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The Theory and Design of a Class of Perfect Reconstruction Modified DFT Filter Banks with IIR filters

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ABSTRACT

This paper proposes a theory and design method for a class of PR causal-stable modified discrete Fourier transform (MDFT) filter bank (FB) with IIR filters. The prototype filter of the MDFT FB is assumed to have identical denominator in order to simplify the PR condition. A new model reduction technique is proposed for deriving a nearly PR (NPR) MDFT FB from a PR MDFT FB with FIR prototype filter. With these NPR IIR MDFT FBs as initial guess, PR IIR MDFT FBs with very good frequency characteristics can be obtained by solving a constrained nonlinear optimisation problem. Because the location of the poles can be approximately determined through model reduciton, the efficiency and reliability of the design method is significantly improved. Design examples are given to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Perfect reconstruction filter banks (PR FBs) have important applications in speech, audio, image and array processing. The discrete Fourier transform (DFT) filterbank is well-known for its computational efficiency in realizing channels with equal bandwidth. The condition for which such DFT FBs is perfect reconstruction was reported in [1,2] and the resultant DFT FB is called the modified discrete Fourier transform (MDFT) filter banks. Like the cosine modulated filter banks (CMFB) [4], they offer good frequency characteristics while offering very low design and implementation complexity. Previous study of CMFB has shown that CMFB with IIR filters have the potential advantage of realizing PR system with lower system delay, sharper cutoff and higher stopband attenuation over their FIR counterparts [6]. The design of IIR CMFBs and MDFT-FBs, however, is complicated by the highly nonlinear objective function and PR constraints. In [6], the polyphase components of the prototype filter are assumed to have identical denominator so that the PR constraints can be simplified considerably. PR IIR CMFBs with very good frequency characteristics and lower system delay can be obtained. However, when the number of variables and constraints increases, the optimization procedure is rather sensitive to the initial guess of the prototype filter. The IIR DFT-FB proposed in [3] requires that the polyphase components to be all-pole filters.

In this paper, we introduce a new method for designing NPR and PR IIR MDFT-FBs. A PR FIR MDFT-FB with similar specification is first designed by nonlinear optimization. The PR FIR prototype filter is then model reduced to an NPR IIR CMFB by modifying a model reduction technique proposed by Brandenstein and Unbehauen [5]. The NPR IIR CMFB has a reasonably good reconstruction error and it is employed as the initial guess to constrained nonlinear optimisation software such as fmincon from MATLAB for designing the PR IIR MDFT-FB. Design results show that both NPR and PR IIR MDFT-FBs with good frequency characteristics and different system delays can be obtained readily by the proposed method.

General speaking, the design of PR FIR DFT-Fbs also involves nonlinear constrained optimisation. However, it is considerably simpler due to the absence of the poles and satisfactory results are usually obtained without much problem, unless the filter is very long. After the PR FIR CMFB has been obtained, there are two different ways to design the target IIR MDFT-FB. One is to employ the FIR prototype filter as initial guess and initialise all the poles to zero. The second is to model reduce the FIR prototype to another IIR filter as initial guess. The latter has the advantage that the highly nonlinear poles locations can be approximately located, giving better convergent speed and reliability. Also, an NPR IIR CMFB with reasonable good reconstruction error is immediately obtained. However, straightforward use of conventional model reduction method will lead to an IIR prototype filter with non-identical denominator in its polyphase components. This, as suggested in [5], will complicate considerably the design process and efficient implementation. To overcome this problem, we modify the model reduction method in [7] so that the denominator of the final IIR filter has the form $D(z^M)$. In other words, the polyphase components of the prototype filter have identical denominator and it considerably simplifies the PR constraints. Other advantages of the method are that the stability of the model-reduced filter is guaranteed and the IIR filters so obtained closely approximate the properties of the original FIR filter. By using these NPR IIR prototype filter as initial guesses to the constrained nonlinear optimiser, significantly better converging speed and reliability over the direct nonlinear optimization is achieved. Design examples show that the proposed method is very effective and it yields readily NPR and PR IIR MDFT-FBs with good frequency characteristics.

The paper is organized as follows: the theory of PR MDFT FBs is recalled in section 2 and the PR condition of the proposed IIR MDFT FBs is studied in Sections 3. A design example and comparison are given

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in Section 4 and finally conclusions are drawn in Section 5.

