A DS-CDMA Receiver Using Exponentially Weighted Despreading Waveforms

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Abstract ---- This paper presents a DS-CDMA receiver using exponentially weighted despreading waveforms (WDW). The chip weighting waveforms are designed for the purpose of multiple access interference (MAI) rejection by emphasizing the transitions of the received signal of interest. The WDW is determined only by one parameter, which leads to easy tuning of the WDW in practice to achieve the best performance. As a result, we show that the proposed receiver can reject MAI without knowing co-user's spreading codes, timing, and phase, and hence increase system capacity. Analysis and numerical results show that the proposed receiver outperforms the conventional receiver especially when MAI is significant. Finally, a discussion on the effect of bandlimited spreading signals is also given on the practical implications of the proposed technique.

I. INTRODUCTION

In a DS-CDMA system with perfect power control, a major limitation to the capacity is due to the multiple access interference (MAI). With the purpose of MAI rejection, the optimum multiuser receiver is studied in [1]-[2]. However, the resulting receiver is extremely complex. Based on the fact that the MAI can be modeled as zero-mean stationary colored Gaussion process, a simple receiver structure called integral equation receiver has been proposed in [3]. The integral equation receiver employs the despreading function which is the solution of a Fredholm integral equation of the second kind, and the resulting despreading function consists of $2N^2$ exponential terms with $N(N+3)$ coefficients (N is processing gain). In practice, it is not easy to tune to the optimum despreading function when the processing gain N is large. Based on the property that the despreading function given in [3] emphasizes the transitions in the received signal of the reference user for MAI rejection, we propose to weight the despreading sequence by exponential chip weighting waveforms[4]. Since the chip weighting waveforms employed is determined only by one parameter, this leads to easy tuning of the WDW in practice to achieve the best performance. In this paper, we evaluate various effects of the proposed WDW on CDMA performance. Numerical results show that when the MAI is significant in a DS-CDMA system, the proposed receiver performs far better than the conventional receiver. Finally, effects of the WDW on bandlimited CDMA performance are discussed.

II. SYSTEM MODEL

Suppose there are K CDMA users accessing the channel. User k transmits a data signal $b_k(t)$ and employs a spreading signal $a_k(t)$ to spread each data bit. The spreading and data signals for the k th user are given by $a_k(t)=\sum_{j=-\infty}^{\infty} a_j^{(k)} P_{T_c}(t-jT_c)$ and $b_k(t)=\sum_{j=-\infty}^{\infty} b_j^{(k)} P_{T_b}(t-jT_b)$ respectively where $P_x(y)=1$ for $0<y<x$ and 0 otherwise. In our study, $b_j^{(k)}$ and $a_j^{(k)}$ are modeled as independent random variables taking values -1 or $+1$ with equal probabilities. It is assumed that there are N chips of a spreading sequence in the interval of each data bit T_b and the spreading sequence has period much larger than $N=T_b/T_c$. The transmitted signal for the k th user is

$$S_k(t)=\sqrt{2P}b_k(t)\,a_k(t)\cos(\omega_0 t+\theta_k) \tag{1}$$

where the transmitted power P and the carrier frequency ω_0 are common to all users, and θ_k is the phase angle of the k th transmitter. For an asynchronous DS-CDMA system the received signal at the base station can be represented as

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} S_k(t-\tau_k)+n(t) \\ &= \sqrt{2P}\sum_{k=1}^{K} b_k(t-\tau_k)a_k(t-\tau_k)\cos(\omega_0 t+\phi_k)+n(t) \end{aligned} \tag{2}$$

where K denotes the number of active users, τ_k and $\phi_k(=\theta_k-\omega_0\tau_k)$ for $1\le k\le K$ are random time delays and phases along the communications links between the K transmitters and the particular receiver, respectively, and $n(t)$ is additive white Gaussian noise (AWGN) with two-side spectral density $N_0/2$. The random variables $\{\tau_k\}$ and $\{\phi_k\}$ are independent of one another and uniformly distributed in $[0, T_b]$ and $[0,\ 2\pi]$, respectively.

