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The Impact of Propagation Time Difference in Transmit Diversity

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Abstract -- **Transmit diversity has been shown to be an efficient method to improve system performance without the increase of the mobile terminal complexity in wireless communications. In this paper, the impact of propagation time difference between transmit antennas on the performance of a transmit diversity scheme has been studied under Rayleigh fading channel. It has been shown that performance of transmit diversity degrades significantly when the propagation time difference increases.**

I. INTRODUCTION

Transmit diversity has been studied extensively in the recent years as an effective technique for achieving spatial diversity in fading channels with an antenna array at the base station rather than the remote mobile terminals. It is developed to meet the requirements for the next generation wireless systems to provide high bit rate data service under high speed mobile environment with small lightweight pocket communicators.

Due to its significant advantages, the Space Time Block Coding (STBC) based transmit diversity scheme proposed by Alamouti [1] has been adopted by 3GPP standards [2], as well as a very similar one by 3GPP2 [3]. In the design and analysis of such scheme it is generally assumed that the channel is static during one space-time coding block and signals from multiple transmit antennas can arrive at the receiving antenna exactly at the same time. However, in reality, the signals from different transmit antennas do arrive at different time. For example, the multipath propagation in indoor environments. In this work we take into account the time difference between the propagation delays of different wireless links from the multiple transmit antennas to the receiving antenna, and assess the resultant performance degradation comparing with the ideal situation.

II. SYSTEM MODEL

In order to achieve expected diversity, usually the antennas deployed in the basestation are separated apart enough so that multiple propagation wireless channels with independent fading can be observed. This is a bit of difference from the smart antenna techniques, which on the other hand take use of the characteristics of cross-correlation among the elements of the antenna array. Therefore, it is inevitable that in STBC based transmit diversity the propagation time delay of transmitted signals from two different antennas to receiving antenna are somewhat different, which will incur a certain amount of arrival time difference in symbol level. It violates the a priori condition on which the normal STBC scheme is derived and applied, thus will deteriorate the performance significantly. The problem might be much worse when transmit diversity is applied to the high bit rate data transmission under indoor environment. The Fig. 1 shows the STBC scheme with concerned propagation time delay difference.

Assuming the signal transmitted from the first antenna arrives at receiving antenna earlier and setting it as the origin timing point, $t_R = 0$ and $|\tau_1 - \tau_0| = \tau$. In other words, the timing system of receiver is synchronized with the arrival time of the symbol signals transmitted from the first transmit antenna. During the first symbol period $0 \sim T_S$, the total received signal at the remote terminal is

$$r^{(0)}(t) = a_0 \cdot h_0^{(0)} + b_{-1} \cdot h_{-1}^{(1)} + n^{(0)} \quad \text{as } t \in (0, \tau)$$

$$r^{(1)}(t) = a_0 \cdot h_0^{(0)} + b_0 \cdot h_0^{(1)} + n^{(1)} \quad \text{as } t \in (\tau, T_S)$$

During the second symbol period $T_S \sim 2T_S$, the total received signal at the remote terminal is

$$r^{(2)}(t) = a_1 \cdot h_1^{(0)} + b_0 \cdot h_0^{(1)} + n^{(2)} \quad \text{when } t \in (T_s, T_s + \tau)$$

$$r^{(3)}(t) = a_1 \cdot h_1^{(0)} + b_1 \cdot h_1^{(1)} + n^{(3)} \quad \text{when } t \in (T_s + \tau, 2T_s)$$

where $h^{(i)}$ ($i = 0,1$) is the independent time-variable channel impulse response associated with two different transmit antenna. And τ_i ($i = 0,1$) stands for two distinct propagation time delay travelling from the corresponding transmit antenna to the receiving antenna.

