## A DECENTRALIZED APPROACH FOR OPERATING RESERVE PROCUREMENT

Chi Yiu CHAN

Ken T.K. CHAN

Yixin NI

Felix F. WU

chiychan@eee.hku.hk h0000622@eee.hku.hk yxni@eee.hku.hk ffwu@eee.hku.hk Dept. of Electrical and Electronic Engineering, The University of Hong Kong

## ABSTRACT

In power markets, operating reserve procurement is usually centrally handled according to certain reliability rules and aiming at minimization of total cost for reserve procurement. In this approach, there are no customers' choices, no incentives for ISO to minimize the total cost or for reserve suppliers to commit promised reserve capacity. It may leads to severe inefficiency. In this paper, we propose a decentralized approach for operating reserve procurement and in the meantime, to discover the market price for it. In the approach, each participant selects the optimal decision to maximize his or her own interests. Insurance theory is applied; which allows consumers to transfer their risk for financial loss of outage to the ISO, and induces incentive for the ISO to manage the entire amount of reserve capacity in an efficient manner. A penalty system is introduced, which improves the liability for the genco to provide operating reserve. Detailed math model and solution procedure are presented. It is also shown that with properly defined reserve market's structure, the decentralized approach can yield same optimal solution as its centralized counterpart. Numerical example results illustrate the effectiveness of the suggested approach.

## 1. INTRODUCTION

Operating reserves is an ancillary service, which is extra generation available capacity that can be generated immediately if required [1], [2]. With the operating reserve, the bulk-power system has the capability to arrest the contingency: include generation outage or transmission outage.

Currently, an independent system operator (ISO) is introduced, who responses to ensure the security and reliability of the network. To improve the efficiency, competition between reserve providers was introduced in operating reserve procurement [3]. The ISO acts as a single buyer, with the objective to minimize the entire procurement cost, with a defined operating reserve requirement. Those approaches minimize the consumers' interruption. However, it lacks consumers' preference consideration and cannot guarantee the efficiency of the reserve procurement.

In order to maximize the social welfare, customers' preference should be considered. With the economic theory, the best way to include customers' preference is to allow customers' choice under a decentralized decision environment, which can maximize the economic efficiency.

With the application of insurance policy [4]-[6], the authors [7]. [8] conducted a conceptual study on decentralized reserve procurement and pricing. The works [7], [8] proved, with a simple case, that insurance policy can be used to design a decentralized market for reserve procurement, which allows the customers' choice, induces incentives and enforces the liability for the ISO to provide reserve capacity. The optimal decision maximize the economic efficiency, and the pool-based approach is cost effective than the contract-based approach.

A detail model of a centralized optimization decision making, for deregulated market environment with the consideration of the individual welfare, was developed to determine the price and the optimal quantity of operating reserve for individual [9]. It is proven that, in order to maximize the social welfare, different consumers should have a different reserve requirement.

In this paper, a decentralized operation environment will be developed for operating reserve procurement. (Hereafter, reserve capacity is referred for operating reserve only.) Assuming that there is a pure competitive market, each participant has the ability to make decision on the optimal supply/demand quantity of the reserve capacity at the market reserve price. Microstructure, math model and optimal process for the decentralized approach will be presented in section 2, 3 and 4 respectively. Moreover, the paper will be finalized with a discussion and a conclusion.

# 2. DECENTRALIZED DECISION PARADIGM FOR RESERVE MARKET

Assuming that there are N gencos  $(G_i, i=1 \sim N)$  and M Consumers  $(C_j, j=1 \sim M)$  in the market, an independent system operator (ISO) coordinates the reserve capacity procurement. Under a decentralized decision paradigm, the ISO will no longer make centralized decision. It acts as a profit-neutral coordinator in the reserve market. The N-1 security assumption is held, i.e. only one genco is in forced-outage at a time. Transmission network impact is not taken into account. For the studied time interval, say one-hour, we know from the spot market that genco  $G_i$  supplies electricity  $S_{Gi}$  and consumer  $C_j$  consumes electricity  $Q_G$  with

$$S_{\Sigma} = \sum_{i=1}^{N} S_{Gi} = Q_{\Sigma} = \sum_{j=1}^{M} Q_{Cj}$$
.

For the reserve market, genco  $G_i$  provides reserve capacity  $R_{Gi}$  and consumer  $C_i$  demands reserve

capacity  $R_{C_i}$ , there should be  $\sum_{i=1}^{N} R_{C_i} = \sum_{j=1}^{M} R_{C_j} = R_{\Sigma}$ .

