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Robust H_∞ Filtering with Error Variance Constraints for Uncertain Discrete-Time Systems

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Abstract

The robust H_∞ filtering problem is considered for discrete-time systems subject to norm-bounded parameter uncertainties in both the state and the output matrices of the state-space model. Sufficient conditions for the filter to satisfy prescribed H_∞ performance and steady-state estimation error variance constraints are given in terms of two discrete algebraic Riccati inequalities. The filter obtained does not depend on the perturbation parameter which is assumed to be unmeasured. An example is provided to illustrate the use of the results for filter design.

Key Words

Robust H_∞ filtering; error variance constraints; uncertain systems; discrete algebraic Riccati inequality.

1. Introduction

In Kalman filtering, the variance of the estimation error is minimized under the assumption that the spectral densities of noise processes are known (e.g. see [1]). In contrast, H_∞ filtering provides a bound for the worst-case estimation error without the need for knowledge of noise statistics (e.g. see [9]). There have been attempts to combine the performance requirements of the Kalman filter and the H_∞ filter into a mixed H_2/H_∞ filtering problem [14,15]. Earlier works on filtering theory often assume that an exact model is known for the system whose states are to be estimated. In most practical problems, an exact model of the system may not be available and the robust performance of the filter in the face of system parameter uncertainties becomes an important issue. The robust filtering problem of state estimation for systems with norm-bounded uncertainties has been considered in the context of both Kalman filtering [7,8,10,11,12] and H_∞ filtering [2,3,6,13].

In this paper, we will consider the problem of robust filtering with mixed H_2/H_∞ performance requirements for a discrete linear time-invariant system subject to parameter uncertainties. Our problem formulation is similar to that of Wang et al.[14], except that we do not require any assumption on the measurability of the perturbation. Although the Riccati equations given in the solution of [14] do not depend on the perturbation parameter, the robust H_2/H_∞ filter is based on a model of the uncertain system and hence requires the perturbation to be measurable. Although our filter design is derived from an H_∞ criterion, provisions are given for error variance constraints to be verified as part of the filter specifications. Our results differ from those of [14] in that our filter has a more general form independent of the perturbation parameter and hence does not require the perturbation to be measurable. Consequently, the sufficient conditions given in this paper for robust filtering is more involved requiring the solution of two unilaterally coupled discrete algebraic Riccati inequalities (*DARI*).

The paper is organized as follows. In Section 2, the robust H_∞ filtering problem with error variance constraints for discrete-time systems subject to norm-bounded parameter uncertainty is formulated. Sufficient conditions for solving the robust H_∞ filtering problem are developed in Section 3. An example is provided in Section 4 to illustrate the use of the sufficient conditions for the design of a robust filter.

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Some concluding remarks are given in Section 5.

2. Problem formulation

Consider the following class of uncertain systems (Wang et al. 1997):

$$\begin{cases} x(k+1)=(A+\Delta A)x(k)+D_1 w(k) \\ y(k)=(C+\Delta C)x(k)+D_2 w(k) \\ z(k)=Lx(k) \end{cases} \quad (1)$$

where $x(k)\in R^n$ is the state, $y(k)\in R^p$ is the measured output, $z(k)\in R^m$ represents an unmeasured output to be estimated, $w(k)\in R^q$ is a zero mean Gaussian white noise process with variance bounded by I, and A, C, D_1, D_2, L are known constant matrices describing the nominal system, whereas ΔA and ΔC are perturbation matrices representing parameter uncertainties. We will assume that ΔA and ΔC are time-invariant of the form

$$\begin{bmatrix} \Delta A \\ \Delta C \end{bmatrix}=\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}\Gamma N \quad (2)$$

where M_1, M_2 and N are known constant matrices of appropriate dimensions, and $\Gamma\in R^{i\times j}$ is a perturbation matrix which satisfies

$$\Gamma^T\Gamma\le I \quad (3)$$

The uncertainties ΔA and ΔC are said to be admissible if they satisfy conditions (2) and (3). The following assumptions will be made throughout this paper.