Based on the Alamouti's scheme [1], we can further elaborate and focus our discussion on two consecutively transmitted symbols s_0 and s_1 with respect to the timing origin at receiver, i.e.

$$a_0 = s_0,\ b_0 = s_1$$
$$a_1 = -s_1^*,\ b_1 = s_0^* \tag{1}$$

It is assumed that the complex multiplicative distortion coefficients of channels remain invariable during each block of every two consecutive symbol periods of time, nominated as a STBC block. Furthermore, the above channel response can be replaced by a complex constant value within the considering STBC block, which varies from block to block. The channels are modeled as independent complex Gaussian distributed random variables $CN(0,\sigma^2)$ with common zero mean and variance, i.e. slow flat Rayleigh fading.

$$h_0^{(i)} = h_1^{(i)} = c_i \qquad (i = 0,1) \tag{2}$$

In this work, we assume the transmit pulse-shaping filter and receiving matched filter are both normalized rectangular within the symbol period of time, i.e.,

$$h(t) = \begin{cases} 1/\sqrt{T_s} & 0 < t < T_s \\ 0 & otherwise \end{cases} \tag{3}$$

Therefore, the total received signal at the remote terminal during the first symbol period $0 \sim T_S$ can be obtained by sampling the output of the matched filter at the time instant of T_S, i.e.,

$$r_0(t) = \int_0^{T_s} \frac{1}{\sqrt{T_s}} \cdot S_0 \cdot c_0 \cdot dt + \int_\tau^{T_s} \frac{1}{\sqrt{T_s}} \cdot S_1 \cdot c_1 \cdot dt + \int_0^{\tau} \frac{1}{\sqrt{T_s}} \cdot b_{n-1} \cdot h_{n-1}^{(1)} \cdot dt + \int_0^{T_s} \frac{1}{\sqrt{T_s}} \cdot n_0(t) \cdot dt \tag{4}$$

where n_0 and n_1 represent noise and interference. Noticing that the transmitted data symbols passing through the shaping filter are

$$S_0 = \frac{1}{\sqrt{T_s}} \cdot s_0 \qquad S_1 = \frac{1}{\sqrt{T_s}} \cdot s_1 \tag{5}$$

further we can get

$$r_0(t) = s_0 \cdot c_0 + \frac{T_s - \tau}{T_s} \cdot s_1 \cdot c_1 + \frac{\tau}{T_s} \cdot b_{n-1} \cdot h_{n-1}^{(1)} + \frac{1}{\sqrt{T_s}} \int_0^{T_s} n_0(t) \cdot dt \tag{6}$$

similarly, during the second symbol period $T_S \sim 2T_S$ it is

$$r_1(t) = -s_1^* \cdot c_0 + \frac{T_s - \tau}{T_s} \cdot s_0^* \cdot c_1 + \frac{\tau}{T_s} \cdot s_1 \cdot c_1 + \frac{1}{\sqrt{T_s}} \int_0^{T_s} n_1(t) \cdot dt \tag{7}$$

Assuming the channel state information can be estimated perfectly through some methods, such as two specific pilot channels, we can construct two received symbol decision variables $\hat{s}_0$ and $\hat{s}_1$ as follows,

$$\begin{aligned} \hat{s}_0 &= r_0 \cdot c_0^* + r_1^* \cdot c_1 \\ &= s_0 \cdot (|c_0|^2 + \frac{T_s - \tau}{T_s} \cdot |c_1|^2) + \frac{\tau}{T_s} \cdot s_1^* \cdot |c_1|^2 \\ &\quad - \frac{\tau}{T_s} \cdot s_1 \cdot c_0^* \cdot c_1 + \frac{\tau}{T_s} \cdot b_{n-1} \cdot h_{n-1}^{(1)} \cdot c_0^* + VS_0 \\ \hat{s}_1 &= r_0 \cdot c_1^* - r_1^* \cdot c_0 \\ &= s_1 \cdot (|c_0|^2 + \frac{T_s - \tau}{T_s} \cdot |c_1|^2) - \frac{\tau}{T_s} \cdot s_1^* \cdot c_0 \cdot c_1^* \\ &\quad + \frac{\tau}{T_s} \cdot s_0 \cdot c_0 \cdot c_1^* + \frac{\tau}{T_s} \cdot b_{n-1} \cdot h_{n-1}^{(1)} \cdot c_1^* + VS_1 \end{aligned} \tag{8}$$

