A SUBSPACE METHOD FOR CHANNEL ESTIMATION OF MULTI-USER MULTI-ANTENNA OFDM SYSTEM

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ABSTRACT

A subspace based blind method is proposed for estimating the channel responses of a multi-user and multi-antenna OFDM uplink system. It gives estimations to all channel responses subject to a scalar matrix ambiguity, and does not need precise channel order information (only an upper bound for all orders is required). Furthermore, the scalar ambiguity matrix can be easily resolved by using only one pilot OFDM block given that the number of users is smaller than the number of symbols in the pilot symbol block. Simulations show that the method is effective and robust.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) and multiple input multiple output (MIMO) have emerged as two major techniques in the future fourth generation (4G) communications [7]. In recent years, systems using OFDM and/or multiple transmit and receive antennas have been studied extensively [2, 8, 9, 10]. However, most of these studies concentrate on single user with one or more transmitting and receiving antennas, sometimes with space-time coding. In [9], a multi-user and multi-antenna OFDM system (MUMA-OFDM) is proposed, in which many users can share the same frequency band if multiple antennas are installed at the basestation. Each user's signal is modulated via OFDM. A joint detection method is presented based on the assumption that channel conditions are completely known. Unfortunately no method to estimate the channel responses is given, which is known to be one of the most difficult task in a MIMO system.

Among various blind channel estimation methods, the subspace (SS) method [1, 5, 6] is believed to be one of the best. The SS method has a simple structure and achieves good performance for SIMO system [5], but it requires precise knowledge of the channel order [6, 3, 4], which is very difficult to obtain in practice. Also, the extension of it to general MIMO system is not successful because it generally can only estimate the channels subject to a polynomial matrix ambiguity [1]. Unlike SIMO case, it is much more difficult to estimate channel orders in MIMO system, because not just one but many orders need to be estimated. Therefore, SS method is not practical for general MIMO channel estimation. In this paper, we propose a subspace method for estimating the MIMO channels in the uplink MUMA-OFDM system, where the zero-padding OFDM (ZP-OFDM) [4, 8] other than the classical cyclic prefix OFDM (CP-OFDM) is used. We shall show that, making use the property of zero-padding, the proposed subspace method no longer needs precise order information (only upper bounds for the orders are required), and it can accurately estimate the channels subject to a scalar matrix ambiguity. Furthermore, the ambiguity matrix is easily resolved by using only one pilot OFDM block if the number of users is smaller than the length of a OFDM block. Simulations show that the method is effective and robust.

Some notations are used in the following. Superscripts T, $\dagger$ and $*$ stand for transpose, transconjugate, and conjugate, respectively. $\mathbf{I}_q$ is the identity matrix of order q, and $\otimes$ is the Kronecker product of matrices.

2. MUMA-OFDM SYSTEM MODEL

The MUMA-OFDM system is first proposed in [9]. Assume that there are K users who share the same frequency band, and J omni-directional receiving antennas in the basestation. Each antenna can receive signals from every user in the cell using the basestation. The received signals of the J antennas are sent to a central unit (CU) for processing. The goal of the processing is to recover the transmitted K signals. We assume that each user uses zero-padding OFDM (ZP-OFDM) [8, 4] other than cyclic prefix OFDM (CP-OFDM) used in [9], because ZP-OFDM avoids inter-block interference (IBI) and therefore simplifies channel estimation and equalization [4, 8]. In zero-padding OFDM, the symbols to be transmitted is grouped into blocks with each block having N symbols, each block is transformed by the inverse discrete Fourier transform (IDFT), and then L ($L \leq N$) zeros are added to the tail of each transformed block (zero-padding), where cyclic prefix is no longer needed. Each user transmit its OFDM modulated signal. Assume that all users are synchronized at block level. Let $\mathbf{s}_i^{(k)}$ be the block symbol to be transmitted by user k at time i (before OFDM modulation), where

$$\mathbf{s}_i^{(k)} = (\mathbf{s}_i^{(k)}(0), \mathbf{s}_i^{(k)}(1), \cdots, \mathbf{s}_i^{(k)}(N-1))^T,$$

and its IDFT is $\mathbf{u}_i^{(k)}$. $\mathbf{u}_i^{(k)}$ is zero-padded with L zeros and then transmitted. Let $\mathbf{h}^{(j,k)}(l)$ $(l = 0, 1, \cdots, L_{j,k})$ be the channel response (including the transmitting and receiving filters) from user k to antenna j, where $L_{j,k}$ is the channel order, $L_{j,k} \leq L$. Then the received ith block at antenna j

