Robust Control of Robot Manipulators using Hybrid H^∞/Adaptive Controller

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ABSTRACT

A robust hybrid control method for robot manipulators is proposed which integrates an H^∞ controller and an adaptive controller. The H^∞ controller is used to minimize the effect of parameter uncertainties of the robot model on the tracking performance, while the adaptive controller continuously adjusts the model parameters to reduce the model error. Simulations show that disturbances generated from the model error will be quickly compensated and so small tracking errors can be achieved.

1. INTRODUCTION

The popularly known computed-torque method [1] that makes use of a robot model to synthesise the controller has been shown to be effective for controlling robot manipulators. However the control scheme relies on an accurate robot model which is not always available. Uncertainties of model parameters can be a serious obstacle in robot control. Various methods have been proposed to combat robot model uncertainties.

In general, these methods can be divided into two major categories - the adaptive control scheme and the robust control scheme. Adaptive control methods usually rely on an adaptive algorithm which changes the robot model parameters on an on-line basis [2][4]. The estimated model parameters are adjusted based on the tracking error of the robot. Theoretically, tracking errors will converge to zero asymptotically for any initial condition. Persistent excitation is usually needed for convergence.

Another control scheme used is the robust control such as variable-structure control (VSC) [6][7][8][14][15] and H^∞ control [19]. In variable-structure control, the tracking error of the joint motion will be forced to zero by applying a non-linear switching torque. Tracking errors will "slide" along the "sliding surface" and reach the origin of the sliding plane despite input disturbances and model errors. The major disadvantage of VSC is the chattering motion generated by the applied switching torque and the finite switching frequency. One remedy is to introduce a boundary layer in the switching controller [5] which reduces the change of the applied torque when the error is small.

In another approach, H^∞ control with robust properties [9] is used to minimize the effect of parameter uncertainties on the tracking error. The advantage of this kind of controller is its simplicity and fixed structure. No adaptation is needed and so the on-line computational requirement is modest. The disadvantage, however, is the lack of self-adjustment of model parameters to deal with large environmental changes such as a sudden change of the payload.

In this paper, a hybrid control scheme is proposed to control a robot in the face of uncertainties. The controller is composed of an adaptive controller and an H^∞ controller. Using the H^∞ optimization method, a linear controller is synthesized to reject the effect of model errors represented in the form of input disturbances. On the other hand, the adaptive controller adjusts the estimates of the model parameters continuously according to the tracking errors. Theoretically, the tracking errors will converge to zero and the overall system is robust against input disturbances. The results will be illustrated by means of a simulation study.

2. MODEL-BASED ROBOT CONTROLLER

The basic model-based robot controller (computed-torque method) relies on feedback linearization of the robot model. Let the robot manipulator be modelled as n-serial-chain rigid body [1] in the following form:

$$T = M(\Theta)\ddot{\Theta} + N(\Theta,\dot{\Theta}) \tag{2.1}$$

where T is the $n \times 1$ vector of the generalized joint torques supplied by the actuators, Θ is the $n \times 1$ vector of joint angle variables, M is the $n \times n$ inertia matrix of the robot and N is the combined $n \times 1$ vector of the Coriolis, centrifugal, gravitational and frictional torque components.

By means of feedback linearization, the required applied torque vector of the actuators is given by

$$T^{*} = \hat{M}(\Theta)u + \hat{N}(\Theta,\dot{\Theta}) \tag{2.2}$$

where $\hat{M}$, $\hat{N}$ are the estimated matrix and vector of M and N respectively, and

$$u = \ddot{\Theta}_d + K_v\dot{e} + K_p e \tag{2.3}$$

where Θ_d is the desired joint angle vector, $e = \Theta_d - \Theta$ is the joint angle error vector and K_v and K_p are constant diagonal matrices which are selected by the designer.