Assumption 1: The system (1) is stable for all admissible perturbations ΔA and ΔC. □

Assumption 2: The matrix A of the system (1) has no zero eigenvalues, i.e. A is nonsingular. □

Assumption 1 is necessary for the robust filtering problem to be meaningful. If Assumption 2 is not satisfied, we can always perturb A to a nonsingular matrix and absorb the perturbation into ΔA. Hence Assumption 2 can be made without loss of generality.

Now consider the following filter for the system (1)

$$\begin{cases} \hat{x}(k+1)=F\hat{x}(k)+G\,y(k) \\ \hat{z}(k)=L\hat{x}(k) \end{cases} \quad (4)$$

where $\hat{x}(k)\in R^n$ is the estimated state, $\hat{z}(k)\in R^m$ is an estimate for $z(k)$, and F and G are the filter parameters to be determined. In Wang et al. (1997), the filter is modeled on the system (1). As a result, F and G will depend on ΔA and ΔC and this requires the uncertainties to be measurable. In this paper, we will however considered the robust filtering problem under the assumption that ΔA and ΔC are unknown.

Define the state estimation error $e(k)$ and the output estimation error $e_z(k)$ by

$$e(k)=x(k)-\hat{x}(k) \quad (5)$$

$$e_z(k)=z(k)-\hat{z}(k) \quad (6)$$

A state-space model describing the augmented system formed from the system (1) and the filter (4) can be expressed in terms of the state vector

$$x_e(k)=\begin{bmatrix} e(k) \\ x(k) \end{bmatrix}$$

as

$$\begin{cases} x_e(k+1)=(A_e+\Delta A_e)x_e(k)+B_e w(k) \\ e_z(k)=C_e x_e(k) \end{cases} \quad (7)$$

where

$$A_e=\begin{bmatrix} F & A-GC-F \\ 0 & A \end{bmatrix} \quad (8a)$$

$$\Delta A_e=\begin{bmatrix} M_1-GM_2 \\ M_1 \end{bmatrix}\Gamma\begin{bmatrix} 0 & N \end{bmatrix}=:M_e\Gamma N_e, \quad (8b)$$

$$B_e=\begin{bmatrix} D_1-GD_2 \\ D_1 \end{bmatrix},\quad C_e=\begin{bmatrix} L & 0 \end{bmatrix} \quad (8c)$$

Our objective in this paper is to design a filter (4) such that for all admissible perturbations ΔA and ΔC, the following three requirements are simultaneously satisfied:

($F1$) The filter (4) is asymptotically stable.

($F2$) Given $\gamma>0$, the H_∞ norm of the transfer function

$$H(z)=C_e(zI-A_e-\Delta A_e)^{-1}B_e$$

from $w(k)$ to $e_z(k)$ satisfies

$$\|H(z)\|_\infty<\gamma \quad (9)$$

($F3$) The state estimation error

$e(k) = [e_1(k) \quad e_2(k) \quad . \quad . \quad . \quad e_n(k)]^T$ has variances satisfying

$$\mathrm{var}[e_i] = \lim_{k\to\infty} E[e_i(k)e_i^T(k)] < \sigma_i^2 \tag{10}$$

for some $\sigma_i^2 > 0$ ($i = 1, 2, \ldots, n$).

A filter satisfying these conditions will be referred to as a robust H_∞ filter with error variance constraints.

3. Robust H_∞ filtering with error variance constraints

In this section, we will show how one may construct a robust H_∞ filter with error variance constraints. The next two lemmas will be required for developing the main results.