2. PR FIR MDFT FBs

The theory of MDFT FBs will be recalled briefly here. Interested readers are referred to [1,2] for more detail. Assume all analysis and synthesis filters in this paper are derived from an identical real-valued low-pass prototype filter $h(n)$, $n = 0,..., N-1$, which has a transition band from $-\pi / M$ to π / M. The complex analysis and synthesis filters for type-I and type-II MDFT FBs are:

$$h_k(n) = f_k(n) = \sqrt{2}h(n)\exp\left(j\frac{\pi k}{M}\left(n - \frac{D}{2}\right)\right), \quad (1)$$

and

$$h_k(n) = \sqrt{2}h(n)\exp\left(j\frac{\pi k}{M}\left(n - \frac{D+M}{2}\right)\right),$$
$$f_k(n) = \sqrt{2}h(n)\exp\left(j\frac{\pi k}{M}\left(n - \frac{D-M}{2}\right)\right), \quad (2)$$

respectively, where $k = 0,...,2M-1$ and $n = 0,..., N-1$. It is clear that two types of MDFT FBs are different in the phase of the modulation and the way in taking the real and imaginery parts to form the subbands (see Fig.1). For simplicity, we consider a $2M$-channel MDFT FB with an overall delay of $D = (2s+1)M - 1$. It is easy to extend to the cases with different overall delays. For the case $D = (2s+1)M - 1$, the PR condition on the polyphase components of the prototype filters for the two types of MDFT FBs are the same:

$$G_k(z)G_{2M-1-k}(z) + G_{M+k}(z)G_{M-1-k}(z) = cz^{-s}, \quad (3)$$

where $0 \le k \le M/2 - 1$, c is a constant, s is an integer and $G_k(z)$ are the type-I order-$2M$ polyphase components of the prototype filter $h(n)$. It is equivalent to the PR condition for an M-channel biorthoginal cosine modulated filter banks with a system delay of $D_{cmfb} = 2sM - 1$. If $s = N/(2M)$, the prototype filter $h(n)$ is linear phase.

3. PR IIR MDFT FBs

Following [6] for PR IIR CMFB, we assume that the polyphase components of the IIR prototype filter take on the following form:

$$G_k(z) = \frac{N_k(z)}{D(z)}, \qquad k = 0,..., M/2 - 1, \quad (4)$$

that is, they have identical denominator. Hence, the PR condition in (3) will reduce to

$$N_k(z)N_{2M-1-k}(z) + N_{k+M}(z)N_{M-1-k}(z) = cz^{-s}D^2(z), \quad (5)$$

where $k = 0,..., M/2 - 1$. It is equivalent to the PR condition for CMFBs with IIR filters given in [5]. To ensure that analysis and synthesis filters are stable, all the roots of $D(z)$ shall remain inside the unit circle.

Similar to CMFBs, MDFT FBs are obtained frequency shifting of the prototype filter. To achieve a good frequency characteristic, the stopband error of the prototype filter needs to be minimized. This leads to the following objective function:

$$\Phi = \int_{\omega_s}^{\pi} \left|H(e^{j\omega})\right|^d d\omega \quad (6)$$

where ω_s is the stopband cutoff frequency of the prototype filter. For $d = 2$, the objective function is the familiar least square design criterion. If approximate equip-ripple passband and stopband erros are desired, the value of d can be chosen as 4.

Let the IIR prototype filter is:

$$H(z) = \frac{\sum_{n=0}^{N_n-1} a(n)z^{-n}}{\sum_{n=0}^{N_d-1} b(n)z^{-n}}, \quad (7)$$

where N_n and N_d are respectively the lengths of the numerator and the denominator of the prototype filter. It is easy to show that if the type-I polyphase components ($G_k(z)$) of the prototype filter have identical denominator, then $b(n) = 0$ whenever $n \ne 2kM + 1$, and

$$H(z) = \sum_{k=0}^{M-1} z^{-k} \frac{N_k(z)}{D(z)} = \frac{\sum_{k=0}^{M-1} z^{-k} N_k(z)}{D(z)}. \quad (8)$$

The PR condition in (5) suggests that the length of denominator in the polyphase components should not be longer than the length of the numerator, otherwise it would be very difficult to balance the various powers of z on both sides of (5). The design problem can be formulated as a constrained optimisation problem where (6) is minimized subject to the PR and stability constraints in (5). For FIR MDFT FBs, $D(z) = 1$ and the design problem is considerably simplified.

