Performance analysis of BPSK and DPSK systems in the presence of Nakagami-*m* Fading and Noisy Phase Reference

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Abstract : This paper presents closed form expressions to evaluate the average bit error rate (BER) for coherent binary phase shift keying (BPSK) and differential PSK (DPSK) modulations over slow Nakagami-*m* fading channel with noisy phase reference. The performance degradation due to the noisy phase reference are then investigated with respect to the channel fading parameter *m* and the phase error θ. The phase error is assumed to be uniformly distributed with maximum phase error ϕ. At the average BER of 10^{-3}, results show that the error performance of the BPSK with $\phi > 44^0$ is worse than the DPSK with ϕ=0 for *m*=9, i.e., the advantage of employing the BPSK over the DPSK is vanished.

Keywords: Nakagami-*m* fading, Noisy phase reference, Phase shift keying

Introduction

For coherent communication systems, error performance are usually evaluated by assuming that a perfect phase reference is available in the receiver for demodulation [1,2]. In practice, this local phase reference is however reconstructed from a noise-corrupted version of a received signal, and thus a phase error, θ, is usually resulted. The immediate effect of the phase error is degradation of detection performance of the coherent systems.

Over the years, many researchers have investigated the error performance of binary phase shift keying (BPSK) and differential PSK (DPSK) systems over an additive white gaussian noise (AWGN) channel in the presence of noisy phase reference [2-8]. For BPSK systems, [9-13] have evaluated the bit error rate (BER) in the presence of Rayleigh fading and noisy phase reference. The corresponding error performance in Rician and Lognormal fading channels have also been investigated in [9] and [12]. Amongst the various fading channels, Nakagami-*m* fading distribution is considered to be the most versatile, since it is better in modelling mobile channels than the Rayleigh, Rician and the Lognormal fading distributions. To the best of the authors' knowledge, no error performance of the BPSK in the presence of Nakagami-*m* fading and noisy phase reference has been reported. In addition, no performance analysis for the DPSK over fading channels with noisy phase reference has been found.

This paper thus derives closed form expressions of the average BER for the BPSK and the DPSK systems in the presence of Nakagami-*m* fading and noisy phase reference. Also, the analysis presented in this paper may be used to characterize the performance of an acquisition scheme for many wireless systems. In Section II, the derivation of the average BER for the BPSK are presented and the corresponding derivation for the DPSK are given in Section III. Numerical results and discussions are given in Section IV.

Derivation of the average BER for the BPSK

The average BER for the BPSK system in the presence of Nakagami-*m* fading and noisy phase reference is considered in this section. For fading channels, the conditional BER for the BPSK with phase error is given by [2]

$$P_e(\gamma,\theta)=\frac{1}{2}erfc(\sqrt{\gamma}\cos\theta) \qquad (1)$$

where *erfc*(.) is the complementary error function [14] and γ is the instantaneous signal to noise ratio (SNR) per bit of the received signal. The phase error θ is assumed to be uniformly distributed in a range of $(-\phi,\phi)$ as in [13] and the probability density function (pdf) of it is given by

$$p(\theta)=1/2\phi \ . \qquad (2)$$

In addition, the pdf of γ for the Nakagami-*m* fading channel is given by [15]

$$p(\gamma)=\left(\frac{m}{\Omega}\right)^{m}\frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(\frac{-m\gamma}{\Omega}\right) \quad (3)$$

where m is the fading severity parameter with values from 0.5 to ∞ and Ω is the average SNR per bit. The notation $\Gamma(m)$ denotes the gamma function [14]. The average BER can then be obtained using (1) to (3) as [13]

$$\overline{P}_e=\int_0^{\infty}\int_{-\phi}^{\phi}P_e(\gamma,\theta)p(\theta)p(\gamma)\,d\theta\,d\gamma\ . \quad (4)$$

Since the conditional BER in (1) is an even function of θ, the average BER in (4) can also be written as

$$\overline{P}_e=\frac{1}{\phi}\int_0^{\phi}\int_0^{\infty}P_e(\gamma,\theta)p(\gamma)\,d\gamma d\theta\ . \quad (5)$$