Lemma 1: For any matrices X and Y of appropriate dimensions and any constant $\alpha > 0$,

$$(X^{-1} - \alpha^{-1}Y^TY)^{-1} = X + XY^T(\alpha I - YXY^T)^{-1}YX \tag{11}$$

□

Lemma 2: For any matrices A_e, M_e, N_e of appropriate dimensions and Γ such that $\Gamma^T\Gamma \le I$, if there exist a constant $\alpha > 0$ and a positive definite matrix X such that $\alpha I - N_eXN_e^T > 0$, then

$$\begin{aligned}&(A_e + M_e\Gamma N_e)X(A_e + M_e\Gamma N_e)^T \le \\ &A_e(X^{-1} - \alpha^{-1}N_e^TN_e)^{-1}A_e^T + \alpha M_eM_e^T\end{aligned} \tag{12}$$

□

In the next lemma, we establish a condition to guarantee the asymptotic stability as well as an H_∞ norm bound for the augmented system (7) in terms of a discrete algebraic Riccati inequality. The Lemma can be proved in a way analogous to that given in Lemma 1 of Geromel et al. [4].

Lemma 3: Given $\gamma > 0$. If the discrete algebraic Riccati inequality (*DARI*)

$$\begin{aligned}&(A_e + \Delta A_e)X_e(A_e + \Delta A_e)^T - X_e + (A_e + \Delta A_e)X_eC_e^T \\ &(\gamma^2 I - C_eY_eC_e^T)^{-1}C_eX_e(A_e + \Delta A_e)^T + B_eB_e^T < 0\end{aligned} \tag{13}$$

has a positive definite solution $X_e > 0$ such that $\gamma^2 I - C_eX_eC_e^T > 0$, then the augmented system (7) is asymptotically stable and $\|H(z)\|_\infty < \gamma$. □

The inequality (13) depends on the unknown perturbation Γ through ΔA and ΔC and is therefore not readily applicable. This dependency can be eliminated by means of Lemmas 1 and 2 yielding a sufficient condition for ($F1$) and ($F2$), as stated in the next result.

Theorem 1: Suppose there exists $\alpha > 0$ such that the *DARI*

$$\begin{aligned}&A_e(X_e^{-1} - \gamma^{-2}C_e^TC_e - \alpha^{-1}N_e^TN_e)^{-1}A_e^T - X_e + \\ &B_eB_e^T + \alpha M_eM_e^T < 0\end{aligned} \tag{14}$$

has a positive definite solution $X_e > 0$ such that $X_e^{-1} - \gamma^{-2}C_e^TC_e - \alpha^{-1}N_e^TN_e > 0$, then ($F1$) and ($F2$) are satisfied for all admissible perturbations ΔA and ΔC.

Proof:

Under the assumption of the Theorem and using Lemma 1 and Lemma 2, the expression on the left of (13) can be written

$$\begin{aligned}&(A_e + \Delta A_e)X_e(A_e + \Delta A_e)^T - X_e + (A_e + \Delta A_e)X_eC_e^T \\ &(\gamma^2 I - C_eY_eC_e^T)^{-1}C_eX_e(A_e + \Delta A_e)^T + B_eB_e^T \\ &= (A_e + \Delta A_e)(X_e^{-1} - \gamma^{-2}C_e^TC_e)^{-1}(A_e + \Delta A_e)^T - X_e + B_eB_e^T \\ &= (A_e + M_e\Gamma N_e)(X_e^{-1} - \gamma^{-2}C_e^TC_e)^{-1}(A_e + M_e\Gamma N_e)^T - X_e + B_eB_e^T \\ &\le A_e(X_e^{-1} - \gamma^{-2}C_e^TC_e - \alpha^{-1}N_e^TN_e)^{-1}A_e^T + \alpha M_eM_e^T - X_e + B_eB_e^T < 0\end{aligned}$$

Hence, Lemma 3 implies that the augmented system (7) is asymptotically stable and $\|H(z)\|_\infty < \gamma$ for all ΔA and ΔC.

This implies that ($F1$) and ($F2$) are satisfied for all admissible perturbations and completes the proof of Theorem 1. □

Now we present our main result. The following theorem provides a filter which ensures that the prescribed H_∞ performance is satisfied along with the steady-state error variance constraints.

