Proceedings of the 2005 IEEE
Engineering in Medicine and Biology 27th Annual Conference
Shanghai, China, September 1-4, 2005

Novel Approach for Time-Varying Bispectral Analysis of Non-Stationary EEG Signals

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Abstract-A novel parametric method, based on the non-Gaussian AR model, is proposed for the partition of non-stationary EEG data into a finite set of third-order stationary segments. With the assumption of piecewise third-order stationarity of the signal, a series of parametric bispectral estimations of the non-stationary EEG data can be performed so as to describe the time-varying non-Gaussian nonlinear characteristics of the observed EEG signals. A practical method based on the fitness of third-order statistics of the signal by using the non-Gaussian AR model, together with an algorithm with CMI is presented. The experimental results with several simulations and clinical EEG signals have also been investigated and discussed. The results show successful performance of the proposed method in estimating the time-varying bispectral structures of the EEG signals.

I. INTRODUCTION

Numerous practical medical signals have their statistical properties changing with time in nature. In many application, a great deal of methods such as FFT and AR modeling for analyzing random processes assume that the underlying time series have satisfied with the stationarity. For a non-stationary process, it is problematic to give a correct interpretation to the results based on these methods. Without testing the first or second-order stationarity of the given series, the analysis of non-stationary process by using the traditional methods will lead to misunderstand the information of the process. In fact, partition of non-stationary time series into a series of isolated segments is needed so that the signal properties within each segment can be characterized for our purposes. Due to the complexity of many biomedical signals, the automatic segmentation and temporal spectral analysis of the signals has been still a difficult and yet interesting task.

To investigate the time-varying spectral characteristics of the underlying processes, most methods often begin by computing the time variation of the common statistical properties of the process [4-6]. However, these methods are all assumed only the piecewise first-order or second-order stationarity is satisfied for each segment of the observation after segmentation. In practice, many medical signals show their significant nonlinear and non-Gaussian characteristics, such as the presences of the nonlinear effect of phase coupling among the signal frequency components [7-10]. The methods based on the spectral analysis fail to properly deal with the nonlinearity and non-Gaussianility of the processes, but HOS allows us to effectively process these kinds of signals to obtain their higher-order statistics. Bispectral estimation has shown to be a very useful tool for extracting the degree of quadratic phase coupling (QPC) between individual frequencies of the process. There have been some researches on bispectral analysis of medical signals, all with the assumption that the finite observations satisfy third-order stationarity so that both parametric and classical bispectral estimations can be performed. From a practical point of view, however, the problem becomes even more complicated when taking account of the non-stationarity, non-Gaussianility and the nonlinearily of the medical processes under investigation since we must concern explicitly with the time variation of more subtle and the nonlinear properties of the process. To extract various nonlinear and non-Gaussian features in different periods of the non-stationary processes, we must take into consideration of the time-varying characteristics of the bispectral structures and try to provide the corresponding bispectral estimation of the processes. Thus, the precondition for extracting the time-varying non-Gaussian features of the process is to properly partition the non-stationary processes into a series of segments which are third-order stationary so that the time-varying bispectral structures of the processes can be described for the purpose of closely tracking the temporal behaviors of the EEG characteristics.

In attempting to provide an effective way for investigating the time-varying bispectral features of the processes, one is motivated to develop a practical segmentation method so that each third-order stationary segment can be identified for subsequent accurate bispectral estimation. This paper proposes a novel method showing how to partition the non-stationary time series into a set of non-overlapping third-order stationary segments and provide the time-varying bispectral estimation of the process. Our aim is to find out the procedure to detect all the change instants in bispectrum of the processes such that the different nonlinear structures involved in the processes can be obtained from the bispectral estimation in each third-order stationary segment. The method developed here is directly derived from a parametric approach.