For BPSK modulation, the structure of the proposed coherent receiver with WDW for the k th user is shown in

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Fig. 1 where $\varsigma_k(\kappa)$ is the decision variable and $\hat{a}_k(t)$ is the WDW which will be stated in detail below.

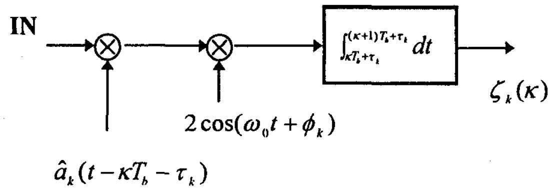

Fig. 1 The structure of a coherent receiver with weighted despreading waveforms.

III The Weighted Despreading Waveforms

In a DS-CDMA system, the MAI is the sum of many independent co-user signals so that it can be modeled as zero-mean colored stationary Gausssian process. When MAI and AWGN occur simutaniously, the optimum CDMA receiver in this case for MAI rejection is called integral equation receiver [3] in which the despreading function $q(t)$ for the k th receiver should be the solution to the integral equation

$$\int_0^{T_b} C_n(t-\tau)q(t)d\tau = -N_0 q(t) + a_k(t)\,,\ 0\le t\le T_b \qquad (3)$$

where $C_n(\tau)$ is the autocorrelation function of the MAI. Eq.(3) is known as Fredholm integral equation of the second kind. The integral equation (3) has been solved in [3] for the case in which the spreading pulses are rectangular and resulting despreading function $q(t)$ consists of $2N^2$ exponential terms with $N(N+3)$ coefficients. However it is not easy to tune to the optimum despreading function $q(t)$ for MAI rejection when processing gain N is large. To overcome the tuning problem, we proposed a WDW for the k th receiver [4,5] as follows

$$\hat{a}_k(t) = \sum_{j=0}^{\infty} a_j^{(k)} w_j^{(k)}(t-jT_c \mid \{c_j^{(k)}, c_{j+1}^{(k)}\}) P_{T_c}(t-jT_c) \qquad (4)$$

where $c_j^{(k)} = a_{j-1}^{(k)} a_j^{(k)}$ and $w_j^{(k)}(t \mid \{c_j^{(k)}, c_{j+1}^{(k)}\})$ for $0\le t\le T_c$ is the j th chip weighting waveform conditioned on the status of three consecutive chips $\{a_{j-1}^{(k)}, a_j^{(k)}, a_{j+1}^{(k)}\}$. In our analysis, we choose the j th chip weighting waveforms as

$$w_j^{(k)}(t \mid \{c_j^{(k)}, c_{j+1}^{(k)}\}) = \begin{cases} cw_1(t) & if\ c_j^{(k)} = +1\ and\ c_{j+1}^{(k)} = +1 \\ cw_2(t) & if\ c_j^{(k)} = -1\ and\ c_{j+1}^{(k)} = -1 \\ cw_3(t) & if\ c_j^{(k)} = -1\ and\ c_{j+1}^{(k)} = +1 \\ cw_4(t) & if\ c_j^{(k)} = +1\ and\ c_{j+1}^{(k)} = -1 \end{cases} \qquad (5)$$

where $cw_p(t)$ for $p\in[1,2,3,4]$ are the chip weighting waveforms which will be described in detail below. Based on the property that the optimum despreading function given in [3] emphasizes the transitions of the received signal of interest user, we therefore define the elements of the chip weighting waveform vector $\{cw_1(t), cw_2(t), cw_3(t), cw_4(t)\}$ as follows

