Investigating the Diversity Performance of Multiuser Spatially Correlated MIMO Broadcast Fading Channels

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***Abstract*— This paper's aim is to investigate the diversity performance of spatially correlated multiple-input multiple-output (MIMO) broadcast channels where multiple communications occur simultaneously in the same frequency band and time slot from the base station (BS) to many mobile stations (MS). To deal with a broadcast system, we employ a previously developed orthogonal space division multiplexing (OSDM) method to decompose the channel into many uncoupled single-user systems so that the diversity can be readily characterized by the second-order statistics of the resultant channel coefficients. Simulation results reveal that good performance can still be achieved with correlation as high as 0.4. Most intriguingly, unlike single-user MIMO channels, the diversity is more sensitive to the spatial correlation on the transmitter than receiver sides.**

***Index Terms*—Broadcast channels, Diversity, MIMO, Multiuser communications, Orthogonal space division multiplexing, Separable channels, Spatial correlation.**

I. Introduction

Recently, considerable attention has been gained to the research on promoting spectral resource reuse in broadcast channels where multiple communications occur in the same frequency band and time slot from one base station (BS) to many mobile stations (MS). From the information-theoretic point of views, this can be accomplished by dirty-paper coding (DPC) [1]–[5], which can be understood as the counterpart of multiuser detection in multiple-access channels (MAC or from many MS to one BS) [6], [7]. Based on so-called known interference pre-cancellation at the transmitter side, DPC encodes the data in a way that the codes align themselves as much as possible with each other so as to maximize the sum-capacity of a broadcast channel.

Practical techniques attempting to realize the broadcast channel capacity have also been presented [8]–[11]. In [8], [9], the broadcast channel is made block diagonal so that co-channel interference (CCI) is eliminated by placing nulls at the antennas of all the unintended MS. Subsequently in [10], [11], methods that jointly optimize the BS and MS antenna weights to perform orthogonal space division multiplexing (OSDM) are proposed. In OSDM, not only that each MS receives no CCI from other users, but also that for a particular MS, its activated spatial modes are self-orthogonal as well. It has been demonstrated in [12] that under the assumption of IID (independent and identically distributed) channels, extraordinary performance gains can be achieved.

However, real channel is hardly perfectly correlated, nor perfectly uncorrelated. It has been shown in [13]–[15] that for a single-user multiple-input multiple-output (MIMO) link, the capacity is very susceptible to the keyhole phenomenon of the channel, but not so sensitive to the spatial correlation among the antennas. Thus far, it is very well known how the diversity of a MIMO channel behaves against the spatial correlation among the antennas. It is much less understood how spatial correlation impacts the diversity performance of a multiuser MIMO broadcast system.

In this paper, we aim to investigate the diversity performance of a spatially correlated MIMO broadcast channel. To deal with a broadcast system, we employ an orthogonal space division multiplexing (OSDM) method in [11] to decompose the channel into many uncoupled single-user systems so that the diversity can be readily characterized by the second-order statistics of the resultant channel coefficients. In particular, we analyze the diversity orders in twofold: 1) the average of λ^2 (denoted by Ω) and 2) the inverse of the normalized variance of λ^2 (denoted by Ψ) where λ is the resultant channel gain of the spatial mode of the OSDM system.

The paper is organized as follows. In Section II, we introduce the system model of a multi-user MIMO broadcast system. Section III presents the channel model we use to characterize the spatial correlation. A brief review on the OSDM method is included in Section IV. Simulation results are presented in Section V. Finally, we make some concluding remarks in Section VI.

II. MIMO Broadcast System Model

For an M-user MIMO system where n_T antennas are located at the BS and $n_{R,m}$ antennas are located at the mth MS, at each symbol duration, multiple spatial modes (spatial modes are the channels created from space by distinguishing the signals received from different locations), denoted by N_m, can be supported for the mth user. The value of N_m is not necessarily equal to the number of antennas at the mth MS, which however depends the number of antennas at both the BS and MS (for details, see [11]).