II. NON-GAUSSIAN AR MODEL

Let the noiseless third-order stationary non-Gaussian process $y(k)$ be described as [11-12]

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$$y(k)-\sum_{j=1}^{P}a_j y(k-i)=w(k) \tag{1}$$

where $w(k)$ denotes a non-Gaussian, independent identically distribution (i.i.d.) input sequence with zero-mean and finite moments. The signal $z(k)$ is observed in additive noise as $z(k)=y(k)+n(k)$, in which the noise $n(k)$ is zero-mean, colored Gaussian, and statistically independent of $y(k)$. Since the l^{th}-order cumulants $C_l(m,n)$ of Gaussian processes will vanish for $l\geq 3$, we have the third-order cumulants relation as $C_{3z}(m,n)=C_{3y}(m,n)$. The goal is to identify the model parameters $\{a_j\}(j=1,\cdots,P)$ and estimate the parametric bispectrum from a finite set of noisy observation based on the estimation of the third-order cumulants of $z(k)$ only. It follows from (1) that

$$\sum_{j=0}^{P}a_j C_{3z}(j-m,j-n)=\beta\delta(m,n) \tag{2}$$

where $C_{3z}(.,.)$ denote third-order cumulants of the sequence $z(k)$. β is the third-order cumulants of the $w(k)$ and δ represents the Kronecker delta. A simpler solution can be used to estimate $\{a_j\}$ by solving (2) for $m=n=0,1,\cdots,p$, the diagonal slice of the third-order cumulant sequences of $z(k)$. In addition, as expressed in (1) and (2), since the higher-order cumulants of the additive Gaussian noise $n(k)$ is identically zero, the method for detecting the change instants of the processes based on the HOS techniques can greatly remain effective in low signal-to-noise ratio (SNR) systems. Thus, the presented segmentation method is asymptotically insensitive to additive noise.

Like most cases for order determination in parametric modeling, the order P in model (1) needs to be estimated so that the process can be properly described by the parametric model. Some methods for order selection of non-Gaussian AR model have been reported. For the noisy AR model in equation (1), singular value decomposition (SVD) based order estimation is adopted in this paper to estimate the order of the non-Gaussian AR model.

III. THE PROPOSED SCHEME

It is assumed that the EEG can be dealt with as the piecewise stationary series. We need to prior properly segment the non-stationary EEG into a finite set of third-order stationary segments. To obtain a better bispectral estimation with the finite length data, the EEG series is modeled as the non-Gaussian AR model with piecewise constant parameters. The task for segmentation in the sense of third-order stationarity is reduced to detect all the change instants of the model parameters so that the model can be fitted between two such adjacent instants.

Given an N-length series $z(k)$, suppose there are M independent third-order stationary segments in the series and we have therefore $(M-1)$ change instants $\{t_i; i=1,\cdots,M-1\}$ to be estimated. For the i^{th} segment, we define the squared prediction error of the non-Gaussian AR model with order P as a penalty function for estimating the corresponding model parameters

$$e_i^2=\sum_{k=t_{i-1}+1+P}^{t_i}\left[z(k)-\sum_{j=1}^{P}a_{ij}z(k-j)\right]^2 \tag{3}$$

For all M segments, the model parameters $\boldsymbol{\theta}=\{a_{ij}; i=1,\cdots,M-1; j=1,\cdots,P\}$ can be estimated by minimizing the total errors

$$\hat{\boldsymbol{\theta}}(t_1,\cdots,t_{M-1})=\arg\min_{\theta\in\Theta}\sum_{i=1}^{M-1}e_i^2 \tag{4}$$

Thus, $(M-1)$ change instants of $y(k)$ can be estimated by minimizing the following joint penalized function (JPF)

$$JPF_z(t_1,\cdots,t_{M-1})=\sum_{i=1}^{M-1}e_i^2+\lambda\cdot M \tag{5}$$

where λ is a adjustable parameter with positive value. The first term of (3) measures the model fitness to each segment, whereas the second term is related to the number of the change instants.

For the given N-length series $z(k)$, the change instants may occur at each time from t_1 to t_N. This proves to be a complex computation. To avoid such a complex problem, a practical algorithm based on the modified simulated annealing algorithm is developed to seek the global optimal solution so that the JPF is minimized. A stepsize ΔT is preset so that $z(k)$ can be divided into K sequential third-order stationary intervals in which K ($K\geq M$) is the integer of $N/\Delta T$. In totally there are ($K-1$) possible change-points at $t=t_1,t_2,\cdots,t_{K-1}$ to be tested which is the change instant between two such adjacent intervals. Thus, $(M-1)$ change instants can be extracted by applying the JPF. If $t=t_m$ is not a change instant, the corresponding JPF can be expressed as

$$JPF_{z1}=\sum_{\substack{i=1\\ i\neq m,m+1}}^{M-1}e_i^2+PF_z(t_{m-1},t_{m+1};\theta)+\lambda\cdot M \tag{6}$$