Theorem 2: Consider the uncertain system (1). Suppose there exists $\alpha > 0$ and $\gamma > 0$ such that the following two discrete algebraic Riccati inequalities (15) and (16) have positive definite solutions $Q_1 > 0$, $Q_2 > 0$ satisfying (17):

$$\begin{aligned} &A_1(Q_1^{-1} - \gamma^{-2}L^TL)^{-1}A_1^T - Q_1 + R_{11} + R_{11}R_2R_{11}^T \\ &-[A_1(Q_1^{-1} - \gamma^{-2}L^TL)^{-1}C_1^T + R_{11}R_2R_{12} + R_{12}]R^{-1} \\ &[A_1(Q_1^{-1} - \gamma^{-2}L^TL)^{-1}C_1^T + R_{11}R_2R_{12} + R_{12}]^T < 0 \end{aligned} \tag{15}$$

$$AQ_2A^T - Q_2 + AQ_2N^T(\alpha I - NQ_2N^T)^{-1}NQ_2A^T + R_{11} < 0 \tag{16}$$

$$\gamma^2 I - LQ_1L^T > 0, \qquad \alpha I - NQ_2N^T > 0 \tag{17}$$

where

$$R_{11} = D_1D_1^T + \alpha M_1M_1^T, \quad R_{12} = D_1D_2^T + \alpha M_1M_2^T \tag{18a}$$

$$R_{22} = D_2D_2^T + \alpha M_2M_2^T, \; R_1 = (Q_2^{-1} - \alpha^{-1}N^TN)^{-1}A^T \tag{18b}$$

$$R_2 = R_1^{-1}(Q_2^{-1} - \alpha^{-1}N^TN)^{-1}(R_1^{-1})^T \tag{18c}$$

$$A_1 = A + R_{11}R_1^{-1}, \; C_1 = C + R_{12}^TR_1^{-1} \tag{18d}$$

$$R = C_1(Q_1^{-1} - \gamma^{-2}L^TL)^{-1}C_1^T + R_{12}^TR_2R_{12} + R_{22} \tag{18e}$$

Then, the filter (4) with

$$F = A_1 - GC_1 \tag{19}$$

$$G = [A_1(Q_1^{-1} - \gamma^{-2}L^TL)^{-1}C_1^T + R_{11}R_2R_{12} + R_{12}]R^{-1} \tag{20}$$

satisfies ($F1$) and ($F2$) for all admissible perturbations ΔA and ΔC. Furthermore, ($F3$) is satisfied provided $\sigma_i^2 \geq [Q_1]_{ii}$ ($i = 1, 2, \dots, n$).

Proof:

By Theorem 1, a sufficient condition for ($F1$) and ($F2$) is

$$\begin{aligned} S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} &= A_e(X_e^{-1} - \gamma^{-2}C_e^TC_e - \alpha^{-1}N_e^TN_e)^{-1}A_e^T \\ &- X_e + B_eB_e^T + \alpha M_eM_e^T < 0 \end{aligned} \tag{21}$$

where the partitioning of S is compatible with that given in (8), and $X_e > 0$ satisfies

$$X_e^{-1} - \gamma^{-2}C_e^TC_e - \alpha^{-1}N_e^TN_e > 0 \tag{22}$$

If X_e is assumed to be block diagonal, i.e.,

$$X_e = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} > 0 \tag{23}$$

then, substituting (8a), (8b) and (23) into (21), we have

$$\begin{aligned} S_{11} &= F(Q_1^{-1} - \gamma^{-2}L^TL)^{-1}F^T + (A - GC - F) \\ &(Q_2^{-1} - \alpha^{-1}N^TN)^{-1}(A - GC - F)^T - Q_1 + (D_1 - GD_2) \\ &(D_1 - GD_2)^T + \alpha(M_1 - GM_2)(M_1 - GM_2)^T \end{aligned} \tag{24}$$

$$\begin{aligned} S_{12} &= (A - GC - F)(Q_2^{-1} - \alpha^{-1}N^TN)^{-1}A^T + \\ &(D_1 - GD_2)D_1^T + \alpha(M_1 - GM_2)M_1^T \end{aligned} \tag{25}$$

$$S_{22} = A(Q_2^{-1} - \alpha^{-1}N^TN)^{-1}A^T - Q_2 + D_1D_1^T + \alpha M_1M_1^T \tag{26}$$