The reserve capacity will be generated in the event of outage occurs. (The generated reserve capacity is referred as reserve energy.) However, when a is in generator  $G_{h}$ forced-outage, corresponding reserve capacity will not be available, which cause the entire available reserve capacity less than the entire demand of reserve capacity. Therefore, a supplementary reserve provider  $G_{\Delta}$  is introduced to provide reserve capacity and corresponding reserve energy, which should originally be provided by genco  $G_h$ . The supplementary reserve capacity  $R_{GA}$  is defined as a certain percentage  $(r_{\Delta})$  of total reserve capacity  $R_{\Sigma}$ , i.e.  $R_{G\Delta} = r_{\Delta}R_{\Sigma}$ , and  $r_{\Delta}$  should be large enough to make  $R_{G\Delta} = r_{\Delta}R_{\Sigma} \ge R_{Gi}$  for all i.

The basic idea of reserve capacity market is as follows. The ISO announces an initial guess on reserve capacity prices for buying it from gencos at  $P_R^G$  and from the supplementary reserve provider  $P_R^\Delta$  and selling it to consumers at  $P_R^G$ . According to the prices, each genco determines how much reserve capacity  $(T_{GI})$  it will sell based on its fixed production cost and each consumer determines the amount it will buy  $(T_{CI})$  based on its loss under

supply interruption. If  $\sum_{i=1}^{N} T_{Gi} = \sum_{j=1}^{M} T_{Gj} = T_{\Sigma}$ , the market task is implemented; if not, the price should be tuned properly until the balance is set up. In this paper, "T" means contracted reserve capacity and "R" means committed reserve capacity. If the microstructure of reserve market is well defined, "R" should be equal to "T".

Two policies are introduced for operating reserve procurement, namely, insurance policy and penalty policy.

# 2.1. Insurance policy between ISO (insurer) and consumers (insured)

Consumers buy reserve capacity  $T_{C_j}$  at reserve capacity price  $P_R^C$  from ISO and pay a premium to the ISO for insurance policy in order to shift the risk for financial loss of outage to the ISO. ISO procures reserve capacity  $R_{C_j}$  (may be different from  $T_{C_j}$ ) for consumer  $C_j$ . When an outage occurs at genco  $G_k$ , the ISO arranges reserve energy  $\hat{R}_{C_j,h}$  to consumer  $C_j$  according to certain rules (see (2)) and pays a claim  $g_{C_j,h}(\hat{R}_{C_j,h})$  (see (1)) to the consumer  $C_j$  to compensate his financial loss according to insurance policy. Suppose that  $g_{C_j,h}(\hat{R}_{C_j,h})$  takes the form

$$g_{Cj,h}(\hat{R}_{Cj,h}) = K_j \cdot [OC_{Cj,h}(\hat{R}_{Cj,h}) + P_{RE,h} \cdot \hat{R}_{Cj,h}], 0 \le K_j \le 1$$
 (1)

where  $\hat{R}_{G_i,h}$  is arranged based on the loses electricity  $(Q_{C_i}/Q_{\Sigma}) \cdot S_{Gh}$  and  $R_{C_i}$  available, i.e.

$$\hat{R}_{C_J,h} = \begin{cases} R_{C_J} & R_{C_J} \le \left( \mathcal{Q}_{C_J} / \mathcal{Q}_{\Sigma} \right) \cdot S_{Gh} \\ \left( \mathcal{Q}_{C_J} / \mathcal{Q}_{\Sigma} \right) \cdot S_{Gh} & R_{C_J} > \left( \mathcal{Q}_{C_J} / \mathcal{Q}_{\Sigma} \right) \cdot S_{Gh} \end{cases} \tag{2}$$

 $OC_{Cj,h}(\hat{R}_{Cj,h})$  denotes the outage cost of consumer  $C_j$ , who loses electricity  $(Q_{Cj}/Q_{\Sigma})\cdot S_{Cj,h}$  and having reserve energy  $\hat{R}_{Cj,h}$  when outage occurs at genco  $G_h$ .  $P_{RE,h}$  is reserve energy price at genco  $G_h$  outage.  $K_j$  denotes the percentage of the outage cost that consumer  $C_j$  can claim from the ISO in the event of outage occurs. It is proven in [7] that, rational consumers will decide to purchase a full-cover insurance, i.e.  $K_{j,opt}=1$ . We shall take  $K_j=1$  in our study. Expression for  $OC_{Cj,h}(\hat{R}_{Cj,h})$  will be given later.