If an accurate robot model can be obtained, $\hat{M}$ and $\hat{N}$ will be equal to M and N respectively. In this case the overall error dynamics of the linearized system satisfies

$$\ddot{e} + K_v\dot{e} + K_p e = 0 \tag{2.4}$$

By selecting appropriate matrices for K_v and K_p, good error dynamics can be obtained and the tracking error will converge to zero. However, if the model is not accurate, model error will appear in the tracking error dynamics. From (2.1) and (2.2), we have

$$\begin{aligned}
\hat{M}(\Theta)u + \hat{N}(\Theta,\dot{\Theta}) &= M(\Theta)\ddot{\Theta} + N(\Theta,\dot{\Theta}) \\
\hat{M}(\Theta)\left[\ddot{\Theta}_d + K_v\dot{e} + K_p e\right] &= M(\Theta)\ddot{\Theta} + \left(N - \hat{N}\right) \\
\hat{M}(\Theta)\left[\ddot{e} + K_v\dot{e} + K_p e\right] &= \left(M - \hat{M}\right)\ddot{\Theta} + \left(N - \hat{N}\right) \\
\ddot{e} + K_v\dot{e} + K_p e &= \underbrace{\hat{M}^{-1}\left\{\left(M - \hat{M}\right)\ddot{\Theta} + \left(N - \hat{N}\right)\right\}}_{\eta}
\end{aligned}$$

$$\ddot{e} + K_v\dot{e} + K_p e = \eta \tag{2.5}$$

This shows that the tracking error dynamics is forced by a "disturbance" η generated by the differences between the estimated model parameters and the true parameters. In the next section, we will show how the effect of model error can be reduced by H^{∞} control so that better tracking results can be obtained.

3. DESIGN OF H^{∞} CONTROLLER FOR ROBOT MANIPULATORS

We modify the control torque from (2.3) to :

$$u = \ddot{\Theta}_d + K_v\dot{e} + K_p e + v \tag{3.1}$$

where v is the output of an H^{∞} controller and all other terms in (2.3) are kept unchanged. As a result, the error dynamics becomes

$$\ddot{e} + K_v\dot{e} + K_p e + v = \eta \tag{3.2}$$

The problem is then to find an H^{∞} controller such that the effective transfer function from η to e is minimal.

Algorithms for solving H^{∞} optimization problems have been obtained by various authors [9][13]. To synthesize an H^{∞} controller, we transform the problem into the standard configuration for H^{∞} controller design. Fig. 1 shows a generalized plant $P(s)$ and a controller $K(s)$ connected together in the form of a linear fractional transformation.

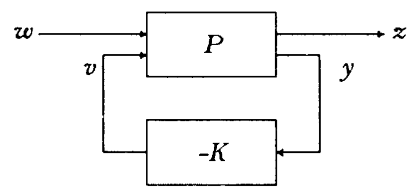

Fig. 1. Generalized linear fractional feedback configuration.

Let $P(s)$ be partitioned conformally with the partitions of the input and output vectors as shown in Figure 1 :

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \tag{3.3}$$

Using the packed matrix notation, let $P(s)$ have a minimal state-space realization given by

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{3.4}$$

in which

$$P_{ij}(s) = C_i(sI - A)^{-1} + D_{ij} \in \mathbb{R}(s)^{p_i \times m_j} \tag{3.5}$$

In order to use the H^{∞} optimization algorithm developed in [13], several assumptions have to be satisfied.

Assumptions :

(a) (A, B_2) is stabilizable and (A, C_2) is detectable.

(b) The realizations for $P_{12}(s)$ and $P_{21}(s)$ have no Smith zeros on the $j\omega$-axis.

(c) (A,B_1) is controllable and (A,C_1) is observable..

(d) D_{12} is column orthogonal, i.e. $D_{12}^T D_{12} = I$.

Assumptions (c) and (d) are not strictly required. Some transformations can be used to render assumptions (c) and (d) valid if they are not satisfied [13].

To express our problem in state-space format, we define the error vector x and the disturbance vector w to be

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \quad \text{and} \quad w = \begin{bmatrix} \eta \\ w_{\Theta} \\ w_{\dot{\Theta}} \end{bmatrix} \tag{3.6}$$

where w_{Θ} and $w_{\dot{\Theta}}$ are the observation errors of the joint angle vector and velocity vector respectively. Expressing the tracking error dynamics in state-space form gives

$$\dot{x} = Ax + B_1 w + B_2 v \tag{3.7}$$

where

$$A = \begin{bmatrix} 0 & I_n \\ -K_p & -K_v \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ I_n & 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ -I_n \end{bmatrix} \tag{3.8}$$