On the other hand, if $t=t_m$ is included as a change instant, then the corresponding JPF is

$$JPF_{z2}=\sum_{\substack{i=1\\ i\neq m,m+1}}^{M-1}e_i^2+JPF_z(t_{m-1},t_m;\theta) \tag{7}$$
$$+JPF_z(t_m,t_{m+1};\theta)+\lambda\cdot(M+1)$$

Considering both (4) and (5), $t=t_m$ is detected as a change instant only if

$$JPF_{z1}-JPF_{z2}>0 \tag{8}$$

Evidently, a change instant exists at $t=t_m$ only if the model parameters or the bispectral structures before and after t_m are significantly changed. The parameter λ is regarded as a control threshold that gives the segmentation resolution level required by the user.

IV. RESULTS AND DISCUSSION

A simulation has been conducted to test the performance of the parametric technique based on the non-Gaussian AR model. A realization having four segments with different third-order stationarity was generated by a non-Gaussian AR(2) model with parameters: a_{11} = -1.5, a_{12} = 0.8 in the first segment, a_{21} = -0.9, a_{22} = 0.2 in the second segment, a_{31} = -0.7, a_{32} =0.2 in the third segment and a_{41} =-0.3, a_{42} = 0.65 in the fourth segment. The energy of the non-Gaussian i.i.d noise driving the non-Gaussian AR model was normalized. There were 2000 samples points in each realization, and the three change instants were set at 480, 1200 and 1800 seconds. A typical realization with four segments of the AR(2) processes is shown in Fig. 1 (a). Three change instants were detected by the proposed parametric segmentation technique and were shown in Fig. 1 (b). In this test, the threshold λ was set between 0.5 and 0.68, and 120 points data iterative stepsize was used. The parametric bispectral estimations corresponding to the data for each segment are shown in Fig. 2, which indicates that all four segments have very different bispectral structure. This example verifies that the values of λ indeed control the resolution level of the segmentation. Different segmentation results may be obtained from the same observed data for threshold parameters set at different ranges. As mentioned before, the value λ is directly compared with the prediction errors of the model in (9).

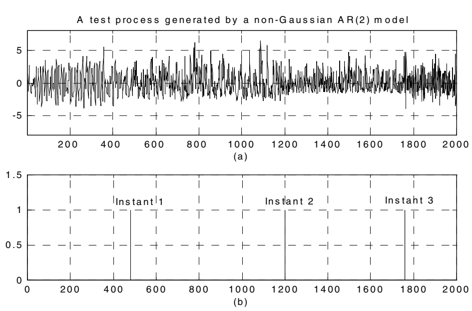

Fig. 1 Detection of changes in parametric bispectral estimation of a time-varying non-Gaussian AR(2) process. The detecting threshold is between 0.5 and 0.68 and the stepsize is set at 120 data points.

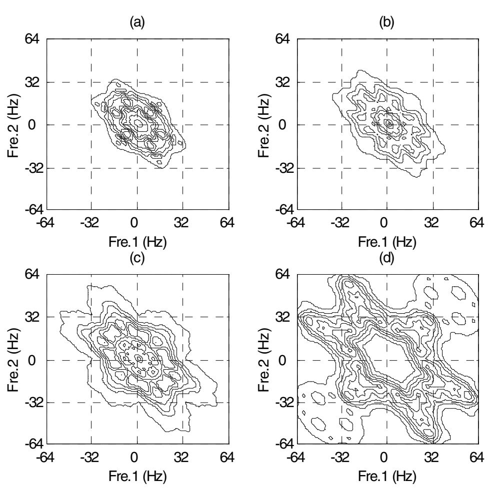

Fig. 2. Four contour map of parametric bispectral estimation, corresponding to the four segments. The model parameters for bispectral estimation are estimated from the third-order stationary series in each segment.