By substituting (1) and (3) into (5) and using a series representation for the $erfc(.)$ as [4]

$$erfc(\sqrt{\gamma}\cos\theta)=1-\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(n+1/2)}{(2n+1)!}\gamma^{n+1/2}$$

$$\times{}_1F_1\left(n+\frac{1}{2};2n+2;-\gamma\right)\cos(2n+1)\theta \quad (6)$$

the average BER in (5) becomes

$$\overline{P}_e=\frac{1}{2}-\frac{1}{\pi\phi}\int_0^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(n+1/2)}{(2n+1)!}\gamma^{n+1/2}$$

$$\times{}_1F_1\left(n+\frac{1}{2};2n+2;-\gamma\right)\int_0^{\phi}\cos(2n+1)\theta\,d\theta\,p(\gamma)d\gamma \quad (7)$$

where ${}_1F_1(a;b;x)$ is the confluent hypergeometric function [14]. The two integrals in (7) can be evaluated as

$$\overline{P}_e=\frac{1}{2}-\frac{1}{\pi\phi}\sum_{n=0}^{\infty}\frac{(-1)^n\Gamma(n+1/2)}{(2n+1)!\Gamma(m)}\frac{\sin(2n+1)\phi}{2n+1}\left(\frac{m}{\Omega}\right)^m$$

$$\times\int_0^{\infty}\frac{\gamma^{m+n+1/2}}{\exp(m\gamma/\Omega)}{}_1F_1\left(n+\frac{1}{2};2n+2;-\gamma\right)d\gamma \quad (8)$$

$$=\frac{1}{2}-\frac{1}{\pi\phi}\sum_{n=0}^{\infty}\frac{(-1)^n\sin((2n+1)\phi)\Gamma(m+n+0.5)\Gamma(n+0.5)}{(2n+1)!\,\Gamma(m)(2n+1)(\Omega/m)^{-n-1/2}}$$

$$\times{}_2F_1\left(m+n+\frac{1}{2},n+\frac{1}{2};2n+2;-\frac{\Omega}{m}\right) \quad (9)$$

where ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [14]. Finally, the average BER in (9) can be written in closed form as

$$\overline{P}_e=\frac{1}{2}-\frac{1}{\pi\phi}\sum_{n=0}^{\infty}\frac{(-1)^n\sin(2n+1)\phi}{(2n+1)^2}\left(\frac{\Omega}{m}\right)^{n+1/2}$$

$$\times\frac{{}_2F_1(m+n+1/2,n+1/2;2n+2;-\Omega/m)}{{}_2F_1(-n-1/2,m-n-1/2;m;1)}\ . \quad (10)$$

Derivation of the average BER for the DPSK

The case of the DPSK is now considered. The conditional BER for the DPSK is also an even function of θ and given by [2]

$$P_e(\gamma,\theta)=\frac{1}{2}\exp(-\gamma\cos^2(\theta))\ . \quad (11)$$

By substituting (3) and (11) into (5), the average BER for the DPSK in the presence of Nakagami-m fading and phase noise can be written as

$$\overline{P}_e=\frac{1}{\phi}\int_0^{\infty}\frac{1}{2}\left(\frac{m}{\Omega}\right)^m\frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(\frac{-m\gamma}{\Omega}\right)$$

$$\times\int_0^{\phi}\exp\left(-\frac{\gamma}{2}(1+\cos^2(\theta))\right)d\theta\,d\gamma\ . \quad (12)$$

By using the following identity [5]

$$\exp\left(-\frac{\gamma}{2}\cos(2\theta)\right)=I_0\left(-\frac{\gamma}{2}\right)+2\sum_{k=1}^{\infty}I_k\left(-\frac{\gamma}{2}\right)\cos(2k\theta) \quad (13)$$

the average BER in (12) becomes

$$\overline{P}_e=\frac{1}{2}\left(\frac{m}{\Omega}\right)^m\frac{1}{\Gamma(m)}\int_0^{\infty}\gamma^{m-1}\exp\left(-\gamma\left(\frac{1}{2}+\frac{m}{\Omega}\right)\right)$$

$$\times I_0\left(-\frac{\gamma}{2}\right)d\gamma+\frac{1}{\phi}\left(\frac{m}{\Omega}\right)^m\frac{1}{\Gamma(m)}\sum_{k=1}^{\infty}\frac{\sin(2k\phi)}{2k}$$

$$\times\int_0^{\infty}\gamma^{m-1}\exp\left(-\gamma\left(\frac{1}{2}+\frac{m}{\Omega}\right)\right)I_k\left(-\frac{\gamma}{2}\right)d\gamma \quad (14)$$

where $I_v(a)$ is the modified Bessel function of the first kind of order v [14]. The two integrals can be evaluated as in [16] and the (14) can then be written as

$$\overline{P}_e=\frac{1}{2}\left(\frac{m}{\Omega}\right)^m\left(\frac{1}{2}+\frac{m}{\Omega}\right)^{-m}{}_2F_1\left(\frac{m}{2},\frac{m+1}{2};1;\frac{1}{4(1/2+m/\Omega)^2}\right)$$