A sufficient condition for (21) is $S_{12} = 0$, $S_{11} < 0$ and $S_{22} < 0$. Setting $S_{12} = 0$ in (25), we obtain

$$\begin{aligned} F &= A - GC + [D_1D_1^T + \alpha M_1M_1^T - G(D_2D_1^T + \alpha M_2M_1^T)]R_1^{-1} \\ &= A_1 - GC_1 \end{aligned} \tag{27}$$

Denoting

$$\tilde{Q}_1 = (Q_1^{-1} - \gamma^{-2}L^TL)^{-1}$$

and substituting (27) into (24), we have

$$\begin{aligned} S_{11} &= (A_1 - GC_1)\tilde{Q}_1(A_1 - GC_1)^T + (R_{11} - GR_{12}^T)R_2 \\ &(R_{11} - GR_{12}^T)^T - Q_1 + (D_1 - GD_2)(D_1 - GD_2)^T \\ &+ \alpha(M_1 - GM_2)(M_1 - GM_2)^T \\ &= A_1\tilde{Q}_1A_1^T - Q_1 + R_{11} + R_{11}R_2R_{11}^T + G[C_1\tilde{Q}_1C_1^T + R_{12}^TR_2R_{12} \\ &+ R_{22}]G^T - G[C_1\tilde{Q}_1A_1^T + R_{12}^TR_2R_{11}^T + R_{12}^T] - \\ &[A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} + R_{12}]G^T \\ &= A_1\tilde{Q}_1A_1^T - Q_1 + R_{11} + R_{11}R_2R_{11}^T - [A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} + R_{12}] \\ &R^{-1}[A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} + R_{12}]^T + \{G - [A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} \\ &+ R_{12}]R^{-1}\}R\{G - [A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} + R_{12}]R^{-1}\}^T \end{aligned} \tag{28}$$

If G is chosen according as (20), then the condition $S_1 < 0$ can be written as

$$\begin{aligned} &A_1\tilde{Q}_1A_1^T - Q_1 + R_{11} + R_{11}R_2R_{11}^T - [A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} + R_{12}] \\ &R^{-1}[A_1\tilde{Q}_1C_1^T + R_{11}R_2R_{12} + R_{12}]^T < 0 \end{aligned} \tag{29}$$

which is (15). Applying Lemma 1 to (26), the condition $S_{22} < 0$ can be written as (16). In terms of Q_1 and Q_2, the condition (22) can be written as (17). It follows that (15), (16) together with (17) imply that the conditions of Theorem 1, and hence ($F1$) and ($F2$), are satisfied.

Next, consider the steady-state error variance of the augmented system (7) defined as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} := \lim_{k\to\infty} \begin{bmatrix} e(k)e^T(k) & e(k)x^T(k) \\ x(k)e^T(k) & x(k)x^T(k) \end{bmatrix} \quad (30)$$

Since $A_e + \Delta A_e$ is asymptotically stable, P exists and satisfies the Lyapunov inequality[5]

$$(A_e + \Delta A_e)P(A_e + \Delta A_e)^T - P + B_e B_e^T \geq 0 \quad (31)$$

where we have made use of the fact that the variance of $w(k)$ is bounded by I. Combining (14) and (31) yields

$$\begin{aligned} &(A_e + \Delta A_e)(X_e - P)(A_e + \Delta A_e)^T - (X_e - P) + \\ &A_e(X_e^{-1} - \gamma^{-2}C_e^T C_e - \alpha^{-1}N_e^T N_e)^{-1} A_e^T + \\ &\alpha M_e M_e^T - (A_e + \Delta A_e)X_e(A_e + \Delta A_e)^T < 0 \end{aligned} \quad (32)$$

Note that

$$\begin{aligned} &A_e(X_e^{-1} - \gamma^{-2}C_e^T C_e - \alpha^{-1}N_e^T N_e)^{-1} A_e^T \\ &\geq A_e(X_e^{-1} - \alpha^{-1}N_e^T N_e)^{-1} A_e^T \end{aligned} \quad (33)$$

