac{1}{F_s}\right)dx = F_x - \lambda_p\sum P_x + \frac{1}{F_s}\int_a^b \omega_c dx$$

$$[17b]\quad \eta_1\int_a^b \sigma_0\left(1 + s'\psi\frac{1}{F_s}\right)dx + \eta_2\int_a^b \lambda_p\sigma_p\left(1 + s'\psi\frac{1}{F_s}\right)dx = F_y + \lambda_p\sum P_y + \frac{1}{F_s}\int_a^b s'\omega_c dx$$

$$[17c]\quad F_s = \frac{\eta_1\int_a^b \sigma_0\psi r_\tau\, dx + \eta_2\int_a^b \lambda_p\sigma_p\psi r_\tau\, dx - \int_a^b r_\tau\,\omega_c dx}{-\eta_1\int_a^b \sigma_0 r_\sigma\, dx - \eta_2\int_a^b \lambda_p\sigma_p r_\sigma\, dx + M_c + \lambda_p\sum M_p}$$

The above equations are rearranged as

$$[18a]\quad \eta_1\left(A_1 + \frac{1}{F_s}A_1'\right) + \eta_2\left(A_2 + \frac{1}{F_s}A_2'\right) = A_3 + \frac{1}{F_s}A_3'$$

$$[18b]\quad \eta_1\left(B_1 + \frac{1}{F_s}B_1'\right) + \eta_2\left(B_2 + \frac{1}{F_s}B_2'\right) = B_3 + \frac{1}{F_s}B_3'$$

$$[18c]\quad F_s = \frac{D_1\eta_1 + D_2\eta_2 + D_3}{E_1\eta_1 + E_2\eta_2 + E_3}$$

in which

$$[19a]\quad A_1 = -\int_a^b s'\sigma_0 dx; \qquad A_1' = \int_a^b \psi\sigma_0 dx$$

$$[19b]\quad A_2 = -\int_a^b s'\lambda_p\sigma_p dx; \qquad A_2' = \int_a^b \psi\lambda_p\sigma_p dx$$

$$[19c]\quad A_3 = F_x - \lambda_p\sum P_x; \qquad A_3' = \int_a^b \omega_c dx$$

$$[19d]\quad B_1 = \int_a^b \sigma_0\, dx; \qquad B_1' = \int_a^b s'\psi\sigma_0 dx$$

$$[19e]\quad B_2 = \int_a^b \lambda_p\sigma_p dx; \qquad B_2' = \int_a^b s'\psi\lambda_p\sigma_p dx$$

$$[19f]\quad B_3 = F_y + \lambda_p\sum P_y; \qquad B_3' = \int_a^b s'\omega_c dx$$

$$[19g]\quad D_1 = \int_a^b \sigma_0\psi r_\tau dx; \qquad D_2 = \int_a^b \lambda_p\sigma_p\psi r_\tau dx; \qquad D_3 = -\int_a^b r_\tau\omega_c dx$$

$$[19h]\quad E_1 = -\int_a^b \sigma_0 r_\sigma dx; \qquad E_2 = -\int_a^b \lambda_p\sigma_p r_\sigma dx; \qquad E_3 = M_c + \lambda_p\sum M_p$$

Equations [18a]–[18c] can be analytically resolved, resulting in an explicit solution to the factor of safety (F_s) as follows:

$$[20]\quad F_s = \frac{t_2}{3} + \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

where p, q, and t can be computed with the parameters shown in eqs. [19a]–[19h]. The brief derivation of eq. [20] is presented in Appendix A; for details, see Zhu et al. (2003).

Solution to required anchor loads

In the design of measures for stabilizing failed slopes or slopes having unacceptable stability conditions, the magnitude of the required anchor loads is often needed. In this case, the magnitude of the required anchor loads can be calculated by trial and error using eq. [20] until the slope attains the specified factor of safety. It can also be directly computed using another explicit expression, the derivation of which is given below.