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is

$$\mathbf{x}_i^{(j)}(n) = \sum_{k=1}^{K}\sum_{l=0}^{L}\mathbf{h}^{(j,k)}(l)\mathbf{u}_i^{(k)}(n-l) + \eta_i^{(j)}(n), \quad (1)$$

$$n = 0, 1, \cdots, M-1,$$

where $M = N+L$, $\mathbf{h}^{(j,k)}(l)$ is zero-padded for $L_{j,k}+1 \le l \le L$, and $\eta_i^{(j)}(n)$ is the channel noise. Notice that $\mathbf{u}_i^{(k)}(n) = 0$, if $n < 0$ or $N \le n < M$.

We assume that $J > K$. By defining

$$\begin{aligned}
\mathbf{x}_i(n) &= (\mathbf{x}_i^{(1)}(n), \mathbf{x}_i^{(2)}(n), \cdots, \mathbf{x}_i^{(J)}(n))^T \\
\mathbf{h}^{(k)}(l) &= (\mathbf{h}^{(1,k)}(l), \mathbf{h}^{(2,k)}(l), \cdots, \mathbf{h}^{(J,k)}(l))^T \\
\eta_i(n) &= (\eta_i^{(1)}(n), \eta_i^{(2)}(n), \cdots, \eta_i^{(J)}(n))^T \\
\mathbf{h}(l) &= (\mathbf{h}^{(1)}(l), \mathbf{h}^{(2)}(l), \cdots, \mathbf{h}^{(K)}(l)) \quad (2)\\
\mathbf{u}_i(n) &= (\mathbf{u}_i^{(1)}(n), \mathbf{u}_i^{(2)}(n), \cdots, \mathbf{u}_i^{(K)}(n))^T
\end{aligned}$$

we can express (1) into vector form as

$$\mathbf{x}_i(n) = \sum_{l=0}^{L}\mathbf{h}(l)\mathbf{u}_i(n-l) + \eta_i(n), \; n = 0, 1, \cdots, M-1. \quad (3)$$

Now let

$$\begin{aligned}
\mathbf{u}_i &= (\mathbf{u}_i(0)^T, \mathbf{u}_i(1)^T, \cdots, \mathbf{u}_i(N-1)^T)^T, \\
\mathbf{x}_i &= (\mathbf{x}_i(0)^T, \mathbf{x}_i(1)^T, \cdots, \mathbf{x}_i(M-1)^T)^T, \\
\eta_i &= (\eta_i(0)^T, \eta_i(1)^T, \cdots, \eta_i(M-1)^T)^T,
\end{aligned}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & & \\ \vdots & \ddots & \\ \mathbf{h}(L) & \ddots & \\ & \ddots & \mathbf{h}(0) \\ & \ddots & \vdots \\ & & \mathbf{h}(L) \end{bmatrix} \quad (4)$$

where $\mathbf{H}$ is a $JM \times KN$ block lower triangular Toeplitz matrix with first block column being

$$(\mathbf{h}^T(0), \mathbf{h}^T(1), \cdots, \mathbf{h}^T(L), 0, \cdots, 0)^T.$$

Then (3) is turned to

$$\mathbf{x}_i = \mathbf{H}\mathbf{u}_i + \eta_i, \; i = 0, 1, \cdots. \quad (5)$$

3. EXISTENCE OF ZERO-FORCING EQUALIZER

Given that the noises can be ignored in (5), $\mathbf{u}_i$ is completely recoverable from $\mathbf{x}_i$ if and only if matrix $\mathbf{H}$ is of full column rank. From the structure of the matrix, we have the following theorem about the rank of matrix $\mathbf{H}$.

