AN APPROACH FOR IDENTIFICATION OF NON-GAUSSIAN LINEAR SYSTEM WITH TIME-VARYING PARAMETERS

Minfen Shen* Rong Song* K. H. Ting** Francis H. Y. Chan**
*Science Research Center, Shantou University, Guangdong 515063, China
E-mail: mfshen@stu.edu.cn
** Department of Electronic Engineering, Hong Kong University, Hong Kong, China

Abstract: A new approach for identification of non-Gaussian linear system with time-varying parameters is addressed in this paper. The proposed method is based on the application of higher-order spectra (HOS) and wavelet analysis. In order to solve the problem and identify the characteristics of the time-varying linear system, a time-varying parametric model is proposed as non-Gaussian AR model. The model parameters that characterize the time-varying system are functions of time and can be represented by a family of wavelet basis functions, of which the corresponding basis coefficients are invariant. This method can well track the changes of the model parameters, and the results show its effectiveness of the proposed approach.

Key words: Time-varying linear system, wavelet basis, higher-order spectra

1. Introduction

Many of the existing methods for system identification and parameters estimation are exclusively based on the stationarity assumption. In spite of its many successful applications, aforementioned methods do not fulfill the nonstationary applications. Now more and more growing focus is put on the nonstationary environments, whose non- stationarity are close related with physiologic accommodation. The instantaneous information of those kinds of signals and systems are hard to be identified and predicted. There are many useful methods applied to nonstationary physical situations.

The most popular approach to estimate the nonstationary signals is to employ an adaptive algorithm and assume that the change of the signals is shown in [2]. Marc Lavielle solved the problem by presuming that the process is locally stationary over a relatively short time interval but globally nonstationary. Then we look on this kind of signals as piece-stationary signals and the most important thing is to find the instants of change [3-4]. More people construct the nonstationary models directly: Satoru Goto present cumulant-based methods for time-varying AR model parameters estimation [5,6,8], but this method is limited in certain AR parameters. A novel Bayesian formulation is developed to identify the system parameters and estimate the models. Other report [7] uses a wavelet basis for the identification of time-varying (TV) system, and TV parameters can be expanded onto a finite set of wavelet basis sequences. Its flexibility in capturing the system's characteristics at different scales is at the cost of computational complexity. Several papers [1][6][7] involve the basis function. With the application of basis vectors such as Legendre polynomials and Fourier series, the TV model can be represented by a family of basis vectors, and the basis coefficients are invariant. We combine wavelet basis functions and the higher-order statistics, and propose a new method to estimate the coëfficients of time-varying AR model, which is better than former methods. Section 2 of this paper introduces the parametric model of our method. Section 3 represents the experimental results and compares the method with Fourier method. Finally, some simulations are demonstrated.

0-7803-7490-8/02/$17.00©2002 IEEE.

2. The Proposed Model

We build a TV linear system or TV AR(p) model to extract the feature of the characteristics of the nonstationary signal. When $X = (X_1, X_2, \cdots X_n)$ is a nonstationary real process, the TV parameters of the model is employed, which is described by the following difference equation:

$$x(n) = -\sum_{k=1}^{P} a_k(n)x(n-k) + v(n) \qquad (1)$$

where $v(k)$ is an independent identically distributed (i.i.d.) stationary, non-Gaussian process, with zero-mean and a finite nonzero cumulant. $a_k(n)$ are a TV parameter, which can be represented by a linear combination of a number of known functions:

$$a_k(n) = \sum_{j=0}^{q} a_{kj} u_j(n) \qquad (2)$$

where $u_j(n)$ is the orthonormal basis functions. The TV AR coefficients are represented on the space spanned by the basis functions. TV AR Coefficients are constants in this space. The functions are all known, so we can get $a_k(n)$ if we know the coefficients of the functions.