## 2.2. Penalty policy between genco and ISO

With the penalty system, the ISO pays genco  $G_i$  for contracted amount of reserve capacity  $T_{Gi}$  and monitors the actual amount of available reserve capacity  $R_{Gi}$ . If an outage occurs at  $G_h$ , genco  $G_i$  will be asked to generate the reserve energy  $\hat{R}_{Gi,h}$ . If  $R_{Gi} < T_{Gi}$ , genco  $G_i$  should pay a penalty  $f_{Gih}(R_{Gi}, T_{Gi})$  to ISO.  $f_{Gih}(R_{Gi}, T_{Gi})$  is defined as a quadratic function:

$$f_{Gih}\left(R_{Gi}, T_{Gi}\right) = \frac{P_{N}^{G}}{\sum_{i=1}^{N} OP_{i}} \left[T_{Gi} - R_{Gi} + \left(T_{Gi} - R_{Gi}\right)^{2}\right], \left(T_{Gi} > R_{Gi}\right)$$
(3)

The penalty function has a factor  $P_R^G / \sum_{l=1}^N OP_l$ . If ISO

pays a higher reserve capacity price, he should be able to receive a higher penalty in case a genco does not commit the provision. Secondly, the probability of reserve energy usage will be reduced with respect to the reduction of the outage probability. The expected penalty will be reduced consequently. Therefore, the penalty should be increased to maintain the incentive for gencos to commit on providing their contracted reserve capacity.

# 3. MATHEMATICAL FORMULATION FOR INDIVIDUALS

The mathematical models for the market participants are similar to the centralized decision model in [9]. The reserve capacity payment collected from consumers is equal to the reserve capacity payment distributed to gencos and supplementary reserve provider, i.e.  $P_R^{\ C} \cdot R_{\Sigma} = P_R^{\ G} \cdot R_{\Sigma} + P_R^{\ \Delta} \cdot R_{G\Delta}$ . According to  $R_{G\Delta} = r_{\Delta}R_{\Sigma}$ , the prices' relationship can be expressed as:

$$P_{R}^{C} = P_{R}^{G} + r_{\Lambda} \cdot P_{R}^{\Delta} \tag{4}$$

## 3.1. Math model for gencos

For genco  $G_i$ , it has fixed production cost function  $C_{Ri}(R_{Gi})$  (of opportunity cost nature) in providing reserve capacity  $R_{Gi}$ , and reserve energy production cost  $PC_{Gih}(\hat{R}_{Gih})$  at genco  $G_h$  outage:

$$PC_{G_i,h}\left(\hat{R}_{G_i,h}\right) = C_{G_i}\left(S_{G_i} + \hat{R}_{G_i,h}\right) - C_{G_i}\left(S_{G_i}\right), (i = 1 \sim N, i \neq h)$$
 (5) where  $C_{G_i}\left(\cdot\right)$  is genco  $G_i$ 's variable production cost function (mainly fuel cost) and genco  $G_i$  really produces reserve energy at  $\hat{R}_{G_i,h}$  with  $S_{G_i}$  as spot market scheduled electricity production.

Supplementary reserve provider  $G_{\Delta}$  will take over the outage genco  $G_h$ 's duty to provide reserve energy, i.e.  $\hat{R}_{G\Delta,h} = \hat{R}_{Gh,h}$ . The corresponding reserve energy production cost  $PC_{G\Delta,h}(\hat{R}_{G\Delta,h})$  should be paid by the outage genco  $G_h$  to  $G_{\Delta}$  i.e. we assumed  $G_{\Delta}$  is profit-neutral in reserve energy production and

$$PC_{Gh,h}(\hat{R}_{Gh,h}) = PC_{G\Delta,h}(\hat{R}_{G\Delta,h})$$
(6)

With the penalty policy, genco  $G_i$  has the following expected-profit  $(EP_{Gi})$  maximization problem

$$\begin{aligned} & \underset{R_{Gi}, T_{Gi}}{Max} EP_{Gi}\left(R_{Gi}, T_{Gi}\right) = P_{R}^{G} \cdot T_{Gi} - C_{Ri}\left(R_{Gi}\right) \\ & + \sum_{h=1}^{N} OP_{h} \cdot \left[P_{RE,h} \cdot \hat{R}_{Gi,h} - PC_{Gi,h}\left(\hat{R}_{Gi,h}\right) - f_{Gi,h}\left(R_{Gi}, T_{Gi}\right)\right] \end{aligned}$$
 (7)   
 
$$s.t. \ R_{Gi,max} \ge R_{Gi}$$

Assuming that the supplementary reserve provider is reliable without penalty being enforced, his expected-profit  $(EP_{G\Delta})$  maximization problem can be expressed as

$$\max_{G_{\Delta}} EP_{G\Delta}(R_{G\Delta}) = P_{R}^{\Delta} \cdot R_{G\Delta} - C_{G\Delta}(R_{G\Delta}) 
s.t. R_{G\Delta, \max} \ge R_{G\Delta}$$
(8)

There is no reserve energy relevant item in (8) since genco  $G_h$  will pay the amount,  $PC_{G\Delta,h}(\hat{R}_{G\Delta,h})$ , to genco  $G_{\Delta}$ .