Denoting $z_n^{(m)}$ as the symbol sent on the nth spatial mode by the mth user (the time index is omitted for conciseness), we can write

$$\mathbf{z}_m = \left[z_1^{(m)}\ z_2^{(m)}\ \cdots\ z_{N_m}^{(m)}\right]^T \tag{1}$$

as the symbol vector for user m where the superscript T denotes the transpose operation. The overall system can be written as

[11]

$$\hat{\mathbf{z}}_m = \mathbf{R}_m^{\dagger}\left(\mathbf{H}_m \sum_{m'=1}^{M} \mathbf{T}_{m'}\mathbf{z}_{m'} + \mathbf{n}_m\right) \quad \forall m \tag{2}$$

where $\mathbf{z}_{m'} \in \mathbb{C}^{N_{m'}}$ denotes the symbol vector transmitted from the m'th user, $\mathbf{T}_{m'} \in \mathbb{C}^{n_T \times N_{m'}}$ denotes the linear transmitter processing for the m'th user's symbols,

$$\mathbf{H}_m = \begin{bmatrix} h_{1,1}^{(m)} & h_{1,2}^{(m)} & \cdots & h_{1,n_T}^{(m)} \\ h_{2,1}^{(m)} & h_{2,2}^{(m)} & & \vdots \\ \vdots & & \ddots & \\ h_{n_{R_m},1}^{(m)} & \cdots & & h_{n_{R_m},n_T}^{(m)} \end{bmatrix} \tag{3}$$

is the IID channel matrix from the BS to the mth MS in which $h_{\ell,k}^{(m)}$ denotes the fading coefficient from the BS antenna k to the ℓth antenna of MS m, $\mathbf{R}_m \in \mathbb{C}^{n_{R_m} \times N_m}$ denotes the linear receiver processing for the mth user's symbols, and $\mathbf{n}_m \in \mathbb{C}^{n_{R_m}}$ is the noise vector at the receive antennas of MS m, whose entries are zero-mean complex Gaussian random variables with variance of $N_0/2$ per dimension. Likewise, $\hat{\mathbf{z}}_m \in \mathbb{C}^{N_m}$ is the mth user signal vector that contains the soft-output estimates of the transmitted symbols, and the superscript † represents conjugate transposition.

We also find it useful to define the multiuser transmit weight matrix as

$$\mathbf{T} = [\mathbf{T}_1\ \mathbf{T}_2\ \cdots\ \mathbf{T}_M] \in \mathbb{C}^{n_T \times \sum_{m=1}^{M} N_m} \tag{4}$$

and the multiuser transmitted symbol vector as

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_M \end{bmatrix} \in \mathbb{C}^{\sum_{m=1}^{M} N_m}. \tag{5}$$

Consequently, (2) can be conveniently written as

$$\hat{\mathbf{z}}_m = \mathbf{R}_m^{\dagger}(\mathbf{H}_m\mathbf{T}\mathbf{z} + \mathbf{n}_m) \quad \forall m. \tag{6}$$

III. Spatially Correlated Multiuser MIMO Channel Model

For IID channels in (3),

$$\left\langle h_{\ell_1,k_1}^{(m_1)}, h_{\ell_2,k_2}^{(m_2)} \right\rangle = 0 \tag{7}$$

if $m_1 \neq m_2$ or $k_1 \neq k_2$ or $\ell_1 \neq \ell_2$ where $\langle x, y\rangle = \mathrm{E}[xy^*]$. To model spatial correlation among the antenna elements at the BS and MS, we assume that the correlation among receiver and transmitter array elements is independent from one another [14], [15]. This can be justified from the fact that in most situations, only immediate surroundings of the antenna array impose the correlation between array elements and have no impact on correlations observed between the elements of the array at the other end of the link. In what follows, the channel is considered to be *separable*.