Although the use of identical denominator greatly simplifies the design procedure, the optimization procedure is rather sensitive to the initial guess of the prototype filter when the number of variables and constraints increases. To overcome this problem, a PR FIR MDFT FB with similar specification is first designed. The PR FIR prototype filter are then model reduced to an NPR IIR MDFT FBs by modifying a model reduction technique proposed by Brandenstein and Unbehauen [7]. The resulting NPR IIR CMFB has a similar frequency characteristic and reasonably good reconstruction error and it is employed as the initial guess to constrained nonlinear optimisation software such as fmincon from MATLAB for designing the PR IIR MDT FB. Design results, to be presented in section 4, shows that efficiency and reliability can be improved significantly by using the proposed approach. In particular, it is found that the initial guess so obtained requires much less iterations than conventional optimisation technique using $D(z) = 1$ as initial guess. The IIR FB so obtained has a better performance than its FIR filter, especially when the transition bandwidth and system delay are reduced. In these cases, the arithmetic complexity will also be reduced considerably.

Direct application of conventional model reduction to the FIR prototype will lead to an IIR filter having the form of (7). To convert it to the form of (8), a common method is to multiply the numerator and denominator by appropriate factors, which usually necessarily increases the orders of the filter. Here, we employ the method in [5], which is a modified version of the model reduction method in [7]. The advantage is that it yields a reduced model in the form of (8) directly, which simplifies considerably its multirate realization. Due to page limitation, interested readers are referred to [5] for more detail. The IIR filter so obtained can be described as:

$$P_0(z) = \frac{N(z)}{D(z)} = \frac{\sum_{n=0}^{N_n-1} a(n)z^{-n}}{\sum_{n=0}^{\lambda} b(n)z^{-2nM}}, b(0) = 1, \qquad (9)$$

where N_n is the length of the numerator, λ corresponds to the non-zero coefficients of $D(z)$, excluding $q(0)$.

4. DESIGN PROCEDURE and EXAMPLES

Design Procedure:

We now summarize the design procedure for the $2M$-channel PR IIR MDFT FB:

1. Design a $2M$-channel PR low-delay FIR MDFT FB using some nonlinear optimisation. The PR constraints and the objection function are described in (3) and (6), respectively.
2. Model reduce the FIR prototype to obtain a nearly PR (NPR) IIR MDFT FB using the using the modified model reduction proposed in [5].
3. Use the NPR prototype filter obtained in step 2 as initial guess to the nonlinear constraints problem which minimizes the objective function (6) subject to the PR condition in (5) and the stability constraints. To ensure that the analysis and synthesis filter are stable, all the roots of $D(z)$ shall remain inside the unit circle.

In this paper, the nonlinear constrained optimisation problmes mentioned above are solved using the function fmincon in MATLAB. Since model reduction usually leads to a lower system order, the number of variables in step 3 is usually lower than that in step 1, although the problem is more nonlinear. Fortunately, the model reduction technique provides very good initial values for locating the position of the poles.

Design Examples:

We now present several design examples to illustrate the effectiveness of the proposed method. Since the two types of MDFT FBs only differ in the phase of the modulation, results for Type-1 MDFT FBs will be given. Fig.2 shows the frequency responses of two 8-channel PR FIR MDFT FBs with the same magnitude specification, but different filter orders. Fig. 2(a) corresponds to a linear phase prototype filter with an order of 55, and the overall system delay 59 samples. Fig. 2(b) corresponds to a FB with a lower system delay of 59 samples and a longer filter order of 63. Fig. 3 shows the magnitude responses of two IIR MDFT FBs. Fig. 3(a) shows the NPR IIR MDFT FB obtained by model reducing the PR FIR MDFT FB in Fig. 2(b). The violation in PR constraints of this NPR IIR MDFT FBs is of the order 10^{-3}. Using this NPR prototype as initial guess, the 8-channel PR IIR MDFT FB in Fig. 3(b) is obtained. It has the same system delay as the PR FIR MDFT FB in Fig.2(b), but a lower filter order of 55. All PR constraints remain at 10^{-15}. The coefficients of filters for MDFT FBs are complex, even if the prototype filters are real-valued. Fig.4 gives the magnitude responses of the real part and imaginary part of the filters in Fig. 3(b).

5. CONCLUSION

The theory and design of a class of PR causal-stable MDFT FB with IIR filters are presented. The prototype filter of the MDFT FB is assumed to have identical denominator in order to simplify the PR condition. A new model reduction technique is proposed for deriving a NPR MDFT FB from a PR MDFT FB with FIR prototype filter. With these NPR IIR MDFT FBs as initial guess, PR IIR MDFT FBs with very good frequency characteristics can be obtained by solving a constrained nonlinear optimisation problem. Design examples are given to demonstrate the usefulness of the proposed approach.