The model can be described as the following:

$$x(n) = -\sum_{k=1}^{p} a_k(n)x(n-k) + v(n) = \sum_{k=1}^{p}[\sum_{j=0}^{q} a_{kj} u_j(n)]x(n-k) + v(n) \qquad (3)$$

or

$$\mathbf{x}(n) = -\sum_{k=1}^{p} a_k^T . X(n-k) + v(n) \qquad (4)$$

where $a_k^T = [a_{k0,} a_{k1,} \cdots, a_{kq}]^T$ and

$$X(n) = [u_0(n)x(n), u_1(n)x(n), \cdots\cdots, u_q(n)x(n)]^T$$

The expression (4) can be evolved into a system of q+1 equations:

$$u_0(n)x(n) = -\sum_{k=1}^{p} u_0(n)a_k^T \cdot X(n-k) + u_0(n)v(n)$$
$$u_1(n)x(n) = -\sum_{k=1}^{p} u_1(n)a_k^T \cdot X(n-k) + u_1(n)v(n)$$
$$\vdots$$
$$u_q(n)x(n) = -\sum_{k=1}^{p} u_q(n)a_k^T \cdot X(n-k) + u_q(n)v(n)$$

where

$$A(k) = [u_0(n)a_k^T, u_1(n)a_k^T, \cdots\cdots, u_q(n)a_k^T]^T$$

$$V(n) = [u_0 v(n), u_1 v(n), \cdots\cdots, u_q v(n)]^T$$

We also know that the mth order cumulants sequence of X(n) satisfies the following recursive equation [5]:

$$\sum_{k=0}^{p} A(k) C_{m,X}(\tau_1, \cdots; \tau_{m-2}, \tau - k) \equiv 0, \quad \tau > 0 \qquad (5)$$

We assume m=3, then (5) can be changed into :

$$\sum_{k=1}^{p} \begin{bmatrix} u_0(n)a_k^T \\ u_1(n)a_k^T \\ \vdots \\ u_q(n)a_k^T \end{bmatrix} \begin{bmatrix} c_{j,1,1}, \cdots c_{j,q+1,1} \\ c_{j,1,2}, \cdots c_{j,q+1,2} \\ \vdots \quad \vdots \\ c_{j,1,q+1}, \cdots c_{j,q+1,q+1} \end{bmatrix} \equiv 0 \qquad (6)$$

where $c_{j,1,1} = E[x_j(n)x_1(n+\tau_1)x_1(n+\tau-k)]$

and $x_j(n)$ is the jth element of X(n)

3. Wavelet Basis Function

A wavelet orthonormal basis of the usual Lebesque square integrable function space L^2 satisfies the following formula:

$$<\psi_{k,n},\psi_{l,m}>=\delta_{kl}\cdot\delta_{nm},\quad k,n,l,m\in Z$$

if $f\in L^2(R)$, we can get:

$$f(x)=\sum_{j,k\in Z}c_{j,k}\psi_{j,k}(x) \qquad (7)$$

So we can choose some kinds of wavelet basis to represent the AR coefficients, such as Harr and Daubechies basis, because of their good performance in transient change.

4. The Results and Discussion

We assume that the AR order p=2, then:

$$x(n)=a_1(n)x(k-1)+a_2(n)x(k-1)+v(n)$$

where v(k) is an independent identically distributed stationary, non-Gaussian process, whose variance $E[v^2(k)]=Q$ and $E[v^3(k)]=\beta\neq 0$.{ $a_1(k)$, $a_2(k)$} are TV AR(2) model parameters changing abruptly in the following manner:

$$a_1(k)=-1.5, a_2(k)=0.8$$

$$k\in[1,N/4]\cup[N/2+1,3N/4],$$

$$a_1(k)=-0.9, a_2(k)=0.2$$

$$k\in[N/4+1,N/2]\cup[N3/4+1,N],$$

The TV process is generated with N=6000 samples in Fig.1 and 2, which are shown by the rectangle blue wave. Both Fourier basis and Harr basis are used to estimate the model parameters, and compare the results with the original values.

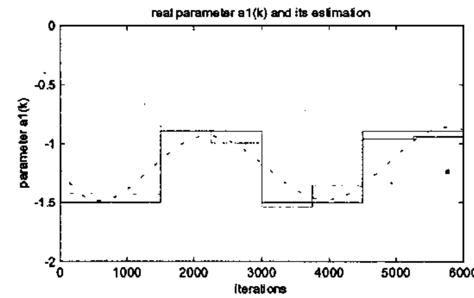

Fig. 1 The estimated result of the TV $a_1(k)$. The estimated result of Fourier basis is shown as the dotted line, and the estimated result of Harr basis is shown as the solid line.