$$\begin{aligned} cw_1(t) &= e^{-\gamma/2} P_{T_c}(t) \\ cw_2(t) &= e^{-\gamma t/T_c} P_{T_c/2}(t) + e^{-\gamma(1-t/T_c)} P_{T_c/2}(t-T_c/2) \\ cw_3(t) &= e^{-\gamma t/T_c} P_{T_c/2}(t) + e^{-\gamma/2} P_{T_c/2}(t-T_c/2) \\ cw_4(t) &= e^{-\gamma/2} P_{T_c/2}(t) + e^{-\gamma(1-t/T_c)} P_{T_c/2}(t-T_c/2) \end{aligned} \qquad (6)$$

where $\gamma\in[0,\infty)$ is a parameter of the chip weighting waveforms. When $\gamma=0$ in (6), the chip weighting waveforms $cw_p(t)$ for $p\in[1,2,3,4]$ reduce to the same rectangular pulse $P_{T_c}(t)$.

IV. Performance Analysis

We arbitrarily choose the i th user as the reference user and analyze the performance of the proposed coherent receiver for data symbol $b_\lambda^{(i)}$. After demodulation, the conditional output random variable of the reference user's receiver , denoted by $\zeta_i(\lambda)$, can be expressed as

$$\zeta_i(\lambda) = \int_{\lambda T_b+\tau_i}^{(\lambda+1)T_b+\tau_i} \{2r(t)\hat{a}_i(t-\lambda T_b-\tau_i) \cdot \cos(\omega_0 t+\phi_i)\} dt \qquad (7)$$

Since the carrier frequency f_0 is much large than T_b^{-1} in practical system, the double-frequency terms in (7) can be ignored and Eq.(7) reduces to

$$\zeta_i(\lambda) = S_i(\lambda \mid \{c_j^{(i)}\}) + N_i(\lambda \mid \{c_j^{(i)}\}) + \sum_{k=1,(k\ne i)}^{K} Y_{\{c_j^{(i)}\}}^{(k,i)} \qquad (8)$$

where the first, second, and third components are the conditional desired, noise, and multiple access interference components. The desired signal term of the $S_i(\lambda \mid \{c_j^{(i)}\})$ in (8) , conditioned on $\{c_j^{(i)}\}$, is given by [5]

$$S_i(\lambda \mid \{c_j^{(i)}\}) = b_\lambda^{(i)} \sqrt{2P}\left[2\hat{N}_i T_c\left(1-e^{-\gamma/2}\right)/\gamma + (N-\hat{N}_i) T_c e^{-\gamma}\right] \qquad (9)$$

where $\hat{N}_i$ is a random variable which represents the number of times of occurrence that $c_j^{(i)} = -1$ for all $j \in[0, N-1]$ in the interval of data symbol $b_\lambda^{(i)}$. The variance of the white noise term $N_i(\lambda \mid \{c_j^{(i)}\})$ in (8), conditioned on $\{c_j^{(i)}\}$ and denoted by $\sigma^2_{N\mid\{c_j^{(i)}\}}$, is given by [5]

$$\sigma^2_{N\mid\{c_j^{(i)}\}} = N_0\left[\hat{N}_i T_c\left(1-e^{-\gamma/2}\right)/\gamma + (N-\hat{N}_i) T_c e^{-\gamma/2}\right] \qquad (10)$$

The third term in (8) is the multiple access interference term of the reference receiver with the conditional variance, conditioned on $\{c_j^{(i)}\}$, given by [5]

$$\sigma^2_{I\mid\{c_j^{(i)}\}} = \frac{NT_c^2(K-1)P}{\gamma^2} e^{-\gamma} \Xi^{(e)}\left(\Gamma^{\{c_j^{(i)}\}}, \gamma\right) \qquad (11)$$

where

$$\Xi^{(e)}\left(\Gamma^{\{c_j^{(i)}\}},\gamma\right)=\frac{1}{N}\left\{\Gamma^{(i)}_{\{-1,-1,-1\}}\left[4+\frac{12}{\gamma}-\frac{16}{\gamma}e^{\gamma/2}+\frac{4}{\gamma}e^{\gamma}\right]\right.$$