We define the output z to be

$$z = C_1 x + D_{12} v \tag{3.9}$$

where

$$C_1 = \begin{bmatrix} r_1 I_n & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \\ r_2 I_n \end{bmatrix} \tag{3.10}$$

in which r_1 and r_2 are constant weights which are to be chosen in the controller design process. Let the observation output y be

$$y = C_2 x + D_{21} w \tag{3.11}$$

where

$$C_2 = I_{2n}, \qquad D_{21} = \begin{bmatrix} 0 & I_{2n} \end{bmatrix} \tag{3.12}$$

The overall state-space representation of the error system can be written as :

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 v \\ z &= C_1 x + D_{12} v \\ y &= C_2 x + D_{21} w \end{aligned} \tag{3.13}$$

In (3.10), the weights r_1 and r_2 are constants. For frequency-dependent weights, we have to rewrite the

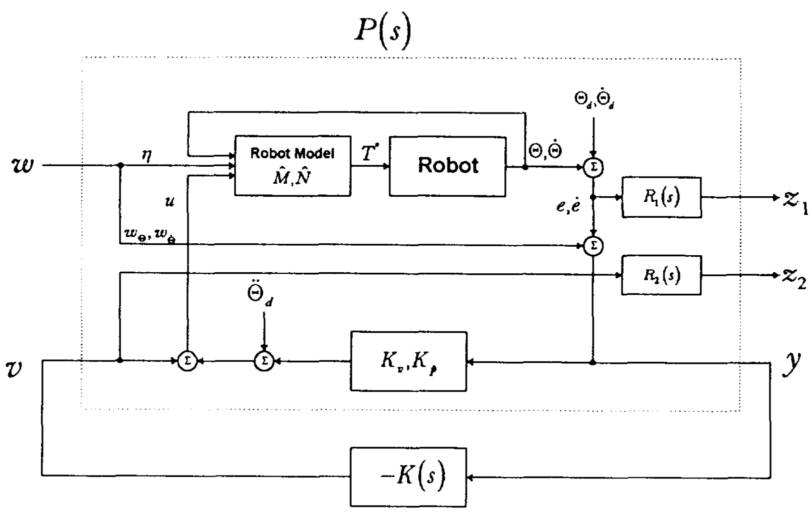

Fig. 2. Structure of H^∞ model-based controller

state-space equations of the system model to incorporate state-space models for the weights. Figure 2 shows the structure of the H^∞ model-based controller with frequency-dependent weights.

Let $R_1(s)$ and $R_2(s)$ be the frequency-dependent weights applied to the controlled output z of the system, with minimal state-space realization given by :

$$R_1(s)=\left[\begin{array}{c|c} E_1 & F_1 \\ \hline G_1 & H_1 \end{array}\right] \tag{3.14}$$

and $$R_2(s)=\left[\begin{array}{c|c} E_2 & F_2 \\ \hline G_2 & H_2 \end{array}\right] \tag{3.15}$$

Incorporating (3.14) and (3.15) into the overall state-space equation gives

$$\begin{bmatrix} \dot{x} \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}=\begin{bmatrix} A & 0 & 0 \\ F_1 & E_1 & 0 \\ 0 & 0 & E_2 \end{bmatrix}\begin{bmatrix} x \\ q_1 \\ q_2 \end{bmatrix}+\begin{bmatrix} B_1 \\ 0 \\ 0 \end{bmatrix}w+\begin{bmatrix} B_2 \\ 0 \\ F_2 \end{bmatrix}v$$

$$z=\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} H_1 & G_1 & 0 \\ 0 & 0 & G_2 \end{bmatrix}\begin{bmatrix} x \\ q_1 \\ q_2 \end{bmatrix}+\begin{bmatrix} 0 \\ H_2 \end{bmatrix}v \tag{3.16}$$

Since the format of the overall system is the same as (3.13), we will develop out results without frequency-dependent weights in the remaining sections.

From Figure 2, we have that

$$v=-Ky \tag{3.17}$$

We wish to find a controller $K \in \mathbb{S}^*$, where $\mathbb{S}^*$ is the set of all stabilizing controllers for the plant $P(s)$, such that the transfer function from w to z has a minimal (or bounded) H^∞ norm.

i.e. $$\exists \gamma>0, \text{ s.t. } \|T_{zw}\|\le\gamma \tag{3.18}$$

Using the H^∞ optimization method, we can obtain the controller by an iterative algorithm which requires the solution of two Riccati equations. The reader is referred to [9][11][12][13] for algorithmic details.