Now, we begin by defining the correlation coefficient between two elements of two distinct mobile receivers as

$$\rho_{\ell_1,\ell_2}^{(m_1,m_2)} \triangleq \left\langle h_{\ell_1,k}^{(m_1)}, h_{\ell_2,k}^{(m_2)} \right\rangle \tag{8}$$

where it is apparent that the receiver correlation coefficient is independent of the transmit antenna element, k (due to the separability assumption of the channel). Further, as the distance between different MS would be large compared to the wavelength of radiation, it is reasonable to assume there is no spatial correlation between elements of different MS, i.e.,

$$\rho_{\ell_1,\ell_2}^{(m_1,m_2)} = \begin{cases} 0 & \text{if } m_1 \neq m_2, \\ \rho_{\ell_1,\ell_2}^{(m_1)} & \text{if } m_1 = m_2 \end{cases} \tag{9}$$

for all ℓ_1, ℓ_2. Following this, a matrix of the receiver correlation coefficients can be constructed as

$$\Gamma_R = \mathrm{diag}(\Gamma_{R_1}, \Gamma_{R_2}, \ldots, \Gamma_{R_M}) \tag{10}$$

where

$$\Gamma_{R_m} \triangleq \begin{bmatrix} \rho_{1,1}^{(m)} & \rho_{1,2}^{(m)} & \cdots & \rho_{1,n_{R_m}}^{(m)} \\ \rho_{2,1}^{(m)} & \rho_{2,2}^{(m)} & & \vdots \\ \vdots & & \ddots & \\ \rho_{n_{R_m},1}^{(m)} & \cdots & & \rho_{n_{R_m},n_{R_m}}^{(m)} \end{bmatrix}. \tag{11}$$

Similarly, we define the transmitter correlation matrix as

$$\Gamma_T \triangleq \begin{bmatrix} \tau_{1,1} & \tau_{1,2} & \cdots & \tau_{1,n_T} \\ \tau_{2,1} & \tau_{2,2} & & \vdots \\ \vdots & & \ddots & \\ \tau_{n_T,1} & \cdots & & \tau_{n_T,n_T} \end{bmatrix} \tag{12}$$

in which

$$\tau_{k_1,k_2} \triangleq \left\langle h_{\ell,k_1}^{(m)}, h_{\ell,k_2}^{(m)} \right\rangle \tag{13}$$

which is independent of both m and ℓ.

From the definitions of (10) and (12), we can effect the spatial correlation by post-multiplying the channel transfer matrix $\mathbf{H}_m$ described before in (3) by the transmitter correlation matrix, $\Gamma_T^{1/2}$ and pre-multiplying by the receiver correlation matrix, $\Gamma_R^{1/2}$ so that

$$\tilde{\mathbf{H}} = \Gamma_R^{1/2}\mathbf{H}(\Gamma_T^{1/2})^{\dagger} \tag{14}$$

where

$$\mathbf{H} \equiv \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_M \end{bmatrix}. \tag{15}$$

Now, the entries of $\tilde{\mathbf{H}}$ are correlated to model the spatial correlation among the antenna elements.

With this model (14), implicitly, the correlation between two channels from the BS to the same MS m can be characterized by the product of the transmitter and receiver correlation coefficients, i.e.,

$$\left\langle h_{\ell_1,k_1}^{(m)}, h_{\ell_2,k_2}^{(m)} \right\rangle = \rho_{\ell_1,\ell_2}^{(m)}\tau_{k_1,k_2}. \tag{16}$$

To use the model, the correlation matrices, Γ_T and Γ_R, have to be defined, either arbitrarily or empirically. The selection of the correlation coefficients may vary from different communication environments. In order to make the analysis tractable, we use the idea of the single-parameter correlation model in [16] to determine Γ_T and Γ_R as a function of a single-parameter, γ_T or γ_{R_m}. As such, we have

$$\Gamma_T = \begin{bmatrix} 1 & \gamma_T & \gamma_T^4 & \cdots & \gamma_T^{(n_T-1)^2} \\ \gamma_T & 1 & \gamma_T & \ddots & \vdots \\ \gamma_T^4 & \gamma_T & 1 & \ddots & \gamma_T^4 \\ \vdots & \ddots & \ddots & \ddots & \gamma_T \\ \gamma_T^{(n_T-1)^2} & \cdots & \gamma_T^4 & \gamma_T & 1 \end{bmatrix} \tag{17}$$