Theorem 1 If $\mathbf{h}(0)$ is of full column rank, then $\mathbf{H}$ is of full column rank.

To save space, we omit the proof here. Noticing that $\mathbf{h}(0)$ is a $J \times K$ ($J > K$) tall matrix defined in (2), we see that the full column rank property of $\mathbf{h}(0)$ is almost surely guaranteed because signal propagation from each of the K users scattered in the cell is most likely independent. In the following, we assume that this condition is held. Even if $\mathbf{h}(0)$ is not of full column rank, it is still possible that $\mathbf{H}$ is of full column rank.

If the channel responses are known, we can use the joint detection method proposed in [9] or other means to recover the transmitted symbols. Unfortunately, channel estimation in MIMO system is usually a very difficult task, and [9] does not discuss the problem. We will address the problem in the following.

4. SUBSPACE ALGORITHM

Before discussing the algorithm, we make the following assumptions for the statistical properties of transmitted symbols $s_l^{(k)}(n)$ and channel noise $\eta_i^{(j)}(n)$.

(A1) Noises are white and uncorrelated, that is,

$$\mathbf{E}(\eta_i^{(j)}(n)(\eta_l^{(k)}(m))^*) = \begin{cases} \sigma_\eta^2, & (i,j,n) = (l,k,m) \\ 0, & (i,j,n) \ne (l,k,m) \end{cases}$$

(A2) Noises and transmitted signals are uncorrelated, that is, $\mathbf{E}(\eta_i^{(j)}(n)(s_l^{(k)}(m))^*) = 0$.
Here $\mathbf{E}(y)$ means the mathematical expectation of a random variable y.

4.1. Theoretical development

Now we consider the statistical auto-correlation matrices of $\mathbf{x}_i$. Based on the assumptions (A1-2), we can verify that

$$\mathbf{R}_x = \mathbf{E}(\mathbf{x}_i\mathbf{x}_i^\dagger) = \mathbf{H}\mathbf{R}_u\mathbf{H}^\dagger + \sigma_\eta^2\mathbf{I}_{MJ}, \quad (6)$$

where $\mathbf{R}_u = \mathbf{E}(\mathbf{u}_i\mathbf{u}_i^\dagger)$ is a positive definite matrix.

The smallest eigenvalue of matrix $\mathbf{R}_x$ is σ_η^2. Since the rank of $\mathbf{H}\mathbf{R}_u\mathbf{H}^\dagger$ is NK, there are $q = MJ - NK$ co-orthogonal eigenvectors corresponding to the smallest eigenvalue. These eigenvectors are denoted by β_i ($i = 0, 1, \cdots, q-1$). Based on a simple mathematical derivation which is used in standard subspace method [1, 5], we know that

$$\beta_i^\dagger\mathbf{H} = 0, \; i = 0, 1, \cdots, q-1, \quad (7)$$

that is, β_i ($i = 0, 1, \cdots, q-1$) spans the left null space of $\mathbf{H}$. Having known the left null space, we can determine the range space, denoted by $\mathbf{span}(\mathbf{H})$, which is all possible linear combinations of the column vectors of $\mathbf{H}$. In general, knowing $\mathbf{span}(\mathbf{H})$ cannot determine matrix $\mathbf{H}$. However, for some matrix with special structure, it is possible to determine $\mathbf{H}$ by $\mathbf{span}(\mathbf{H})$ up to certain ambiguity. The matrix $\mathbf{H}$ here is a block lower triangular Toeplitz full column rank matrix. The following theorem states that $\mathbf{H}$ is uniquely determined by $\mathbf{span}(\mathbf{H})$ subject to a $K \times K$ matrix ambiguity.