Assuming

$$[21a]\quad \omega_x = -s' + \psi\frac{1}{F_s}; \qquad \omega_y = 1 + s'\psi\frac{1}{F_s}; \qquad \omega_r = r_\sigma + r_\tau\psi\frac{1}{F_s}$$

$$[21b]\quad \omega_u = \frac{1}{F_s}(u\psi - c); \qquad \omega_b = \frac{1}{F_s}\bar{B}\Delta\sigma_1\psi$$

eqs. [15a]–[15c] are written in matrix form as

$$[22]\quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} \eta_1 \\ \lambda_p\eta_2 \\ \lambda_p \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

in which

$$[23a]\quad a_{11} = \int_a^b \sigma_0\omega_x dx, \qquad a_{12} = \int_a^b \sigma_p\omega_x dx, \qquad a_{13} = -\int_a^b \omega_b dx + \sum P_x$$

$$[23b]\quad a_{21} = \int_a^b \sigma_0\omega_y dx, \qquad a_{22} = \int_a^b \sigma_p\omega_y dx, \qquad a_{23} = -\int_a^b s'\omega_b dx - \sum P_y$$

$$[23c]\quad a_{31} = \int_a^b \sigma_0\omega_r dx, \qquad a_{32} = \int_a^b \sigma_p\omega_r dx, \qquad a_{33} = -\int_a^b r_\tau\omega_b dx - \sum M_p$$

$$[23d]\quad c_1 = F_x + \int_a^b \omega_u \mathrm{d}x, \qquad c_2 = F_y + \int_a^b s'\omega_u \mathrm{d}x,$$

$$c_3 = M_c + \int_a^b r_c \omega_u \mathrm{d}x$$

The solution to eq. [22] follows the Cramer rule, with

$$[24a]\quad \lambda_p = \frac{\Delta_3}{\Delta}$$

$$[24b]\quad \eta_1 = \frac{\Delta_1}{\Delta}$$

$$[24c]\quad \eta_2 = \frac{\Delta_2}{\Delta_3}$$

in which

$$[25a]\quad \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$[25b]\quad \Delta_1 = \begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix}$$

$$[25c]\quad \Delta_2 = \begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix}$$

$$[25d]\quad \Delta_3 = \begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}$$

Illustrative examples

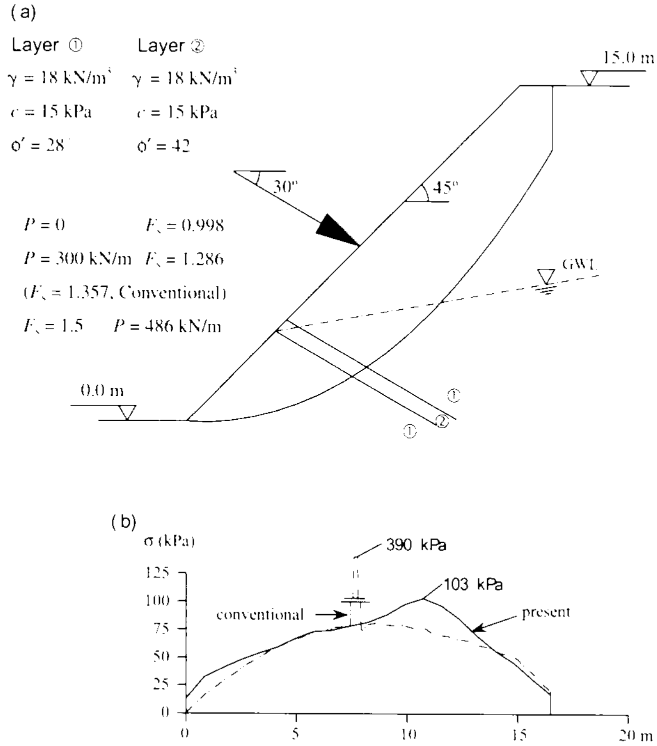
Example 1

A slope with a height of 15 m and an inclination of 45° is shown in Fig. 3*a*. The slope mass consists of two types of soils, whose parameters are presented in Fig. 3*a*. The anchor is to be applied at the half height of the slope with an inclination of 30° to the horizontal.

Before the anchor is applied, the factor of safety for this slope is 0.998, calculated with the Spencer method. When an anchor load of 300 kN per unit length is applied to the slope and a drained condition is assumed (i.e., $\overline{B} = 0$), the factor of safety of the slope is increased to 1.286 in the present approach. The normal stress distribution over the slip surface after the application of the anchor load is shown in Fig. 3*b*. It can be seen that under the action of the anchor load, the normal stress on the slip surface is continuous and fairly smooth in shape, with a maximum value of 103 kPa occurring in close proximity to the point of action of the anchor load. If a minimum factor of safety is required for the slope, then the minimum anchor load can be directly computed by using eq. [24*a*] with a value of 485 kN/m.