Let the resultant controller be $K(s)=\left[\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right]$, the controller dynamics can be represented by

$$\begin{aligned} \dot{p}&=ap+by \\ v&=cp+dy \end{aligned} \tag{3.19}$$

Applying (3.19) to (3.13), a state-space model for the closed-loop system with H^∞ control is

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix}=\begin{bmatrix} A+B_2dC_2 & B_2c \\ bC_2 & a \end{bmatrix}\begin{bmatrix} x \\ p \end{bmatrix}+\begin{bmatrix} B_1+B_2dD_{21} \\ bD_{21} \end{bmatrix}w$$

$$z=\begin{bmatrix} C_1+D_{12}dC_2 & D_{12}c \end{bmatrix}\begin{bmatrix} x \\ p \end{bmatrix}+D_{12}dD_{21}w \tag{3.20}$$

This will be written in a more concise notation as

$$\begin{aligned} \dot{X}&=A'X+B'w \\ z&=C'X+D'w \end{aligned} \tag{3.21}$$

4. INCORPORATION OF MODEL-BASED ADAPTIVE CONTROLLER

Figure 3 shows the block diagram of a robot manipulator controlled by a hybrid controller. The adaptive regulator is used to tune the model parameters on an on-line basis to reduce the estimated

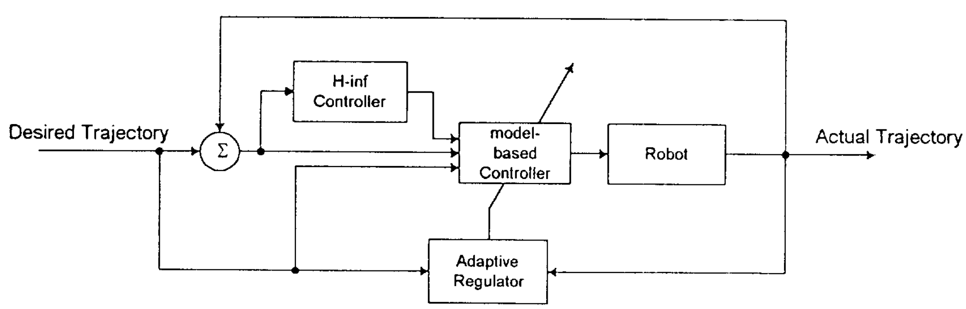

Fig. 3. Block diagram of the hybrid H^∞/adaptive controller

model error. The purpose of adaptation is to enable the system to adapt to parameter changes.

Different adaptation algorithms may be used provided that linear error dynamics can be maintained. In the present study, an algorithm which has been used in purely adaptive manipulator control is considered for the hybrid control scheme. The result is an even more robust controller with reduced manipulator tracking errors.

In order to incorporate an adaptive regulator into the robot controller obtained in section 3, we rewrite the overall error dynamics to exclude the observation noise of the joint angle vector and joint velocity vector as (c.f. equation (2.5)):

$$\begin{aligned}\dot{X} &= A'X + B^{*}\eta \\ &= A'X + B^{*}\hat{M}^{-1}\left[\left(M-\hat{M}\right)\ddot{\Theta} + N - \hat{N}\right] \\ &= A'X + B^{*}\hat{M}^{-1}W\left(\Theta,\dot{\Theta},\ddot{\Theta}\right)\Phi \end{aligned} \tag{4.1}$$

where B^{*} is the matrix extracted from B' to exclude w_{Θ} and $w_{\dot{\Theta}}$, Φ is the difference between the true model parameter vector and the corresponding estimate, and W is the matrix formed after parameterization. We can exclude the observation noise because it is assumed that no observation noise is present in the synthesis of the adaptive controller and so we can set w_{Θ} and $w_{\dot{\Theta}}$ to zero.