Herein

$$VS_0 = \frac{1}{\sqrt{T_s}} \int_0^{T_s} [n_0(t) \cdot c_0^* + n_1^*(t) \cdot c_1] \cdot dt$$
$$VS_1 = \frac{1}{\sqrt{T_s}} \int_0^{T_s} [n_0(t) \cdot c_1^* - n_1^*(t) \cdot c_0] \cdot dt \tag{9}$$

III. PERFORMANCE ANALYSIS

In this section we derive the conditional Probability Density Function (PDF) of Signal-to-Noise Ratio (SNR) expressions upon different cases, and achieve the closed form bit error probability representations eventually.

A. Expressions of Conditional SNR

For the uncoded BPSK modulation format, the final decision can be conducted as follows,

$$\hat{s}_n = \begin{cases} +1 & \mathrm{Re}[\hat{s}_n] > 0 \\ -1 & \mathrm{Re}[\hat{s}_n] < 0 \end{cases} \qquad (n = 0, 1) \tag{10}$$

Given transmitted symbols s_n ($n = 0, 1$) and channel fading coefficients c_n ($n = 0, 1$), the decision variables in (8) can be modeled as complex Gaussian distribution conditioned on s_n and c_n ($n = 0, 1$). Next we inspect the decision variable $\hat{s}_0$ in terms of SNR with the given s_n and c_n ($n = 0, 1$). As for the case of $\hat{s}_1$, the result is just similar.

Considering the noise term VS_0, its expected value and variance are

$$E\{VS_0\} = \frac{1}{\sqrt{T_s}}\int_0^{T_s} E\{n_0(t)\cdot c_0^* + n_1^*(t)\cdot c_1\}\cdot dt = 0 \quad (11)$$

$$Var\{VS_0\} = E\{|VS_0|^2\} = N_0\cdot(|c_0|^2 + |c_1|^2) \quad (12)$$

Where it is assumed that the noise components of $n_0(t)$ and $n_1(t)$ are uncorrelated zero-mean circularly symmetric complex Gaussian random variables with the common variance $\sigma_n^2 = \frac{1}{2}N_0$ in each dimension. Since

$$\mathrm{Re}\{n_0\cdot c_0^* + n_1^*\cdot c_1\} = \mathrm{Re}(n_0)\mathrm{Re}(c_0) + \mathrm{Im}(n_0)\mathrm{Im}(c_0) + \mathrm{Re}(n_1)\mathrm{Re}(c_1) + \mathrm{Im}(n_1)\mathrm{Im}(c_1) \quad (13)$$

It is obviously that

$$E\{\mathrm{Re}[VS_0]\} = 0 \quad (14)$$

$$Var\{\mathrm{Re}[VS_0]\} = E\{\mathrm{Re}^2[VS_0]\} = \frac{N_0}{2}\cdot(|c_0|^2 + |c_1|^2) \quad (15)$$

For the sake of simplicity, we further assume $h_{n-1}^{(1)} = c_1$, that is, the channel remains invariable during the adjacent symbol period of time across the boundary of STBC coding blocks. Thus the signal term can be simplified as,

$$ES_0 = s_0\cdot|c_0|^2 + |c_1|^2\cdot(s_0 - \frac{\tau}{T_s}\cdot s_0 + \frac{\tau}{T_s}\cdot s_1^*) + \frac{\tau}{T_s}\cdot(b_{n-1} - s_1)\cdot c_0^*\cdot c_1 \quad (16)$$

It is assumed that the BPSK modulated symbols s_0, s_1 and b_{n-1} are chosen from the set $\{+\sqrt{E_b}, -\sqrt{E_b}\}$ with equal probability. Here E_b is the energy of the transmitted signal. Therefore, we can discriminate followed four different cases to evaluate previous signal expression.