For comparison purposes, the Spencer method, with conventional treatment of anchor loads, is also used in this example, and the corresponding results are shown in Fig. 3*a*. In this case, the factor of safety for the slope with the anchor load of 300 kN/m is 1.357, which is 6% larger than that provided in the above solution. From the practical point of view, such a difference is rather small. The associated normal stress distribution on the slip surface is also shown in Fig. 3*b*. It can be seen that the normal stress on the slip surface increases abruptly at the point immediately under the point of action of the anchor load. This is quite unreasonable from the static point of view, and thus one cannot ensure that the conventional procedure is valid for anchor loads in all cases (Krahn 2003).

Fig. 3. Slope profile and normal stresses on the slip surface for example 1. (*a*) Slope profile and soil parameters. (*b*) Normal stresses on slip surface computed by conventional and present methods. GWL, groundwater level.



Example 2

The slope profile of another example and the soil parameters are shown in Fig. 4. Three anchors are to be applied to stabilize this slope. For a slope without a predefined failure surface, the stabilization measure should ensure that all potential slip surfaces have factors of safety greater than a specified value, say 1.2 for this example. All local critical slip surfaces with factors of safety of <1.2 are located by using the critical slip field method (Zhu 2001). A total of 11 critical slip surfaces are plotted in Fig. 4. The values of factors of safety (F_{s0}) corresponding to these slip surfaces without anchor loads are presented in the second column of Table 1. To evaluate the effect on slope stability of possible excess pore-water pressure induced by abrupt application of the anchor load, we assume that the pore pressure parameter ($\overline{B}$) varies between 0 and 1.0.

The factors of safety with anchor loads ($P_1 = P_2 = P_3 = 1000$ kN/m) and the load factors required by the specified factor of safety of 1.2 are presented in Table 1 for $\overline{B}$ = 0.00, 0.25, 0.50, 0.75, and 1.00. It can be seen from Table 1 that

Fig. 4. Slope profile and soil parameters for example 2.

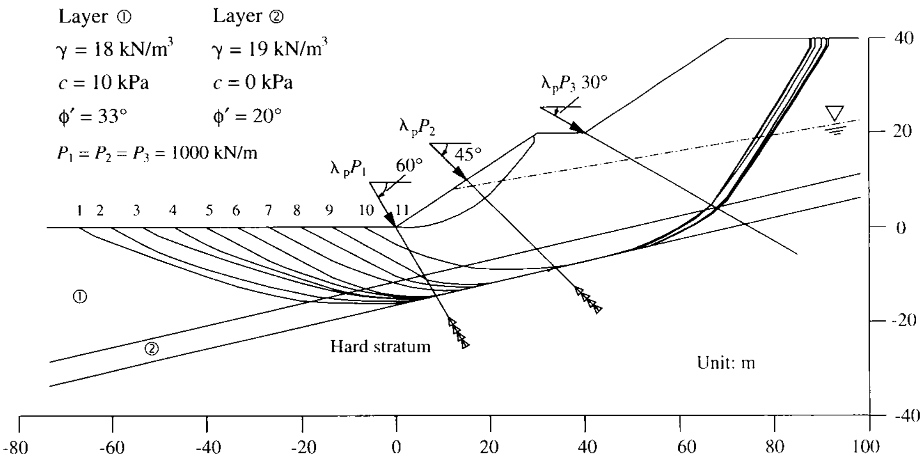


Table 1. Values of factors of safety and required load factors.