## 3.2. Math model for consumers

In spot market, consumer  $C_j$  obtains benefit  $B_{C_j}(Q_{C_j})$  with electricity  $Q_{C_j}$  purchased from the spot market at market clearing price  $P_{MC}$ . With the pool-based approach, each consumer  $C_j$  will lose a small amount of electricity  $(Q_{C_j}/Q_{\Sigma}) \cdot S_{Ch}$  if genco  $G_h$  outage. The corresponding outage loss is defined as the loss of benefit, so-called outage cost and denoted as  $OC_{C_i,h}(\hat{T}_{C_i,h})$ 

$$OC_{G,h}(\hat{T}_{G,h}) = \left[B_{G}(Q_{G}) - P_{MC} \cdot Q_{G}\right] - \left[B_{G}(Q_{G} - \frac{\varrho_{G}}{Q_{C}} \cdot S_{Gh} + \hat{T}_{G,h}) - P_{MC} \cdot \left(Q_{G} - \frac{\varrho_{G}}{Q_{C}} \cdot S_{Gh}\right)\right]$$

$$(9)$$

where  $\hat{T}_{CJ,h}$  is real reserve energy designated for consumer  $C_j$  at  $G_h$  outage based on the contracted amount of reserve capacity  $T_{CJ}$ . The first term of (9) is consumer  $C_j$  's utility without outage and the second term is new utility with  $G_h$  outage with obtained reserve energy  $\hat{T}_{CJ,h}$  considered. The charge for reserve energy  $\hat{T}_{CJ,h}$  (i.e.  $P_{RE,h} \cdot \hat{T}_{CJ,h}$ ) is not included in (9). For a rational consumer, he will purchase a full-cover insurance policy [7], therefore, the expected claim is expressed as

$$g_{Cj,h}(\hat{T}_{Cj,h}) = OC_{Cj,h}(\hat{T}_{Cj,h}) + P_{RE,h} \cdot \hat{T}_{Cj,h}$$
 (10)

With the insurance policy, consumer  $C_j$  has the following expected-cost  $\left(EC_{C_j}\right)$  minimization problem

$$\begin{aligned}
& \underset{T_{C_j}}{Min} \ EC_{C_j} \left( T_{C_j} \right) = P_R^{\ C} \cdot T_{C_j} + \sum_{h=1}^{N} OP_h \cdot g_{C_j,h} \left( \hat{T}_{C_j,h} \right) \\
&+ \sum_{h=1}^{N} OP_h \cdot \left[ OC_{C_j,h} \left( \hat{R}_{C_j,h} \right) + P_{RE,h} \cdot \hat{R}_{C_j,h} - g_{C_j,h} \left( \hat{R}_{C_j,h} \right) \right]
\end{aligned} \tag{11}$$

where similar  $\hat{R}_{Cl,h}$  (see (2)), we have

$$\hat{T}_{C_{G,h}} = \begin{cases} T_{C_{G}} & T_{C_{G}} \le (Q_{C_{G}}/Q_{\Sigma}) \cdot S_{Gh} \\ (Q_{C_{G}}/Q_{\Sigma}) \cdot S_{Gh} & T_{C_{G}} > (Q_{C_{G}}/Q_{\Sigma}) \cdot S_{Gh} \end{cases}$$
(12)

 $g_{G,h}(\hat{R}_{G,h})$  and  $g_{G,h}(\hat{T}_{G,h})$  are defined in (1) (with  $K_j = 1$ ) and (10) similarly.  $OC_{G,h}(\hat{R}_{G,h})$  is real outage cost of  $C_j$  at  $G_h$  outage in (9) with replaced  $\hat{T}_{G,h}$  by  $\hat{R}_{G,h}$ . With full-cover insurance (10), the consumer's expected net outage cost should be equal to zero.

## 3.3. Math model for the ISO

ISO pays reserve capacity payments to the gencos and the supplementary reserve provider, receive premiums from consumers and pays claims to the consumers in the event of outage occurs. All the reserve energy payments received from consumers will be distributed to gencos. ISO solves for reserve capacity requirement for consumers  $R_{G_i}$ ,  $j=1 \sim M$ 

and  $R_{\Sigma} = \sum_{j=1}^{M} R_{Cj}$  to minimize the entire reserve related cost. The objective function can be expressed as

$$\begin{aligned} & \underset{R_{G}, R_{\Sigma}}{\text{Min}} EC_{I}\left(R_{CI}, R_{\Sigma}\right) = P_{R}^{C} \cdot R_{\Sigma} + P_{R}^{\Delta} \cdot r_{\Delta} \cdot R_{\Sigma} \\ & - \sum_{j=1}^{M} \left[P_{R}^{C} \cdot T_{CJ} + \sum_{h=1}^{N} OP_{h} \cdot g_{CJ,h}\left(\hat{T}_{CJ,h}\right)\right] + \sum_{h=1}^{N} \sum_{j=1}^{M} OP_{h} \cdot g_{CJ,h}\left(\hat{R}_{CJ,h}\right) \\ & - \sum_{h=1}^{N} \left[OP_{h} \cdot P_{RE,h} \cdot \sum_{j=1}^{M} \hat{R}_{CJ,h}\right] + \sum_{h=1}^{N} \left[OP_{h} \cdot P_{RE,h} \cdot \sum_{i=1}^{N} \hat{R}_{Gi,h}\right] \\ & s.t. \ R_{\Sigma} = \sum_{i=1}^{M} R_{CJ} \end{aligned}$$