$$ES_0^{(A)} = \pm\sqrt{E_b}\cdot(|c_0|^2 + |c_1|^2) \quad (17)$$

$$ES_0^{(B)} = \pm\sqrt{E_b}\cdot[|c_0|^2 + (1 - \frac{2\tau}{T_s})\cdot|c_1|^2 + \frac{2\tau}{T_s}\cdot c_0^*\cdot c_1] \quad (18)$$

$$ES_0^{(C)} = \pm\sqrt{E_b}\cdot(-|c_0|^2 - \frac{T_s - 2\tau}{T_s}\cdot|c_1|^2) \quad (19)$$

$$ES_0^{(D)} = \pm\sqrt{E_b}\cdot(-|c_0|^2 - |c_1|^2 + \frac{2\tau}{T_s}\cdot c_0^*\cdot c_1) \quad (20)$$

Based on above four kinds of expression of signal term, the final decision variables $\mathrm{Re}[\hat{s}_0]$ can be constructed with different cases. In order to investigate the error probability eventually, we need to derive PDF of four kinds of SNR per bit representations as follows

$$SNR^{(A)} = \frac{2E_b}{N_0}\cdot(|c_0|^2 + |c_1|^2) \quad (21)$$

$$SNR^{(B)} = \frac{2E_b}{N_0}\cdot(|c_0|^2 + \frac{T_s - 4\tau}{T_s}\cdot|c_1|^2) \quad (22)$$

$$SNR^{(C)} = \frac{2E_b}{N_0}\cdot[|c_0|^2 + |c_1|^2 - \frac{2\tau}{T_s}\cdot(c_0 c_1^* + c_0^* c_1)] \quad (23)$$

$$SNR^{(D)} = \frac{2E_b}{N_0}\cdot[|c_0|^2 + \frac{T_s - 4\tau}{T_s}\cdot|c_1|^2 + \frac{2\tau}{T_s}\cdot(c_0 c_1^* + c_0^* c_1)] \quad (24)$$

It is notified that the terms with high order coefficients have been simply omitted during the derivation process of formula (22) to (24), which will be explained more in detail later.

B. PDF of SNR expressions

Four kinds of instantaneous SNR per bit expressions can be derived as follows.

a) $$\gamma_b^{(A)} = \frac{2E_b}{N_0}\cdot(|c_0|^2 + |c_1|^2) \quad (25)$$

Recalling that c_0 and c_1 are modeled as independent complex Gaussian distributed random variables, that is, the channels follow Rayleigh flat fading. Therefore, the SNR in (25) actually is chi-square-distributed with 4 degrees of freedom and corresponding PDF is

$$p(\gamma_b^{(A)}) = \frac{\gamma_b^{(A)}}{\bar{\gamma}_b^2}\cdot\exp(-\frac{\gamma_b^{(A)}}{\bar{\gamma}_b}) \quad (26)$$

where $\bar{\gamma}_b$ is the average SNR per bit, which is assumed to be identical for both of two channels. By definition,

$$\bar{\gamma}_b = \frac{2E_b}{N_0}\cdot E(|c_0|^2) = \frac{2E_b}{N_0}\cdot E(|c_1|^2) = \frac{2E_b}{N_0}\cdot 2\sigma^2 \quad (27)$$

b) $$\gamma_b^{(B)} = \frac{2E_b}{N_0}\cdot(|c_0|^2 + \frac{T_s - 4\tau}{T_s}\cdot|c_1|^2) \qquad (28)$$

This is the sum of two independent chi-square-distributed random variables with 2 degrees of freedom. Its PDF can be deduced as follows [4]

$$\begin{aligned} p(\gamma_b^{(B)}) = \frac{1}{4\tau_p\cdot\bar{\gamma}_b}\cdot\{\exp(-\frac{\gamma_b^{(B)}}{\bar{\gamma}_b}) \\ -\exp[-\frac{\gamma_b^{(B)}}{(1-4\tau_p)\bar{\gamma}_b}]\} \end{aligned} \qquad (29)$$

where $\bar{\gamma}_b$ is the average SNR per bit defined as above. Moreover, we introduce a new parameter, named as Relative Time-delay Factor (RTF),

$$\tau_p = \frac{\tau}{T_s} \qquad (30)$$

where τ is the absolute time delay and T_s is the symbol period. Thus, we can discuss the problem without concerning the actual transmission data rate.