With the H^{∞} controller, it is guaranteed that A' is a stable matrix. According to Lyapunov's stability theory [10],

$$\exists P,Q>0 \text{ s.t. } \quad A'P + PA'^{T} = -Q \tag{4.2}$$

To obtain an update law which can be used to control the tracking error to zero, we introduce a Lyapunov function candidate

$$V(X) = X^{T}PX + \Phi\Gamma^{-1}\Phi \tag{4.3}$$

in which Γ is a diagonal gain matrix selected by the designer. So we have

$$\begin{aligned}\dot{V}(X) &= \dot{X}^{T}PX + X^{T}P\dot{X} + 2\Phi^{T}\Gamma^{-1}\dot{\Phi} \\ &= X^{T}\left(A'^{T}P + PA'\right)X + \\ &\qquad 2\Phi^{T}\left(W^{T}\hat{M}^{-T}B^{*T}PX + \Gamma^{-1}\dot{\Phi}\right)\end{aligned} \tag{4.4}$$

Let $C^{*} = PB^{*}$ and $E_{1} = C^{*T}X$. If we set

$$\dot{\Phi} = -\Gamma W^{T}\hat{M}^{-1}E_{1} \tag{4.5}$$

then

$$\begin{aligned}\dot{V}(X) &= -X^{T}QX \\ &\leq 0\end{aligned} \tag{4.6}$$

So with the update law (4.5), the state vector X will converge to zero and therefore $e \to 0$ asymptotically.

5. Simulation of the Hybrid Controller on a Two-Link Robot

The proposed hybrid H^{∞}/adaptive control scheme is verified by means of simulations. For simplicity, a two-link robot is used in the simulations. Only the case with frequency-dependent weights is illustrated for brevity. The results for the case with constant weights are slightly inferior.

Fig. 4 shows the simulation results of robot motion controlled by hybrid controller. Fig.5 to 6 show the corresponding results for the H^{∞} controller and the adaptive controller respectively. In the graphs, θ_1 and θ_2 are the joint angles, e_1 and e_2 are the joint angle errors, and τ_1 and τ_2 are the applied torques.

The simulation results show that the best performance is achieved by the hybrid controller. Errors occuring at the turning point of the trajectory are due to the direction change of the Coulomb friction. With the adaptive controller, such errors can be smoothed out and so gives a better trackiing properties.

6. Conclusion

A hybrid H^{∞}/adaptive control scheme is proposed to control robot manipulators in the face of uncertainties. By using frequency-dependent weights, the controlled system can be made robust against external disturbances and internal model uncertainties. Simulations show that the hybrid control scheme has better performance than the basic adaptive control scheme for trajectory tracking. It also excels the variable-structure control scheme in terms of chatter-free motion.

From the computational point of view, on-line computation required is about the same as the basic adaptive control scheme. As a large fraction of computation is used to implement the update law for adaptation, the additional computation of the H^{∞} controller is negligible.

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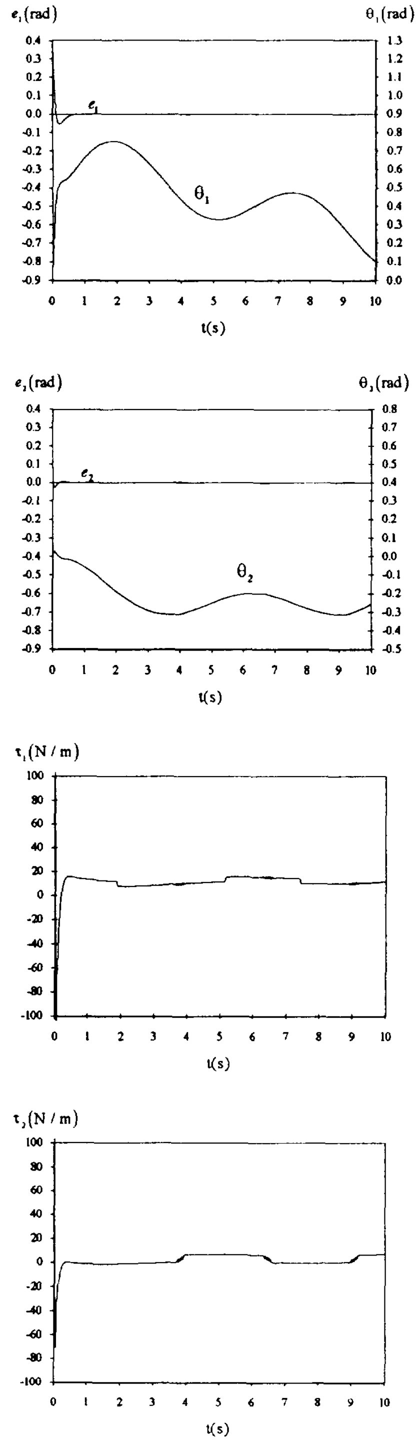