		$\overline{B}$ = 0.0		$\overline{B}$ = 0.25		$\overline{B}$ = 0.5		$\overline{B}$ = 0.75		$\overline{B}$ = 1.0	
Slip surface	F_{s0}	F_s	λ_p	F_s	λ_p	F_s	λ_p	F_s	λ_p	F_s	λ_p
1	1.173	1.331	0.182	1.307	0.215	1.283	0.260	1.258	0.332	1.234	0.456
2	1.147	1.306	0.352	1.281	0.413	1.257	0.502	1.232	0.637	1.208	0.874
3	1.093	1.254	0.684	1.228	0.805	1.203	0.977	1.177	1.244	1.152	1.712
4	1.081	1.239	0.770	1.214	0.903	1.189	1.091	1.165	1.377	1.141	1.867
5	1.071	1.229	0.832	1.204	0.971	1.180	1.164	1.156	1.453	1.133	1.934
6	1.058	1.218	0.897	1.193	1.048	1.168	1.261	1.143	1.582	1.119	2.122
7	1.026	1.205	0.977	1.178	1.129	1.151	1.339	1.125	1.643	1.099	2.127
8	1.031	1.229	0.868	1.201	0.996	1.173	1.167	1.145	1.410	1.118	1.781
9	1.059	1.289	0.641	1.259	0.729	1.229	0.845	1.199	1.006	1.169	1.241
10	1.055	1.343	0.537	1.306	0.612	1.268	0.712	1.230	0.850	1.192	1.054
11	0.996	3.069	0.134	2.308	0.212	1.515	0.499	0.439	−1.383	0.519	−0.29

the most critical slip surface is a shallow surface (No. 11) passing through the toe of the slope. However, in the case of $\overline{B}$ = 0.00 (i.e., drained condition) for this shallow slip surface, the increased factor of safety is the largest, and the required load factor is the least. In other words, this most critical slip surface without anchor load is the least critical after the application of anchor loads. If the slope is to meet the prescribed stability conditions, the anchor loads should be designed with due consideration to the second most critical slip surface (No.7), which passes below the toe of the slope: it is associated with the lowest factor of safety for the given anchor loads, and it also requires the largest anchor loads to attain the specified factor of safety. It is evident from Table 1 that with an increase in pore pressure parameters ($\overline{B}$), the factor of safety decreases and the required load factor increases. It should be noted that the locations and inclinations of the anchors shown in Fig. 3 are selected only for the purposes of illustration. In practical application, it is recommended that an optimization process be performed to determine an optimum combination of anchors. The procedure proposed here would serve as a useful tool for this purpose.

Conclusions

The limit equilibrium methods of slices have been widely used for analysing the stability of slopes without the action of concentrated forces. Although the extension of the conventional methods to include anchor loads is straightforward, an unreasonable normal stress distribution on the slip surface would arise as a result. An alternative procedure is proposed in this paper for a more rational analysis of anchor-reinforced slopes. With this procedure, the normal stress on the slip surface is assumed to be a linear combination of two parts involving two auxiliary unknowns: one part corresponds to the unreinforced slope obtained using conventional methods; the other part is induced solely by the anchor loads, with an approximate closed-form solution. Solving the three equilibrium equations yields explicit solutions to the factor of safety with given anchor loads and to the required anchor loads with a specified factor of safety. The disadvantages of conventional procedures in dealing with anchor loads can thus be overcome. This method can serve as a promising tool for the design of stabilization measures involving anchors or soil nail and geotextile reinforcements for failed slopes and for those having unacceptable stability conditions.

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References

Bishop, A.W. 1955. The use of the slip circle in the stability analysis of earth slopes. Géotechnique, **5**(1): 7–17.

Bromhead, E.N. 1994. The stability of slopes. 2nd ed. Blackie Academic & Professional, London, UK.

Duncan, J.M. 1996. State of the art: limit equilibrium and finite-element analysis of slopes. Journal of Geotechnical Engineering, ASCE, **122**(7): 577–596.

Fellenius, W. 1936. Calculation of the stability of earth dams. *In* Transactions of the 2nd Congress on Large Dams, Washington, D.C. Vol. 4, pp. 445–462.

Fredlund, D.G., and Krahn, J. 1977. Comparison of slope stability methods of analysis. Canadian Geotechnical Journal, **14**(3):429–439.

Hobst, L., and Zajic, J. 1983. Anchoring in rock and soil. 2nd ed. Elsevier Scientific Publishing Company, Amsterdam, The Netherlands. Developments in Geotechnical Engineering, No. 33.

Hutchinson, J.N. 1977. Assessment of the effectiveness of corrective measures in relation to geological conditions and types of slope movement. *In* General Report on Theme 3, Symposium on Landslides and Other Mass Movements, Prague. IAEG Bulletin, **16**: 131–155.

Janbu, N. 1973. Slope stability computations. *In* Embankment dam engineering, Casagrande memorial volume. *Edited by* E. Hirschfield and S. Poulos. John Wiley & Sons, New York. pp. 47–86.