and

$$\Gamma_{R_m} = \begin{bmatrix} 1 & \gamma_{R_m} & \gamma_{R_m}^4 & \cdots & \gamma_{R_m}^{(n_{R_m}-1)^2} \\ \gamma_{R_m} & 1 & \gamma_{R_m} & \ddots & \vdots \\ \gamma_{R_m}^4 & \gamma_{R_m} & 1 & \ddots & \gamma_{R_m}^4 \\ \vdots & \ddots & \ddots & \ddots & \gamma_{R_m} \\ \gamma_{R_m}^{(n_{R_m}-1)^2} & \cdots & \gamma_{R_m}^4 & \gamma_{R_m} & 1 \end{bmatrix}. \tag{18}$$

IV. Orthogonal Space Division Multiplexing

To deal with a broadcast system, here, we propose an OSDM scheme to decompose the channel into many uncoupled single-user systems. Specifically, the objective is to find the weight matrices, $\mathbf{T}, \mathbf{R}_1, \ldots, \mathbf{R}_M$ jointly that can ensure interference-free at all the signal outputs of the spatial modes of the system and that the resultant channel gains of the spatial modes are maximized. Mathematically, this can be written as [11]

$$(\mathbf{T}, \mathbf{R}_1, \ldots, \mathbf{R}_M)_{\rm opt} = \arg \max_{\mathbf{T}, \mathbf{R}_1, \ldots, \mathbf{R}_M} \sum_{m=1}^{M} \|\mathbf{\Lambda}_m\|^2 \tag{19}$$

where $\|\cdot\|$ denotes the Frobenius norm of the input matrix [17] and $\mathbf{\Lambda}_m$ is defined by

$$\mathbf{R}_m^\dagger \mathbf{H}_m \mathbf{T} = [\mathbf{0}_1 \ \cdots \ \mathbf{0}_{m-1} \quad \underbrace{\mathbf{\Lambda}_m}_{\substack{m\text{th} \\ \text{sub-block} \\ \text{matrix}}} \quad \mathbf{0}_{m+1} \ \cdots \ \mathbf{0}_M] \tag{20}$$

where

$$\mathbf{\Lambda}_m = \operatorname{diag}\left(\lambda_1^{(m)}, \lambda_2^{(m)}, \ldots, \lambda_{N_m}^{(m)}\right) \tag{21}$$

is of dimension $N_m \times N_m$ and $\lambda_n^{(m)}$ corresponds to the resultant channel gain for the nth spatial mode (or the nth signal stream) of the mth MS. Likewise, $\mathbf{0}_{m'}$ is the m'th sub-block zero matrix of dimension $N_m \times N_{m'}$. From (20), each sub-block matrix corresponds to the signals transmitted from each user to the MS m and hence by making all of them zero except for the mth user, the CCI can be completely eliminated.

To achieve (19) and (20) simultaneously, an iterative algorithm is proposed as follows:

1. Initialize $\mathbf{R}_m = \mathbf{I} \ \forall m$ where $\mathbf{I}$ is an identity matrix of appropriate dimensions.
2. For each m, form the matrix

$$\mathbf{H}_e^{(m)-} \triangleq \begin{bmatrix} \mathbf{R}_1^\dagger \mathbf{H}_1 \\ \vdots \\ \mathbf{R}_{m-1}^\dagger \mathbf{H}_{m-1} \\ \mathbf{R}_{m+1}^\dagger \mathbf{H}_{m+1} \\ \vdots \\ \mathbf{R}_M^\dagger \mathbf{H}_M \end{bmatrix} \in \mathbb{C}^{\left(\sum_{m'=1, m'\neq m}^{M} N_{m'}\right) \times n_T}. \tag{22}$$

   and obtain the nullspace of $\mathbf{H}_e^{(m)-}$, $\mathbf{Q}_m$. Then, compute the singular value decomposition (SVD) of $\mathbf{H}_m \mathbf{Q}_m = \mathbf{U}_m \mathbf{\Lambda}_m \mathbf{V}_m^\dagger$. Afterwards, find $\mathbf{T}_m$ and $\mathbf{R}_m$, respectively, by