c)

$$\gamma_b^{(C)} = \frac{2E_b}{N_0}\cdot[|c_0|^2 + |c_1|^2 - \frac{2\tau}{T_s}\cdot(c_0 c_1^* + c_0^* c_1)] \qquad (31)$$

This actually is one special case of real quadratic expression formed by two independent complex-valued Gaussian distributed random variables c_0 and c_1. The PDF can be yielded through the inverse Fourier-transformation of its characteristic function [5], [6] as follows

$$\begin{aligned} p(\gamma_b^{(C)}) = \frac{1}{4\tau_p\cdot\bar{\gamma}_b}\cdot\{\exp[-\frac{\gamma_b^{(C)}}{(1+2\tau_p)\cdot\bar{\gamma}_b}] \\ -\exp[-\frac{\gamma_b^{(C)}}{(1-2\tau_p)\cdot\bar{\gamma}_b}]\} \end{aligned} \qquad (32)$$

d)

$$\begin{aligned} \gamma_b^{(D)} = \frac{2E_b}{N_0}\cdot[|c_0|^2 + \frac{T_s-4\tau}{T_s}\cdot|c_1|^2 \\ +\frac{2\tau}{T_s}\cdot(c_0 c_1^* + c_0^* c_1)] \end{aligned} \qquad (33)$$

This is also one special case of real quadratic expression formed in c_0 and c_1. The PDF can be yielded through the inverse Fourier-transformation of its characteristic function [5], [6] as follows

$$\begin{aligned} p(\gamma_b^{(D)}) = \frac{1}{4\sqrt{2}\,\tau_p\cdot\bar{\gamma}_b} \\ \cdot\{\exp\{-\frac{\gamma_b^{(D)}}{[1+(2\sqrt{2}-2)\tau_p)]\cdot\bar{\gamma}_b}\} \\ -\exp\{-\frac{\gamma_b^{(D)}}{[1-(2\sqrt{2}+2)\tau_p)]\cdot\bar{\gamma}_b}\}\} \end{aligned} \qquad (34)$$

C. Derivation of Probability of Bit Error

As mentioned before, the final decision variable $\mathrm{Re}[\hat{s}_0]$ is conditional complex Gaussian distributed upon c_n $(n = 0, 1)$. Thus, the probability of bit error conditioned on a fixed set of fading channel c_n can be obtained first [6],

$$P_B(\gamma_b) = Q(\sqrt{2\gamma_b}) \qquad (35)$$

where γ_b is the instantaneous SNR per bit conditioned on fading channel. The next step is to average the conditional error probability over the PDF of the instantaneous SNR. That is, we evaluate the integral

$$P_B = \int_0^\infty P_B(\gamma_b)\cdot p(\gamma_b)\cdot d\gamma_b \qquad (36)$$

Therefore, upon conditional PDF of four kinds of instantaneous SNR expressions obtained previously, we can correspondingly conduct four representations for probability of bit error as follows [7], respectively

$$P_B^{(A)} = \frac{1}{4}(\mu^3 - 3\mu + 2) \qquad (37)$$

where, by definition,

$$\mu = \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}} \qquad (38)$$

$$\begin{aligned} P_B^{(B)} = \frac{1}{2} - \frac{1}{8\tau_p}\sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}} \\ +\frac{1-4\tau_p}{8\tau_p}\sqrt{\frac{(1-4\tau_p)\cdot\bar{\gamma}_b}{1+(1-4\tau_p)\cdot\bar{\gamma}_b}} \end{aligned} \qquad (39)$$

$$\begin{aligned} P_B^{(C)} = \frac{1}{2} + \frac{1-2\tau_p}{8\tau_p}\sqrt{\frac{(1-2\tau_p)\cdot\bar{\gamma}_b}{1+(1-2\tau_p)\cdot\bar{\gamma}_b}} \\ -\frac{1+2\tau_p}{8\tau_p}\sqrt{\frac{(1+2\tau_p)\cdot\bar{\gamma}_b}{1+(1+2\tau_p)\cdot\bar{\gamma}_b}} \end{aligned} \qquad (40)$$