$$+\frac{1}{\phi}\left(\frac{m}{\Omega}\right)^m\frac{1}{\Gamma(m)}\sum_{k=1}^{\infty}\frac{\sin(2k\phi)}{2k}\left(\frac{-1}{4}\right)^k\left(\frac{1}{2}+\frac{m}{\Omega}\right)^{-m-k}$$

$$\times\frac{\Gamma(m+k)}{\Gamma(1+k)}{}_2F_1\left(\frac{m+k}{2},\frac{m+k+1}{2};k+1;\frac{1}{4(1/2+m/\Omega)^2}\right) \quad (15)$$

After further manipulation, the average BER can be simplified as

$$\overline{P}_e=\frac{1}{2}\left(1+\frac{\Omega}{2m}\right)^{-m}{}_2F_1\left(\frac{m}{2},\frac{m+1}{2};1;\frac{1}{4(1/2+m/\Omega)^2}\right)$$

$$+\sum_{k=1}^{\infty}\frac{\sin(2k\phi)}{2k\phi}\left(\frac{-1}{4}\right)^k\left(\frac{m}{\Omega}\right)^m\left(\frac{1}{2}+\frac{m}{\Omega}\right)^{-m-k}{}_2F_1(m-1,k;m+k;1)$$

$$\times{}_2F_1\left(\frac{m+k}{2},\frac{m+k+1}{2};k+1;\frac{1}{4(1/2+m/\Omega)^2}\right)\ . \quad (16)$$

Numerical results and discussions

The performance degradation for the BPSK and the DPSK systems over Nakagami-*m* fading channels with noisy phase reference are presented. Fig. 1 depicts the average BER for the BPSK with the channel fading parameter *m* ranging from 1 to 9 and the maximum phase error $\phi = 30^0$. The case of the DPSK is shown in Fig. 2. Furthermore, Table 1 shows the performance degradation at the average BER of 10^{-3} for the BPSK and the DPSK with *m* and ϕ ranging from 3 to 9 and from 10^0 to 60^0, respectively. For $\phi = 0^0$, the average SNR at the average BER of 10^{-3} for different values of *m* are presented in the last row of Table 1.

For the BPSK system over channel having *m* increasing from 1 to 9, the performance degradation due to ϕ=30^0 is increased as shown in Fig. 1. From Table 1, it shows that the degradation are 2.69dB and 3.08dB as ϕ is increased from 10^0 to 60^0 for *m*=1 and 9, respectively. As ϕ is increased, more degradation is therefore resulted for higher values of *m*. For low values of ϕ, the degradation are similar and equal to about 0.6dB for all values of *m*. When ϕ is large, the degradation are different as *m* is varying, for example, the degradation with ϕ=60^0 are 2.73dB and 3.14dB for *m*=3 and 9, respectively. In addition, for all values of *m*, the degradation are approximately 1dB and 3dB for $\phi = 40^0$ and 60^0. Similar conclusions can be drawn for the DPSK as shown in Fig. 2 and Table 1.

The advantage of using the DPSK over the BPSK is that the DPSK has a lower probability of having a phase error. On the other hand, the BPSK has superior performance over the DPSK when there is no phase error. Therefore, we are interested in the value of ϕ that makes the error performance of the BPSK with phase error equal to the DPSK with no phase error. From the last row of Table 1, it shows that the performance of the BPSK is superior than the DPSK by 1.87dB and 1.38dB for *m*=1 and 9, respectively. The error performance of both modulations are therefore equal when $\phi \cong 52^0$ and 44^0 for *m*=1 and 9, respectively. Hence, the advantage of using the BPSK is vanished with $\phi > 44^0$ for the fading channel with *m*=9.