$$+\left(\Gamma^{(i)}_{\{-1,-1,1\}}+\Gamma^{(i)}_{\{1,-1,-1\}}\right)\left[\frac{5}{2}-\frac{\gamma}{4}+\frac{\gamma^2}{24}+\frac{19}{2\gamma}+e^{\gamma/2}-\frac{12e^{\gamma/2}}{\gamma}+\frac{5e^{\gamma}}{2\gamma}\right]$$

$$+\left(\Gamma^{(i)}_{\{-1,1,1\}}+\Gamma^{(i)}_{\{1,1,-1\}}\right)\left[-\frac{3}{2}-\frac{3}{4}\gamma+\frac{19}{24}\gamma^2-\frac{1}{2\gamma}+e^{\gamma/2}+\frac{e^{\gamma}}{2\gamma}\right]$$

$$+\Gamma^{(i)}_{\{-1,1,-1\}}\left[-3-\frac{3}{2}\gamma+\frac{7}{12}\gamma^2-\frac{1}{\gamma}+2e^{\gamma/2}+\frac{e^{\gamma}}{\gamma}\right]$$

$$\left.+\Gamma^{(i)}_{\{1,-1,1\}}\left[1-\frac{\gamma}{2}+\frac{\gamma^2}{12}+\frac{7}{\gamma}+2e^{\gamma/2}-\frac{8}{\gamma}e^{\gamma/2}+\frac{e^{\gamma}}{\gamma}\right]+\gamma^2\Gamma^{(i)}_{\{1,1,1\}}\right\} \quad (12)$$

and $\Gamma^{(i)}_{\{\upsilon_1,\upsilon_2,\upsilon_3\}}$ is the number of times of occurrence that $\{c^{(i)}_{j-1},c^{(i)}_j,c^{(i)}_{j+1}\}=\{\upsilon_1,\upsilon_2,\upsilon_3\}$ for all $j\in[0,N-1]$ in the reference user's spreading sequence of data symbol $b^{(i)}_\lambda$ duration and each υ_n, $n\in[1, 2, 3]$, is a random variable of elements of $\{+1,-1\}$. It is clear that $\sum_{\{\upsilon_1,\upsilon_2,\upsilon_3\}}\Gamma^{(i)}_{\{\upsilon_1,\upsilon_2,\upsilon_3\}}=N$.

By definition, the conditional signal to interference plus noise ratio of the decision variable is expressed as

$$SINR_i=\left\{\frac{\sigma^2_{N|\{c_j^{(i)}\}}}{\left[S\left(\lambda\,|\,\{c_j^{(i)}\}\right)\right]^2}+\frac{\sigma^2_{I|\{c_j^{(i)}\}}}{\left[S\left(\lambda\,|\,\{c_j^{(i)}\}\right)\right]^2}\right\}^{-1} \quad (13)$$

where $S(\lambda\,|\,\{c_j^{(i)}\})$, $\sigma^2_{N|\{c_j^{(i)}\}}$ and $\sigma^2_{I|\{c_j^{(i)}\}}$ are expressed by (9), (10) and (11), respectively. Substituting (9), (10) and (11) into (13), the conditional signal to interference plus noise ratio, conditioned on $\{c_j^{(i)}\}$, is given by

$$SINR_i=\left\{\frac{\gamma\left[\chi\left(1-e^{-\gamma}\right)+\gamma(1-\chi)e^{-\gamma}\right]}{2\bar{\gamma}_b\left[2\chi\left(1-e^{-\gamma/2}\right)+\gamma(1-\chi)e^{-\gamma/2}\right]^2}+\frac{(K-1)\,\Xi^{(e)}\left(\Gamma^{\{c_j^{(i)}\}},\gamma\right)}{2N\left[2\chi(e^{\gamma/2}-1)+\gamma(1-\chi)\right]^2}\right\}^{-1} \quad (14)$$

where $E_b=PT_b$, $\bar{\gamma}_b=E_b/N_0$ and $\chi=\hat{N}_i/N$.