$$\mathbf{T}_m = \mathbf{Q}_m \mathbf{V}_m|_{1 \leftrightarrow N_m} \tag{23}$$

   and

$$\mathbf{R}_m = \mathbf{U}_m|_{1 \leftrightarrow N_m} \tag{24}$$

   where the notation $\cdot|_{1 \leftrightarrow N_m}$ collects the column vectors of the input matrix that correspond to the N_m largest singular values.
3. Compute $\epsilon = \text{off}\,(\mathbf{H}_e \mathbf{T})$ where

$$\mathbf{H}_e \triangleq \begin{bmatrix} \mathbf{R}_1^\dagger \mathbf{H}_1 \\ \vdots \\ \mathbf{R}_M^\dagger \mathbf{H}_M \end{bmatrix} \in \mathbb{C}^{\left(\sum_{m=1}^{M} N_m\right) \times n_T} \tag{25}$$

   and

$$\text{off}(\mathbf{A}) \triangleq \sum_{k,\ell} |a_{k,\ell}|^2. \tag{26}$$

   in which $|\cdot|$ takes the modulus of a complex number. If $\epsilon \leq \epsilon_{Th} (= 10^{-12}$ typically), proceed to Step 4; otherwise, go back to Step 2.
4. The convergence is said to be achieved and normalization is then performed for all columns of $\mathbf{T}$ to satisfy the power constraint.

Numerical results have demonstrated that the proposed iteration can numerically achieve OSDM whenever

- the number of BS antennas is no smaller than the total number of activated spatial modes of the system, i.e., $n_T \geq \sum_{m=1}^{M} N_m$, and
- the number of receive antennas at MS m is no smaller than the number of activated spatial modes of that MS, i.e., $n_{R_m} \geq N_m$.

Throughout this paper, we shall refer to the above algorithm as an iterative nullspace-directed SVD (Iterative Nu-SVD).

V. Simulation Results

A. Setup

We study the diversity performance of OSDM (obtained by Iterative Nu-SVD) in terms of both average bit-error-rate (BER) and the second and fourth-order statistics of the resultant channel gains by Monte Carlo simulations. The results are plotted against various spatial correlation coefficients, γ_T and γ_R. Perfect channel information is assumed to be available at the transmitter (BS) and all the receivers (MS). The channel model we use in our simulations is a quasi-static narrow-band Rayleigh fading channel. We assume 4-QAM is used for all transmission. Additionally, we assume that the spatial correlation among different users is all the same (i.e., $\gamma_{R_1} = \cdots = \gamma_{R_M} = \gamma_R$).

To investigate the diversity gains, two statistical parameters are examined. They are the average of the squared channel gain (second-order statistic)

$$\Omega = \mathrm{E}[\lambda^2]. \tag{27}$$

and the inverse of the normalized variance of the squared channel gain (fourth order statistic)

$$\Psi = \frac{\Omega^2}{\mathrm{E}[(\lambda^2 - \Omega)^2]} \tag{28}$$

where λ represents the resultant channel gain of a spatial mode. The diversity orders of the system can be measured in terms of Ω and Ψ relative to a system without diversity with $\Omega = \Psi = 1$. Consequently, Ω can be readily considered as the diversity order obtained in terms of received power, and Ψ can be considered as the diversity order obtained for reducing the effect of fading (i.e., when $\Psi \to \infty$, channel becomes AWGN).

For convenience, we use $\{n_T, [n_{R_1}(N_1), \ldots, n_{R_M}(N_M)]\}$ to denote a broadcast MIMO system where an n_T-element BS is communicating to M MS and each MS m has n_{R_m} receive antennas and supports N_m spatial modes.