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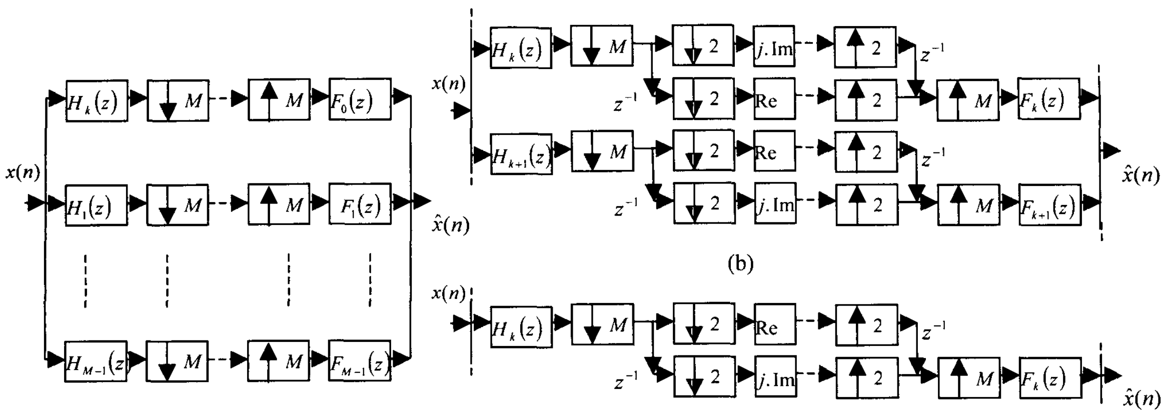
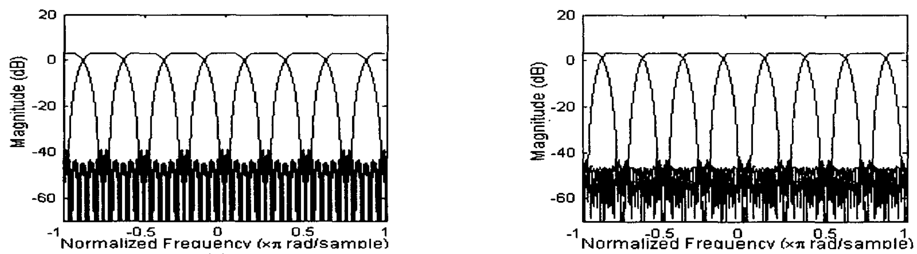


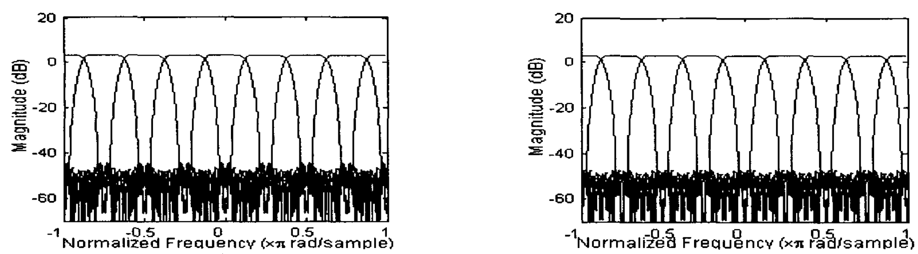
(a) (c)

Fig.1. (a) M-channel maximally-decimated filter bank; (b) structure of type-1 MDFT FB and (c) structure of type-2 MDFT FB.



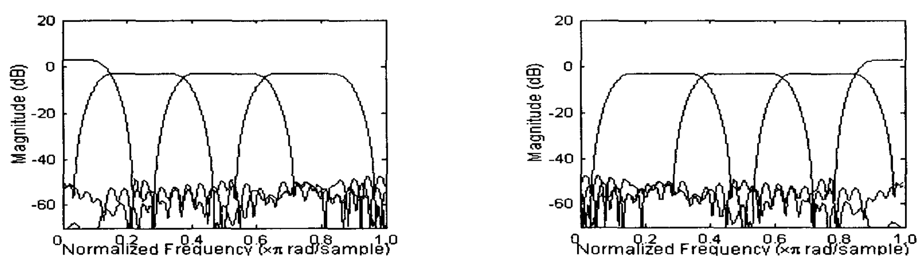
(a) (b)

Fig.2. Analysis filters for 8-channel PR FIR MDFT FBs with (a) the linear-phase prototype filter; (b) low-delay prototype filter.



(a) (b)

Fig.3. Analysis filters for (a) 8-channel NPR IIR MDFT FB, (b) 8-channel PR IIR MDFT FB.



(a) (b)

Fig.4. (a) Real part of Fig.3(b), (b) imaginary part of Fig.3(b).