$$(13)$$

with  $\sum_{i=1}^{N} \hat{R}_{Gi,h} = \sum_{j=1}^{M} \hat{R}_{G_j,h}$  and eqn. (4), we can see that the ISO has a cost-neutral characteristic if he maintaining  $R_{G_j}$ , which is equal to  $T_{G_j}$ , i.e.

 $EC_I(R_{C_I}) = 0$  for  $R_{C_I} = T_{C_I}$ . It is proven in [7]; insurance policy induces incentives for the ISO to maintain reserve capacity at  $R_{C_I} = T_{C_I}$  and  $\hat{R}_{C_I,h} = \hat{T}_{C_I,h}$  for all  $j = 1 \sim M$  and  $h = 1 \sim N$ .

Although  $\sum_{i=1}^{N} \hat{R}_{Gi,h} = \sum_{j=1}^{M} \hat{R}_{Cj,h}$  and the reserve energy allocation does not affect the ISO's cost function, the ISO is responsible to optimize  $\hat{R}_{Gi,h}$  to minimize the social production cost  $\sum_{i=1}^{N} PC_{Ri,h}(\hat{R}_{Gi,h})$ 

for society. The minimization problem for reserve energy selection can be expressed as

$$\min_{\hat{R}_{Gl,h}} \left[ \sum_{i=1}^{N} PC_{Rl,h} \left( \hat{R}_{Gl,h} \right) \right] \\ s.t. T_{Gl} \ge \hat{R}_{Gl,h}, \sum_{i=1}^{N} \hat{R}_{Gl,h} = \sum_{j=1}^{M} \hat{R}_{Cj,h} \right]$$
 (14)

Assuming that the gencos bid the electricity for the spot market according to the marginal cost of electricity, the marginal production cost function for reserve energy should be equal to the marginal cost function of the electricity. (see (5))

$$\frac{\partial PC_{Ri,h}\left(\hat{R}_{Gi,h}\right)}{\partial \hat{R}_{Gi,h}} = \frac{\partial C_{Gi}\left(S_{Gi} + \hat{R}_{Gi,h}\right)}{\partial \left(S_{Gi} + \hat{R}_{Gi,h}\right)}$$
(15)

Therefore, without the bidding for reserve energy, ISO should be able to solve (14) with the electricity bidding information.

## 4. DECENTRALIZED DECISION PROCESS

## 4.1. Individual Optimal Conditions

The optimization problem of gencos, supplementary reserve provider, consumers and ISO can be solved by the Lagrangian relaxation method. The corresponding optimal conditions are summarized as follows. (Assumed all the inequality constraints are satisfied and the corresponding Lagrangian multiplier are neglected)

1) Genco's optimal conditions for solving  $R_{Gi}$ ,  $T_{Gi}$  are:

$$R_{Gi} = T_{Gi} \tag{16}$$

$$P_{R}^{G} = \frac{\partial C_{Ri}(R_{Gi})}{\partial R_{Gi}} + \sum_{h=1}^{N} OP_{h} \cdot \left[ \frac{\partial PC_{Gi,h}(\hat{R}_{Gi,h})}{\partial R_{Gi}} - P_{RE,h} \frac{\partial \hat{R}_{Gi,h}}{\partial R_{Gi}} \right] \quad (17)$$

Eqn. (16) shows that, the penalty function induces incentives for the gencos to maintain an amount of reserve capacity same as the amount they bided.

2) Supplementary reserve provider's optimal Condition for solving  $R_{GA}$  is:

$$\frac{\partial C_{R\Delta}(R_{G\Delta})}{\partial R_{G\Delta}} = P_R^{\Delta} \tag{18}$$

Assuming that the ISO has a contract with supplementary reserve provider, where the quantity is determined by the ISO and the price  $P_R^{A}$  is

determined according to (18).

3) Consumer's optimal condition for solving  $T_{Ci}$  is:

$$P_{R}^{C} = -\sum_{h=1}^{N} OP_{h} \cdot \left[ \frac{\partial OC_{CJ,h} \left( \hat{T}_{CJ,h} \right)}{\partial T_{CJ}} + P_{RE,h} \frac{\partial \hat{T}_{CJ,h}}{\partial T_{CJ}} \right]$$
(19)

4) ISO's optimal conditions:

The optimal condition for solving  $R_{Cj}$  is

$$P_{R}^{C} = -\sum_{h=1}^{N} OP_{h} \cdot \left[ \frac{\partial OC_{C_{J},h} \left( \hat{R}_{C_{J},h} \right)}{\partial R_{C_{J}}} + P_{RE,h} \frac{\partial \hat{R}_{C_{J},h}}{\partial R_{C_{J}}} \right]$$
(20)

By comparing (20) with (19), we can conclude that the ISO will solve an optimal amount of reserve capacity  $R_{C_j}$ , which is same as the optimal amount of reserve capacity  $T_{C_j}$  selected by the consumers.