However, if we want to detect small changes in bispectrum of the data, a range of small λ values should be chosen. Each range of thresholds provides a resolution level for the segmentation. The determination of the threshold range became an important factor for segmentation by detecting the changes in bispectrum. One principle for selecting the threshold can be inferred that larger λ values can be adopted as the control threshold parameter if the prediction errors of the observed data between two instants can be approximately expressed as a white non-Gaussian noise, which means the error process in each segment will be approximately uncorrelated with each other. Moreover, we have to estimate a reasonable range of λ to provide the segmentation result, avoiding the risk of over-segmentation or missing the change instants.

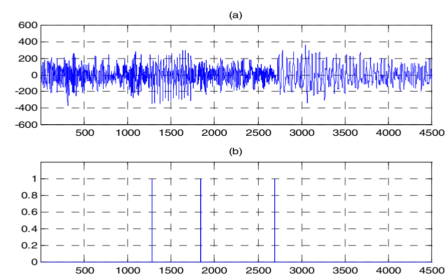

Fig. 3 The segmentation result of a record of clinical EEG data with spikes-waves and other discharges. (a) A 120-second piece of EEG series extracted from one of 14 channel clinical EEG record. In total four change instants were deteced at 24, 44, 54 and 94 seconds, indicating by 4 arrowheads. The stepsize was chosen as 2 seconds, and the threshold is adjusted to be between 0.24 and 0.72 in this example.

As an application, clinical EEG signals were recorded with a PC-based system through a standard commercial electroencephalograph (Model EEG 4400A, by Nihon Kohden Corporation). 14 electrodes were placed on the subjects' scalp according to the international 10-20 system [17]. 14-channel EEG data were collected from elctrodes placed at Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2, F7, F8, T5 and T6, and linked earlobes were used as the refernece potential. One clinical EEG siganls was used for testing the proposed method. A record of EEG series recorded from a schizophrenianic subject with temporal spikes and other discharges patterns was used to investigate the behavior of the instants detection based on the changes in the bispectrum of the data corresponding to the brain functional states. The EEG data sets with 4500 data points are shown in Fig.3 (a).

In order to distinguish the different nonlinear interactions and their bispectral structures of the EEG observation in different periods, the proposed procedure were also applied to the EEG record by choosing the iterative stepsize as 218 data points. Non-Gaussian AR model was employed to detect the change instants so that we can preprocess the seizure to recognize the seizure pattern from the patients suffering from temporal lobe epilepsy, which had been manually segmented by a neurologist. The segmentation result is shown in Fig. 3 (b) with change instants indicated by vertical solid lines. In total 3 change instants were detected at 1280, 1840 and 2696 data points by selecting the threshold range between 0.36 and 0.67. Thus 4 segments with different third-order stationarity were clearly separated, each segment corresponding one brain functional state. To show the effectiveness of the temporal nonlinear interaction of the EEG series, the EEG data after segmentation were employed to estimate their time-varying bispectrum in each segment, respectively. Clearly, the proposed procedure based on the non-Gaussian parametric modeling successfully segment the EEG series with alternating brain functional states to identify the degree of quadratic nonlinear coupling in different periods. The results demonstrate that the EEG signal is a typical time-varying non-Gaussian process due to the change of the brain functional states, normal or abnormal. By partition the non-stationary EEG into a finite set of third-order stationary segments in advance, the corresponding localized bispectral estimation can be successfuly performed so that to provide the global time-varying nonlinear structure of the signals.

The bispectral structures of both segments 1 and 3 were found to be similar. The normal patterns of EEG can be seen in theses two segments, having the QPC phenomena between 13 Hz frequency components. On the other hand, both segment 2 and 4 corresponding to the spike discharged periods have similar bispectral patterns, the bispectral peaks happened at (3Hz, 3Hz) in the bifrequency domain. The same physiological state of the brain having similar model parameters confirms that third-order stationarity and the inherent bispectral structure were approximately indentified in each segment, which enables clearly separation of different brain functional states. The segments are in agreement with the result of the visual stationarity indicated by the EEG expert. The temporal nonlinear interaction among the EEG components can be taken advantage of partitioning different EEG discharges patterns and identifying some interesting abnormal discharges such as spikes and waves, and burst waves. The local nonlinear interaction between the EEG rhythms can be therefore distinguished through the proposed procedure.