Fig. 4. Hybrid controller

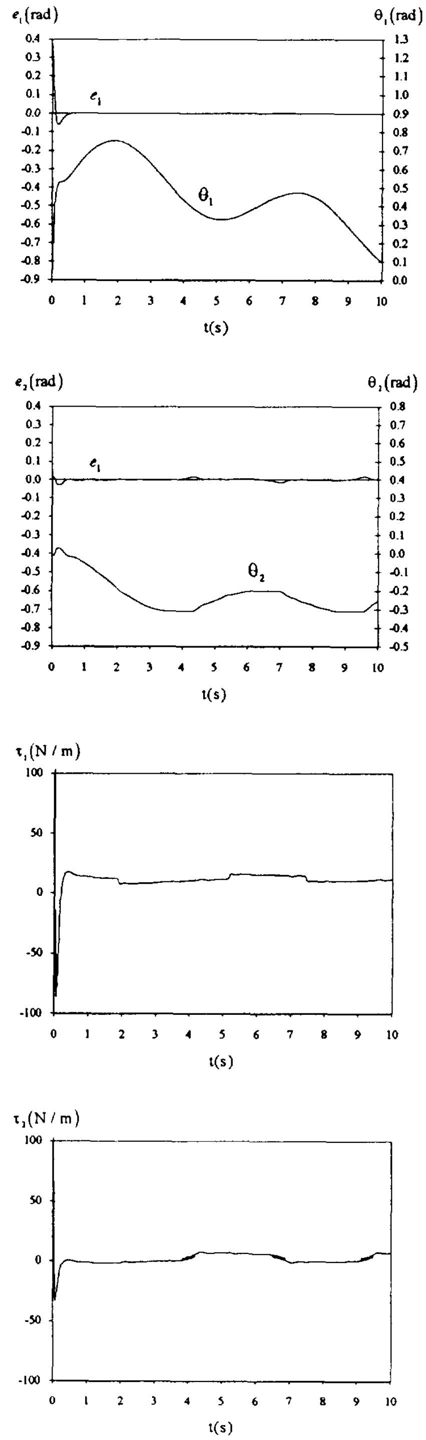

Fig. 5. H^{∞} controller

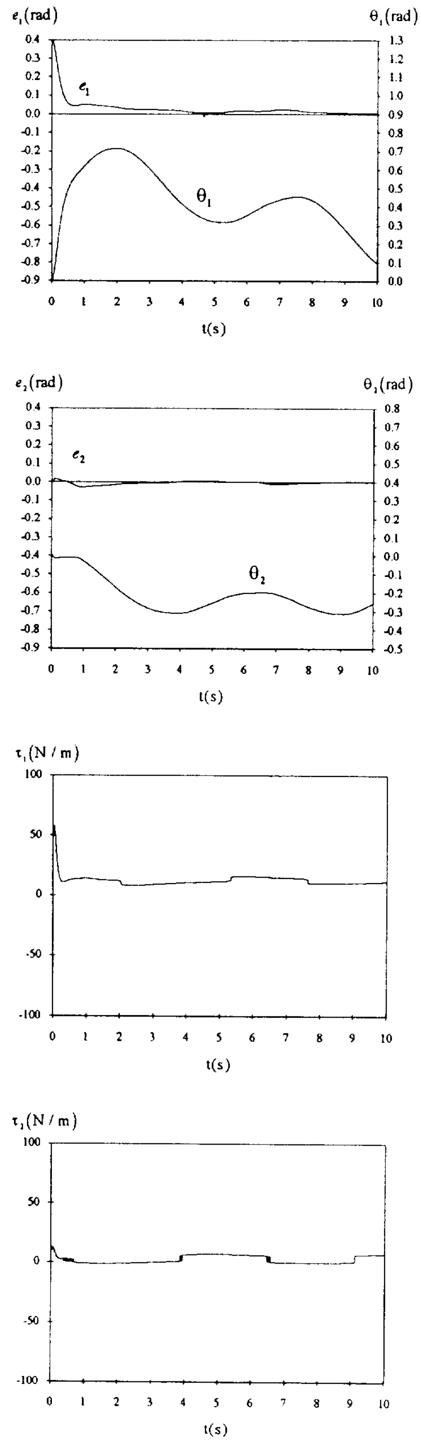

Fig. 6. MRAC controller