The optimal condition for solving  $\hat{R}_{Gih}$  is:

$$\frac{\partial PC_{Ri,h}(\hat{R}_{Gi,h})}{\partial \hat{R}_{Gi,h}} = \alpha_h, (h = 1 \sim N)$$
 (21)

where  $\alpha_h$  be the shadow price of reserve energy when genco  $G_h$  outages.

## 4.2. Global Solution Process

Fig. 1 shows the information flows between market participants.

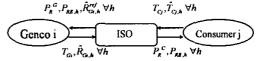


Fig. 1 Information flows in the Reserve Market The corresponding solution procedures can be summarized as follows. Firstly, the ISO announces an initial guess on reserve capacity price for selling it to consumers at  $P_R^C$  and reserve energy price at  $P_{RE,h}$ . According to the information provided by ISO, each consumer solves optimization problem (11) for  $T_{Cj}^{(t)}$ . The optimal solution of  $T_{Cj}^{(t)}$  together with calculated  $\hat{T}_{Cj,h}^{(t)}$ ,  $(h=1 \sim N)$  by (12) will be sent back to the ISO.

With the demand of reserve capacity, the ISO sets  $R_{C_j}^{(t)} = T_{C_j}^{(t)}$  and  $\hat{R}_{C_j,h}^{(t)} = \hat{T}_{C_j,h}^{(t)}$  for all j and h, and solves (14) for the reference value of reserve energy  $\hat{R}_{G_i,h}^{ref}$  Then ISO calculates  $R_{G_A}^{(t)}$  ( $R_{G_A}^{(t)} = r_A R_{\Sigma}$ ) and price  $P_R^{\Delta(t)}$ ,  $P_R^{G(t)}$  (using (18), (4)), and sends  $P_R^{G(t)}$ ,  $P_{RE,h}^{(t)}$  and  $\hat{R}_{G_i,h}^{ref}$  to gencos.

Genco solves the optimization problem (7) for  $T_{Gi}^{(t)}$ ,  $R_{Gi}^{(t)}$  and determines  $\hat{R}_{Gi,b}^{(t)}$  (see (22)) and sends the solution  $T_{Gi}^{(t)}$  and  $\hat{R}_{Gi,b}^{(t)}$  to the ISO.

$$\hat{R}_{Gi,h}^{(t)} = \hat{R}_{Gi,h}^{ref} (t) \quad \text{for } \hat{R}_{Gi,h}^{ref} (t) < T_{Gi}^{(t)}$$

$$\hat{R}_{Gi,h}^{(t)} = T_{Gi}^{(t)} \quad \text{for } \hat{R}_{Gi,h}^{ref} (t) = T_{Gi}^{(t)}$$
(22)

If 
$$\sum_{i=1}^{N} T_{G_i} = \sum_{j=1}^{M} T_{C_j}$$
 and  $\sum_{i=1}^{N} \hat{R}_{G_i,h}^{(t)} = \sum_{j=1}^{M} \hat{R}_{C_j,h}^{(t)}$  for

 $h=1\sim N$ , the market task is implemented; if not, the prices  $P_R^C$ ,  $P_{RE,h}$  will be tuned properly (see (23), (24)) until the balance is set up.

$$P_{R}^{C(t-1)} = P_{R}^{C(t)} + \delta_{1} \cdot \left( \sum_{j=1}^{M} R_{Cj}^{(t)} - \sum_{i=1}^{N} R_{Ci}^{(t)} \right)$$
 (23)

$$P_{RE,h}^{(t+1)} = P_{RE,h}^{(t)} + \delta_2 \cdot \left( \sum_{j=1}^{M} \hat{R}_{Cj,h}^{(t)} - \sum_{j=1}^{N} \hat{R}_{Gj,h}^{(t)} \right) \text{ for all h}$$
 (24)

#### 5. FEATURES AND FURTHER DISCUSSIONS

## 5.1. Centralized vs. Decentralized Approach

The social cost (SC) for the reserve procurement will be the summation of the expected costs of all individuals with their payments and two policies canceled/deleted, and with  $R_{Gi} = T_{Gi}$  for all i and  $R_{Gi} = T_{Gi}$  for all j. We have (see (7), (8), and (11))

$$SC = \sum_{i=1}^{N} C_{Ri}(R_{Gi}) + C_{R\Delta}(R_{G\Delta})$$

$$+ \sum_{i=1}^{N} \sum_{b=1}^{N} OP_{h} \cdot PC_{Ri,h}(\hat{R}_{Gi,h}) + \sum_{i=1}^{M} \sum_{b=1}^{N} OP_{h} \cdot OC_{Gi,h}(\hat{R}_{Gi,h})$$
(25)

Eqn. (25) includes the reserve capacity and energy costs and the consumers' outage losses. It is clear that the objective function in (25) should be the same as the centralized optimization through maximizing social welfare or minimizing social cost [9].