V NUMERICAL RESULTS

In this section, we present numerical results on the performance of the proposed coherent receiver. In all the bit error rate (BER) curves, the BER for the data symbol $b^{(i)}_\lambda$ is defined as $BER=Q\left(\sqrt{SINR_i}\right)$ where

$$Q(x)=(2\pi)^{-1}\int_x^{\infty}\exp(-t^2/2)dt \quad (15)$$

In the following, it is assumed that, in the reference user's random spreading sequence of data symbol $b^{(i)}_\lambda$ duration, $N=127$, $\hat{N}_i=62$, $\Gamma^{(i)}_{\{-1,-1,-1\}}=10$, $\Gamma^{(i)}_{\{-1,-1,1\}}+\Gamma^{(i)}_{\{1,-1,-1\}}=36$, $\Gamma^{(i)}_{\{-1,1,1\}}+\Gamma^{(i)}_{\{1,1,-1\}}=32$, $\Gamma^{(i)}_{\{-1,1,-1\}}=18$, $\Gamma^{(i)}_{\{1,-1,1\}}=16$, and $\Gamma^{(i)}_{\{1,1,1\}}=15$.

In Fig. 2, $SINR_i$ is plotted using the parameters γ as the independent variables for four values of the signal to white noise ratios $\bar{\gamma}_b$ when $K=37$. It is clear that the WDW reduces to rectangular spreading function when $\gamma=0$. Therefore, $SINR_i$ at $\gamma=0$ takes the value equal to the $SINR_i$ derived in the case that the chip waveforms of the reference despreading function are rectangular. Fig. 2 shows that the parameter γ should be tuned with respect to each $\bar{\gamma}_b$ in order to maximize the $SINR_i$ when $\bar{\gamma}_b$ is relatively large. From this figure one can see that the maximum value of $SINR_i$ obtained by tuning the parameter γ is very close to the value of $SINR_i$ at $\gamma=0$ when the MAI is insignificant. For example, when $\bar{\gamma}_b\le 5\ dB$, the maximum value of $SINR_i$ obtained by tuning the parameter γ can simply be replaced by the value of $SINR_i$ at $\gamma=0$ with a negligible loss of $SINR_i$. In other words, the change to the despreading function has little effect on system $SINR_i$ when the AWGN is significant.

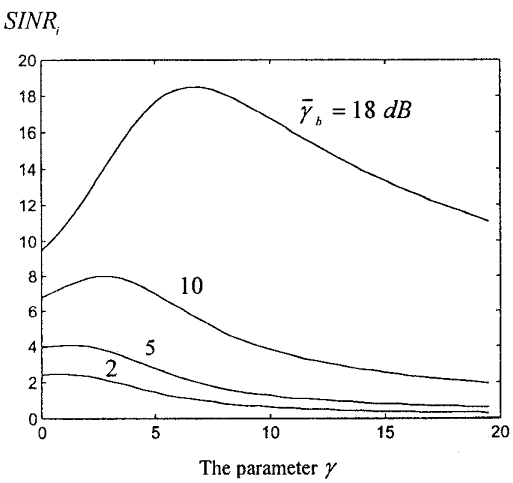

Fig. 2 $SINR_i$ versus the parameter γ for four values $\bar{\gamma}_b$ when $N=127$ and $K=37$.

Fig.3 shows the BER performance of the proposed coherent receiver for the cases of $\gamma=0$ and γ tuned to maximize the $SINR_i$. The dashed line is the BER for the receiver when the despreading sequence is the rectangular spreading function ($\gamma=0$). The solid line corresponds to the BER for the receiver in which the parameters γ of the WDW is tuned explicitely to each $\bar{\gamma}_b$ to maximize the $SINR_i$. It is clear that the performance of the receiver with the WDW over the conventional receiver with the rectangular despreading function is dramatic, although the WDW is not exactly the solution of the integral equation.