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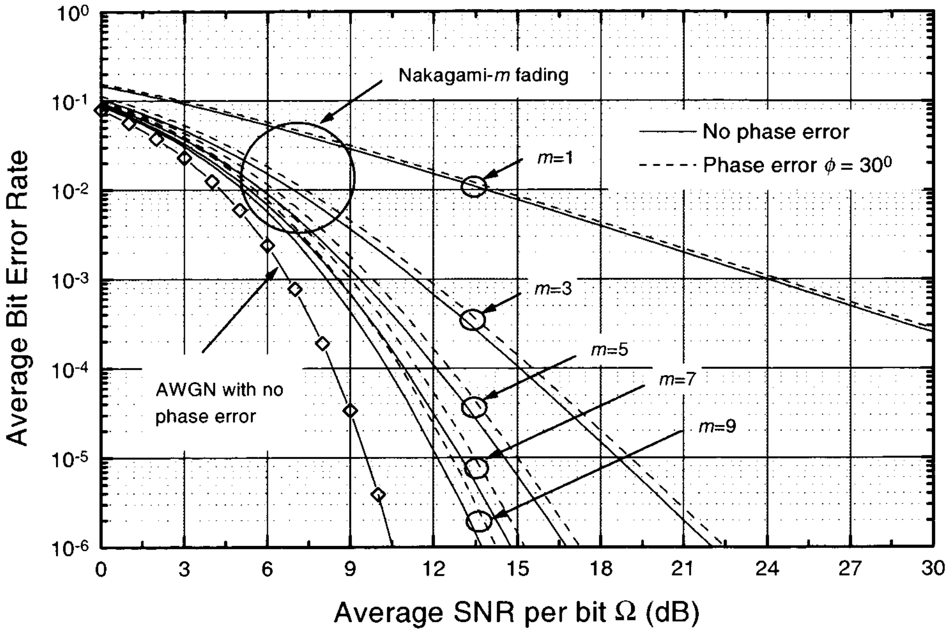
Table 1. The performance degradation of the BPSK and the DPSK at the average BER of 10^{-3} for m and ϕ ranging from 3 to 9 and 10^0 to 60^0, respectively. The average SNR with $\phi = 0^0$ are also given for m increasing from 3 to 9.

Average BER at 10^{-3}	*Degradation due to ϕ (dB)*							
	BPSK				**DPSK**			
maximum phase error ϕ deg.	m=3	m=5	m=7	m=9	m=3	m=5	m=7	m=9
10	0.04	0.06	0.06	0.06	0.04	0.05	0.05	0.05
20	0.19	0.19	0.20	0.19	0.18	0.20	0.19	0.20
30	0.44	0.47	0.48	0.48	0.44	0.46	0.46	0.50
40	0.89	0.92	0.95	0.96	0.90	0.93	0.95	0.98
50	1.59	1.68	1.75	1.77	1.60	1.72	1.79	1.82
60	2.73	2.96	3.07	3.14	2.78	3.08	3.20	3.31
	Average SNR at 10^{-3} (dB)							
ϕ = 0	11.31	9.32	8.53	8.12	13.18	10.90	10.00	9.50

Fig.1 The average BER performance of the BPSK with ϕ=0^0, 30^0 and m=1,3,5,7,9.



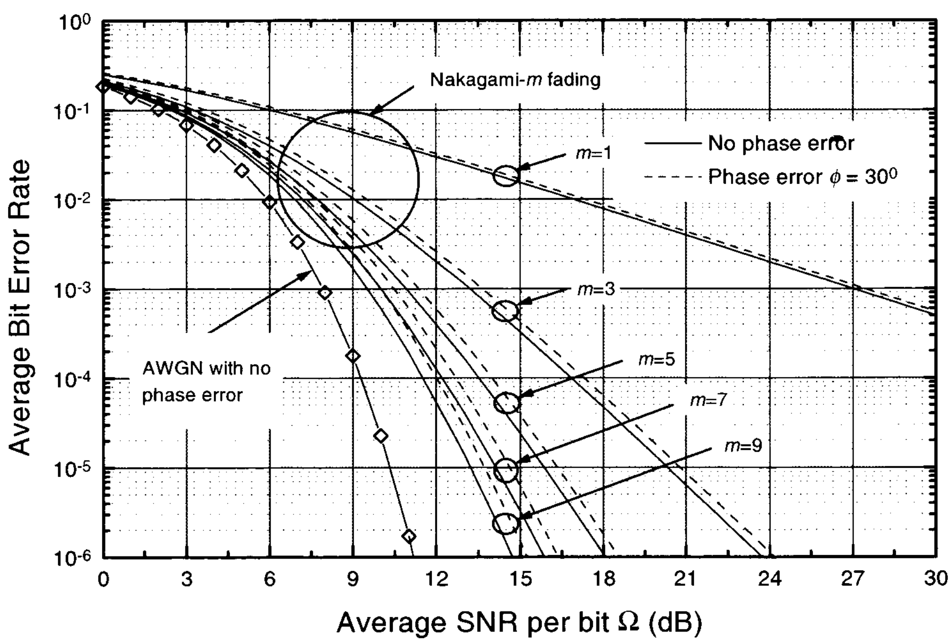


Fig. 2. The average BER performance of the DPSK with ϕ=0^0, 30^0 and m=1,3,5,7, 9.