Using lemma 2, we have

$$\begin{aligned} &A_e(X_e^{-1} - \alpha^{-1}N_e^T N_e)^{-1} A_e^T + \alpha M_e M_e^T \\ &\geq (A_e + \Delta A_e)X_e(A_e + \Delta A_e)^T \end{aligned} \quad (34)$$

In view of (33) and (34), (32) implies that

$$(A_e + \Delta A_e)(X_e - P)(A_e + \Delta A_e)^T - (X_e - P) < 0 \quad (35)$$

Since $A_e + \Delta A_e$ is asymptotically stable, we conclude from (35) that $X_e > P$, which in turn implies

$$Q_1 - P_{11} = \begin{bmatrix} I & 0 \end{bmatrix}(X_e - P)\begin{bmatrix} I \\ 0 \end{bmatrix} > 0 \quad (36)$$

Thus, under the assumption that $\sigma_i^2 \geq [Q_1]_{ii}$ ($i = 1, 2, \ldots, n$),

$$\text{var}[e_i] = [P_{11}]_{ii} < [Q_1]_{ii} \leq \sigma_i^2 \quad (37)$$

which proves ($F3$). This completes the proof of Theorem 2. ▯

Theorem 2 provides a sufficient condition for solving the robust H_∞ filtering problem with the steady-state error variance constraints for systems with parameter uncertainty. Since the two discrete algebraic Riccati inequalities (15) and (16) are unilaterally coupled, we can solve for Q_2 from (16) first, and then Q_1 from (15).

4. Numerical example

Consider linear discrete-time system described by (1) with

$$A = \begin{bmatrix} 0.5 & 0.01 \\ 0 & -0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix},$$

subject to admissible perturbations given by

$$\Delta A = M_1 \Gamma N = \begin{bmatrix} 0.1 & 0.5 \\ -0.2 & 0.1 \end{bmatrix} \Gamma \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta C = M_2 \Gamma N = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix} \Gamma \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$

where Γ is a perturbation matrix satisfying $\Gamma^T \Gamma \leq I$. We wish to design an asymptotically stable H_∞ filter such that $\|H(z)\|_\infty < \gamma = 0.3$, and $\text{var}[e_1] \leq \sigma_1^2 = 0.5$ and $\text{var}[e_2] \leq \sigma_2^2 = 0.5$. Selecting $\alpha = 0.1$, the discrete algebraic Riccati inequalities (15) and (16) subject to constraints (17) can be solved using LMI techniques, giving

$$Q_2 = \begin{bmatrix} 0.1367 & 0.0016 \\ 0.0016 & 0.0397 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.0985 & -0.0180 \\ -0.0180 & 0.2515 \end{bmatrix}.$$

By (19) and (20), the H_∞ filter parameters are given by

$$F = \begin{bmatrix} 0.2148 & -0.0064 \\ 0.0470 & -0.0801 \end{bmatrix}, \quad G = \begin{bmatrix} 0.4314 & -0.2052 \\ 0.0467 & -1.3341 \end{bmatrix}.$$

The above example shows how Theorem 2 can be used to design an asymptotically stable filter such that the prescribed H_∞ performance is satisfied for all admissible perturbations. The steady-state estimation error variance constraints are satisfied since $[Q_1]_{11} \leq \sigma_1^2$ and $[Q_1]_{22} \leq \sigma_2^2$.

5. Conclusion

In this paper, the problem of robust H_∞ filtering with error variance constraints is considered for discrete-time systems subject to unmeasured parameter perturbations. Sufficient conditions for the filter to satisfy steady-state estimation error variance constraints as well as prescribed H_∞ performance are given in terms of two unilaterally coupled algebraic Riccati inequalities which can be solved using LMI techniques. The filter is independent of the unknown perturbation, but its performance is robust against all admissible uncertainties. An area for further work is to extend the results of this paper to discrete time-varying systems with parameter uncertainty.

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