VI DISCUSSION

Assuming that system bandwidth is infinite, performance analysis and numerical results are shown in previous sections. However, it is difficult to imagine that a real CDMA system can operate with infinite bandwidth. As a result, it is important to see how the proposed receiver performs when the received signals are

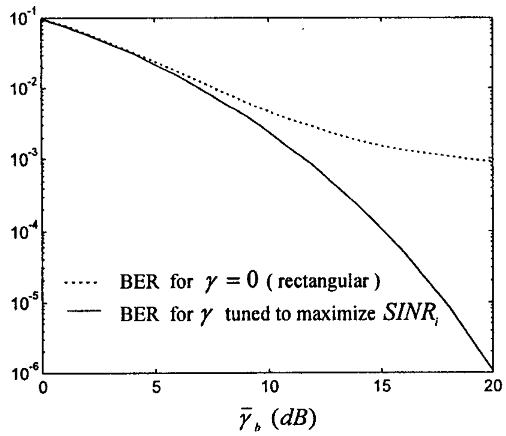

Fig. 3 BER versus $\bar{\gamma}_b$ for both rectangular and adjustable despreading function when $N = 127$ and $K = 37$.

bandlimited. For bandlimited CDMA, a appropriate measure of system performance is the in-band portion of the cross power spectrum of a pair of spreading and weighted despreading signals [6]. To obtain the cross power spectrum we need to evaluate the cross-correlation function of $a_k(t)$ and $\hat{a}_k(t)$, defined by

$$R_{cross}(t,\tau) \equiv E[a_k(t)\hat{a}_k(t-\tau)] \quad (16)$$

which depends on t as well as on τ. By assuming the variable t in (16) is uniformly distributed in $[0,T_c]$, we can remove the variable t in (16) and obtain $R_{cross}(\tau)$ which is independent of t. Then by making the Fourier transform of $R_{cross}(\tau)$, we can obtain the cross-correlation function, denoted by $S_{cross}(f,\gamma)$, which is a function of the frequency f as well as the parameter γ of the WDW. Assuming that the system bandwidth is constrained to satisfy $|f| \leq 1/T_c$, Fig. 4 shows $S_{cross}(f,\gamma)$ versus the frequency f for two values of γ. From this figure, one can see that the in-band portion of $S_{cross}(f,\gamma)$ can be flattened to a certain degree by simply tuning γ. This agrees with our intuitive explanation and we expect the performance of bandlimited CDMA system can also be improved by using the proposed approach.

VII. CONCLUSIONS

In this paper, we have introduced a coherent CDMA receiver in which the despreading function is weighted by adjustable exponentially chip waveforms. To achieve the maximum value of the system $SINR_i$, we can simply tune the parameter γ of the WDW based on the relative strength between MAI and AWGN. When the MAI is significant, the receivers using the WDW employed outperforms greatly the conventional receiver using the rectangular despreading function without increasing a lot in complexity. Finally, the performance of the proposed system for bandlimited spreading signals are discussed to reveal the practical implications.

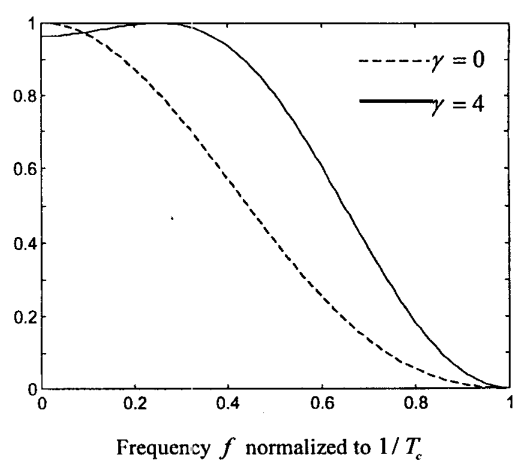

Fig. 4 Normalized cross power spectral density $S_{cross}(f,\gamma)/\max[S_{cross}(f,\gamma)]$ for two values of γ.

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