V. CONCLUSIONS

A new procedure by using the non-Gaussian AR modeling scheme is employed to permit a sequential detection of the changes in bispectral structure of the observed finite data set. The method is based on the procedure for off-line processing for searching the possible changing instants of the non-stationary time series by measuring the differences of the parameters deduced from the non-Gaussian AR modeling of the corresponding pre-set fixed intervals. An algorithm depend on the penalty function is also presented for rapidly implementing the separation and the temporal bispectral estimation of the process. The segmentation algorithm in terms of the parameters of the non-Gaussian AR model is well suited for detecting the changes in bispectrum and evaluating the change instants based on third-order stationarity in each segment. The performances of the proposed method based on simulations and clinical EEG signals have also been provided and discussed. The experimental results show that the new method is capable of detecting changes in the bispectrum of the data. In general, the method presented is also effective and applicable to nonstationary, non-Gaussian and nonlinear random signals in other medical signal processing.

Compared with other conventional methods, the non-Gaussian parametric modeling method is an effective method suitable for detecting changes in the bispectral structures of EEG data even though it is computational more complex. Further testing of the segmentation of EEG signal and comparing the results with those from EEG experts can help to simplify the selection of the threshold parameters.

Acknowlegement: This work is partly supported by the Natural Science Foundation of China (60271023).

REFERENCES

[1] I. Clark, Multiresolution decomposition of nonstationary EEG signals: A preliminary study, *Comput. Bio. Med.*, Vol. 25, No. 4, pp. 373-382, 1995.

[2] M. Shen, L. Sun and F. H. Y. Chan, Method for extracting time-varying rhythms of electroencephalograph via wavelet packet analysis. *IEE Proceedings in Science, Measurement and Technology*, Vol.148, No.1, pp.23-27.January 2001.

[3] C. E. D'Attellis, Detection of epileptic events in electroencephalograms using wavelet analysis, *Annals of Biomedical Engineering*, Vol. 25, pp. 286-293, 1997.

[4] Roberto D. and et al, Segmentation of brain electrical activity into microstates：model estimation and validation, *IEEE Trans. on Biomedical Engineering*, Vol.42,No.7, pp.658-665. July 1995.

[5] A. Y. Kaplan, Segmental structure of EEG more likely reveals the dynamic multistability of the brain tissue than the continual plasticity one. *Proceedings of ICONIP'99*, Perth, Australia, pp.633-638. 1999.

[6] H. Laurent and C. Doncarli, Stationarity index for abrupt changes detection in the time–frequency plane. *IEEE Signal Processing Letters,* Vol.5, No. 2, pp.43-45, February 1998.

[7] M. Shen, F. H. Y. Chan, L, Sun and B.J. Beadle, Parametric bispectral estimation of EEG signals in different functional states of the brain. *IEE Proceedings in Science, Measurement and Technology,* Vol.147, No.6, pp.374-377. November 2000.

[8] T. Ning, "Nonlinear analysis of the hippocampal subfields of CA1 and the dentate gyrus," *IEEE Transactions on Biomedical Engineering*, Vol.40, No.9, pp.870-876. 1993,

[9] T. Ning, J.D. Bronzino. Autoregressive and bispectral analysis techniques: EEG applications. *IEEE Engineering in Med. and Bio. Magazine.* Vol.9, No.1, pp.47-50, March 1990.

[10] P. Husar and G. Henning, "Bispectrum analysis of visually evoked potentials," *IEEE Engineering in Medicine and Biology* , pp.57-63. January/February, 1997,

[11] Boualem Boashash, Edward J. Powers. *Higher-Order Statistical Signal Processing*. Wiley Halsted Press, 1996.

[12] J. Vidal et al., Causal AR modeling using a linear combination of cumulant slices, *Signal Processing*, Vol. 36, pp. 329-340, 1994.

[13] J. K. Tugnait "Linear Model validation and order selection using higher order statistics", *IEEE Transaction on Signal processing*, Vol. 42, No. 7, pp. 1728-1735, July 1994.

[14] G. Giannakis and J. M. Mendel "Cumulant-based order determination of non-Gaussian ARMA models", *IEEE Transaction on Acoustics, Speech, and Signal processing*, Vol. 38, No. 8, pp. 1411-1423, August 1990.