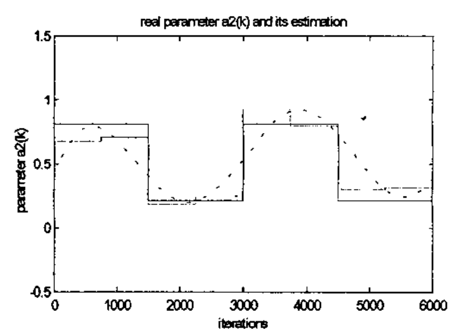

Fig. 2 The result of the estimation of the TV $a_2(k)$. The estimated result based on Fourier basis is shown as the dotted line, and the estimated result based on Harr basis is shown as the solid line.

Both Fig.1 and Fig.2 depict the true AR(2) parameters $a_1(k)$ and $a_2(k)$, and the estimated results based on Fourier basis and Harr basis, respectively. We can see from the results that Harr basis is much better than Fourier basis and we can get the same result by the error criterion. The error is estimated by minimizing a penalized contrast function of the form [3]

$$d_0(x_1,x_2,\cdots,x_N;a_1,a_2\cdots a_p)=\sum_{i=P+1}^{N}[X_i-\sum_{j=1}^{P}\hat{a}_j\hat{X}_{i-j}]^2/N \quad (8)$$

The error of wavelet basis with $d_{01}=1.7332$ is less than the error of Fourier basis with $d_{02}=2.0256$, which reflect that the wavelet basis is more suitable than the conventional method in the identification of the TV linear systems.

5. Conclusion

The aim of the proposed method was to investigate the problem of the identification of the time-varying linear systems described by an non-Gaussian AR model. The model parameters that characterize the time-varying system are functions of time and can be represented by a family of wavelet basis functions. A comparison between wavelet basis and Fourier basis of cumulants-based method is also given. The results in the presented method show the applicability and the effectiveness of the procedures, while some signal processing techniques is needed to apply to minimize the estimated error.

Acknowledgements

The research is supported by: The Key Grant of Guangdong Natural Science Foundation 2002.

References

[1] J. J. Rajan , Peter J. W. Rayner *"Generalized Feature Extraction For Time-varying Autoregressive Models"*, IEEE Trans. on Signal Processing. vol. 44, No. 10, October 1996.

[2] D. Aboutajdine *"Fast Adaptive Algorithms for AR Parameters Estimation Using Higher Order Statistics"* IEEE Trans. on Signal Processing. Vol. 44, No.8, Aug. 1996.

[3] Marc Lavielle *"Optimal Segmentation of Random Processes,"* IEEE Trans. on Signal Processing. vol. 46, No.5, May 1998

[4] D. Geman, *"Random fields and inverse problems in imaging,"* in Lecture Notes in Mathematics. New York: Springer -Verlag,1990

[5] Lee M. Garth *"A comparison of optimized higher-order spectral detection techniques for non-Gaussian signals,"* IEEE Trans. on Signal Processing. vol. 44, No.5, May 1996.

[6] M. Bakrim, D. Aboutajdine, and M. Najim, *"New cumulant-based approaches for non-Gaussian time varying AR models,"* Signal Processing, vol. 39, nos. 1/2, pp. 107-115, Sept. 1994.

[7] M. K. Tsatsanis and G.. B Giannakis. *"Time-varying System Identification and Model Validation Using Wavelets."* IEEE Trans. on Signal Processing, 41(12): 3512-3523, 1993.

[8] M. Martone *"Non-Gaussian Multivariate Adaptive AR Estimation Using the Super Exponential Algorithm,"* IEEE Trans. on Signal Processing, vol. 44, No.10, Oct. 1996.

[9] G. Thonet, Jean-Marc Vesin *"Stationarity Assessment with Time-varying Autoregressive Modeling"* Signal Processing Laborator, pp. 3721-3724, 1997.