Theorem 2 Let $\mathbf{H}$ be the matrix defined in (4) and

$$\hat{\mathbf{H}} = \begin{bmatrix} \hat{\mathbf{h}}(0) & & \\ \vdots & \ddots & \\ \hat{\mathbf{h}}(L) & & \ddots \\ & \ddots & \hat{\mathbf{h}}(0) \\ & & \ddots & \vdots \\ & & & \hat{\mathbf{h}}(L) \end{bmatrix},$$

where $\mathbf{h}(l)$ and $\hat{\mathbf{h}}(l)$ $(l = 0, 1, \cdots, L)$ are $J \times K$ matrices, and $\mathbf{h}(0)$ and $\hat{\mathbf{h}}(0)$ are of full column rank. If $\mathbf{span}(\mathbf{H})$=$\mathbf{span}(\hat{\mathbf{H}})$, there exists a $K \times K$ invertible matrix b such that

$$\hat{\mathbf{h}}(l) = \mathbf{h}(l)b, \quad l = 0, 1, \cdots, L. \tag{8}$$

Proof. It is easy to verify that $\mathbf{span}(\mathbf{H})$=$\mathbf{span}(\hat{\mathbf{H}})$ if and only if there exists an $KN \times KN$ invertible matrix $\mathbf{B}$ such that $\hat{\mathbf{H}} = \mathbf{HB}$. We section $\mathbf{B}$ into $K \times K$ blocks and express it as

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \cdots & \mathbf{B}_{1N} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \cdots & \mathbf{B}_{2N} \\ \vdots & \vdots & & & \vdots \\ \mathbf{B}_{N1} & \mathbf{B}_{N2} & \cdots & \cdots & \mathbf{B}_{NN} \end{bmatrix} \tag{9}$$

where $\mathbf{B}_{ij}$ are $K \times K$ matrices.

Firstly, we want to prove that $\mathbf{B}$ must be a block lower triangular matrix and all its diagonal blocks are the same. We use the mathematical induction. The ith row of $\hat{\mathbf{H}}$ is the multiplication of the ith row of $\mathbf{H}$ and $\mathbf{B}$. Considering the first row, we have

$$\mathbf{h}(0)\mathbf{B}_{11} = \hat{\mathbf{h}}(0), \ \mathbf{h}(0)\mathbf{B}_{1j} = 0, \ j = 2, \cdots, N.$$

Since $\mathbf{h}(0)$ and $\hat{\mathbf{h}}(0)$ are of full column rank, we know that $\mathbf{B}_{11}$ must also be of full rank, that is, $\mathbf{B}_{11}$ is invertible. Also, $\mathbf{B}_{1j} = 0$, $j = 2, \cdots, N$.

Now assuming that $\mathbf{B}_{ij} = 0$ and $\mathbf{B}_{ii} = \mathbf{B}_{11}$, where $i = 1, \cdots, k$, and $j \geq i+1$, we want to show that $\mathbf{B}_{k+1,j} = 0$ $(j \geq k+2)$ and $\mathbf{B}_{k+1,k+1} = \mathbf{B}_{11}$. Considering the $(k+1)$th row, based on the assumption we have

$$\mathbf{h}(0)\mathbf{B}_{k+1,k+1} = \hat{\mathbf{h}}(0), \ \mathbf{h}(0)\mathbf{B}_{k+1,j} = 0, \ j = k+2, \cdots, N.$$

Since we also have $\mathbf{h}(0)\mathbf{B}_{11} = \hat{\mathbf{h}}(0)$, it is obvious that $\mathbf{B}_{k+1,k+1} = \mathbf{B}_{11}$. Furthermore, the full column rank property of $\mathbf{h}(0)$ means that $\mathbf{B}_{k+1,j} = 0$, $j = k+2, \cdots, N$. Based on the mathematical induction law, we know that $\mathbf{B}$ is a block lower triangular matrix with identical diagonal blocks.