Krahn, J. 2003. The 2001 R.M. Hardy Lecture: The limits of limit equilibrium analyses. Canadian Geotechnical Journal, **40**(3): 643–660.

Morgenstern, N.R., and Price, V.E. 1965. The analysis of the stability of general slip surfaces. Géotechnique, **15**(1): 79–93.

Skempton, A.W. 1954. The pore pressure coefficients *A* and *B*. Géotechnique, **36**(3): 425–447.

Spencer, E. 1967. A method of analysis of the stability of embankments assuming parallel interslice forces. Géotechnique, **17**(1): 11–26.

Timoshenko, S.P., and Goodier, J.N. 1970. Theory of elasticity. 3rd ed. McGraw-Hill Book Company, New York.

Zhu, D.Y. 2001. A method for locating critical slip surfaces in slope stability analysis. Canadian Geotechnical Journal, **38**(2): 328–337.

Zhu, D.Y., Lee, C.F., Qian, Q.H., Zou, Z.S., and Sun, F. 2001. A new procedure for computing the factor of safety using the Morgenstern–Price method. Canadian Geotechnical Journal, **38**(3): 882–888.

Zhu, D.Y., Lee, C.F., and Jiang, H.D. 2003. Generalized framework of limit equilibrium methods for slope stability analysis. Géotechnique, **53**(4): 377–395.

Appendix A. Solution for the factor of safety

Solving eqs. [18*a*] and [18*b*] for η_1 and η_2, one obtains

$$[\mathrm{A1}a]\quad \eta_1 = \frac{T_0 + \frac{1}{F_s}T_1 + \frac{1}{F_s^2}T_2}{G_0 + \frac{1}{F_s}G_1 + \frac{1}{F_s^2}G_2}$$

$$[\mathrm{A1}b]\quad \eta_2 = \frac{S_0 + \frac{1}{F_s}S_1 + \frac{1}{F_s^2}S_2}{G_0 + \frac{1}{F_s}G_1 + \frac{1}{F_s^2}G_2}$$

where

$$[\mathrm{A2}a]\quad T_0 = A_3B_2 - A_2B_3;\quad T_1 = A_3B'_2 + A'_3B_2 - A_2B'_3 - A'_2B_3;\quad T_2 = A'_3B'_2 - A'_2B'_3$$

$$[\mathrm{A2}b]\quad S_0 = A_1B_3 - A_3B_1;\quad S_1 = A_1B'_3 + A'_1B_3 - A_3B'_1 - A'_3B_1;\quad S_2 = A'_1B'_3 - A'_3B'_1$$

$$[\mathrm{A2}c]\quad G_0 = A_1B_2 - A_2B_1;\quad G_1 = A_1B'_2 + A'_1B_2 - A_2B'_1 - A'_2B_1;\quad G_2 = A'_1B'_2 - A'_2B'_1$$

Substituting eqs. [A1*a*] and [A1*b*] into eq. [18*c*] and rearranging yields a cubic function of F_s, as follows:

$$[\mathrm{A3}]\quad F_s^3 + t_2F_s^2 + t_1F_s + t_0 = 0$$

where

$$[\mathrm{A4}a]\quad t_0 = -\frac{D_1T_2 + D_2S_2 + D_3G_2}{E_1T_0 + E_2S_0 + E_3G_0}$$

$$[\mathrm{A4}b]\quad t_1 = \frac{E_1T_2 + E_2S_2 + E_3G_2 - D_1T_1 - D_2S_1 - D_3G_1}{E_1T_0 + E_2S_0 + E_3G_0}$$

$$[\mathrm{A4}c]\quad t_2 = \frac{E_1T_1 + E_2S_1 + E_3G_1 - D_1T_0 - D_2S_0 - D_3G_0}{E_1T_0 + E_2S_0 + E_3G_0}$$

Equation [A3] is rewritten as:

$$[\mathrm{A5}]\quad \left(F_s - \frac{t_2}{3}\right)^3 + p\left(F_s - \frac{t_2}{3}\right) + q = 0$$

where

$$[\mathrm{A6}a]\quad p = -\frac{t_2^2}{3} + t_1$$

$$[\mathrm{A6}b]\quad q = -\frac{1}{27}t_2^3 - \frac{1}{3}t_1t_2 + t_0$$

Solving eq. [A5] gives the expression for the factor of safety F_s as in eq. [20].