Assuming that the reserve capacity market is pure competitive and there is a unique optimal solution for both centralized optimization problem and decentralized optimization problem. With the spot pricing theory and the first order conditions of optimal problems (both minimization problems have the same set of optimal conditions), the shadow prices solved in the centralized approach should be equivalent to the prices be solved in the decentralized approach, and both approaches must converge to the same optimal solutions. We can conclude that, the proposed decentralized approach can yield the optimal condition, which will maximize the economic efficiency as its centralized counterpart.

## 5.2. Discussion

Key features of the new approach are summarized as follows:

- All participants including gencos, consumers are attempting to solve the reserve capacity supply and demand and to optimize their own benefits with ISO announcing key information on reserve capacity.
- Two policies are introduced, namely, insurance policy and penalty policy. Insurance policy allows consumers to purchase reserve capacity and transfers risk of outage loss to the ISO simultaneously. Insurance policy induces incentives for the ISO to maintain the amount of

reserve capacity same as the optimal amount requested by consumers. Penalty policy penalizes the gencos if they do not commit the reserve provision, which induces incentives for the gencos to commit the reserve capacity provision same as the contracted amount.

- The decentralized approach can maximize the social welfare same as the centralized approach with an objective to maximize social welfare.
- The system reliability is held, because the quantity of reserve capacity plus the amount of load shedding is greater than the loss of generation.

## 6. COMPUTER TEST RESULTS

Modified IEEE RTS-96 one-area system [10] is used to demonstrate the suggested approach. Assume top 14 thermal units in table 7 of [10] are in operation, where oil price is \$ 6/Mbtu and coal price is \$ 4/Mbtu. The generation cost function is supposed to be a quadratic equation:

$$C_{Gi}(S_{Gi}) = a_i + b_i \cdot S_{Gi} + c_i' \cdot S_{Gi}^{2}$$
 (26)

Based on the incremental heat rate data (see Table 9 of [10]), the coefficients  $a_i$ ,  $b_i$  and  $c_i$  in eq. (26) are worked out and listed in Table 1 together with the forced outage probability  $OP_i$ .

Table 1. Generator data

Genco	Unit Size (MW)	$\mathbf{a}_{i}$	b <sub>i</sub>	c <sub>i</sub>	OP <sub>i</sub>
G <sub>1</sub> - G <sub>4</sub>	20	1285.6686	-42.5466	3.2629	0.10
G5- G8	76	531.1360	31.6040	0.1238	0.02
G9- G11	100	758.6000	46.0620	0.0635	0.04
$G_{12}$ - $G_{14}$	197	1136.7203	45.9591	0.0298	0.05

Suppose there are 20 consumers in the market, each of them has a quadratic benefit function as eq. (27) with the coefficients are assumed and listed in Table 2

$$B_{C_j}(Q_{C_j}) = d_j \cdot Q_{C_j} + e_j \cdot Q_{C_j}^{2}$$
 (27)

Table 2. Consumer data

Consumer $C_j$	dj	ej
C <sub>1</sub> -C <sub>5</sub>	300	-5.0
C5-C10	400	-5.0
$C_{11}$ - $C_{15}$	600	-4.0
C <sub>16</sub> -C <sub>20</sub>	800	-4.0

Assume the electricity market has completed before the reserve market starts to work. With the cost and benefit functions given above, the market clearing price  $P_{MC}$  is solved as  $P_{MC} = $55.9274/MWh$ .

Assume the reserve capacity cost (opportunity cost) functions for gencos are quadratic function as eqn. (28) with the coefficients listed in

Table 3

$$C_{Ri}(R_{Gi}) = f_i \cdot R_{Gi} + g_i \cdot R_{Gi}^2$$
 (28)

Table 3. Reserve capacity cost data

Genco G <sub>i</sub>	fi	gi
G <sub>1</sub> - G <sub>14</sub>	1.5	0.2
$G_{\Delta}$	1.0	0.1

And the supplementary reserve provider has a quadratic reserve energy production cost function as

$$PC_{R\Delta J}(\hat{R}_{G\Delta J}) = 56 \cdot \hat{R}_{G\Delta J} + 0.3 \cdot \hat{R}_{G\Delta J}^{2}$$
 (29)

The suggested decentralized optimal approach is tested with initial tuning factors  $\delta_i = 0.025$ , and  $\delta_2 = 0.15$ , and  $r_{\lambda} = 0.15$ . The optimal solutions for both decentralized approach and centralized approach are equal to each other and summarized in Table 4, with  $R_{GA} = 12.01MW$ ,  $P_R^C = \$3.28/MW$ ,  $P_R^G = \$2.77 / MW$ ,  $P_R^\Delta = \$3.40 / MW$ . The result of the new proposed decentralized approach is compared with the N-1 security approach (i.e.  $R_{\Sigma} = \max(S_{Gi})$ ). The result is summarized in Table 5.