Secondly, we prove that $\hat{\mathbf{h}}(l) = \mathbf{h}(l)\mathbf{b}$ $(l = 0, 1, \cdots, L)$ with some $\mathbf{b}$. For simplicity, we use $\mathbf{b}$ to denote the matrix $\mathbf{B}_{11}$. It is shown above that $\mathbf{B}$ is a block lower triangular matrix and all its diagonal blocks are $\mathbf{b}$. Comparing the last block column of $\hat{\mathbf{H}}$ and $\mathbf{H}$, we finally get

$$\hat{\mathbf{h}}(l) = \mathbf{h}(l)\mathbf{b}, \quad l = 0, 1, \cdots, L.$$

4.2. Implementation of the subspace method

Equation (7) can be expressed equivalently as

$$\mathbf{H}^{\dagger}\beta_i = 0, \ i = 0, 1, \cdots, q-1. \tag{10}$$

By sectioning the vector β_i into blocks as

$$\beta_i = (\beta_i^T(M-1), \beta_i^T(M-2), \cdots, \beta_i^T(0))^T, \tag{11}$$

where $\beta_i(m)$ $(m = 0, 1, \cdots, M-1)$ are $J \times 1$ vectors, it is easy to turn (10) into

$$\sum_{l=0}^{L} \mathbf{h}^{\dagger}(l)\beta_i(n+L-l) = 0, \ n = 0, 1, \cdots, N-1, \tag{12}$$

or, equivalently,

$$\sum_{l=0}^{L} \beta_i^{\dagger}(n+L-l)\mathbf{h}(l) = 0, \ n = 0, 1, \cdots, N-1. \tag{13}$$

Denote two matrices $\mathbf{G}_i$ and $\bar{\mathbf{H}}$ as

$$\mathbf{G}_i = \begin{bmatrix} \beta_i^{\dagger}(L) & \beta_i^{\dagger}(L-1) & \cdots & \cdots & \beta_i^{\dagger}(0) \\ \beta_i^{\dagger}(L+1) & \beta_i^{\dagger}(L) & \cdots & \cdots & \beta_i^{\dagger}(1) \\ \vdots & \vdots & & & \vdots \\ \beta_i^{\dagger}(M-1) & \beta_i^{\dagger}(M-2) & \cdots & \cdots & \beta_i^{\dagger}(N-1) \end{bmatrix}, \tag{14}$$

$$\bar{\mathbf{H}} = (\mathbf{h}(0)^T, \mathbf{h}(1)^T, \cdots, \mathbf{h}(L)^T)^T \tag{15}$$

Then (13) is equivalent to

$$\mathbf{G}_i\bar{\mathbf{H}} = 0, \ i = 0, 1, \cdots, q-1. \tag{16}$$

(16) can also be expressed as

$$\mathbf{G}\bar{\mathbf{H}} = 0, \tag{17}$$

where $\mathbf{G}$ is a $qN \times J(L+1)$ matrix defined as

$$\mathbf{G} = (\mathbf{G}_0^T, \mathbf{G}_1^T, \cdots, \mathbf{G}_{q-1}^T)^T. \tag{18}$$

Based on Matlab notation, we use $\bar{\mathbf{H}}(:,j)$ to denote the jth column of the matrix $\bar{\mathbf{H}}$. Then

$$\mathbf{G}\bar{\mathbf{H}}(:,j) = 0, \ j = 1, 2, \cdots, K, \tag{19}$$

that is, $\bar{\mathbf{H}}(:,j)$ belongs to the right null space of $\mathbf{G}$. From Theorem 2, it is easy to show that the dimension of right null space of $\mathbf{G}$ is K. So, $\bar{\mathbf{H}}(:,j)$ $(j = 1, 2, \cdots, K)$ is actually a basis of the null space. Therefore, matrix $\bar{\mathbf{H}}$ can be obtained by finding a basis for the null space, which can be achieved by solving the eigenvectors of eigenvalue 0 for matrix $\mathbf{G}$ or singular value decomposition (SVD). In practice, due to the errors in computing the auto-correlation matrix and roundoff errors, (19) will not hold precisely, and therefore we should choose K right singular vectors corresponding to the K least singular values respectively to form the columns of $\bar{\mathbf{H}}$.

The channel responses can also be determined by minimizing a target function $\epsilon(\bar{\mathbf{H}}) = \sum_{i=0}^{q-1} ||\mathbf{G}_i\bar{\mathbf{H}}||_F^2$, where $||\cdot||_F$ means the Frobenius norm for matrix.

The subspace method is summarized as follows.