B. Results

Figures 1 and 2 provide the results of BER and diversity orders of $\{2, [2(1)2(1)]\}$ with particular focus on systems where users supporting only one stream (or mode). The average received signal-to-noise ratio (SNR) is set to 16 dB. Analysis is done by varying one value of spatial correlation coefficient $\gamma_T(\gamma_R)$ while the other $\gamma_R(\gamma_T)$ is fixed. As expected, observing from the results in Figure 1, the BER gets worse if the spatial correlation increases. Intriguingly, the performance degradation is more severe on the transmit correlation factor than the receive correlation factor (It is worth-noting that this is contrary to the known results of the single-user MIMO system that the transmit and receive correlation factors have the same effect on the system performance.). In particular, when γ_T approaches 0.99 (perfectly correlated in space), BER becomes 0.5 indicating that the multiuser system actually breaks down. Otherwise however, when γ_R tends to 0.99, the BER performance though degrades considerably, still stays around 10^{-3}. This can be explained by recognizing that the orthogonal property of the system is largely provided by the difference of the channels seen by the transmit antenna array. As a consequence, when γ_T increases, the channels of the users quickly become non-distinguishable while the effect of increasing γ_R goes only to the loss of receive diversity at the users.

More can be observed from the results in Figure 2 where diversity orders are given. As can be seen, when transmit correlation factor increases (while γ_R is fixed to 0), both channel gain Ω and diversity gain Ψ degrade, and will approach to 0 as γ_T tends to 0.99. By contrast, when the receive correlation factor increases (while γ_T is fixed to 0), the diversity gain Ψ degrades but the channel gain Ω almost stays the same. Actually, it can be easily shown that when the receive antennas are entirely correlated, the multiuser system $\{2, [2(1), 2(1)]\}$ is equivalent to the single-user system $\{1, [2(1)]\}$ with perfect receive correlation where $\Omega = 2$ and $\Psi = 1$.

In Figures 3–5, BER and diversity results are, respectively, plotted for systems with multiple spatial modes per user. The system configuration we consider is $\{4, [3(2), 3(2)]\}$ and SNR is still 16 dB. Results in Figure 3 demonstrate that the BER performances of all the modes generally degrade when the correlation factor increases. The degradation is not sensitive even when the spatial correlation is as large as 0.4. Particularly, when γ_T tends to 0.99 (while γ_R is fixed to 0), the BER of both streams approaches 0.5. Furthermore, when γ_R tends to 0.99, the BER performance of stream 2 approaches to 0.5 whereas the BER performance of stream 1 is about 10^{-4}. This indicates that with high spatial correlation, essentially, at most one stream can be supported for each user. Similar conclusions can be drawn from the results in Figures 4 and 5.

VI. Conclusions

To summarize, this paper has studied the effect of spatial correlation on the performance of the multiuser MIMO antenna system in broadcast channels. A simple single-parameter spatial correlation MIMO channel model is introduced to investigate the effect. Simulation results have shown that the system performance including BER and diversity is more sensitive to the transmit correlation than the receive correlation. Furthermore, the performance is not sensitive to the spatial correlation and it has been demonstrated that performance degradation is small even when the spatial correlation is as large as 0.4.

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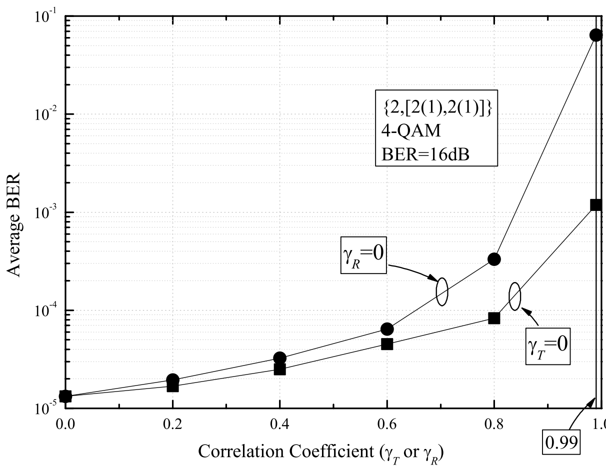

Fig. 1. BER performance of $\{2, [2(1), 2(1)]\}$ against various transmit and receive correlation coefficients.