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$$P_B^{(D)} = \frac{1}{2} - \frac{1+(2\sqrt{2}-2)\tau_p}{8\sqrt{2}\,\tau_p}\cdot\left\{\frac{[1+(2\sqrt{2}-2)\tau_p]\cdot\bar{\gamma}_b}{1+[1+(2\sqrt{2}-2)\tau_p]\cdot\bar{\gamma}_b}\right\}^{\frac{1}{2}} + \frac{1-(2\sqrt{2}+2)\tau_p}{8\sqrt{2}\,\tau_p}\cdot\left\{\frac{[1-(2\sqrt{2}+2)\tau_p]\cdot\bar{\gamma}_b}{1+[1-(2\sqrt{2}+2)\tau_p]\cdot\bar{\gamma}_b}\right\}^{\frac{1}{2}} \quad (41)$$

Finally, the overall performance can be investigated through averaging all four kinds of probability of bit error.

$$P_B = \frac{1}{4}\cdot(P_B^{(A)} + P_B^{(B)} + P_B^{(C)} + P_B^{(D)}) \quad (42)$$

IV. NUMERICAL RESULTS

In this section we compare the probability distribution and probability of bit error performance for the different RTF parameters defined above. Generally, it is reasonable to believe that the relative timing-delay difference will be limited within a certain relatively small range. For example, the propagation distance difference might be tens of meters in a WCDMA cell with a radius of 1 km, in which the chip rate is about 5Mbps. While in an indoor WLAN system operating on 50Mbps, the propagation distance difference usually is less than several meters. Therefore, the RTF parameter could be practically remained less than 0.2. The probability of bit error corresponding to different cases of SNR expression varies diversely upon the RTF parameters. The overall performance of STBC transmit diversity with the difference of propagation time delay is shown in Fig. 2 with different RTF parameters. When τ_d is relative small, the degradation of the performance is not too severe. But as $\tau_d = 0.2$, the overall performance deteriorates nearly 4 dB when the concerned probability of bit error is around 10^{-3}. It can be seen that performance degradation is significant when the delay is large (i.e. 20%).

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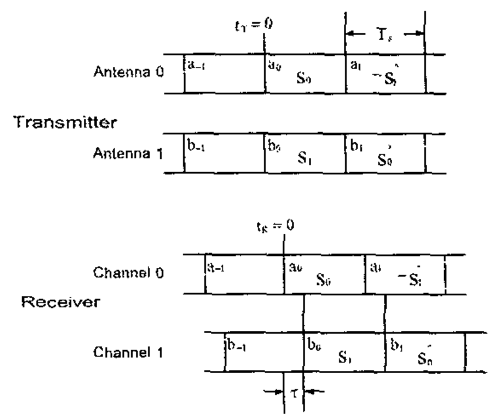

Fig. 1. STBC with propagation time-delay difference

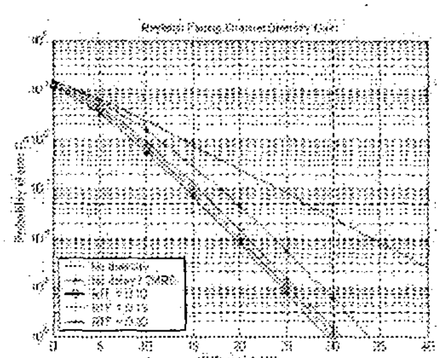

Fig. 2. Overall probability of bit error with different RTF parameter