Table 4. Reserve Market Data

$G_i$	$R_{Gi}$ (MW)	С,	R <sub>cj</sub> (MW)	$R_{c_i}/Q_{c_i}$
G <sub>1</sub> - G <sub>4</sub>	3.17	C <sub>1</sub> - C <sub>5</sub>	0.54	2.22%
G5- G8	0.00	C <sub>6</sub> - C <sub>10</sub>	1.60	4.64%
G <sub>9</sub> - G <sub>11</sub>	11.12	C11 - C15	5.03	7.40%
G <sub>12</sub> - G <sub>14</sub>	11.33	C <sub>16</sub> - C <sub>20</sub>	8.84	9.50%
	$R_{\Sigma} = 80.04$		$R_{\Sigma} = 80.04$	$R_{\Sigma}/Q_{\Sigma}$ =7.28%

Table 5. Decentralized approach vs. N-1 approach

	New Decentralized Approach	N-1 Approach
$R_{\Sigma}$	80.04	167.25
$R_{\Sigma}/Q_{\Sigma}$	7.28%	15.22%
Total Social Cost	\$3215.15	\$3802.55

From the computer results presented, we can know that consumers with difference demand functions will have different reserve capacity requirements in quantity or in percentage according to their own outage losses. From the numerical test results, the reserve capacity required by consumers have the range of 2.22% to 9.50% with an average of 7.28% (see Table 4), and the new proposed approach can save \$587.4 (15.4%) of the social cost as compare with the traditional N-1 approach (see Table 5).

#### 7. CONCLUSION

In this paper, a decentralized optimal approach is proposed for operating reserve market. The formulation of the optimal problem formulations and the solutions procedures are presented in details.

The result shows that, some participants may prefer to curtail a part of electricity supply rather than to buy expensive reserve capacity, in order to maximizing individual benefits. This fact shows clearly the drawbacks of the traditional reliability-rule-based decision, since it might against individual consumers' wills and lead to inefficiency. Therefore, the reserve market should encourage decentralized optimal decision and use centrally provided information to lead to the best social welfare.

#### 8. ACKNOWLEDGMENTS

The research is supported by RGC of Hong Kong Government and National Key Basic Research Special Fund (No. G1998020305), P. R. China, to whom sincere acknowledgements are expressed.

## 9. REFERENCES

- [1] C.Y. Chan, Y. Ni, F.F. Wu, "An Overview On Operating Reserve Assessment and Pricing In Power System", Proceeding of International Conference on Power Systems. Sep 3-5, 2001, Wuhan. China.
- [2] E. Hirst, B. Kerby, "Ancillary Service Details Operating Reserve". OAK Ridge National Laboratory, Tennessee, Tech. REP. ORNL/CON-452, Nov. 1997.
- [3] H. Singh, A. Papalexopoulos, "Competitive Procurement of Ancillary Services by an Independent System Operator", IEEE Transactions on Power Systems, Vol. 14, No. 2, May 1999. pp. 498-504.
- [4] M. Manove, "Provider Insurance", Bell Journal of Economics, 1983, pp. 327-330.
- [5] K.H. Borch, "The mathematical theory of insurance: an annotated selections of papers on insurance published", 1960-1972. Lexington, Mass.: Heath, 1974.
- [6] K. Borch, "Economics of Insurance", Amsterdam: North-Holland, 1990
- [7] C.Y. Chan, T. Bi, F. Wen, Y. Ni, F.F. Wu, "Discussion on Decentralized Ancillary Service Trading Based on Insurance Theory", Automation of Electric Power System, Vol. 26, No. 20, Oct 2002, pp. 18-23.
- [8] C.Y. Chan, Y. Ni, F.F. Wu, "Fundamentals of Operating Reserve Procurance Using Decentralized Approach with Insurance Theory Application", Electric Power Systems Research, Vol. 63, 2002, pp. 229-238
- [9] C.Y. Chan, Y. Ni, F. Wen, F.F. Wu, "A Study on Optimization of Operating Reserve through Maximizing Social Welfare", Submitted to IEEE transaction for peer review.
- [10] "The IEEE Reliability Test System 1996". IEEE Transactions on Power Systems, Vol. 14, No. 3, August 1999, pp. 1010-1018.