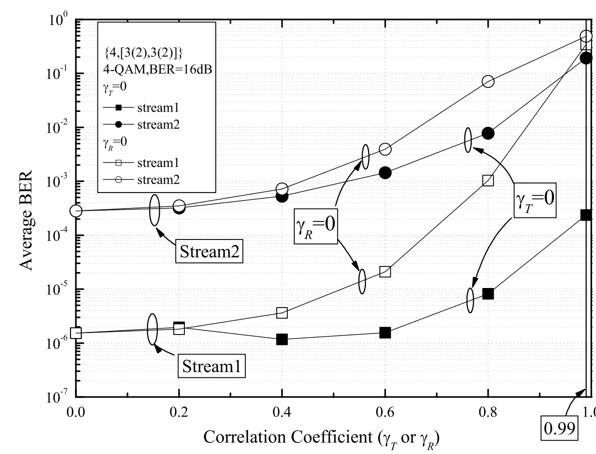

Fig. 3. BER performance of $\{4, [3(2), 3(2)]\}$ against various transmit and receive correlation coefficients.

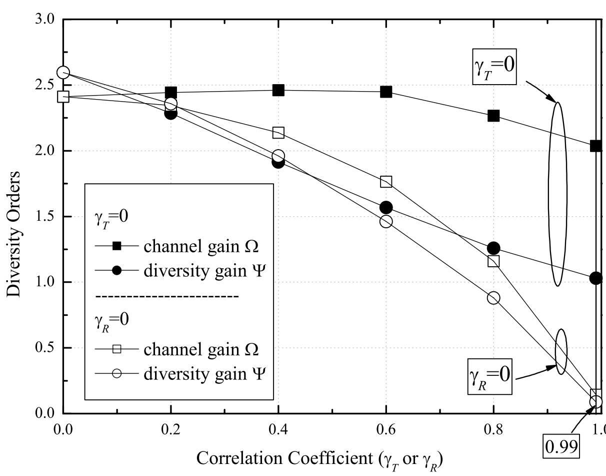

Fig. 2. Diversity performance of $\{2, [2(1), 2(1)]\}$ against various transmit and receive correlation coefficients.

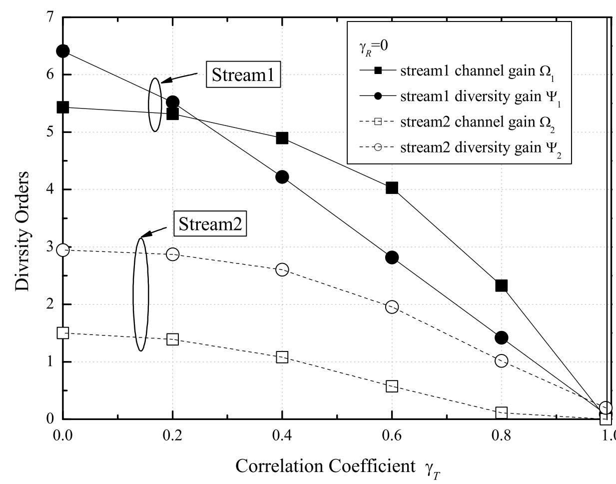

Fig. 4. Diversity performance of $\{4, [3(2), 3(2)]\}$ against the transmit correlation coefficient.

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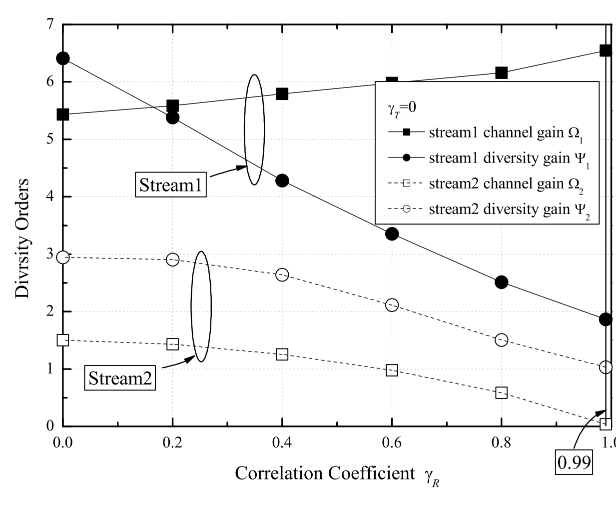

Fig. 5. Diversity orders of $\{4, [3(2), 3(2)]\}$ against the receive correlation coefficient.