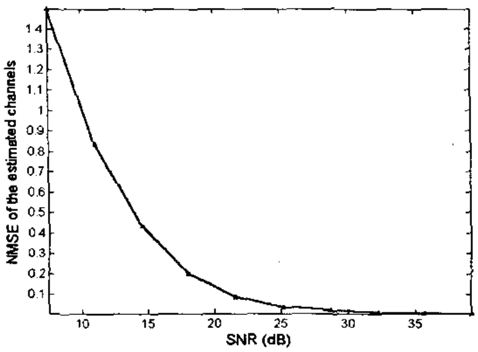

Figure 1: NMSE of the estimated channel

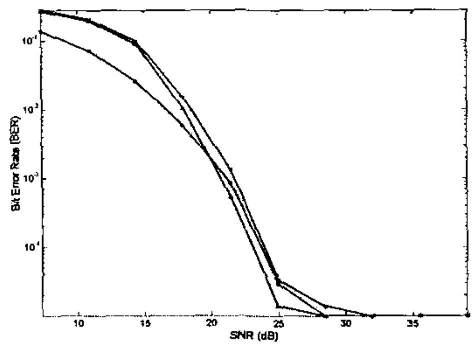

Figure 2: Bit Error Rate versus SNR (16-QAM)

Algorithm 1 Subspace Method for Channel Estimation of MUMA-OFDM System

Step 1. Compute $\mathbf{R}_x = \mathbf{E}(\mathbf{x}_i\mathbf{x}_i^\dagger)$.

Step 2. Find $q = JM - KN$ co-orthogonal eigenvectors, β_i $(i = 0, 1, \cdots, q-1)$, corresponding to the least q eigenvalues of matrix $\mathbf{R}_x$.

Step 3. Form the matrix $\mathbf{G}$ defined in (18) from β_i and compute the SVD of $\mathbf{G}$. Choose K right singular vectors corresponding to the K least singular values respectively to form the columns of $\tilde{\mathbf{H}}_0$.

Step 4. The channel matrix is $\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_0 b$, where $\mathbf{b}$ is a $K \times K$ invertible matrix to be determined.

5. SIMULATIONS

In the following, signal-noise-ratio (SNR) means the ratio of the average received signal power to the average noise power as $\text{SNR} = \mathbf{E}(||\mathbf{x}_i(n) - \eta_i(n)||^2)/\mathbf{E}(||\eta_i(n)||^2)$. Simulations show that the method is very effective. Some examples are shown below.

(1) Channel estimation The system parameters are as follows: $J = 3$, $K = 2$, $N = 32$, $L = 9$. The symbol constellation is 16-QAM. In order to resolve the ambiguity matrix, we assume that the first transmitted OFDM block is a pilot. Based on this known block, the ambiguity matrix can be solved. 150 blocks are used for computing the matrix $\mathbf{R}_x$. The normalized mean square error (NMSE) between the estimated and true channel responses is defined as

$$\text{NMSE} = \sqrt{\frac{\sum_{j=1}^{J}\sum_{k=1}^{K}||\hat{\mathbf{h}}^{(j,k)} - \mathbf{h}^{(j,k)}||^2}{\sum_{j=1}^{J}\sum_{k=1}^{K}||\mathbf{h}^{(j,k)}||^2}}. \tag{20}$$

Figure 1 shows the NMSE.

(2) Bit Error Rate The system parameters are the same as in the last example. In Figure 2, the bit error rates (BER) are shown as the SNR varies from 5 to 40 dB, where lines with marks $\circ$, $\triangle$ and ∇ are for frequency domain equalization (FDE) with true channel responses, time domain equalization (TDE) and FDE with estimated channel responses respectively. The BER is obtained by Monte-Carlo test on 1000 symbol blocks.

6. CONCLUSIONS

A subspace method has been proposed for estimating the channel responses of a MUMA-OFDM uplink system. It gives estimations to all channel responses subject to a scalar matrix ambiguity. Unlike the subspace method for general MIMO systems, the proposed method does not need precise channel order information, and only requires an upper bound for the orders. The scalar ambiguity matrix is resolved by using few pilot symbols provided that the number of users is smaller than that of pilot symbols. Simulations have shown that the method is effective and robust.

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